## LECTURE 6: HANC, TANK, AND HANK

Heterogeneous Agents in Macroeconomics

**Rustam Jamilov** 

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## KRUEGER, MITMAN, AND PERRI (2016)

Workhorse Heterogeneous Agent Neo Classical (HANC) model.

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## TECHNOLOGY

Discrete time. Recursive notation.

Production:

$$Y = Z^* F(K, N) \tag{1}$$

TFP:

$$Z^* = ZC^{\omega}, \quad \omega \ge 0$$
 (2)

Z is first-order Markov with transition matrix  $\pi(Z'|Z)$ .

If  $\omega = 0$  then TFP is exogenous and driven only by Z. If  $\omega > 0$  then  $C^{\omega}$  represent aggregate demand externalities. TFP (and output) are then partially demand-determined.

Benchmark: two aggregate states -  $Z \in \{Z_{low}, Z_{high}\}$ 

# LIFE CYCLE

- Unit mass continuum of households.
- Households are either young-workers or old-retirerees. Household j is  $j \in \{W, R\}$ .
- Young households have a constant probability of retiring  $1 \theta \in [0, 1]$ .
- Old households have a constant probability of dying  $1 \nu \in [0, 1]$ .
- Deceased retirees are replaced by new young-workers.
- Ergodic distribution of the age groups:

$$\Pi_{W} = \frac{1 - \theta}{(1 - \theta) + (1 - \nu)}$$
(3)  
$$\Pi_{R} = \frac{1 - \nu}{(1 - \theta) + (1 - \nu)}$$
(4)

## PREFERENCES

- Preferences over stochastic consumption u(c). No value from leisure.
- Potentially (ex-ante) heterogeneous discount factor  $\beta$ .
- Labor supply of *W* households is exogenous and normalzied to 1.
- Incomplete markets. Idiosyncratic, uninsured shocks of two forms.
- Shock one: unemployment risk with employment status  $s \in S = \{u, e\}$ . First-order Markov with transition  $\pi(s'|s, Z', Z)$  which is aggregate state-dependent.
- Shock two: conditional on being employed, labor productivity risk  $\gamma \in Y$ , first-order Markov, with transition  $\pi(\gamma'|\gamma)$  which is aggregate state-independent.

## DISTRIBUTION

Households save in physical capital *a*.

Denote  $\Pi_Z(s)$  the deterministic fraction of households with employment state *s*.

Denote  $\Pi(\gamma)$  the deterministic cross-sectional distribution over productivity, by assumption aggregate state-invariant.

Denote  $\phi$  the full cross-sectional distribution of employment *s*, productivity  $\gamma$ , asset holdings *a*, and discount factors  $\beta$ .

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#### **UNEMPLOYMENT INSURANCE**

Balanced budget unemployment insurance with benefits *b* and wages  $\gamma w$  earnings:

$$b(\gamma, Z, \Phi) = \rho \times \gamma w(Z, \Phi)$$
(5)

where  $\rho$  is the replacement rate parameter.  $\rho = 0$  means no insurance.

Benefits are paid to the unemployed s = u and financed by labor taxes  $\tau(Z, \Phi)$  that are levied on both earnings and unemployment benefits.

The budget constraint of the unemployment insurance system reads:

$$\Pi_{Z}(u)\sum_{\gamma}\Pi(\gamma)b(\gamma,Z,\Phi) = \tau(Z,\Phi)\left[\sum_{\gamma}\Pi(\gamma)(\Pi_{Z}(u)b(\gamma,Z,\Phi) + (1-\Pi_{Z}(u))w(Z,\Phi)\gamma)\right]$$

Since the cross-sectional distribution over  $\gamma$  is the same for s = u and s = w, can write:

$$\Pi_{Z}(u)\rho = \tau(Z,\Phi) \left[\Pi_{Z}(u)\rho + (1 - \Pi_{Z}(u))\right]$$
(6)

