Lecture 4: Regions

Heterogeneous Agents in Macroeconomics

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Mendoza, Quadrini, and Ríos-Rull (2009)

Two countries, $i \in \{1, 2\}$. Continuum of agents of unity mass.

Ex-ante homogeneous preferences, technology, and productivity.

Locally incomplete markets. Cross-country risk sharing. No capital accumulation.

Ex-ante heterogeneity in insurability of shocks and enforceability of financial contracts ϕ_i.

Preferences and Technology

Agents maximize $\mathbb{E}\sum_{t=0}^{\infty}\beta^t\mathcal{U}(c_t)$. $\mathcal{U}(0) = -\infty$ and $\mathcal{U}' > 0, \mathcal{U}'' < 0, \mathcal{U}''' > 0$.

Unit supply of internationally immobile asset k_t, traded at price P_{it}.

Production with decreasing returns to scale $\nu < 1$ and investment shocks z_{t+1}: $y_{t+1} = z_{t+1}k_t^{\nu}$.

Idiosyncratic stochastic endowment ω_{it} that is first-order Markov.

No aggregate uncertainty.

Budget Constraint

Define $s_t \equiv (\omega_t, z_t)$ as the exogenous state and $g(s_t, s_{t+1})$ its Markov transition process.

Agents buy state-contingent claims $b(s_{t+1})$ that are priced with $q_{it}(s_t, s_{t+1}) = \frac{g(s_t, s_{t+1})}{1+r_{it}}$ with r_{it} the equilibrium real interest rate.

With a_t the end-of-period net worth the budget constraint is:

$$a_t = c_t + k_t P_{it} + \sum_{s_{t+1}} b(s_{t+1}) q_{it}(s_t, s_{t+1}) \tag{1}$$

Law of motion of net worth:

$$a(s_{t+1}) = \omega_{t+1} + k_t P_{i,t+1} + z_{t+1} k_t^{\nu} + b(s_{t+1}) \tag{2}$$

Financial Market

Regional heterogeneity in local financial market depth ϕ_i:

$$b(s_n) - b(s_1) \geq -\phi_i \left[(\omega_n + z_n k_t^{\nu}) - (\omega_1 + z_1 k_t^{\nu}) \right] \tag{3}$$

for all $n \in \{1, \ldots, N\}$ where N is the number of all possible realizations. s_1 is the lowest realization.

Full insurance limit: ϕ_i large and as-if complete markets case.

Risk-free debt limit: $\phi_i = 0$ and only non-state-contingent claims feasible.

Limited liability:

$$a(s_n) \geq 0 \tag{4}$$

Optimization Problem

Let $\{P_{i\tau}, q_{i\tau}(s_\tau, s_{\tau+1})\}_{\tau=t}^{\infty}$ be a deterministic sequence of local prices.

Capital mobility: prices equalized internationally. Agents indifferent between domestic and foreign capital.

Individual agent solves:

$$V_{it}(s,a) = \max_{\{c,k,b(s')\}} \left\{ \mathcal{U}(c) + \beta \sum_{s'} v_{i,t+1}(s', a(s'))g(s,s') \right\} \tag{5}$$

subject to (1), (2), (3), and (4).

Policy rules: $c_{it}(s,a)$, $k_{it}(s,a)$, and $b_{it}(s,a,s')$. Distribution: $M_{it}(s,k,b)$.

Equilibrium

Equilibrium is defined by sequences of agents' policies $\{c_{i\tau}(s,a), k_{i\tau}(s,a), b_{i\tau}(s,a,s')\}_{\tau=t}^{\infty}$, value functions $\{V_{i\tau}(s,a)\}_{\tau=t}^{\infty}$, prices $\{P_{i\tau}, r_{i\tau}, q_{i\tau}(s,s')\}_{\tau=t}^{\infty}$, and distributions $\{M_{i\tau}(s,k,b)\}_{\tau=t}^{\infty}$ such that: (i) policy rules solve the optimization problem, (ii) value functions are associated with the solution, and ...

