The Regional Keynesian Cross†

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September 14, 2023

Abstract

Does regional heterogeneity matter for the transmission of monetary policy in a currency union? We build a heterogeneous agents New Keynesian model of a monetary union with two-layered regional heterogeneity. Regions are heterogeneous in (i) the local intertemporal Marginal Propensity to Consume (iMPC) and (ii) the size of the non-tradable sector. At the regional level, a Keynesian multiplier operates through the non-tradable sector. The magnitude of the regional multiplier is increasing non-linearly in (i) and (ii). We show that, because of this non-linearity, the joint distribution over space of our two layers of regional heterogeneity shapes the nation-wide transmission of monetary policy. We provide empirical support for our theory using detailed county-level data from the US and construct a novel county-level measure of MPCs. We find that MPCs and the size of the non-tradable sector are the most important drivers of the regional heterogeneity of the employment response to monetary policy. Our sufficient statistic approach suggests that regional heterogeneity amplifies the national response to monetary policy in the context of the US economy.

†We thank Greg Mankiw and Ludwig Straub for very valuable discussions in the early stages of the project. We also thank Klaus Adam, Adrien Auclet, Bence Bardóczy, Christian Bayer, Adrien Bilal, Ambrogio Cesa-Bianchi, Luis Calderon (discussant), Jeff Campbell (discussant), Edouard Challe (discussant), Giancarlo Corsetti, Wouter Den Haan, Matthias Doepke, Jan Eeckhout, Axelle Ferriere, Jordi Gali, Manuel García-Santana, Joe Hazell, Tomer Ifergane, Ethan Ilzetzki, Michael McMahon, Isabelle Mejean, Kurt Mitman, Ben Moll, Tommaso Monacelli, Gernot Müller, Dima Mukhin, Steve Pischke, Ricardo Reis, Nicolò Rizzotti, Adi Soenarjo, Iván Werning and seminar participants at various venues for useful comments and suggestions. Oliver Seager provided excellent research assistance.

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1 Introduction

A long-standing strand of literature pioneered by Mundell (1961), McKinnon (1963) and Kenen (1969) has emphasized the importance of openness to trade for the transmission of monetary policy and the design of monetary unions. A more recent literature, building on ideas dating back to Keynes (1936), has stressed the importance of household heterogeneity and intertemporal Marginal Propensities to Consume (iMPCs) for the propagation of aggregate shocks via a Keynesian multiplier effect. So far, the insights from these two strands of literature have remained largely separated. In this paper, we bring them together to study the transmission of monetary policy across regions, both theoretically and empirically. We do so by building a multi-region heterogeneous agents New Keynesian (HANK) model of a monetary union featuring two-layered regional heterogeneity. First, we introduce regional heterogeneity in intertemporal Marginal Propensities to Consume. Second, we allow regions to differ in the relative size of the non-tradable sector. Thus, our modeling framework features both household heterogeneity within regions as well as iMPCs and trade openness heterogeneity between regions. In this setting a regional variant of the Keynesian multiplier is present.

To understand this, it is useful to study a simple static Keynesian framework and consider a shock $\varepsilon$ to aggregate income in a given region. As usual, a fraction MPC of the shock is going to be passed-through to local consumption. However, when the region is atomistic, the extra spending in tradable goods will be lost to the rest of the nation. Only the fraction of income spent in the non-tradable sector is going to remain within the region and translate in additional income, thus feeding further increases in consumption. Letting $\rho$ denote the relative size of the non-tradable sector, this means that the local output response to the shock is $dY = \rho \times \text{MPC} \times \varepsilon$, with the regional Keynesian multiplier given by $\frac{1}{1 - \rho \times \text{MPC}}$. It is immediate to see that this regional multiplier is non-linearly increasing in $\rho \times \text{MPC}$. This has two implications. First, keeping fixed the size of the shock $\varepsilon$, regions with different $\rho$ and MPC are going to have different output responses $dY$. Second, because of the non-linear nature of the multiplier, the joint distribution of $\rho$ and MPC over space is going to matter for the national output response.

We formalize these results by deriving a general equilibrium regional variant of the canonical Keynesian-cross-like representation, which we label the regional Keynesian cross. Our formula shows that the first-order response of local employment to changes in interest rates can be decomposed into three channels: exposure to regional fluctuations, exposure to national fluctuations, and expenditure switching. We express these channels analytically as a simple function of iMPCs and the non-tradable share of the wage bill using a
sequence space representation.

We move beyond the regional Keynesian cross representation by aggregating the continuum of counties in our economy into an expression that we label the national Keynesian cross. The national Keynesian cross showcases how the nation-wide response to monetary policy is shaped by the joint distribution of iMPCs and openness to trade across regions, as a result of the non-linearities present in the regional Keynesian multiplier. We also provide an intuitive and tractable decomposition of the response of aggregate employment to monetary shocks into four components: a “representative region” Keynesian cross, heterogeneity in the openness to national trade, heterogeneity in local iMPCs, and a complementarity term that links openness to trade and household MPCs. We argue that a complete characterization of the national macroeconomic response is only possible in a model that jointly captures MPCs and openness to trade heterogeneity. In general, failure to incorporate either of these two elements leads to model mis-specification because the complementarity term –which is essential both in the data and in our theory– would then be lost. However, we also provide conditions under which an “as-if” result (Werning, 2015) applies and the distribution of iMPCs across regions is irrelevant for nation-wide dynamics. In fact, absent trade frictions, the national response to monetary policy is identical to that of a “representative region”.

Our theoretical results can speak to the intertemporal Keynesian cross (Auclert et al., 2023), the New Keynesian cross (Bilbiie, 2020), and the international Keynesian cross (de Ferra et al., 2020, Auclert et al., 2021b). Crucially, our framework moves beyond the elegant convenience of two agent New Keynesian (TANK) models where a fixed fraction of households are non-Ricardian (Campbell and Mankiw, 1989, Galí et al., 2007, Bilbiie, 2008). Instead, each county in our multi-region economy is populated by a continuum of households, each endogenously featuring different iMPCs like in the standard HANK literature and in line with the powerful finding in Hagedorn et al. (2019) that incomplete markets and a full distribution of households are essential elements to analyze the magnitude of MPCs and of the macroeconomic propagation of shocks.

Empirically, we develop a novel methodology to construct the first measure of county-level MPCs. Our approach is close in spirit to Patterson (2023) and estimates regional MPCs by leveraging the full joint distribution of households’ socio-economic characteristics within each county. We then document that US monetary policy surprises induce local employment responses that vary substantially across US counties. Two characteristics go a long way in accounting for this observable heterogeneity: our novel proxy for regional MPCs, and the local non-tradable to tradable employment ratio. This finding thus provides an empirical backing for the two layers of regional heterogeneity that we introduce
in our model. Finally, we find that counties with either high non-tradable employment or high MPCs are more responsive to monetary policy. Crucially, these two channels are jointly significant, both economically and statistically.

Our theoretical and empirical results are important for at least two reasons. First, it is often assumed in the optimal currency area (OCA) literature that symmetric demand shocks can be handled by the monetary authority, while asymmetric shocks require fiscal stabilization. We contrast this view by showing that monetary policy itself induces asymmetric responses across different regions within a currency area. Moreover, as we show in our data, the ability of the central bank to influence local economic activity is dampened the more open to national trade a region is, i.e., the lower non-tradable employment is. This is in stark contrast to the seminal idea of McKinnon (1963) that openness to trade alleviates the costs of monetary unions and is, in fact, in line with the powerful insights of Farhi and Werning (2016a, 2017): monetary stabilization is more effective if regions are less open to inter-regional trade.

Second, our findings are important for the large and ever-growing literature that bridges together incomplete markets, cross-sectional heterogeneity, and nominal rigidities (Werning, 2015, Auclert, 2019). In particular, the influential HANK literature emphasizes the importance of heterogeneity in households’ MPCs (McKay and Reis, 2016, Kaplan et al., 2018). Our theoretical and empirical results, on the other hand, suggest that in order to rationalize the dynamics which is observed in the data, it is important to account for a second layer of heterogeneity: openness to national trade. As we emphasize throughout the paper, modelling both channels of heterogeneity at the same time leads to multiple novel implications.

**Literature** Our paper contributes to several different literature strands. First and foremost, we contribute to the literature that embeds incomplete-markets economies (Bewley, 1977, Huggett, 1990, Aiyagari, 1994, Imrohoglu, 1996) into environments with nominal rigidities. In particular, we develop a HANK model of a monetary union with two-layered regional heterogeneity and use it to study the regional and aggregate economic effects of demand-driven fluctuations.\(^1\)

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Second, our framework and analysis are conceptually related to the OCA literature (Mundell, 1961, McKinnon, 1963, Kenen, 1969, Alesina et al., 2002, Kenen and Meade, 2008). Specifically, important ideas that we touch upon in the context of US regional dynamics are openness to trade (McKinnon, 1963), factor mobility across counties (Blanchard and Katz, 1992), and fiscal integration and stabilization policies (Farhi and Werning, 2016a, 2017). In doing so, our modelling approach is heavily inspired by Farhi and Werning (2017)’s treatment of fiscal unions. Our framework presents a tractable nexus between the above two literature strands which in large part emphasize, respectively, the role of cross-sectional heterogeneity in MPCs and openness to trade (or, alternatively, home bias). In fact, an absolutely crucial component of our narrative is the interaction between openness to national trade and households’ MPC—a complementarity that we capture both in the model and in the data.

Third, our paper builds on the new open-economy macroeconomics literature (Obstfeld and Rogoff, 1995, Galí and Monacelli, 2005, Corsetti and Pesenti, 2005, Rey, 2013). Our theoretical framework is, methodologically, essentially a continuum of small open counties which are modelled in the spirit of the Galí and Monacelli (2005, 2008) small open economy setup. Our solution characterization is in the sequence space, an approach developed and popularized in the works of Mankiw and Reis (2006), Boppart et al. (2018), and Auclert et al. (2021a). Fourth, our empirical analysis complements studies that elicit macroeconomic and/or partial equilibrium elasticities in response to policy shocks (not limited to those by the monetary authority) from regional data, often in combination with structural modelling (Nakamura and Steinsson, 2014, Chodorow-Reich, 2019).² Last but not least, our paper contributes to a growing body of work that studies how monetary policy operates across space, building on Carlino and Defina (1998) who, to the best of our knowledge, are the first to provide empirical evidence on differential regional effects of US monetary policy.³

2 A Model of the Regional Keynesian Cross

In this section, we build a model of a monetary union composed of a continuum of small open counties (Galí and Monacelli, 2005). Our economy features no aggregate uncertainty

²Some relevant examples include Beraja et al. (2019), Guren et al. (2020), Chodorow-Reich et al. (2021), Holm et al. (2021), Wolf (2021a,b), Dupor et al. (2023), Hazell et al. (2022), Beraja and Wolf (2022), Patterson (2023), McCrory (2022), among others. See Chodorow-Reich (2020) for a comprehensive discussion.

³Other notable studies include Adam and Zhu (2016), Corsetti et al. (2021), Adam et al. (2022), Almgren et al. (2022), De Ridder and Pfajfar (2017), Fornaro and Romei (2022), Hauptmeier et al. (2023), Bergman et al. (2022), Herreño and Pedemonte (2022), and Pica (2023).
and we restrict attention to perfect-foresight transitions in response to zero-probability “MIT shocks”. We model each county as an Heterogeneous Agents New Keynesian (HANK) economy featuring incomplete markets and nominal rigidities. Our regional framework comprises two production sectors. In particular, households’ preferences are defined over two types of consumption goods: a tradable and a non-tradable good. While tradable goods can be produced everywhere and are such that the law of one price holds, non-tradable goods need to be consumed in the same place where they have been produced.

In modeling differences between counties, we bring together insights from two strands of literature which have remained so far largely separated. First, there is a burgeoning literature studying household heterogeneity in Keynesian settings. This literature has emphasized the crucial role that marginal propensities to consume (MPCs) play in shaping both the channels and the magnitude of the propagation of aggregate shocks to the macroeconomy (Bilbiie, 2008, Oh and Reis, 2012, Kaplan et al., 2018, Auclert, 2019, Auclert et al., 2023). Second, following the seminal work of McKinnon (1963), several studies have highlighted the importance of openness to trade for the transmission of aggregate shocks in general, and monetary policy in particular (Mian and Sufi, 2014, Cugat, 2019, Chodorow-Reich et al., 2021, Auclert et al., 2021b, Fornaro and Romei, 2022). We thus introduce two layers of heterogeneity between counties and allow our regions to differ both in the degree of exposure to the tradable and non-tradable sector, as well as in their local MPCs. In Section 5 we show that our focus on these two dimensions of heterogeneity is supported by the data. In fact, empirical proxies for local openness to trade and MPCs also turn out to be important predictors of the heterogeneous transmission of monetary policy across US counties observed in the data.