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#### **UNEMPLOYMENT INSURANCE**

Tax rate that is needed to balanced the budget:

$$\tau(Z,\Phi;\rho) = \left(\frac{\Pi_Z(u)\rho}{1 - \Pi_Z(u) + \Pi_Z(u)\rho}\right) = \left(\frac{1}{1 + \frac{1 - \Pi_Z(u)}{\Pi_Z(u)\rho}}\right) = \tau(Z;\rho) \in (0,1)$$
(7)

The tax rate depends on the generosity of the welfare system  $\rho$  and aggregate state-dependent employed-unemployed ratio  $\frac{1-\Pi_Z(u)}{\Pi_z(u)}$ .

# SOCIAL SECURITY

- Pay as you go, balanced budget system, determined by the constant payroll tax rate  $\tau_{SS}$  that applies only to labor earnings.
- Social security benefits  $b_{SS}(Z, \Phi)$  are aggregate state-dependent but independent of past contributions:

$$b_{SS}(Z,\Phi)\Pi_R = \tau_{SS}\Pi_W \left[\sum_{\gamma} \Pi(\gamma)(1-\Pi_Z(u))w(Z,\Phi)\gamma\right]$$
(8)

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#### **RECURSIVE COMPETITIVE EQUILIBRIUM**

Retired households solve:

$$V_R(a,\beta;Z,\Phi) = \max_{c,\alpha' \ge 0} \left\{ u(c) + \nu\beta \sum_{Z' \in Z} \pi(Z'|Z) V_R(a',\beta;Z',\Phi') \right\}$$

(9)

s.t.

$$c + a' = b_{SS}(Z, \Phi) + (1 + r(Z, \Phi) - \delta)a/\nu$$
  
$$\Phi' = H(Z, \Phi', Z')$$

where H is the law of motion of the distribution.

## **RECURSIVE COMPETITIVE EQUILIBRIUM**

Working households solve:

$$V_{W}(s,\gamma,a,\beta;Z,\Phi) = \left\{ \max_{c,a' \ge 0} u(c) + \beta \sum_{\{Z',s',\gamma'\}} \pi(Z'|Z)\pi(s'|s,Z',Z)\pi(\gamma'|\gamma) \times (\theta V_{W}(s',\gamma',a',\beta;Z',\Phi')) \right\}$$
(10)

s.t.

$$c + a' = (1 - \tau(Z; \rho) - \tau_{SS})w(Z, \Phi)\gamma [1 - (1 - \rho)\mathbf{1}_{s=u}] + (1 + r(Z, \Phi) - \delta)a$$
(11)  
$$\Phi' = H(Z, \Phi', Z')$$
(12)

 $\mathbf{1}_{s=u}$  means earnings equal  $\rho w(Z, \Phi) \gamma$  which equates benefits  $b(\gamma, Z, \Phi)$ .

#### MARKET CLEARING

Factor prices are given by:

$$w(Z,\Phi) = ZF_N(K(Z,\Phi), N(Z,\Phi))$$

$$r(Z,\Phi) = ZF_K(K(Z,\Phi), N(Z,\Phi))$$
(13)
(14)

Market clearing:

$$N(Z,\Phi) = (1 - \Pi_Z(u)) \sum_{\gamma} \gamma \Pi(\gamma)$$

$$K(Z,\Phi) = \int a d\Phi$$
(15)
(16)

## DISTRIBUTION: DATA AND MODELS

% Share held by:	Dat	ta	Models	
	<b>PSID</b> , 06	SCF, 07	Bench	KS
Q1	-0.9	-0.2	0.3	6.9
Q2	0.8	1.2	1.2	11.7
Q3	4.4	4.6	4.7	16.0
Q4	13.0	11.9	16.0	22.3
Q5	82.7	82.5	77.8	43.0
90–95	13.7	11.1	17.9	10.5
95–99	22.8	25.3	26.0	11.8
T1 %	30.9	33.5	14.2	5.0
Gini	0.77	0.78	0.77	0.35

Table 6 Net worth distributions: Data vs models

KS: original Krusell-Smith (1998) model - no life cycle, no  $\beta$  heterogeneity, no idiosyncratic earnings risk. Only aggregate uncertainty and idiosyncratic employment risk.