1. If autarky
   - Prices satisfy: $q_{i\tau} = \frac{g(s,s')}{1+r_{it}}$
   - Asset markets clear $\forall\, i \in [1,2]$ and $\tau \geq t$:

$$\int_{s,k,b} k_{i\tau}(s,a) M_{i\tau}(s,k,b) = 1 \quad (6)$$

$$\int_{s,k,b,s'} b_{i\tau} M_{i\tau}(s,k,b) g(s,s') = 0 \quad (7)$$

EQUILIBRIUM

Equilibrium is defined by sequences of agents' policies $\{c_{i\tau}(s,a), k_{i\tau}(s,a), b_{i\tau}(s,a,s')\}_{\tau=t}^{\infty}$, value functions $\{V_{i\tau}(s,a)\}_{\tau=t}^{\infty}$, prices $\{P_{i\tau}, r_{i\tau}, q_{i\tau}(s,s')\}_{\tau=t}^{\infty}$, and distributions $\{M_{i\tau}(s,k,b)\}_{\tau=t}^{\infty}$ such that: (i) policy rules solve the optimization problem, (ii) value functions are associated with the solution, and ...

1. If globally integrated asset markets
   - Prices satisfy: $q_{1\tau} = \frac{g(s,s')}{1+r_{1t}} = \frac{g(s,s')}{1+r_{2t}} = q_{2\tau}$
   - Asset markets clear $\forall\ \tau \geq t$:

$$\sum_{i=1}^{2} \int_{s,k,b} k_{i\tau}(s,a) M_{i\tau}(s,k,b) = 2 \tag{8}$$

$$\sum_{i=1}^{2} \int_{s,k,b,s'} b_{i\tau} M_{i\tau}(s,k,b) g(s,s') = 0 \tag{9}$$

Net Foreign Asset Position

$$NFA_{i\tau} = \int_{s,k,b,s'} b_{i\tau} M_{i\tau}(s,k,b) g(s,s') \tag{10}$$

$$+ \int_{s,k,b} \left[k_{i\tau}(s,a) - 1\right] P_\tau M_{i\tau}(s,k,b) \tag{11}$$

First term on the right-hand side: net position in contingent claims. "International lending" when positive and "borrowing" when negative.

Second term on the right-hand side: net position in productive assets.

With open borders and capital mobility, assets owned by country i are no longer constrained to equal to assets located in country i. NFA positions generally not zero.

ENDOWMENT ω SHOCKS ONLY, AUTARKY

Assume z is invariant ($z = \bar{z}$). Let $\bar{\phi}$ sufficiently large such that (3) is slack and markets are complete. Let $\phi = 0$ represent the case without state-contingent claims.

Case 1: Autarky, $\phi = \bar{\phi}$. First-order conditions of problem (5) w.r.t. k and $b(\omega')$:

$$\mathcal{U}'(c) = \beta(1 + r_t)\mathcal{U}'(c(\omega')) + (1 + r_t)\lambda(\omega') \quad \forall\omega' \tag{12}$$

$$\mathcal{U}'(c) = \beta R_{t+1}(k, \bar{z})\mathbb{E}\mathcal{U}'(c(\omega')) + R_{t+1}(k, \bar{z})\mathbb{E}\lambda(\omega') \tag{13}$$

where $\lambda(\omega')$ the multiplier on the limited liability constraint. $R_{t+1}(k, \bar{z}) = (P_{t+1} + \nu\bar{z}k^{\nu-1})/P_t$ is the gross return on assets.

If $\phi = \bar{\phi}$: consumption is time-invariant. $R_{t+1} = 1 + r_t$. All agents choose $k_t = k$.

Equilibrium interest rate: $\beta(1 + r) = 1$. Equilibrium price: $P = \nu\bar{z}/r$.

Endowment ω Shocks Only, Autarky

Case 2: Autarky, $\phi = 0$. $b(\omega_1) = \cdots = b(\omega_N) = b$: Assets cannot be state-contingent.

$$\mathcal{U}'(c) = \beta(1+r_t)\mathbb{E}\mathcal{U}'(c(\omega')) + (1+r_t)\mathbb{E}\lambda(\omega') \quad (14)$$

$$\mathcal{U}'(c) = \beta R_{t+1}(k,\bar{z})\mathbb{E}\mathcal{U}'(c(\omega')) + R_{t+1}(k,\bar{z})\mathbb{E}\lambda(\omega') \quad (15)$$

As before, $P = \nu\bar{z}/r$ and $R_{t+1} = 1 + r_t$.