2.1 Setup

Time $t \geq 0$ is discrete. There is a continuum of atomistic counties indexed by $j \in [0, 1]$ and modeled as small open economies à la Gali and Monacelli (2005). There is no aggregate uncertainty and we consider perfect-foresight impulse responses to shocks around the steady-state (“MIT shocks”).

**Households** Each county $j$ is inhabited by a continuum of households $i \in [0, 1]$. As in the standard incomplete markets model, households are ex-ante identical, but face non-insurable idiosyncratic shocks to their labor productivity $e$, which evolves over time ac-
according to some general Markovian process which is county-specific. Allowing for different income processes is a reduced-form way of getting different levels of iMPCs across counties, thus generating our first layer of regional heterogeneity. The preferences of household \( i \) living in county \( j \) are defined over an aggregate consumption good \( c_{jit} \) as well as aggregate labor supply \( \ell_{jit} \), which imply the following time-0 utility:

\[
E_0 \sum_{t \geq 0} \beta^t \{ u(c_{jit}) - v(\ell_{jit}) \}
\]

Agents pay a proportional tax \( \tau_t \) on their real labor income and can imperfectly insure themselves by trading in a nominal risk-free bond with real value \( b_{jit} \) subject to a borrowing limit \( b \leq 0 \). Their budget constraint then reads:

\[
c_{jit} + b_{jit+1} = z_{jit} \ell_{jit} + (1 + r_{jt}) b_{jit}, \quad b_{jit+1} \geq b \tag{1}
\]

In (1) above, we denote by \( z_{jit} \) real gross labor income which is given by:

\[
z_{jit} = \frac{W_{jt}}{P_{jt}} \ell_{jit}
\]

where \( W_{jt} \) and \( P_{jt} \) are respectively the aggregate wage and price index in county \( j \) and \( D_{jt} \) is Federal real transfers. All will be defined momentarily. From (1) it immediately follows that the resource constraint for county \( j \) reads:

\[
P_{jt} C_{jt} + B_{jt+1} = Z_{jt} + (1 + r_{jt}) B_{jt}
\]

Where \( C_{jt} \equiv \int_0^1 c_{jit} \, di \) and \( B_{jt} \equiv \int_0^1 b_{jit} \, di \) respectively denote aggregate consumption and bond holdings in county \( j \).

**Demand Composition** There are two consumption goods in the economy: non-tradables and tradables. The defining feature of non-tradable goods is that they must be consumed in the same county where they have been produced. Tradable goods, on the other hand, can be freely shipped across the nation, so that the location of production is completely decoupled from that of consumption. Tradables and non-tradables are combined into the aggregate consumption basket \( c_{jit} \) according to a constant-elasticity-of-substitution (CES) aggregator

\[
c_{jit} = \left[ \omega^{1/v} \left( c_{jit}^{\text{NT}} \right)^{(v-1)/v} + (1 - \omega)^{1/v} \left( c_{jit}^{\text{T}} \right)^{(v-1)/v} \right]^{v/\nu} \tag{2}
\]
Where $c_{jit}^{NT}$ and $c_{jit}^T$ respectively denote consumption of the non-tradable and tradable good, $\omega$ is a parameter governing households’ preferences for non-tradables and $\nu > 0$ is the elasticity of substitution between the two types of goods. Both of these parameters are constant across counties. In turn, households split their spending between the two types of goods as follows:

$$c_{jit}^{NT} = \omega \left( \frac{P_{jit}^{NT}}{P_{jt}} \right)^{-\nu} c_{jit} \quad \text{and} \quad c_{jit}^T = (1 - \omega) \left( \frac{P_{jt}^T}{P_{jt}} \right)^{-\nu} c_{jit}$$

(3)

Where $P_{jit}^{NT}$ and $P_{jt}^T$ represent, respectively, county $j$’s price index for non-tradable and tradable goods, while $P_{jt}$ is the aggregate price index in county $j$. Because in our model preferences are homothetic and do not depend on the household type $i$, both the price and wage indices as well as the composition of the consumption basket will be identical across household types within one county.\(^5\) Moreover, we assume perfect substitutability between tradable goods produced in different counties:

$$c_{jit}^T = \int_0^1 c_{jit}^T(j')dj'$$

Hence, the law of one price holds nationally for the tradable good, i.e., $P_{jt}^T = P_t^T$ for all $j$.\(^6\) Note that, because tradable goods produced in different counties are perfect substitutes, for a given level of tradable consumption $c_{jit}^T$, the composition of this consumption $\{c_{jit}^T(j')\}_{j' \in [0,1]}$ will be indeterminate. We solve this indeterminacy by assuming that the share of tradable consumption sourced from every county $j' \in [0,1]$ is equal across counties $j'$. This, coupled with the small open county-economy assumption and the absence of home bias, implies that within-county demand for tradable goods produced by the same county will be zero. Finally, the price index corresponding to the preferences represented in (2) is given by:

$$P_{jt} = \left[ \omega \left( \frac{P_{jt}^{NT}}{P_{jt}} \right)^{1-\nu} + (1 - \omega) \left( \frac{P_t^T}{P_{jt}} \right)^{1-\nu} \right]^{1-\nu}$$

(4)

\textbf{Supply Composition}  \quad \text{Similarly to demand, the supply side of each county is comprised of two sectors: one producing the tradable good and one producing the non-tradable good. We follow Berger et al. (2022) to model the supply of labor to the two sectors: individual households’ aggregate labor supply $\ell_{jit}$ is a composite of a measure of labor}

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\(^5\)Clearly, the level of consumption can still differ between households within a county.

\(^6\)Here we are ruling out home bias in the consumption of tradable goods. This simplifies our analysis and does not affect the substance of any of the results.
supplied to the non-tradable sector $\ell_{jit}^{NT}$ and a measure $\ell_{jit}^T$ supplied to the tradable sector. In particular, the labor supply composition is given by the following CES aggregator:

$$\ell_{jit} = \left( \alpha_j^{-\frac{1}{\eta}} (\ell_{jit}^{NT})^{\frac{\eta+1}{\eta}} + (1 - \alpha_j)^{-\frac{1}{\eta}} (\ell_{jit}^T)^{\frac{\eta+1}{\eta}} \right)^{\frac{\eta}{\eta+1}} \tag{5}$$

Where $\eta$ is the elasticity of substitution between the two sectors and is assumed to be constant across counties. This parameter governs how easy it is to reallocate workers between the two sectors. The parameter $\alpha_j$, on the other hand, is county-specific and captures the propensity of county $j$ to produce non-tradable goods. It is going to play a crucial role in our analysis. Given (5), households split their labor supply in the following fashion:

$$\ell_{jit}^{NT} = \alpha_j \left( \frac{W_{jit}^{NT}}{W_{jit}} \right)^{\eta} \ell_{jit}, \quad \text{and} \quad \ell_{jit}^T = (1 - \alpha_j) \left( \frac{W_{jit}^T}{W_{jit}} \right)^{\eta} \ell_{jit} \tag{6}$$

Finally, the wage index corresponding to this labor supply structure is given by:

$$W_{jt} = \left[ \alpha_j (W_{jt}^{NT})^{1+\eta} + (1 - \alpha_j)(W_{jt}^T)^{1+\eta} \right]^{\frac{1}{1+\eta}} \tag{7}$$

**Final Good Producers** Firms in both the tradable and the non-tradable sector produce using a linear production technology: $Y_{st} = L_{st}$, $s \in \{NT, T\}$. Moreover, in both sectors the market for final goods is perfectly competitive. As a result, final prices for the two goods equal the marginal cost, i.e., $P_{jt}^s = W_{jt}^s$, $s \in \{NT, T\}$. Note that because the law of one price holds in the tradable sector, the wage in this sector is going to be equalized across counties, i.e., $W_{jt}^T = W_t^T$ for all $j \in [0, 1]$.

**Labor Markets** Our economy features nominal rigidities in the form of sticky wages. In line with the New Keynesian sticky-wage literature (Erceg et al., 2000, Schmitt-Grohé and Uribe, 2005, Auclert et al., 2023), we assume that the amount of hours worked is determined by labor unions. In particular, there are two sets of labor union. First, in each county $j$ there is a continuum of labor unions $i \in [0, 1]$ which set nominal non-tradable wages in county $j$ subject to quadratic utility cost of wage adjustment in order to maximize the welfare of the average household in that county. Unions then allocate labor among their members in a uniform fashion, i.e., $\ell_{jit}^{NT} = \ell_{jt}^{NT}$. The problem of the labor union

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7The parameter $\alpha_j$ can be equivalently interpreted as governing county $j$’s non-tradable labor endowment.

8The assumptions that the union maximizes the welfare of the average household, as well as the uniform labor allocation rule can be easily relaxed to more general cases.
in the non-tradable sector gives rise to a county-specific Phillips Curve for non-tradable wages. Similarly, there is a continuum of national labor unions $\zeta \in [0, 1]$ which are in charge of setting nominal wages in the tradable setting in order to maximize the welfare of the average household in the nation. As in the case of non-tradable unions, unions in the tradable sector also face quadratic utility costs to wage adjustments and allocate labor among their members in a uniform fashion, so that $\ell^T_{jt} = \ell^T_t$. The problem of the labor union in the tradable sector gives rise to one national Phillips Curve for tradable wages.\textsuperscript{9}

**Monetary policy** Monetary policy follows a real interest rate rule.\textsuperscript{10}

$$i_t = r_t + \pi_{t+1} + \epsilon_t$$

where $\pi_t$ denotes national inflation, $\pi_t = \int_0^1 \pi_{jt} dj$. This rule is a special case of the standard Taylor Rule, with a coefficient of 1 on inflation. This real interest rate rule assumption guarantees that the wage Phillips curve only affects nominal quantities. While this specific form for the monetary rule is not needed for deriving the Regional Keynesian Cross in the next section, it is going to be useful to simplify our derivation of the National Keynesian Cross later on in the text, which will then be a function of real variables alone.

### 2.2 Equilibrium

Because of the fact that each county is atomistic from the perspective of the national economy, throughout the rest of our analysis we will rely on two different equilibrium concepts. In particular, when focusing on a single county in isolation, we will focus on Regional equilibrium. This is equivalent to the notion of equilibrium used in the international macroeconomics literature studying small open economies. In particular, when considering a regional equilibrium, we will take all national variables—including tradable prices—as exogenously given. On the other hand, when focusing on nation-wide responses, we will rely on the concept of National equilibrium, which is the standard equilibrium concept used in closed economy macroeconomics and posits that every single county, as well as the national economy overall, must experience market clearing, and thus solves for tradable prices endogenously.

**Definition 1** (Regional Equilibrium). Given an initial regional distribution $G_{j0}(b, e)$ over bonds $b$ and idiosyncratic labor productivity $e$, an exogenous path for national demand for tradables

\textsuperscript{9}See Appendix A.6 for a detailed derivation of the two sectoral Phillips Curves.

\textsuperscript{10}This type of rule has been used extensively in the literature, see for example Woodford (2011), McKay et al. (2016), Auclert et al. (2023).
and exogenous paths of real interest rates \( \{ r_{jt} \}_{t \geq 0} \), a regional equilibrium for county \( j \) is a path for county \( j \)'s prices \( \{ P_{jt}, P_{NT}^{jt}, W_{jt}, W_{NT}^{jt} \}_{t \geq 0} \), aggregate quantities \( \{ L_{jt}, L_{NT}^{jt}, L_{T}^{jt}, C_{jt}, C_{NT}^{jt}, C_{T}^{jt} \}_{t \geq 0} \), individual allocation rules \( \{ c_{jt}(b,e), b_{jt+1}(b,e) \}_{t \geq 0} \), and joint distributions over assets and productivity levels \( \{ G_{jt}(b,e) \}_{t \geq 0} \), such that households, unions, and firms in county \( j \) optimize, and the market clearing conditions for non-tradables and tradables produced in county \( j \) hold, i.e.:

\[
\begin{align*}
L_{NT}^{jt} &= C_{NT}^{jt} \\
L_{T}^{jt} &= C_{T}^{jt}
\end{align*}
\] (8)

**Definition 2** (National Equilibrium). Given initial regional distributions \( \{ G_{j0}(b,e) \}_{j \in [0,1]} \) and a path for monetary \( \{ r_{t} \}_{t \geq 0} \) a national equilibrium consists of a path for prices \( \{ P_{t}^{T}, W_{t}^{T}, P_{jt}, P_{NT}^{jt}, W_{jt}, W_{NT}^{jt}, r_{jt} \}_{j \in [0,1]} \}_{t \geq 0} \) and aggregate quantities \( \{ \{ L_{jt}, L_{NT}^{jt}, L_{T}^{jt}, C_{jt}, C_{NT}^{jt}, C_{T}^{jt} \}_{j \in [0,1]} \}_{t \geq 0} \), individual allocation rules \( \{ \{ c_{jt}(b,e), b_{jt+1}(b,e) \}_{j \in [0,1]} \}_{t \geq 0} \), and joint distributions over assets and productivity levels \( \{ \{ G_{jt}(b,e) \}_{j \in [0,1]} \}_{t \geq 0} \), such that all counties are in a regional equilibrium and all markets clear, i.e.:

\[
\begin{align*}
L_{NT}^{jt} &= C_{NT}^{jt} \quad \forall j \\
L_{T}^{jt} &= \int_{0}^{1} C_{T}^{jt} dj' \quad \forall j \\
B_{t} &= \int_{0}^{1} \left( \int_{0}^{1} b_{jt} di \right) dj
\end{align*}
\] (10-12)

### 3 The Regional Keynesian Cross

In this section, we start by revisiting the logic of the standard Keynesian multiplier in our regional framework. Next, we provide a sequence space representation for the model described in Section 2. Finally, we derive the Regional Keynesian Cross, a characterization of the local employment response to a monetary policy shock in the sequence space. Throughout the Section, we consider Regional equilibria and focus on a single small open county \( j \) in isolation, thus treating changes in tradable prices and demand from the rest of the nation as exogenous.