### IMPULSE RESPONSE



Fig. 2 Impulse response to aggregate technology shock in two economies: One time technology shock.

## CONSUMPTION ELASTICITY



Fig. 5 Consumption function and wealth distribution: Krusell–Smith (left panel) and benchmark (right panel).

## UNDERSTANDING THE MECHANISM

% Share:	KS	$+\sigma(y)$	+Ret.	$+\sigma(\beta)$	+UI	KS + Top 1%
Q1	6.9	0.7	0.7	0.7	0.3	5.0
Q2	11.7	2.2	2.4	2.0	1.2	8.6
Q3	16.0	6.1	6.7	5.3	4.7	11.9
Q4	22.3	17.8	19.0	15.9	16.0	16.5
Q5	43.0	73.3	71.1	76.1	77.8	57.9
90-95	10.5	17.5	17.1	17.5	17.9	7.4
95-99	11.8	23.7	22.6	25.4	26.0	8.8
T1 %	5.0	11.2	10.7	13.9	14.2	30.4
Wealth Gini	0.350	0.699	0.703	0.745	0.767	0.525
$\Delta C$	-1.9%	-2.5%	-2.6%	-2.9%	-2.4%	-2.0%

Table 12 Net worth distributions and consumption decline: Different versions of the model Models\*

\* The KS model only has unemployment risk and incomplete markets, and thus the first column repeats information from table 6. The column  $+\sigma(\gamma)$  adds idiosyncratic earnings shocks (transitory and permanent) while employed. The column +Ret, adds the basic life cycle structure (positive probability of retirement and positive probability of death, plus social security in retirement). The column  $+\sigma(\beta)$  incorporates preference heterogeneity into the model, and finally the column +UI raises the replacement of the unemployment insurance system from 1% to 50%; the resulting model is therefore the benchmark model, with results already documented in table 6. In all models, the (mean) discount factor is calibrated so that all versions have the same capital-output ratio.

#### Column +UI is the benchmark.

#### TAKEAWAY

The benchmark HANC model delivers amplification of aggregate shocks relative to the standard representative-agent RBC.

Wealth inequality is the reason. Idiosyncratic earnings risk (Transitory and Persistent) and preference heterogeneity, in particular, are responsible for most of the added amplification.

Impatient households consume too much in booms and too little in recessions, which is exacerbated by counter-cyclical employment risk. Idiosyncratic earnings risk increases the share of low-wealth, hand-to-mouth agents in the distribution.

Unemployment insurance has two conflicting effects: for a given distribution, it dampens the recessionary effects; but it also shifts the distribution leftward such that the average consumption decline grows. The net effect is a quantitative question.

## GALÍ, LÓPEZ-SALIDO, AND VALLÉS (2007)

#### Workhorse Two Agent New Keynesian (TANK) model.

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## Households

Unit mass continuum. Fraction  $1 - \lambda$  are Ricardian: can buy and sell capital, have full access to contingent securities.

Fraction  $\lambda$  are "rule-of-thumb" in the sense of Campbell and Mankiw (1989). These households just consume labor income and hold no assets.

Superscript *O* stands for "optimizers". Superscript *r* stands for "rule-of-thumb".

Denote:  $P_t^O$  the price of the final good,  $W_t$  the real wage,  $N_t^O$  labor supply,  $K_t^O$  capital holdings,  $R^k$  capital rental rate,  $B_t^O$  risk-less one-period bonds,  $R_t$  nominal return on bonds,  $D_t^O$  dividends from firm ownership,  $T_t^O$  lump-sum taxes,  $I_t^O$  investment, and  $C_t^O$  consumption.