However, because markets are incomplete we recover the Aiyagari result: $\beta(1+r) < 1$.

Country with lower financial development $(\phi = 0)$ has a lower r and higher P than the more developed $(\phi = \bar{\phi})$ country.

Endowment ω Shocks Only, Integration

Now consider open borders with perfect capital mobility.

Country 1 is more $(\phi_1 = \bar{\phi})$ and Country 2 is less $(\phi_2 = 0)$ financially developed.

Case 3: Integration. Capital prices and interest rates equalize across countries. Country 1 (C1) has no need for precautionary savings. C2 does.

Can show that NFA_1 is negative. Proof in class. Also see Chapter 18.4 in Ljungqvist and Sargent for intuition.

Financial market liberalization $\rightarrow$ countries with lower ϕ accumulate positive NFA positions.

Investment z_{t+1} Shocks Only, Autarky

Now, the opposite situation: z is stochastic but $\omega = \bar{\omega}$ invariant.

Case 1: Autarky, $\phi = \bar{\phi}$. First-order conditions of problem (5) w.r.t. k and $b(z')$:

$$\mathcal{U}'(c) = \beta(1 + r_t)\mathcal{U}'(c(z')) + (1 + r_t)\lambda(z') \quad \forall z' \tag{16}$$

$$\mathcal{U}'(c) = \beta\mathbb{E}R_{t+1}(k, z')\mathcal{U}'(c(z')) + \mathbb{E}R_{t+1}(k, z')\lambda(z') \tag{17}$$

Because of full insurance, consumption is invariant to realizations of z'. $\mathbb{E}R_{t+1}(k, z') = 1 + r_t$. No asset premium. $\beta(1 + r_t) = 1$ holds.

Investment z_{t+1} Shocks Only, Autarky

Case 2: Autarky, $\phi = 0$. First-order conditions of problem (5) w.r.t. k and $b(z')$:

$$\mathcal{U}'(c) = \beta(1+r_t)\mathbb{E}\mathcal{U}'(c(z')) + (1+r_t)\mathbb{E}\lambda(z') \tag{18}$$

$$\mathcal{U}'(c) = \beta\mathbb{E}\left(R_{t+1}(k,z')\mathcal{U}'(c(z'))\right) + \mathbb{E}R_{t+1}(k,z')\lambda(z') \tag{19}$$

As in the case of endowment shocks, $\beta(1+r) < 1$. But also:

$$\mathbb{E}R_{t+1}(k,z') - (1+r_t) = -\frac{\text{Cov}\left[R_{t+1}(k,z'), U'(c(z'))\right]}{\mathbb{E}\mathcal{U}'(c(z'))} \tag{20}$$

which is generally positive since $\mathcal{U}'(c(z'))$ is negatively correlated with $R_{t+1}(k,z')$.

INVESTMENT z_{t+1} SHOCKS ONLY, INTEGRATION

Two countries with $\phi_1 = \bar{\phi}$ and $\phi_2 = 0$ trade with full capital mobility.

Can show that: $NFA_1 < 0$ but $k_1 - 1 > 0$, i.e. positive position in the asset. The average return of C1's assets is greater than the cost of its liabilities. C1 is a global hedge fund.

Concavity of the production function is crucial for the result. Greater asset holdings depress the interest rate. Recall the negative relationship between $R_{t+1}(k, z)$ and k.

With linear technology, the developed country owns all of the world's capital. In turn, the less developed country internalizes this and has no incentive to save.

In the case of $0 < \nu < 1$: the developed country generally owns some of C2's risky capital, earns higher returns, and borrows with cheaper foreign debt.

Endowment and Investment Shocks

Now the state is s as there are both ω and z shocks.

Suppose that $\phi_1 = \bar{\phi}$ and $\phi_2 = 0$. In the steady state with perfect capital mobility, $\beta(1+r) < 1$. C1 has negative NFA position and a positive foreign asset position. C1's average return of foreign ownership is larger than the cost of liabilities. Same as with investment shocks only.