#### 3.1 Revisiting the Keynesian Multiplier

**Sufficient statistics for openness** Before diving into the details of our mechanism, we introduce one object that will be at the core of our analysis.
Definition 3 ($\rho_j$). We define $\rho_j$ as county $j$’s non-tradable share of the wage bill. Formally:

$$\rho_j = \frac{L_j^{NT} W_j^{NT}}{L_j W_j}$$

Since it represents the share of non-tradable labor income, $\rho_j$ is naturally bounded between 0 and 1 and gauges the extent to which county $j$ is exposed to fluctuations in the non-tradable sector, as opposed to fluctuations in the tradable one. This can be seen by log-linearizing real labor income in county $j$:

$$d \ln \left( \frac{W_{jt}}{P_{jt} L_{jt}} \right) = \rho_j \left( d \ln W_{jt}^{NT} + d \ln L_{jt}^{NT} - d \ln P_{jt} \right) + (1 - \rho_j) \left( d \ln W_{Tt} + d \ln L_{jt}^T - d \ln P_{jt} \right)$$

(13)

Importantly, (13) is an accounting equation that simply follows from the definition of the wage index $W_{jt}$. As a result, it is independent of the functional form of the labor and consumption aggregators. Thus, $\rho_j$ represents a sufficient statistic governing county $j$’s pass-through from non-tradable to aggregate real labor income in a general class of models. Moreover, given that counties are atomistic, the equilibrium in the tradable sector is established at the national level, while fluctuations in the non-tradable sector are determined locally. Hence, $\rho_j$ can also be viewed as shaping county $j$’s exposure to local, rather than national, business cycles.

If one is then willing to assume a CES demand structure, it is useful to define one extra object. This is the non-tradable share of consumption expenditure, which we denote by $\xi_j$. Under a CES consumption aggregator as the one we defined in (2), $\xi_j$ represents the elasticity of the local price index $P_{jt}$ to non-tradable prices and one can rewrite (13) just in terms of sectoral wages and employment. Finally, in a balanced trade steady-state it turns out that $\rho_j = \xi_j$. Hence, in the balanced trade steady-state, the real wage channel is inactive and changes in real labor income only come from changes in employment $L_{jt}$. In particular we have:

$$d \ln \left( \frac{W_{jt}}{P_{jt} L_{jt}} \right) = \rho_j d \ln L_{jt}^{NT} + (1 - \rho_j) d \ln L_{jt}^T$$

Around a balance trade steady-state, $\rho_j$ then captures the exposure of local real income to employment fluctuations in the non-tradable, as opposed to the tradable, sector. In what follows, we will focus on perturbations around a balanced trade steady-state.

\[\text{See Appendix A.1 for a derivation of this result.}\]
The regional Keynesian multiplier logic  We are now ready to extend the logic of the Keynesian multiplier to our regional setting. In order to do so, we focus on an individual county \( j \) and consider a rise in local aggregate demand.\(^{12}\) For simplicity, we consider perturbations around county \( j \)’s balanced trade steady-state.\(^{13}\) Figure 1 describes the transmission mechanism of the local demand shock in this setting. First, the local increase in aggregate demand splits into a rise in demand for tradable as well as non-tradable goods. Since the county is atomistic, the increase in tradable demand does not feed back to the local economy, but is instead “lost” to the rest of the nation. On the other hand, because non-tradables can only be produced locally, the rise in non-tradable demand fully feeds back to the home county. Hence, the initial rise in local aggregate demand has an effect on domestic demand for non-tradables only, thus inducing an asymmetric sectoral transmission at the local level.

This asymmetric transmission turns out to have crucial implications. In fact, as already discussed above, \( \rho_j \) determines county \( j \)’s exposure to fluctuations in the non-tradable sector. In particular, the rise in demand for non-tradables induces a rise in non-tradable labor which is passed-through to aggregate real income in proportion to \( \rho_j \). Therefore, \( \rho_j \) represents a central object shaping the pass-through of local demand shocks to regional real income. In turn, the pass-through of real income to consumption is governed by iMPCs, just like in the standard Keynesian multiplier logic. As each iteration of the Keynesian multiplier goes through, the asymmetric sectoral transmission we just described takes place. However, the strength of this transmission is jointly governed by \( \rho_j \) and county \( j \)’s

\(^{12}\)For now, we do not need to take a stance on the origin of the rise in local demand.

\(^{13}\)Note that it is possible to relax this assumption. In particular, the same logic would apply, but an extra real wage channel would be present, governed by the term \( \rho_j - \xi_j \).
iMPC. In other words, our two layers of regional heterogeneity interact in determining the total pass-through from local aggregate demand to consumption and, in turn, in shaping the magnitude of the regional Keynesian multiplier. Our framework therefore nests the standard Keynesian multiplier logic, which is however distorted by the county-specific national trade openness.

Finally, in Figure 1 we also allow for a generic impulse in national aggregate demand, which could be potentially correlated with the rise in local aggregate demand.\footnote{This will be the case, for example, when we will consider a monetary policy shock.} This rise in national aggregate demand induces a rise in demand for tradable goods.\footnote{Clearly, the rise in national demand also generates an increase in demand for non-tradables. However, this increase in non-tradable demand affects county j only to the extent that it shapes the total increase in national demand.} According to the same logic as before, the pass-through from national demand for tradables to local real income in county j is going to be determined by $1 - \rho_j$. The local MPC is then going to shape the response of county j’s consumption to the increase in real income. However, and crucially to our intuition, after the initial impulse in national aggregate demand takes place, all the subsequent iterations of the regional Keynesian multiplier are going to transmit through the non-tradable sector only, and will thus be shaped by the interaction between $\rho_j$ and county j’s MPC.

### 3.2 Sequence Space Representation

Figure 1 provides an intuitive and schematic representation of the mechanism at the basis of our framework. Before being able to further formalize this mechanism and our results, we now derive a sequence space representation of our model (Auclert et al., 2021a, 2023). Throughout the rest of our analysis, we will adopt the following notation. For a generic variable $X_{jt}$, $\bar{X}_j$ denotes its steady-state value. $X_j$ is the vector representing the full sequence $\{X_{js}\}_{s \geq 0}$, i.e., $X_j \equiv (X_{j0}, X_{j1}, \ldots)'$.\footnote{Here the prime notation denotes the transpose operator.} Finally, we denote by $dX_j$ the full sequence of log-deviations of variable $X_{jt}$ from its steady-state value, i.e., $dX_j \equiv \left(\frac{X_{j0} - X_j}{X_j}, \frac{X_{j1} - X_j}{X_j}, \ldots\right)'$. For real interest rates $r_{jt}$, we adopt a slightly different notation and let $dr_j \equiv \left(\frac{r_{j0} - r_j}{1 + r_j}, \frac{r_{j1} - r_j}{1 + r_j}, \ldots\right)'$.

**Regional Consumption Function** Because the labor union allocates labor uniformly across households, we can express idiosyncratic net real income as a function of aggregate...
county-level quantities only. In particular we have:

\[ z_{ijt} e_{ijt} = \frac{W_{jt} L_{jt}}{P_{jt}} e_{ijt} \]

Substituting the expression above into the household’s budget constraint (1), it is easy to see that, given the state \((b, e)\), the path of optimal policy rules \(\{c_{jt}(b, e), b_{jt+1}(b, e)\}_{t \geq 0}\) is entirely pinned down by the sequence of aggregate real income \(\{\frac{W_{jt} L_{jt}}{P_{jt}} e_{ijt}\}_{t \geq 0} \equiv \{Z_{jt}\}_{t \geq 0}'\) together with the sequence of the real interest rate \(\{r_{jt}\}_{t \geq 0}'\). We can then integrate over the states to write aggregate consumption at time \(t\) as a function of the sequence of aggregate real income and real the interest rate:

\[
\int c_{jt}(b, e)dG_{jt}(b, e) = C_{jt} \left( \{Z_{js}\}_{s \geq 0}, \{r_{js}\}_{s \geq 0} \right)
\]

Following Auclert et al. (2023), we denote the Jacobian of \(C_{jt}(\cdot)\) with respect to aggregate real labor income \(Z_{j} \equiv (Z_{j0}, Z_{j1}, \ldots)'\) by \(M_{j}\), which is a matrix whose element \((t, s)\) is given by \(\frac{\partial \ln C_{jt}(\cdot)}{\partial \ln Z_{js}}\). Similarly, we denote by \(M_{rj}\) the matrix of elasticities of \(C_{jt}(\cdot)\) with respect to the interest rate sequence \(r_{j} \equiv (r_{j0}, r_{j1}, \ldots)'\), that is \(\left(M_{rj}\right) \equiv \frac{\partial \ln C_{jt}(\cdot)}{\partial \ln (1+r_{js})}\).

### 3.3 Deriving the Regional Keynesian Cross

As mentioned above, our derivation of the Regional Keynesian Cross relies on the concept of regional equilibrium and hence focuses on a single county in isolation. As a result, we take the response of the rest of the nation \(\{dL_{i}\}_{i \neq j}\) as given for now. We will endogeneize it in the next section, when deriving the National Keynesian Cross.

In order to derive our result, we start by substituting domestic demand for non-tradable goods (3), together with the aggregate consumption function (14), into the non-tradable market clearing condition (8):

\[
L_{jt}^{NT} = \omega \left( \frac{P_{jt}}{P_{jt}} \right)^{-\nu} C_{jt} \left( \{Z_{js}\}_{s \geq 0}, \{r_{js}\}_{s \geq 0} \right)
\]

We then log-linearize (15) around the balanced trade steady-state and consider an unanticipated and exogenous perfect-foresight path for the real interest rate \(dr_{j}\).\footnote{In Appendix A.3 we derive the Regional Keynesian Cross around a steady-state which does not feature balanced trade, so that \(\rho_{j} \neq \xi_{j}\).} Similarly, we can log-linearize the market clearing condition for tradables (9), denoting by \(C_{iT} \equiv \frac{\partial \ln C_{jt}(\cdot)}{\partial \ln (1+P_{jt})}\).
the national demand for tradable goods, which we take as given since we are focusing on a regional equilibrium. We then plug the log-linearized versions of (9) and (15) inside the definition \( dL_j = \rho_j dL_j^{NT} + (1 - \rho_j)dL_j^{T} \) to obtain a fixed point equation for \( dL_j \).

We are now ready to derive one of the main results of this paper.

**Proposition 1** (The Regional Keynesian Cross). The first-order response of employment \( dL_j \) around the balanced trade steady-state to a monetary shock \( dr_j \) satisfying a regional equilibrium solves the following fixed point equation:

\[
dL_j = \rho_j \left( M_j^r dr_j + M_j dL_j \right) + (1 - \rho_j) dC^T - \frac{\nu}{\eta}(1 - \rho_j) \left( dL_j - dC^T \right)
\]

Where \( dC^T \) represents the path for the national demand for tradables, which is taken as given.

**Proof.** See Appendix A.2. \( \square \)

Given a path for the real interest rate \( dr_j \) induced by monetary policy, as well as a path for national tradable demand \( dC^T \), Proposition 1 characterizes the local employment response in county \( j \). In particular, (16) shows how the local employment response to a monetary policy shock is fully governed by the two initial impulses \( dr_j \) and \( dC^T \), together with a few sufficient statistics: three elasticities (\( \rho_j, \nu, \) and \( \eta \)), the iMPCs summarized by \( M_j \), and the intertemporal substitution motives captured by \( M_j^r \).