## **RICARDIAN HOUSEHOLDS**

Ricardian (optimizing) households solve:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^O, N_t^O) \tag{1}$$

s.t.

$$P_t(C_t^O + I_t^O) + \frac{B_{t+1}^O}{R_t} = W_t P_t N_t^O + R_t^k P_t K_t^O + B_t^O + D_t^O - P_t T_t^O$$
(2)

and the law of motion of capital:

$$K_{t+1}^{O} = (1-\delta)K_t^{O} + \phi\left(\frac{I_t^{O}}{K_t^{O}}\right)K_t^{O}$$
(3)

 $\phi\left(\frac{I_t^O}{K_t^O}\right)K_t^O$  are capital adjustment costs such that  $\phi' > 0$ ,  $\phi'' \le 0$ , and  $\phi(\delta) = \delta$ .

## Rule-of-thumb Households

U

No consumption smoothing or intertemporal substitution.

Utility is given by:

$$(C_t^r, N_t^r) \tag{4}$$

Budget constraint:

$$P_t C_t^r = W_t P_t N_t^r - P_t T_t^r \tag{5}$$

Consumption is simply disposable income:

$$C_t^r = W_t N_t^r - T_t \tag{6}$$

Competitive labor market:

$$W_t = C_t^r (N_t^r)^{\varphi} \tag{7}$$

## AGGREGATION

Aggregate consumption is the weighted-average of type-level consumption:

$$C_t = \lambda C_t^r + (1 - \lambda) C_t^O \tag{8}$$

Similarly for employment:

$$N_t = \lambda N_t^r + (1 - \lambda) N_t^O \tag{9}$$

And for aggregate investment and capital stock:

$$I_t = (1 - \lambda)I_t^O \quad K_t = (1 - \lambda)K_t^O \tag{10}$$

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#### FIRMS

Representative, perfectly competitive firm with CRS technology produces the final good:

$$Y_t = \left(\int_0^1 X_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}}\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$
(11)

where  $X_t(j)$  are differentiated intermediate goods that are produced by a unit mass continuum of monopolistically competitive firms. Standard downward-sloping demand schedule:

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$
(12)

Price index:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj\right)^{\frac{1}{1-\epsilon_p}}$$
(13)

#### FIRMS

Production function for intermediate goods firms:

$$Y_t(j) = K_t(j)^{\alpha} N_t(j)^{1-\alpha}$$
(14)

Standard cost minimization yields:

$$\frac{K_t(j)}{N_t(j)} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{W_t}{R_t^k}\right)$$
(15)

The real marginal cost is common to all firms:

$$MC_t = \alpha^{-\alpha} (1 - \alpha)^{-1(1 - \alpha)} (R_t^k)^{\alpha} (W_t)^{1 - \alpha}$$
(16)

# PRICE SETTING

Calvo (1983) staggered price setting by intermediate good firms. Each firm resets its price with probability  $1 - \theta$  each period.

Firm resetting its price in period *t* solves:

$$\max_{P_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \mathbf{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( (P_t^*/P_{t+k}) - MC_{t+k} \right) \right\}$$
(17)

s.t. to the sequence of demand constraints:

$$Y_{t+k}(j) = X_{t+k}(j) = (P_t^*/P_{t+k})^{-\epsilon_p} Y_{t+k}$$
(18)

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## PRICE SETTING

First-order condition:

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k}(j) \left( (P_t^*/P_{t+k}) - \mu^p M C_{t+k} \right) \right\}$$
(19)

where  $\mu^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$  is the frictionless markup and  $\Lambda$  is the household SDF. Law of motion of the aggregate price level:

$$P_{t} = \left[\theta P_{t-1}^{1-\epsilon_{p}} + (1-\theta)(P_{t}^{*})^{1-\epsilon_{p}}\right]^{\frac{1}{1-\epsilon_{p}}}$$
(20)

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## ECONOMIC POLICY

Monetary policy follows the Taylor rule:

$$r_t = \bar{r} + \phi_\pi \pi_t \tag{21}$$

where  $\phi \ge 0$  and  $\bar{r}$  is the nominal interest rate target.