Generally for $0 \leq \phi_2 < \phi_1 < \bar{\phi}$, the NFA position is not necessarily negative and depends on calibration. If the endowment shock is sufficiently large, C1 holds a negative NFA position.

Multiple Countries Generalization

General model includes any finite number of countries $I \geq 2$.

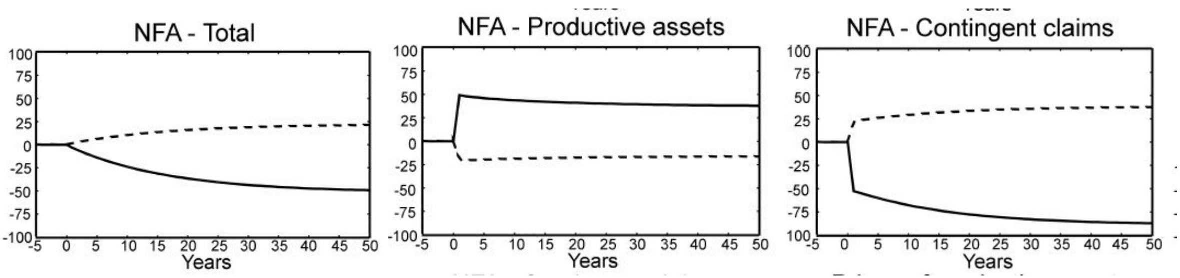
Cross-country diversification of investment risk. Can introduce differences in country (market) size.

Denote $A_{jt} \in [0,1]$ allocation of managerial capital into country j. Total production:

$$y_{t+1} = \sum_{j=1}^{I} z_{j,t+1} A_{jt}^{1-\nu} k_{jt}^{\nu}, \quad \sum_{j} A_{jt} = 1 \tag{21}$$

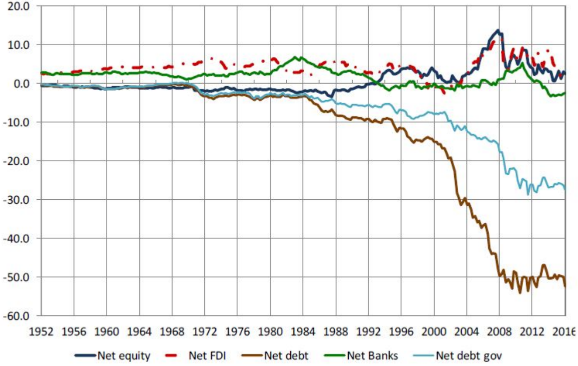
Now managerial capital is divisible. If $z_{j,t+1}$ are imperfectly correlated, integration allows agents to diversity investment risk across regions. Gross positions can also be determined now. The full model is solved numerically.

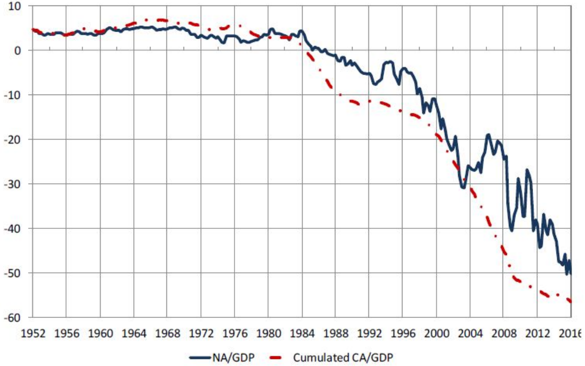
Quantitative Performance



Years after liberalization. Solid line - C1. Dashed line - C2.

Empirical Moment





Source: Gourinchas and Rey (2022).

Other Channels

Supply of assets - Caballero et al. (2008).

Financial development - Maggiori (2017).

Market size - Hassan (2013).

Risk aversion - Gourinchas and Rey (2010).

Disaster insurance - Gourinchas and Rey (2022, revision of the 2010 paper).

Takeaway

Ex-ante heterogeneity in regional financial market development can generate endogenous global imbalances.

Locally incomplete markets. Internationally complete markets.

Financial integration forces developed countries to reduce savings, accumulate more net foreign liabilities.

Portfolio composition: developed country borrows low-risk from abroad and invests in high-return foreign risky assets.

Nesting the Huggett-Aiyagari closed-economy model as a special case.