Note how, given \( dr_j \) and \( dC^T \), the response of real variables in county \( j \) does not depend on nominal rigidities. In particular, the regional Phillips Curve –together with the Fisher Equation– only matters for backing out the monetary impulse \( d\bar{r} \) needed to generate the specific path for \( dr_j \). We share this feature that nominal rigidities do not matter for real outcomes with, among others, Auclert et al. (2021b, 2023). However, in these papers the reason why nominal rigidities do not matter for real outcomes can be traced back to the fact that the central bank targets the real interest rate. In our context, instead, the regional Phillips Curve operates only in the background because we focus on a regional equilibrium and hence consider an atomistic county in isolation, so that the path for monetary policy is exogenous regardless of the rule followed by the central bank. As a result, we can take the path for monetary policy \( d\bar{r} \) as given, and in particular choose it such that it induces the desired \( dr_j \) change in the real interest rate in county \( j \).\(^{18}\)

\(^{18}\)Relatedly, the reason why price rigidities do not affect relative price changes and do not enter (16) is that the tradable Phillips Curve is determined at the national level, so we can take it as given when focusing on regional equilibria.
Before moving to a description of the different channels operating in our Regional Keynesian Cross, and to build intuition for Proposition 1, it is useful to consider two limit cases.

**Corollary 1.** When \( \rho_j \to 1 \), the first-order response of employment \( dL_j \) around the balanced trade steady-state to a monetary shock \( dr_j \) satisfying a regional equilibrium solves a standard intertemporal Keynesian Cross (Auclert et al., 2023):

\[
dL_j = M'_j dr_j + M_j dL_j
\]  

(17)

Corollary 1 considers the limit case in which county \( j \) is a fully closed economy within a monetary union. In this scenario, quite intuitively, the local employment response to a monetary shock is not going to be affected by fluctuations in the tradable sector and hence follows the standard closed-economy intertemporal Keynesian Cross described in Auclert et al. (2023), which is thus nested by our framework.

**Corollary 2.** When \( \rho_j \to 0 \), the first-order response of employment \( dL_j \) around the balanced trade steady-state to a monetary shock \( dr_j \) satisfying a regional equilibrium does not depend on \( j \)'s characteristic, and is fully nationally determined:

\[
dL_j = dC_T
\]  

(18)

Corollary 2 shows how, in the limit case in which a county faces no trade frictions and is thus fully exposed to national business cycles, the county-level response to monetary policy shocks is fully independent of county-specific characteristics. In particular, county-specific iMPCs, as summarized by \( M_j \), do not matter in determining the county specific response. This result is going to be useful later on, once we derive the National Keynesian Cross.

**Decomposing the channels** Proposition 1 provides a decomposition of the local employment response to a monetary policy shock into three channels, each of which is governed by a subset of the sufficient statistics detailed above \( M_j, M'_j, \rho_j, \nu, \) and \( \eta \).

*Regional Equilibrium Effects* – The first term in (16) captures the effect of county \( j \)'s exposure to local fluctuations on the regional employment response. This regional exposure channel is premultiplied by \( \rho_j \) because, as discussed above, this factor exactly captures the pass-through of fluctuations in the non-tradable sector, i.e., local fluctuations, to the local economy. In turn, the regional exposure channel is composed of two terms. First, there is a term capturing the “direct effect” of monetary policy (Kaplan et al., 2018), \( M'_j dr_j \). In
fact, monetary policy generates regional fluctuations in employment first and foremost through its effect on real interest rates, thus inducing households to substitute consumption intertemporally. The matrix $M^j_r$ exactly captures the extent to which local households are both willing and able to engage in this intertemporal substitution and thus propagate the monetary impulse to the local economy. The second term in the regional exposure channel represents the Regional Keynesian Multiplier, $M_j dL_j$. This channel captures the indirect (higher order) effects of the transmission of the original shock. Following the intuition in Figure 1, the increase in labor $dL_j$ necessary to satisfy the original change in demand generates a rise in local real income. In turn, the iMPC matrix $M_j$ determines the pass-through from labor income onto consumption, and back to local demand. However, in our regional setting county $j$ is exposed to its own local economy only through the non-tradable sector. For this reason, only a share $\rho_j$ of the Keynesian multiplier is activated at the regional level.

National Equilibrium Effects – The second object in (16) represents the role played by county $j$’s exposure to national fluctuations on the local employment response. This national exposure channel is premultiplied by a factor $1 - \rho_j$ because, as discussed before, national fluctuations are passed-through to the local economy only through the tradable sector. In particular, national fluctuations affect local employment as an exogenous shifter, and the way they enter the Regional Keynesian Cross (16) is akin to the monetary impulse.

Expenditure Switching – (16) comprises one final channel, which captures the fact that local demand shocks get transmitted asymmetrically to the two sectors: the expenditure switching channel. In particular, the asymmetric sectoral transmission implies that the relative response of local as opposed to national demand, $dL_j - dC^T$ is going to be crucial in shaping the response of relative prices. Intuitively, whenever the local response is larger than the national one, the price of non-tradables will rise more than that of tradables, so that households will substitute away from local non-tradables, thus reducing the local employment response. In addition, the magnitude of the expenditure switching channel is also governed by the relative size of the elasticity of substitution in demand and supply, $\nu$ and $\eta$ respectively. Clearly, when the demand elasticity is large (small) relative to the supply one, the relative price of tradables vs non-tradables will move a lot (little) for a given asymmetric demand shock. The larger the movement in the relative price, the more households are going to substitute consumption between the two sectors.

iMPCs-trade openness complementarity Proposition 1 also shows that our framework predicts a mechanism novel to the HANK literature: a complementarity between openness to national trade and household heterogeneity for the regional transmission of shocks.
To see this, notice that openness to trade and iMPCs interact in shaping the size of the regional Keynesian multiplier, which is given by the term $\rho_j M_j$ in (16). In other words, there is a complementarity between the demand-side channels at work in the standard HANK literature, captured by the iMPC matrix $M_j$, and the supply-side channels of our framework, captured by $\rho_j$. In particular, the effect of MPCs on the multiplier term is increasing in $\rho_j$, because low values of $\rho_j$ discount the role played by iMPCs for the propagation of shocks through higher order effects. This complementarity is not only at play in the context of the regional multiplier term. In fact, similarly to the case of the Keynesian multiplier, the magnitude of the local direct effect of monetary policy, the term $\rho_j M_j^r$ in (16), is also shaped by the interaction between openness to trade and intertemporal substitution motives.\footnote{Because it affects higher order terms, we expect the complementarity between iMPCs and openness to trade for the multiplier channel to quantitatively dominate the one associated to the intertemporal substitution channel.} Hence, within our framework, local openness to trade governs the extent to which iMPCs and intertemporal substitution motives –and, more in general, household heterogeneity– matter for the local employment response to monetary policy. In Section 5 we present empirical evidence that this complementarity is indeed present in the context of the US economy.

4 The National Keynesian Cross

In this section, we look at the implications of our two-layered regional heterogeneity for the aggregate, nation-wide, employment response to monetary policy. First, we abandon the concept of regional equilibrium and fully endogeneize the national response to the common monetary policy shock. Next, we provide an aggregation result by deriving the National Keynesian Cross, a characterization of the national employment response to a monetary policy shock as a function of the joint distribution of our two layers of heterogeneity across regions. Finally, we discuss how our framework relates to different models in the literature.

4.1 Deriving the National Keynesian Cross

We now turn to aggregating the Regional Keynesian Crosses for our continuum of counties $j \in [0,1]$ to derive an expression for the National Keynesian Cross, linking the response of aggregate, country-wide employment to a monetary shock. In particular, this is going to allow us to show how the joint distribution of iMPCs and trade openness across space (regions) matters in shaping the national response. To do so, we endogenize the
national response and integrate the Regional Keynesian Cross (16) over our measure of counties. The country-wide change in employment $dL = \int_0^1 dL_j dj$ is then characterized by the following Proposition:

**Proposition 2** (The National Keynesian Cross). The first-order country-wide response of employment $dL \equiv \int_0^1 dL_j dj$ around the balanced trade steady-state to a monetary shock $dr$ satisfying a national equilibrium is characterized by:

$$
\begin{align*}
\frac{dL}{dL_j} &= \frac{M dL + M^r dr + \text{Cov}(M_j, dL_j) + \text{Cov}(M^r_j, dr_j)}{\text{Representative county}} \\
&\quad + \frac{M \text{Cov}(\rho_j, dL_j) + M^r \text{Cov}(\rho_j, dr_j) + \nu}{\eta} \text{Cov}(\rho_j, dL_j) \\
&\quad + \text{Cov}(M_j, (\rho_j - \rho) dL_j) + \text{Cov}(M^r_j, (\rho_j - \rho) dr_j) \\
&\quad \text{Trade openness heterogeneity} \\
&\quad \text{iMPC-trade openness complementarity}
\end{align*}
$$

(19)

Where $dL_j$ is given by (16), $\rho \equiv \int_0^1 \rho_j dj$, $M \equiv \int_0^1 M_j dj$ and $M^r \equiv \int_0^1 M^r_j dj$.

**Proof.** See Appendix A.5.

**Decomposing the channels** Proposition 2 shows that in our regional framework, the nation-wide response to a monetary policy shock can be decomposed into four different objects: (i) an “as-if” (Werning, 2015) benchmark which abstracts from regional heterogeneity, (ii) a term capturing the role of iMPCs heterogeneity across regions, (iii) one object representing the effect of regional heterogeneity in trade openness, and (iv) a term capturing equilibrium interactions between our two layers of regional heterogeneity. We now describe the role of each of these terms. To do so, we begin with an homogeneous counties benchmark, and sequentially introduce one dimension of regional heterogeneity at a time.

**Homogeneous regions** First, we consider the case in which our monetary union is composed by a continuum of counties $j \in [0, 1]$ which are however homogeneous both in terms of openness to trade, i.e., $\rho_j = \rho$, and in terms of iMPCs, i.e., $M_j = M$ and $M^r_j = M^r$. Then, a natural “as-if” result (Werning, 2015) applies. In particular, in this scenario the National Keynesian Cross (19) only comprises the very first term. Thus, the national response to monetary shocks can be characterized by simply aggregating the continuum of regions into an average, representative, county. Moreover, in this case trade frictions and sectoral composition, as summarized by the national trade openness $\rho$, do
not matter for the national response. The reason for this is that from a country-wide perspective there is no distinction between tradable and non-tradable goods, since all goods produced in the nation need to be effectively consumed within the nation.

**Heterogeneity in regional iMPCs only** – Let’s now turn to the case in which all counties share the same degree of openness to trade, i.e., $\rho_j = \rho$, but are allowed to differ in terms of iMPCs. Then, the national response to monetary shocks is given by the first two terms in (19). This implies that the national response to monetary policy is given by that of the representative region, corrected by the degree of “sorting” between iMPCs and the employment response, as well as intertemporal substitution motives and the real interest rate response, across regions. First, the country-wide employment response is amplified whenever high MPC regions experience large labor responses. The reason for this is that the strength of the regional Keynesian multiplier is increasing in households’ iMPCs, summarized by $M_j$. Thus, whenever regions with high iMPCs experience large employment responses, a large multiplier effect is going to kick-in at the local level, hence amplifying the national response. Similarly, the national response is also going to be increasing in the covariance across space between households’ intertemporal substitution motives –captured by $M_j'$– and the change in the county-specific real interest rate. Finally, differently from the Regional Keynesian Cross (16), the expenditure switching channel does not matter for the national response when trade openness is homogeneous across regions, even in the presence of iMPCs heterogeneity. This is because any substitution of demand from non-tradables to tradables taking place at the local level is still going to be satisfied within the nation, so that the expenditure switching channel cancels out on aggregate.

**Heterogeneity in regional trade openness only** – We now consider the opposite scenario, where counties are allowed to differ in their degree of openness to trade, $\rho_j$, but share an identical structure for the household block, so that iMPCs and intertemporal substitution motives are equalized across space, i.e., $M_j = M$ and $M_j' = M'$. In this case, the national employment response is characterized by the first and third terms in (19). Thus, the National Keynesian Cross will consists of the “as-if” benchmark, augmented by the covariance between counties’ openness to trade and their employment and real interest rate responses. In particular, since the magnitude of the regional Keynesian multiplier is increasing in the share of non-tradable employment $\rho_j$, the country-wide response is going to be amplified whenever high non-tradable intensity regions also experience large employment responses. Moreover, remember that a county’s exposure to regional business cycles is increasing in $\rho_j$. Thus, the national response is going to be larger when counties that are highly exposed to their own fluctuations also experience a large monetary impulse. Finally, note that when trade openness is heterogeneous across regions, it is no
longer true in general that the expenditure switching channel cancels out on aggregate, as captured by the last entry in the “trade openness heterogeneity” term in (19). This is because at the local level the size of the expenditure switching channel is decreasing in $\rho_j$. Hence, whenever regions that experience large employment responses also have high non-tradable employment, the expenditure switching is going to be smaller in exactly those areas that also feature a large multiplier channel. In turn, this contributes to amplify the national employment response.