Government budget constraint is:

$$P_t T_t + \frac{B_{t+1}}{R_t} = B_t + P_t G_t$$
 (22)

where  $T_t = \lambda T_t^r + (1 - \lambda)T_t^O$ 

Government spending G follows an AR(1) process in growth rates g:

$$g_t = \rho_g g_{t-1} + \epsilon \tag{23}$$

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#### MARKET CLEARING

Factor and good markets clear for all time periods:

$$N_t = \int_0^1 N_t(j) dj$$
  

$$K_t = \int_0^1 K_t(j) dj$$
  

$$Y_t(j) = X_t(j), \quad \forall j$$
  

$$Y_t = C_t + I_t + G_t$$

(24)

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## IMPACT MULTIPLIER

A. Competitive Labor Market



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# IMPULSE RESPONSE TO FISCAL SPENDING SHOCK

A. Competitive Labor Market



### TAKEAWAY

The aggregate potency of fiscal policy is enhanced with the presence of hand-to-mouth consumers.

The breakdown of Ricardian equivalence makes fiscal policy powerful in the short run, as h-t-m households are sensitive to transitory income windfalls.

Paper also considers the case of imperfectly competitive labor markets. In this case, wages are set in a centralized manner by an economy-wide union. Hours are determined by firms, given the wage set by the union.

With non-competitive labor markets, the amplifying effects of rule-of-thumb consumers get even stronger and quantitative performance improves.

## KAPLAN, MOLL, AND VIOLANTE (2018)

#### Workhorse Heterogeneous Agent New Keynesian (HANK) model.

## Households

- Continuum of households indexed by holdings of liquid assets b, illiquid assets a, and idiosyncratic labor productivity z.
- Markets are incomplete. z is Markov. Time is continuous.
- Denote the joint distribution as  $\mu_t(da, db, dz)$ .
- Households die with Possion intensity  $\zeta$ . This assumption is useful to generate sufficient number of low-wealth, high-elasticity agents.

- Labor supply is endogenous. Denote  $\rho$  the rate of time pdiscounting:
- No aggregate uncertainty and the law of large numbers applies.

## Households

Households can borrow in liquid assets *b* up to an exogenous limit <u>b</u> at the real interest rate  $r_t^{b-} = r_t^b + \kappa$ .  $\kappa > 0$  is an exogenous wedge between lending and borrowing rates.

Denote as *a* illiquid assets. Households must pay a cost of depositing or withdrawing funds from *a*.

Denote  $d_t$  the deposit flow rate and  $\chi(d_t, a_t)$  the flow cost of depositing at rate  $d_t$  for a household with holdings  $a_t$ .

Illiquid asset transaction cost guarantees that  $r_t^a > r_t^b$  in equilibrium.

## HOUSEHOLD PROBLEM

Households maximize:

$$\mathbf{E}_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t, l_t) dt \tag{1}$$

subject to:

$$\dot{b}_{t} = (1 - \tau_{t})w_{t}z_{t}l_{t} + r_{t}^{b}(b_{t})b_{t} + T_{t} - d_{t} - \chi(d_{t}, a_{t}) - c_{t}$$

$$\dot{a}_{t} = r_{t}^{a}a_{t} + d_{t}$$
(3)
$$b_{t} = b_{t} - c_{t} \ge 0$$
(4)

$$b_t \ge -\underline{\mathbf{D}}, \quad u_t \ge 0$$
 (4)

where  $T_t$  are government transfers and  $\tau_t$  is the labor income tax rate. The functional form for  $\chi$  is:

$$\chi(d,a) = \chi_0|d| + \chi_1 \left| \frac{d}{a} \right|^{\chi_2} a \tag{5}$$

with  $\chi_1 > 0, \chi_2 > 1$  ensuring deposit rates are finite.  $\chi_0 > 0$  creates a region of inaction.