Two-layered regional heterogeneity – Finally, we consider the case in which both layers of regional heterogeneity –namely, heterogeneity in iMPCs and openness to trade– are active at the same time. It turns out that, in order to capture the role of this two-layered regional heterogeneity on the nation-wide response, it is not enough to sum the individual effects of heterogeneity in iMPCs and trade openness. In fact, an extra term, capturing complementarities between iMPCs and trade openness, now appears. The reason for this is that, as discussed before, at the local level the magnitude of the regional Keynesian multiplier and of the direct effect of monetary policy is shaped by the interaction of trade openness with iMPCs and intertemporal substitution motives. Proposition 2 then shows that this complementarity between household heterogeneity and trade openness is still present when considering national aggregates. This result showcases once again how in our framework the joint distribution of trade openness and households’ MPCs across space matters for the aggregate response to a monetary shock.

Representative county After having dissected how different layers of regional heterogeneity affect the national response to monetary shocks, we now present a neutrality result for regional heterogeneity in iMPCs. In particular, the following corollary shows how in the absence of trade frictions heterogeneity in MPCs across counties is irrelevant for the transmission of monetary policy within a currency area.

Corollary 3 (A regional “as-if” benchmark). When $\rho_j \to 0$ for all $j$ heterogeneity in iMPCs across regions does not matter for the nation-wide employment response:

$$dL = M dL + M' dr$$

Corollary 3 follows directly from Proposition 2 and Corollary 2. We see Corollary 3 as an extension of the neutrality result in Werning (2015), in the context of regional –rather than household– heterogeneity. In particular, Corollary 3 is not stating that household heterogeneity does not matter for the response of nation-wide aggregates to monetary shocks. In fact, whether this is the case or not is still going to depend on the conditions
on the cyclicality of income and liquidity spelled out in Werning (2015). What Corollary 3 is instead stating is that, when frictions to trade between regions are removed, the response to monetary policy of a monetary union composed of multiple regions is going to be identical to that of a representative region. In other words, in this case heterogeneity in MPCs (and intertemporal substitution motives) between regions is irrelevant for the nation-wide response. Corollary 3 also directly implies that whenever frictions to trade between regions are present, a single-region model which abstracts from heterogeneity in MPCs is going to deliver somewhat misspecified predictions for the national response to monetary shocks.

4.2 Discussion

We now offer a qualitative discussion of how our framework relates to other prominent and recent advances in the literature. Specifically, an important implication of our key result –Proposition 1– is that our two-layered heterogeneity structure relates to and sometimes nests several existing models and concepts.

**HANK** – When both the preference for non-tradables $\omega$ and the trade openness parameter $\alpha_j$ go to 1, the model collapses to a standard one-industry, one-region economy, since the only active sector is the non-tradable one. In particular, we have $\rho_j = \bar{\varepsilon}_j = 1$. Thus, under this parametrization, the Regional Keynesian Cross (16) simplifies to $dL_j = M_j dL_j + M_r^j dL_j$. This representation corresponds to a monetary version of the intertemporal Keynesian cross described in Auclert et al. (2023), with the driver of the response given by monetary shocks, rather than fiscal shocks.

**TANK** – While the specification of our household block does not directly nest the Two Agent New Keynesian (TANK) model of Bilbiie (2020), our main Propositions 1 and 2 still hold in this setting. In fact, both the Regional as well as the National Keynesian Cross rely on market clearing conditions, which are independent of the household block specification. Thus, (16) and (19) are still valid in a Two-Agents setting, with the Jacobians $M_j$ and $M_r^j$ corresponding to those in the standard TANK framework. Furthermore, if we consider the limiting case of having only tradables ($\omega = \alpha_j = 0$), our framework boils down to a special case of a regional TANK model with no sectoral heterogeneity, such as the one presented in Herreño and Pedemonte (2022).

**RANK** – Note that the matrices $M_j$ and $M_r^j$ capture all household heterogeneity in the economy. In particular, changes to the borrowing limit or to the parametrization of the income process in the economy are going to result in different iMPCs. Thus, if we relax the borrowing limit to the natural one and shut down income volatility, the Jacobians $M_j$
and $M_j^r$ collapse to the ones of a representative agent economy, with $(M_j)_k s = \left(1 - \beta \right) \beta^{s-1}$ and $(M_j^r)_k s = -\sigma^{-1} (\beta^s - 1_{k>s})$. If we also set $\omega = \alpha_j = 1$ as above, our model collapses to the standard representative agent New Keynesian model, as in Galí (2008) and Woodford (2003).

International Keynesian Cross – A recent literature has uncovered novel insights when analyzing household heterogeneity in the context of international economics. de Ferra et al. (2020) develop a small open economy HANK framework and show that portfolio composition and foreign currency borrowing determine the degree of amplification of the domestic macroeconomic response to foreign demand shocks. In a recent paper, Auclert et al. (2021b) derive an insightful result in sequence space: the International Keynesian Cross (IKC), whose key component is the open-economy multiplier of domestic real interest rate shocks, which is governed by home bias. Generally speaking, a small-open-economy extension of our model would yield a generalized multiplier with two very distinct features. First, the degree of openness to national trade, as captured by the regional distribution of non-tradable employment intensity. Second, the extent of openness to international trade which, as in de Ferra et al. (2020), is measured on the intensive margin by the exposure of the domestic population to foreign currency and demand shocks. This generalization would enable general equilibrium quantification of asymmetric regional welfare effects of foreign shocks such as, for example, the China syndrome (Autor et al., 2020), which is very well known to have had a highly unequal impact on US counties. More work in this direction is required.

The Matching Multiplier – A growing literature highlights that sorting of workers across sectors can produce powerful amplification effects of demand shocks (Cugat, 2019, Patterson, 2023). In a recent paper, Patterson (2023) uncovers the “matching multiplier” channel: the transmission of aggregate shocks is amplified by high-MPC individuals sorting themselves into highly cyclical jobs. Similarly, Cugat (2019) finds that working in the tradable versus non-tradable sector is an important determinant of the household-level response to aggregate shocks. She then shows that this channel has important consequences for the propagation of aggregate shocks in a small open economy New Keynesian model with household heterogeneity. This sorting channel is absent from our theoretical framework. However, it is possible to augment our model to include two groups of households working in the two different sectors.\footnote{We thank Edouard Challe for the suggestion.} Our Regional Keynesian Cross would then comprise two types of Jacobians, referring to the two different types of households.
5 Empirical Analysis

In this section we first describe the data used in our analysis. We then present novel estimates of county-level marginal propensities to consume. Next, we document heterogeneity in the regional responses of employment to monetary shocks across U.S. counties. We show that the two characteristics that we considered in our modeling framework can jointly account for the observed geographical heterogeneity: (i) our novel measure of regional MPCs, and (ii) openness to national trade, which we measure as the local ratio of employment in non-tradable industries to employment in tradable industries. Finally, we show that high local MPCs and high non-tradable employment ratios amplify the regional response to monetary policy. Moreover, we find empirical evidence for the MPC-trade openness complementarity predicted by our model.

5.1 Data

Employment Our main data source is the Local Area Unemployment Statistics (LAUS) from the Bureau of Labor Statistics. The LAUS register is a non-survey based dataset which combines multiple data sources to provide monthly employment estimates for different levels of regional disaggregation. In what follows, we focus on county-level employment.\(^{21}\)

We obtain annual county-level employment for 4-digit North American Industry Classification System (NAICS) sectors from the County Business Patterns (CBP) dataset published by the US Census. Data before 1998 are based on the SIC industry classification. Hence, we link SIC sectors to NAICS according to the SIC-NAICS concordance tables provided by the US Census. We then classify 4 digit-NAICS sectors into tradable and non-tradable industries according to the standard definition proposed by Mian and Sufi (2014).

Next, for each county \(j\) and each year \(t\) in our dataset we define our baseline trade openness variable as the non-tradable to tradable employment ratio \(\tilde{\rho}_{jt} \equiv \frac{L_{\text{NT},jt}}{L_{\text{NT},jt} + L_{\text{T},jt}}\), where \(L_{\text{NT},jt}\) represents the total number of people working in non-tradable sectors in county \(j\) and year \(t\), while \(L_{\text{T},jt}\) is the total number of people employed in tradable sectors in the same

\(^{21}\)As of 2020, there were 3,143 counties across the 50 US states. Our dataset comprises a total of 3,120 counties, 92.50% of which are present in all months of the sample.
count-year unit. Figure B.1 in Appendix B.2 plots the distribution of $\tilde{\rho}_{jt}$ across counties, averaged over all years in our sample.

**Monetary policy** In order to capture monetary policy surprises, we follow the high-frequency identification approach. Specifically, following Gurkaynak et al. (2005) and Gertler and Karadi (2015) we use the change in the 3-month ahead Fed Funds futures within a 30 minute window around FOMC announcements as our baseline instrument for monetary shocks. For robustness, we also consider the narrative instrument approach proposed in Romer and Romer (2000) and updated by Miranda-Agrippino and Rey (2020). Throughout the rest of our analysis, we normalize the sign of the measure of monetary shocks $\epsilon_t$ such that positive values are associated with expansionary shocks. Moreover, we also normalize $\epsilon_t$ to have unitary standard deviation.

**Data for regional MPCs** To construct our regional measure of MPCs we rely on two data sources. The first consists of four special survey modules designed by Fuster et al. (2020) and fielded as part of the NY Fed Survey of Consumer Expectations (SCE). These modules are constructed to elicit respondents’ self-reported MPCs out of windfall gains and losses of different magnitudes. A total of 2,586 panelists participated to the survey across four waves between March 2016 and March 2017. We specifically rely on a question asking respondents to report their quarterly MPC out of a windfall loss of 500$. Second, we use the American Community Survey (ACS) 5-year data to obtain county-level information on the socio-demographic characteristic of the local population. In particular, we extract data on the number of households living in any given county, binned by the age and race of the householder as well as household income. This data is at annual frequency and spans the period 2009-2023.

To validate our regional MPC measure we also use county-level data on stock market wealth from Chodorow-Reich et al. (2021). This measure of wealth is obtained by applying an improved version of the canonical capitalization method to data on taxable dividend income aggregated at the county level. We construct an annual measure of stock market wealth per capita, spanning the years 1989-2015, by using county population data from

---

22Note how our empirical $\tilde{\rho}_{jt}$ measure maps pretty closely to the $\rho_j$ measure that we define in the data. In particular, under the assumption that the average wage in tradable and non-tradable industries is the same, it holds that $\tilde{\rho}_{jt} = \frac{\rho_{jt}}{1 - \rho_{jt}}$. We plan to tighten the link between our theoretical and empirical measure even more by constructing $\tilde{\rho}_{jt}$ based on payroll, rather than employment, data.


24We refer the reader to Chodorow-Reich et al. (2021) for a thorough description of the construction of stock wealth data.
5.2 The Geography of MPCs

Marginal propensities to consumer are a challenging object to estimate in the data. First, because identified exogenous variations in household’s income are rare. Second, because it is often difficult to get access to household-level expenditure data at a sufficiently high frequency. Moreover, our research question requires an accurate proxy for MPCs that varies granularly in space. However, since most estimates of MPCs rely on survey data, the sample size is usually not large enough to construct accurate estimates at the local level. We address these challenges by extending the method proposed in Patterson (2023) to a regional setting and construct a novel measure of county-level MPC. In particular, we follow a two-steps approach. In the first step, using data from Fuster et al. (2020), we regress self-reported MPCs on indicator variables for the respondent’s race, age, and household income, as well as a time fixed effect.\(^{25}\) Our estimating equation reads:

\[
MPC_{it} = \alpha + \delta_t + \sum_{s=1}^{5} \beta_{s}^{R} D_{sit}^{R} + \sum_{s=1}^{4} \beta_{s}^{A} D_{sit}^{A} + \sum_{s=1}^{9} \beta_{s}^{Y} D_{sit}^{Y} + u_{it}
\]  

(21)

Where \(\alpha\) is the constant, \(\delta_t\) is a time fixed effect, \(D_{sit}^{R}\) is a race dummy, \(D_{sit}^{A}\) is an age dummy and \(D_{sit}^{Y}\) is an income dummy.\(^{26}\)

Next, from the ACS data we bin households into income × age × race groups as defined in (21). We denote groups by \(g\). For each county \(j\) and year \(t\) we then compute the number of households in each group and construct a group-specific measure of MPC by relying on our estimates from (21):

\[
\hat{MPC}_{g} = \hat{\alpha} + \sum_{s=1}^{5} \hat{\beta}_{s}^{R} D_{gs}^{R} + \sum_{s=1}^{4} \hat{\beta}_{s}^{A} D_{gs}^{A} + \sum_{s=1}^{9} \hat{\beta}_{s}^{Y} D_{gs}^{Y}
\]  

(22)

Finally, we aggregate (22) at the county level by taking the weighted average of the group-level MPC, weighted by the number of households in each group, that is:

\[
MPC_{jt} = \sum_{g} s_{jt} \hat{MPC}_{g}
\]  

(23)

\(^{25}\)In particular, we include 5 race dummies (white, black, asian, latino and other), 4 age dummies (less than 25, 25-44, 45-64, and more than 65 years old), and 9 dummies for household income (less than 20k, 20-30k, 30-40k, 40-50k, 50-60k, 60-75k, 75-100k, 100-150k, and more than 150k USD).

\(^{26}\)The estimates from (21) are reported in Table B.1 in Appendix B.2.
Where  $s_{jtg} \equiv \frac{N_{jtg}}{N_{jt}}$ is the fraction of households in county $j$ year $t$ belonging to group $g$.

Because (21) relies on bins and is hence non-parametric in nature, our methodology does not impose any linearity and does not rely on average measures when estimating MPCs. On the contrary, we account for the full distribution of households along both economic and socio-demographic dimensions. This is consistent with the theoretical insights of heterogeneous agent models, which highlight that the cross-sectional average MPC is not well approximated by the MPC of the average household, since the relationship between an individual’s MPC and her states is in general non-linear. Our procedure is also flexible and portable to different contexts. For example, it is possible to use alternative measures of MPCs coming from different data sources, as well as consider different household characteristics.

Figure 2 shows our estimated quarterly regional MPCs, averaged over the years 2009-2019. Our estimates range from 0.23 to 0.35, with an average of 0.315 and a standard deviation of 0.014. To further validate our methodology, we compute the correlation between our regional measure of MPC and the logarithm of stock market wealth per capita (Chodorow-Reich et al., 2021) which we never relied upon to produce our MPC measure. The two measures display a strong negative correlation of -0.49, as theory predicts.

5.3 Regional Responses to Monetary Shocks

Geographic heterogeneity  We document substantial heterogeneity in the response of employment to monetary shocks across US counties. To do so, we estimate a panel version
Figure 3: Regional Heterogeneity in the Effects of US Monetary Policy

Note: This figure plots the 3-year ahead county-specific cumulative employment responses to a 1 standard deviation expansionary monetary policy shock $\beta_{jh}$, estimated from the panel local projection (24). The coefficients are in percentage points and represent deviations from the (population weighted) average response.

of the Jordà (2005) local projections. In particular, for each county $j$ and month $t$ in our sample, and for horizons $h = 0, \ldots, 36$, we run the following regression:

$$
\Delta \ln(L_{jt+h}) = \alpha_{jh} + \delta_{th} + \beta_{jh} \times \epsilon_t + \sum_{\ell=1}^{12} \gamma_{h\ell} \Delta \ln(L_{jt-\ell}) + u_{jht}
$$

(24)

Where $\Delta \ln(L_{jt+h}) = \ln(L_{jt+h}) - \ln(L_{jt-1})$ represents the $h$-month ahead cumulative change in employment in county $j$, $\alpha_{jh}$ is a county fixed effect, while $\delta_{th}$ denotes a time fixed effect. Finally, $\Delta \ln(L_{jt-\ell}) = \ln(L_{jt-1}) - \ln(L_{jt-\ell-1})$ denotes past county-level employment growth, while $\epsilon_t$ is the monetary surprise.

Figure 3 shows the county-specific coefficients $\beta_{jh}$ estimated from (24) for a 3-year ahead horizon, $h = 36$. Because they represent individual deviations from the population-weighted average response, the coefficients are centered around zero. Figure 3 documents a large degree of cross-county heterogeneity in the employment response to monetary shocks. In particular, some counties experience an increase in employment up to 4.7 percentage points larger than the average response, while for others the change in employment is up to 4.8 p.p. smaller than the average county. Furthermore, the heterogeneity uncovered in Figure 3 does not seem to be randomly distributed across regions. On the contrary, there appears to be some geographical clustering in the distribution of the county-specific response to shocks. For this reason, we next turn to analyzing the potential factors underlying this heterogeneity.

27 For consistency with the rest of the analysis, our regression results are weighted by county population in the year 2000.
Explaining geographic heterogeneity  We now explore which fundamental county-level characteristics are able to account for the observed regional heterogeneity displayed in Figure 3. Guided by economic theory, in Section 2 we focused on MPCs and openness to trade as the two main candidates that could generate such regional heterogeneity in the response to monetary shocks. However, the literature has uncovered many other channels that affect the transmission of monetary policy: some examples include housing and mortgage markets,\(^{28}\) demographic structure,\(^{29}\) fiscal response and automatic stabilizers,\(^{30}\) banking markets,\(^{31}\) firm age and capital structure,\(^{32}\) and price and wage rigidities.\(^{33}\) We now show that the two layers of regional heterogeneity we consider in our modelling framework – local MPCs and openness to trade – turn out to be two crucial drivers of the heterogeneity in the regional response to monetary policy. To measure county-level openness to trade we use the non-tradable to tradable employment ratio \(\tilde{\rho}_{jt}\), as defined previously. For MPCs, we rely on our novel measure obtained with the two-step procedure described above.

Armed with our proxies for county-level MPCs and openness to trade, we then run an empirical horse race between our two preferred channels and several of the other channels proposed in the literature, to assess what are the best predictors of the observed geographical heterogeneity in the response to monetary shocks. To do so, we focus on the estimated \(\hat{\beta}_{j,24}\) in (24). In particular, we rank the estimated 2-year ahead county-specific responses \(\hat{\beta}_{j,24}\) from the smallest to the largest and group them into 50 bins. We then compute the population-weighted average of \(\hat{\beta}_{j,24}\) within each bin. Next, we collect data on a variety of county specific characteristics that have been showed to be potentially important determinants of the transmission of monetary policy.\(^{34}\) For each of these variables, we first compute the county average over the years in our sample, and then take a population-weighted average within each bin. Finally, we regress the within-bin average coefficient on our whole battery of county specific characteristics. Next, we focus on the loss in R-squared that is generated by removing each of our potential explanatory variables individually, one at a time.

Figure 4 plots the results of this exercise. The striking result from this figure is how the

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\(^{28}\)See, e.g., Di Maggio et al. (2017), Beraja et al. (2018), Cloyne et al. (2019), Berger et al. (2021), Eichenbaum et al. (2022).

\(^{29}\)See, e.g., Leahy and Thapar (2022), Bartscher et al. (2022).

\(^{30}\)See, e.g., McKay and Reis (2016, 2021), Kaplan et al. (2018), Alves et al. (2020).

\(^{31}\)See, among others, Drechsler et al. (2017) and Bellifemine et al. (2022).

\(^{32}\)See, e.g., Ottonello and Winberry (2020), Bahaj et al. (2022), Cloyne et al. (2022), Jungherr et al. (2022), Jeenas (2019).

\(^{33}\)See, for example, Olivei and Tenreyro (2007, 2010), De Ridder and Pfajfar (2017), Coglianese et al. (2022)

\(^{34}\)Appendix B.3 describes the data used in Figure 4 in more detail.
Figure 4: Accounting for Regional Heterogeneity

Note: we group the estimated county-specific coefficients $\hat{\beta}_{j,24}$ from (24) into 50 bins. Then, we regress within-bin population weighted averages of coefficients on within-bin population weighted averages of a battery of explanatory variables, all included in the regression. The bars represent the R-squared loss from removing every variable individually from this baseline regression.

R-squared loss induced by ignoring either regional MPCs or the non-tradable to tradable employment ratio are more than two times as large as those from any other variable we consider. Figure B.3 in Appendix B.2 plots results from a complementary exercise, where we consider the gain in R-squared from adding each of our explanatory variables individually, one at a time to the regression of the within-bin population weighted averages of $\hat{\beta}_{j,24}$ on within-bin population average MPC. Even in this case, MPCs and non-tradable employment remain the main predictors of the local response to monetary shocks.

Panel local projections  We now turn to decomposing the regional heterogeneity in the response to monetary policy according to the two crucial characteristics considered in our theory and highlighted by the data in Figure 4: our novel measure of regional MPCs, and openness to trade. In order to do so, for each month in our sample we rank counties in quartiles according to our two variables of interest: MPC$_j$, and the non-tradable employment ratio $\tilde{\rho}_{jt}$. We then construct two indicator variables: $D_{j,\text{NT}}^t$, which equals one when the ratio of non-tradable to tradable employment $\tilde{\rho}_{jt}$ in county $j$ is in the top quartile of the cross-section of counties in the year before period $t$; and $D_{j,\text{M}}^t$, which equals for those counties in the top quartile of the MPC distribution. Notice that, to avoid endogeneity concerns, we lag our indicator $D_{j,\text{NT}}^t$ by one year so that it refers to the year before the monetary shock. However, using contemporaneous variables does not materially af-

\[35\] Since our estimates for regional MPCs are only available from 2009, we consider counties’ average MPC over the period 2009-2019, and do not exploit time variation.
fect any of our results. We then run the following lag-augmented panel local projection (Montiel Olea and Plagborg-Møller, 2021), with errors two-way clustered at the time and county level:

\[
\Delta \ln (L_{jt+h}) = \alpha_{jh} + \delta_{lh} + \beta_{h}^{NT} \times D_{jt}^{NT} \times \varepsilon_{t} + \beta_{h}^{M} \times D_{j}^{M} \times \varepsilon_{t} + \alpha_{h}^{NT} D_{jt}^{NT} + \alpha_{h}^{M} D_{j}^{M} + \sum_{\ell=1}^{12} \gamma_{h \ell} \Delta \ln (L_{jt-\ell}) + u_{jht}
\]  

(25)

where the definition of \( \Delta \ln (L_{jt+h}), \varepsilon_{t}, \) and \( \Delta \ln (L_{jt-\ell}) \) is the same as in (24), \( \alpha_{jh} \) is a county fixed-effect, \( \delta_{lh} \) represents a time fixed effect, while \( D_{jt}^{NT} \) and \( D_{j}^{M} \) are the indicator variables defined above. Notice that, while the time fixed effect \( \delta_{lh} \) absorbs the monetary shock, what we are interested in is the differential response to the shock across counties. For this reason, (25) crucially includes interaction terms between the monetary shock and our newly constructed indicator variables. Because we are interacting the shock with binary variables, the interpretation of the coefficients is straightforward: the baseline group is represented by counties which are in the bottom 75% of the non-tradable to tradable employment distribution \( (D_{jt}^{NT} = 0) \) and in the bottom 75% of the MPC \( (D_{j}^{M} = 0) \). Then, \( \beta_{h}^{NT} \) simply represents the differential response of high non-tradable employment counties (for which \( D_{jt}^{NT} = 1 \) and \( D_{j}^{M} = 0 \)) relative to the baseline group. Similarly, \( \beta_{h}^{M} \) represents the differential response of high MPC, low non-tradable employment counties (for which \( D_{j}^{M} = 1 \) and \( D_{jt}^{NT} = 0 \)) relative to the baseline. Finally, the differential response of counties for which both \( D_{jt}^{NT} = 1 \) and \( D_{j}^{M} = 1 \) is simply given by the sum \( \beta_{h}^{NT} + \beta_{h}^{M} \).

Figure 5 plots the IRF coefficients from (25) in response to a 1 standard deviation expansionary monetary shock \( \varepsilon_{t} \). Panel (a) shows the estimates for the \( \beta_{h}^{NT} \) coefficient. Compared to counties for which \( D_{jt}^{NT} = 0 \) and \( D_{j}^{M} = 0 \), counties which are in the top quartile of the non-tradable to tradable employment distribution tend to respond more to monetary shocks. In fact, these regions experience a cumulative increase in employment up to 0.1% larger relative to the baseline group. To put this estimate in perspective, consider that in our sample a 1 standard deviation expansionary monetary shock corresponds to roughly a 10 basis points cut in the Fed funds rate. Ramey (2016) finds that the 3-year ahead cumulative response of real activity to a 10 basis points cut estimated in the literature lies in the range 0.03%-1.18%, with a mean of 0.53% and a median of 0.43%. Thus, our estimated differential response of 0.1% appears economically sizeable, when compared to the average response.

Similarly, panel (b) displays the estimated \( \beta_{h}^{M} \). High MPC counties experience a larger
employment response to monetary shocks, compared to low non-tradable employment, low MPC areas. In particular, the cumulative change in employment is up to 0.2% larger for those counties for which $D^M_j = 1$, relative to the baseline. Again, this magnitude appears economically large when we compare it to the range of estimates on the aggregate output effects of monetary shocks.