#### FINAL GOODS PRODUCER

Representative, competitive firm produces the final good with intermediate goods as inputs:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0$$
(6)

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Standard cost minimization implies:

$$y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t, \quad P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$
(7)

#### INTERMEDIATE GOODS PRODUCER

Each good is produced by a monopolistically competitive firm:

$$y_{jt} = k_{jt}^{\alpha} n_{jt}^{1-\alpha} \tag{8}$$

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Cost minimization implies a common marginal cost:

$$m_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}$$
(9)

## PRICE SETTING

Intermediate good producer chooses prices to maximize profits subject to Rotemberg (1982) adjustment costs, which are quadratic in the rate of price change:

$$\Theta_t \left(\frac{\dot{p}_t}{p_t}\right) = \frac{\theta}{2} \left(\frac{\dot{p}_t}{p_t}\right)^2 Y_t, \quad \theta > 0$$
(10)

One can show that the solution to the optimization problem of the intermediate good producer characterizes the New Keynesian Phillips curve:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\theta}(m_t - m^*) + \dot{\pi}_t, \quad m^* = \frac{\epsilon - 1}{\epsilon}$$
(11)

where  $1/m^*$  is the flex-price markup.

## COMPOSITION OF ILLIQUID WEALTH

Illiquid wealth can be invested into either physical capital  $k_t$  or equity claims on intermediate firms  $s_t$  that are priced at  $q_t$ .

Individual's illiquid assets can be thus written as  $a_t = k_t + q_t s_t$ .

The law of motion of capital and equity is:

$$\dot{k}_t + q_t s_t = (r_t^k - \delta)k_t + \Pi_t s_t + d_t \tag{12}$$

where  $\Pi_t$  are monopoly profits net of price adjustment costs.

Absence of arbitrage guarantees that returns on equity and capital equilize:

$$\frac{\Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta = r_t^a \tag{13}$$

Thus, market clearing condition for capital will pin down  $r_t^a$ .

## ECONOMIC POLICY

Monetary authority follows a Taylor rule:

$$i_t = \bar{r}^b + \phi \pi_t + \epsilon_t \tag{14}$$

with  $\phi > 1$  and  $\mathbb{E}\epsilon = 0$ .

The Fisher equation relates the nominal interest rate with the real return on liquid assets:

$$r_t^b = i_t - \pi_t \tag{15}$$

The fiscal authority's intertemporal budget constraint is:

$$\dot{b}_t^g + G_t + T_t = \tau_t \int w_t z_t l_t(a, n, z) d\mu_t + r_t^b B_t^g$$
 (16)

where *G* is exogenous government expenditure,  $\tau$  a labor income tax rate,  $T_t > 0$  the lump-sum transfer, and  $B_t^g$  real short-term bonds.

## Equilibrium

The liquid asset market clears:

$$\int bd\mu_t + B_t^g = 0 \tag{17}$$

The illiquid asset market clears, with the total number of shares normalized to 1:

$$K_t + q_t = \int a d\mu_t \tag{18}$$

Labor market clears:

$$N_t = \int z l_t(a, b, z) d\mu_t \tag{19}$$

Goods market clears:

$$Y_t = C_t + I_t + G_t + \underbrace{\Phi_t}_{\text{Price Adjustment Cost}} + \underbrace{\chi_t}_{\text{Transaction Cost}} + \underbrace{\kappa \int \max\{-b, 0\} d\mu_t}_{\text{Borrowing Cost}}$$
(20)