**Testing for the complementarity** We now present evidence that the complementarity between household heterogeneity and trade openness showcased in Proposition 1 is present in the data. To do so, we modify our baseline regression (25) to include a triple interaction between our MPC dummy $D^M_{jt}$, the trade openness dummy $D^{NT}_{jt}$, and the monetary shock $\epsilon_t$. The coefficient on this triple interaction term, $\beta^{NT,M}_{ht}$, can be interpreted as a cross-derivative. It captures the complementarity between the demand-side channel related to MPC and the supply-side channel due to the intensity of non-tradable activity. Hence, the regression specification now becomes:

$$
\Delta \log(L_{jt+h}) = \alpha_{jh} + \delta_{ht} + \beta^{NT}_{ht} \times D^{NT}_{jt} \times \epsilon_t + \beta^M_{ht} \times D^M_{jt} \times \epsilon_t + \beta^{NT,M}_{ht} \times D^{NT}_{jt} \times D^M_{jt} \times \epsilon_t + \sum_{\ell=1}^{12} \gamma_{ht}\Delta \log(L_{jt-\ell}) + u_{jht}
$$

(26)
Figure 6: MPC-Trade Openness Complementarity in the Data

Note: IRFs to a 1 standard deviation expansionary monetary shock. Errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.

Figure 6 plots the IRFs for the three interaction coefficients. The first two panels show that the more flexible specification (26) delivers results very similar to our baseline regression (25) for the coefficients $\beta_{NT}^h$ and $\beta_{M}^h$. Moreover, the estimated coefficient $\beta_{NT,M}^h$ is positive and statistically significant for nearly all horizons $h = 1, \ldots, 36$. This means that the employment response of counties with both $D^M = 1$ and $D^{NT} = 1$ is greater than the sum of the response of counties with $D^M = 1$ and $D^{NT} = 0$ and the response of counties with $D^M = 0$ and $D^{NT} = 1$. Thus, these results suggest that the two channels—MPCs and openness to trade—reinforce each other, in line with the complementarities predicted by our model and discussed in Section 3.

6 Conclusion

We build an empirically-motivated general equilibrium model of a monetary union with two layers of regional heterogeneity: intertemporal MPCs and trade openness. We derive a Regional Keynesian Cross: variation of a canonical formula that characterizes the regional transmission of monetary policy in terms of few, measurable, sufficient statistics. Essential to our mechanism and derivations is a novel iMPC-trade openness complementarity that arises through equilibrium interactions between our two sources of regional heterogeneity. This complementarity is supported by the data, thus validating our modelling approach. We then derive the National Keynesian Cross, an aggregation result showing how the joint distribution of openness to trade and iMPCs across regions shapes the nationwide macroeconomic response to monetary shocks.

Our paper can help to direct a large class of macroeconomic models that study gen-
general equilibrium transmission of shocks through regions. The regional Keynesian cross representation can guide new avenues for empirical tests and applications in a wide variety of settings. A promising application of our approach is represented by the Eurozone context, where intra- as well as cross-country heterogeneity in households’ MPCs and trade openness present another laboratory for policy analysis. Another fruitful extension of our framework could be along the international dimension: permitting households in our economy to invest internationally would generate a generalized Keynesian cross that would combine intra- and inter-national multipliers of domestic and foreign shocks.

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A Model Appendix

A.1 Balanced Trade Steady-State

Note that in steady-state, it is always the case that \( C_{NT}^j = L_{NT}^j \), thus it is also always true that \( W_{NT}^j C_{NT}^j = W_{NT}^j L_{NT}^j \) (remember that \( P_s^j = W_s^j \), for \( s \in \{ NT, T \} \)). In order for \( \rho_j = \xi_j \) it then needs to be the case that \( W_T^j C_T^j = W_T^j L_T^j \) as well. Notice that because of the atomicity assumption, county \( j \)’s exports are simply given by \( W_T^j L_T^j \). Similarly, county \( j \)’s imports are given by \( W_T^j C_T^j \). It thus follows that in a balanced trade steady-state \( W_T^j C_T^j = W_T^j L_T^j \).

Then it is immediate to show that in a balanced trade steady-state:

\[
\rho_j = \frac{W_{NT}^j L_{NT}^j}{W_j L_j} = \frac{W_{NT}^j C_{NT}^j}{P_j C_j} = \xi_j
\]

A.2 Proof of Proposition 1

Linearizing the market clearing for non-tradables and the consumption function:

\[
dL_{NT}^j = -\nu(1 - \xi_j)(dW_{NT}^j - dW^T) + M_j \left[ (\rho_j - \xi_j)(dW_{NT}^j - dW^T) + dL_j \right] + M_j dL_j + M_j d r_j \quad (A.1)
\]

In a balanced trade steady-state it holds that \( \rho_j = \xi_j \) so that:

\[
dL_{NT}^j = -\nu(1 - \rho_j)(dW_{NT}^j - dW^T) + M_j dL_j + M_j d r_j \quad (A.2)
\]

Linearizing the market clearing for tradables and the consumption function:

\[
dL_T^j = dC^T \quad (A.3)
\]

Moreover, combining the log-linearized condition for labor-supply in the tradable and non-tradable sector together with the log-linearized definitions of the wage and price index in county \( j \) gives:

\[
dL_T^j = \eta \rho_j \left( dW^T - dW_{NT}^j \right) + dL_j \quad (A.4)
\]

Next, combining (A.3) with (A.4) and plugging into (A.2):

\[
d ln L_{NT}^j = -\nu \frac{1 - \rho}{\eta \rho_j} d \ln L_j + M_j d \ln L_j + M_j d \ln r_j + \left( 1 + \frac{\nu}{\eta} \right) dC^T \quad (A.5)
\]
Finally, plugging (A.5) and (A.3) inside the condition \( dL_j = \rho_j dL_j^{NT} + (1 - \rho_j) dL_j^T \) gives (16).

### A.3 Regional Keynesian Cross: General Case

To derive (16) linearizing around a generic steady-state, we plug (A.3) together with (A.4) into (A.1) and plug the resulting equation, together with (A.5) inside the condition \( dL_j = \rho_j dL_j^{NT} + (1 - \rho_j) dL_j^T \). The Regional Keynesian Cross (16) now reads:

\[
\begin{align*}
    dL_j &= \rho_j \left( M_j' dr_j + M_j dL_j \right) + (1 - \rho_j) dC^T - \frac{\nu}{\eta} \left( 1 - \xi_j \right) \left( dL_j - dC^T \right) + \frac{\rho_j - \xi_j}{\eta} M_j \left( dL_j - dC^T \right)
\end{align*}
\]

### A.4 National Demand: General Case

Starting from (A.3) note that it needs to be the case that \( dL_j^T = dL_i^T \equiv dL^T \). Moreover, using the definition \( dC^T = \int_0^1 dC^T_{ji} \) we get:

\[
\begin{align*}
    dL^T &= \int \left\{ - \nu \xi_i \left( dW^T - dW^T_i \right) + M_i \left[ (\rho_i - \xi_i) (dW^T_i - dW^T) + dL_i \right] 
    + M_i' dr_i \right\} di
\end{align*}
\]

Plugging (A.4) into (A.6) above gives:

\[
\begin{align*}
    dC^T &= \left( I + \frac{\nu}{\eta} \int \frac{\xi_i}{\rho_i} di + \int M_i \frac{\rho_i - \xi_i}{\eta \rho_i} di \right)^{-1} \left[ \frac{\nu}{\eta} \int \frac{\xi_i}{\rho_i} dL_i di + \int M_i \frac{\rho_i - \xi_i}{\eta \rho_i} dL_i di + \int M_i dL_i di + \int M_i' dr_i di \right]
\end{align*}
\]

(A.7) represents the national response around a generic steady-state, where balanced trade does not necessarily hold.

### A.5 Proof of Proposition 2

Let’s consider again the market clearing condition for tradables:

\[
\begin{align*}
    L_{ji}^T &= \int_0^1 (1 - \omega) \left( \frac{p_t^j}{p_j^t} \right)^{-\nu} C_{ji} j^j
\end{align*}
\]
Note that (A.8) implies that the labor response in the tradable sector will be equalized across counties in every period, i.e., \( dL^T_{jt} = dL^T_{st} \), for all \( j, s \in [0, 1] \). This model prediction is consistent with the empirical results in, for example, Mian and Sufi (2014) and Chodorow-Reich et al. (2021). We can then substitute the aggregate consumption function (14) for every county \( j' \) inside (A.8) and log-linearize around the balanced trade steady-state to obtain:

\[
dL^T = \left( I + \frac{\nu}{\eta} \right)^{-1} \left( \frac{\nu}{\eta} \int_0^1 dL_i di + \int_0^1 M_i dL_i di + \int_0^1 M'_{di} dr_{i} \right) \quad \text{(A.9)}
\]

(A.9) above endogeneizes the national response \( dC^T \) that was taken as given when deriving the Regional Keynesian Cross in Proposition 1:

\[
dC^T = \left( I + \frac{\nu}{\eta} \right)^{-1} \left( \frac{\nu}{\eta} \int_0^1 dL_i di + \int_0^1 M_i dL_i di + \int_0^1 M'_{di} dr_{i} \right) \quad \text{(A.10)}
\]

Incidentally, (A.10) also shows how the joint distribution of employment responses \( dL_j \) and iMPCs \( M_j \) across counties matters in determining the size of the national response. To clarify this point further, we can recast (A.10) in terms of covariances as follows:

\[
dC^T = \left( I + \frac{\nu}{\eta} \right)^{-1} \left[ \frac{\nu}{\eta} dL + MdL + M' dr + \text{Cov}(M_i, dL_i) + \text{Cov}(M_{r_i}, dr_i) \right] \quad \text{(A.11)}
\]

Where \( dL \equiv \int_0^1 dL_i di, \; dr \equiv \int_0^1 dr_{i} di, \; M \equiv \int_0^1 M_i di, \) and \( M' \equiv \int_0^1 M'_{di} di \). As we know from Proposition 1, the local employment response \( dL_j \) is in turn going to depend on the degree of openness to trade \( \rho_j \). Thus, it follows that the joint distribution of our two sources of regional heterogeneity –iMPCs and openness to national trade– is going to matter for the national response. We are going to further discuss this insight momentarily, after deriving the National Keynesian Cross. However, before turning to this, it is useful to formalize the response of local employment to a monetary shock in the context of a national equilibrium:

**Lemma 1** (The Regional Keynesian Cross with Endogenous National Response). The first-order response of employment \( dL_j \) around the balanced trade steady-state to a monetary shock \( dr_j \)

---

36This follows directly from the fact that the law of one price holds for tradable goods and that tradables are sourced in equal proportion from all counties.

37See Appendix A.4 for the case in which we log-linearize around a steady-state without balanced trade.
satisfying a national equilibrium solves the following fixed point equation:

\[
\begin{align*}
\frac{dL_j}{dj} &= \rho_j \left( M'_j dr_j + M_j dl_j \right) - \frac{\nu}{\eta} (1 - \rho_j) \left( dL_j - dL \right) \\
&+ (1 - \rho_j) \left[ M dL + M'_j dr + \text{Cov}(M_i, dL_i) + \text{Cov}(M'_i, dr_i) \right]
\end{align*}
\]

(A.12)

Where \( dL \equiv \int_0^1 dL_i di, dr \equiv \int_0^1 dr_i di, M \equiv \int_0^1 M_i di, \) and \( M' \equiv \int_0^1 M'_i di. \)

To obtain (19) simply integrate (A.12) over \( j \in [0, 1] \):

\[
\int dL_j dj = \int \rho_j \left( M'_j dr_j + M_j dl_j \right) dj - \frac{\nu}{\eta} \int (1 - \rho_j) \left( dL_j - dL \right) dj + \\
\int (1 - \rho_j) \left[ \int M_i dl_i di + \int M'_i dr_i di \right]
\]

(A.13)

Rearranging the equation above gives:

\[
dL = \int \rho_j M'_j dr_j dj + \int \rho_j M_j dl_j dj + \frac{\nu}{\eta} \text{Cov}(\rho_j, dL_j) + (1 - \rho) \int M_j dl_j dj + \int M'_j dr_j dj
\]

(A.14)

Expressing (A.14) in terms of covariances gives (19).