#### MONETARY TRANSMISSION MECHANISM

Write aggregate consumption as a function of the full sequence of equilibrium prices, taxes, and transfers  $\Gamma_t = \{r_t^b, r_t^a, w_t, \tau_t, T_t\}_{t \ge 0}$ :

$$C_t(\{\Gamma_t\}_{t\geq 0}) = \int c_t(a, b, z; \{\Gamma_t\}_{t\geq 0}) d\mu_t$$
(21)

Totally differentiate (21):

$$dC_{0} = \underbrace{\int_{0}^{\infty} \frac{\partial C_{0}}{\partial r_{t}^{b}} dr_{t}^{b} dt}_{\text{Direct Effect}} + \underbrace{\int_{0}^{\infty} \left(\frac{\partial C_{0}}{\partial w_{t}} dw_{t} + \frac{\partial C_{0}}{\partial r_{t}^{a}} dr_{t}^{a} + \frac{\partial C_{0}}{\partial \tau_{t}} d\tau_{t} + \frac{\partial C_{0}}{\partial T_{t}} dT_{t}\right) dt}_{\text{Indirect Effects}}$$
(22)

## MPC HETEROGENEITY

Panel B.  $MPC_1^{$500}(a, b, z)$ 



#### AGGREGATE IMPULSE RESPONSE



Panel A. Monetary shock, interest rate, inflation

Panel B. Aggregate quantities

FIGURE 3. IMPULSE RESPONSES TO A MONETARY POLICY SHOCK (A Surprise, Mean-Reverting Innovation to the Taylor Rule)

## IMPULSE RESPONSE DECOMPOSITION



FIGURE 4. DIRECT AND INDIRECT EFFECTS OF MONETARY POLICY IN HANK

Notes: Returns are shown as annual percentage point deviations from steady state. Real wage and lump-sum transfers are shown as log deviations from steady state.

## THE ROLE OF THE FISCAL REACTION

T adjusts (1)	G adjusts	$\tau$ adjusts	B <sup>g</sup> adjusts
	(2)	(3)	(4)
-0.28	-0.23	-0.33	-0.34
-3.96	-7.74	-3.55	-2.17
-9.43	-14.44	-8.80	-5.07
-2.93	-2.80	-2.75	-1.68
-0.55	-0.60	-0.56	-0.71
19	21	20	42
51	81	62	49
32	-	-	9
_	_	18	_
$^{-2}$	-2	0	0
	$\begin{array}{r} (1) \\ -0.28 \\ -3.96 \\ -9.43 \\ -2.93 \\ -0.55 \\ 19 \\ 51 \\ 32 \\ -2 \\ -2 \end{array}$	$\begin{array}{c ccccc} (1) & (2) \\ \hline -0.28 & -0.23 \\ \hline -3.96 & -7.74 \\ -9.43 & -14.44 \\ \hline -2.93 & -2.80 \\ -0.55 & -0.60 \\ \hline \\ 19 & 21 \\ 51 & 81 \\ 32 & - \\ \hline -2 & -2 \\ \hline \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

TABLE 8-IMPORTANCE OF FISCAL RESPONSE TO MONETARY SHOCK

*Notes:* Average responses over the first year. Column 1 is the baseline specification in which transfers T adjust to balance the government budget constraint. In column 2 government expenditure G adjusts, and in column 3 the labor income tax  $\tau$  adjusts. In column 4 government debt adjusts, as described in the main text.

## TAKEAWAY

Share of hand-to-mouth (high MPC) households is important for the aggregate effects of monetary policy. HANK endogenizes this share with market incompleteness and two-asset structure. Many complementary, sophisticated models now exist.

Generally, high-MPC households respond sharply to changes in both labor income and government transfers that occur in equilibrium in the wake of a monetary shock.

The rise in labor income is a consequence of an expansionary monetary shock that increases demand for final goods.

Transfers rise because the interest payments on government debt fall and because the rise in aggregate income increases tax revenues.

Generally, indirect effects account for 80% of the total consumption response in KMV's HANK.