A.6 Wage Phillips Curves

Non-Tradable Phillips Curve In each county \( j \in [0, 1] \) there’s a continuum of local non-tradable unions \( i \in [0, 1] \) that set their wage \( W^NT_{ji}(i) \) at any time \( t \) to maximize the following problem:

\[
\max_{W^NT_{ji}(i)} \sum_{h \geq 0} \beta^{t+h} \left[ u(C_{ji}) - v(L_{ji}) - \frac{\psi}{2} \left( \frac{W^NT_{ji+h}(i)}{W^NT_{ji+h-1}(i)} - 1 \right)^2 \right]
\]

s.t. \( L^NT_{ji}(i) = \left( \frac{W^NT_{ji}(i)}{W^NT_{ji}} \right)^{-\varepsilon} L^NT_{ji} \)

Where \( \psi \) governs the utility cost of wage adjustments, \( W^NT_{ji} = \left[ \int_0^1 \left( W^NT_{ji}(i) \right)^{1-\varepsilon} dt \right]^{1/\varepsilon} \). Labor supplied by the unions get packed into an aggregate non-tradable labor bundle according to a standard CES aggregator function \( L^NT_{ji} = \int_0^1 L^NT_{ji}(i)^{\varepsilon} \). The FOC to the
union problem reads:

\[ u'(C_{jt}) \frac{\partial C_{jt}}{\partial W_{jt}^{NT}(i)} - v'(L_{jt}) \frac{\partial L_{jt}}{\partial W_{jt}^{NT}(i)} - \psi \left( \frac{W_{jt}^{NT}(i)}{W_{jt-1}(i)} - 1 \right) \frac{1}{W_{jt-1}(i)} + \beta \psi \left( \frac{W_{jt+1}(i)}{W_{jt}^{NT}(i)} - 1 \right) \frac{W_{jt+1}(i)}{W_{jt}^{NT}(i)} \frac{1}{W_{jt}^{NT}(i)} = 0 \]

Which can be rewritten as:

\[ u'(C_{jt})W_{jt}^{NT}(i) \frac{\partial C_{jt}}{\partial W_{jt}^{NT}(i)} - v'(L_{jt})W_{jt}^{NT}(i) \frac{\partial L_{jt}}{\partial W_{jt}^{NT}(i)} - \psi \pi_{i}^{NT}(i) \left( 1 + \pi_{i}^{NT}(i) \right) \]

\[ + \beta \psi \pi_{i+1}^{NT}(i) \left( 1 + \pi_{i+1}^{NT}(i) \right) = 0 \]

Note that \( \frac{\partial C_{jt}}{\partial W_{jt}^{NT}(i)} = \frac{\partial Z_{jt}}{\partial W_{jt}^{NT}(i)} = \frac{\partial W_{jt}}{\partial W_{jt}^{NT}(i)} L_{jt} + \frac{W_{jt}}{\partial W_{jt}^{NT}(i)} \frac{\partial L_{jt}}{\partial W_{jt}^{NT}(i)} \). The following equations are going to be useful when solving the union’s problem:

\[ \frac{\partial L_{jt}^{NT}}{\partial W_{jt}^{NT}(i)} = -\frac{\varepsilon}{W_{jt}^{NT}(i)} L_{jt}^{NT} \]

\[ \frac{\partial L_{jt}}{\partial L_{jt}^{NT}} = \left( \frac{W_{jt}}{\alpha_{j} L_{jt}} \right)^{\frac{\varepsilon}{2}} \frac{W_{jt}^{NT}}{W_{jt}} \]

\[ \frac{\partial W_{jt}}{\partial W_{jt}^{NT}} = \alpha_{j} \left( \frac{W_{jt}^{NT}}{W_{jt}} \right)^{\eta} = \frac{L_{jt}^{NT}}{L_{jt}} \]

\[ \frac{\partial W_{jt}^{NT}}{\partial W_{jt}^{NT}(i)} = \left( \frac{W_{jt}^{NT}(i)}{W_{jt}^{NT}} \right)^{-\varepsilon} = \frac{L_{jt}^{NT}(i)}{L_{jt}^{NT}} \]

Where the first condition comes from exploiting the fact that from the households’ perspective \( L_{jt}^{NT} = \int_{0}^{1} L_{jt}^{NT}(i) \, dt \) and plugging in the labor demand equation for \( L_{jt}^{NT}(i) = \left( \frac{W_{jt}^{NT}(i)}{W_{jt}^{NT}} \right)^{-\varepsilon} \) \( L_{jt}^{NT} \) and the other equations just follow from standard variable definitions. Combining all of the above yields:

\[ \pi_{i}^{NT}(i) + \left( \pi_{i}^{NT}(i) \right)^{2} = \beta \left( \pi_{i+1}^{NT}(i) + \left( \pi_{i+1}^{NT}(i) \right)^{2} \right) \]

\[ + \frac{1}{\psi} \left[ \frac{u'(C_{jt})}{P_{jt}} \left( W_{jt}^{NT}(i) L_{jt}^{NT}(i) - \varepsilon W_{jt}^{NT} L_{jt}^{NT}(i) \right) + \varepsilon v'(L_{jt}) L_{jt}^{NT}(i) \left( \frac{L_{jt}^{NT}}{\alpha_{j} L_{jt}} \right)^{\frac{1}{2}} \right] \]
Imposing symmetry across $i$:

$$\pi^N_{it} + (\pi^N_{it})^2 = \beta \left( \pi^N_{i+1} + (\pi^N_{i+1})^2 \right) + \frac{\epsilon}{\psi} \frac{L^N_{jt}}{\alpha_j L^N_{jt}} \left[ v' \left( \frac{L^N_{jt}}{\alpha_j L^N_{jt}} \right) \left( \frac{L^N_{jt}}{\alpha_j L^N_{jt}} \right)^{\frac{1}{\eta}} - \epsilon - 1 \right] \psi L^N_{jt}$$

Let’s now define $Z^N_{jt} \equiv \frac{W^N_{jt} L^N_{jt}}{P^N_{jt}}$ and $\mu \equiv \frac{\epsilon - 1}{\epsilon}$. Then:

$$\pi^N_{it} + (\pi^N_{it})^2 = \beta \left( \pi^N_{i+1} + (\pi^N_{i+1})^2 \right) + \frac{\epsilon}{\psi} \left[ L^N_{jt} v' \left( \frac{L^N_{jt}}{\alpha_j L^N_{jt}} \right) \left( \frac{L^N_{jt}}{\alpha_j L^N_{jt}} \right)^{\frac{1}{\eta}} - \mu u' \left( C^N_{jt} \right) Z^N_{jt} \right]$$

Note that in SS the following holds:

$$\rho_j L_j v' \left( L_j \right) = \mu u' \left( C_j \right) Z^N_j$$

Let’s take a first order approximation around the zero inflation steady state:

$$\pi^N_{jt} = \beta \pi^N_{jt+1} + \kappa^N_j \left[ \frac{1}{\eta} \left( dL^N_{jt} - dL_{jt} \right) + \frac{1}{\phi} dL_{jt} + \frac{1}{\sigma} dC_{jt} - \left( dZ^N_{jt} - dL^N_{jt} \right) \right]$$

Where $\kappa^N_j \equiv \frac{\epsilon}{\psi} P_j L_j v' \left( L_j \right)$, $\sigma \equiv -\frac{u' \left( C_j \right)}{u'' \left( C_j \right) C_j}$ and $\phi \equiv \frac{v' \left( L_j \right)}{v'' \left( L_j \right) L_j}$. Note that $dZ^N_{jt} - dL^N_{jt} = dW^N_{jt} + dL^N_{jt} - dP_{jt} - dL^N_{jt} = (1 - \rho_j) \left( dW^N_{jt} - dW^T_{jt} \right)$. Moreover, remember that the labor split in the non-tradable sector obeys: $dL^N_{jt} = \eta dW^N_{jt} - \eta dW_{jt} + dL_{jt}$ that is $(1 - \rho_j) \left( dW^N_{jt} - dW^T_{jt} \right) = \frac{1}{\eta} \left( dL^N_{jt} - dL_{jt} \right)$. Thus, it holds that:

$$\left[ \frac{1}{\eta} \left( dL^N_{jt} - dL_{jt} \right) + \frac{1}{\phi} dL_{jt} + \frac{1}{\sigma} dC_{jt} - \left( dZ^N_{jt} - dL^N_{jt} \right) \right] = \left[ \frac{1}{\phi} dL_{jt} + \frac{1}{\sigma} dC_{jt} \right]$$

Finally, we can rewrite county $j$’s New Keynesian Wage Phillips Curve for non-tradables as:

$$\pi^N_{jt} = \beta \pi^N_{jt+1} + \kappa^N_j \left[ \frac{1}{\phi} dL_{jt} + \frac{1}{\sigma} dC_{jt} \right] \quad (A.15)$$

The sequence space formulation of (A.15) above is:

$$\pi^N_{jt} = K^N_j \left( \frac{1}{\phi} dL_{jt} + \frac{1}{\sigma} dC_{jt} \right) \quad (A.16)$$
Where:

\[
K_j^{NT} \equiv \kappa_j^{NT} = \begin{bmatrix}
1 & \beta & \beta^2 & \ldots \\
0 & 1 & \beta & \ldots \\
0 & 0 & 1 & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
\end{bmatrix}
\]  

(A.17)

** Tradable Phillips Curve**  
There is a continuum of federal tradable labor unions \( \zeta \in [0, 1] \) that set their wage \( W_t^T(\zeta) \) at any time \( t \) to maximize the following problem:

\[
\max_{W_{t+h}^T(\zeta)} \sum_{h \geq 0} \beta^{t+h} \left[ u(C_t) - v(L_t) - \frac{\psi}{2} \left( \frac{W_{t+h}^T(\zeta)}{W_{t+h-1}^T(\zeta)} - 1 \right)^2 \right]
\]

s.t. \( L_t^T(\zeta) = \left( \frac{W_t^T(\zeta)}{W_t^F(\zeta)} \right)^{-\epsilon} L_t^F \)

Where \( C_t = \int_0^1 C_j \, dj \) and similarly \( L_t = \int_0^1 L_j \, dj \). Then it follows that \( \frac{\partial C_t}{\partial W_t^T(\zeta)} = \int \frac{\partial C_j}{\partial W_t^T(\zeta)} \, dj \) and \( \frac{\partial L_t}{\partial W_t^T(\zeta)} = \int \frac{\partial L_j}{\partial W_t^T(\zeta)} \, dj \). Which gives:

\[
\pi_t^T(\zeta) + \left( \pi_t^T(\zeta) \right)^2 = \beta \left( \pi_{t+1}^T(\zeta) + \left( \pi_{t+1}^T(\zeta) \right)^2 \right)
\]

\[
+ \frac{\epsilon}{\psi} \left[ v'(L_t) L_t^T(\zeta) \left( L_t^T \right)^{1/\eta} \int \left[ \left( 1 - \alpha_j \right) L_j \right]^{-1/\eta} \, dj - \mu u'(C_t) W_t^T(\zeta) L_t^T(\zeta) \int \frac{1}{P_j} \, dj \right]
\]

Imposing symmetry across \( \zeta \):

\[
\pi_t^T + \left( \pi_t^T \right)^2 = \beta \left( \pi_{t+1}^T + \left( \pi_{t+1}^T \right)^2 \right)
\]

\[
+ \frac{\epsilon}{\psi} \left[ v'(L_t) \left( L_t^T \right)^{1+\frac{1}{\eta}} \int \left[ \left( 1 - \alpha_j \right) L_j \right]^{-\frac{1}{\eta}} \, dj - \mu u'(C_t) W_t^T(\zeta) L_t^T(\zeta) \int \frac{1}{P_j} \, dj \right]
\]

Let’s define \( Z_t^T \equiv \int \frac{W_t^T}{P_j} \, dj = W_t^T(\zeta) L_t^T(\zeta) \int \frac{1}{P_j} \, dj \). Then:

\[
\pi_t^T + \left( \pi_t^T \right)^2 = \beta \left( \pi_{t+1}^T + \left( \pi_{t+1}^T \right)^2 \right) + \frac{\epsilon}{\psi} \left[ v'(L_t) \left( L_t^T \right)^{1+\frac{1}{\eta}} \int \left[ \left( 1 - \alpha_j \right) L_j \right]^{-\frac{1}{\eta}} \, dj - \mu u'(C_t) Z_t^T \right]
\]

Note that in SS the following condition holds:

\[
v'(L) \int \left( 1 - \rho_j \right) L_j \, dj = \mu u'(C) Z_t^T
\]
Taking a first order approximation around the zero inflation SS yields the following New Keynesian Wage Phillips Curve for the tradable sector:

$$\pi^T_t = \beta \pi^T_{t+1} + \kappa^T \left[ \frac{1}{\varphi} dL_t + \frac{1}{\sigma} dC_t + \frac{1}{\eta} \left( dL^T_T - \Phi dL_t \right) - \left( dZ^T_t - dL^T_t \right) \right]$$  \hspace{1cm} (A.18)

Where \( \kappa^T \equiv \frac{\xi}{\varphi} v'(L) \) \( \mathbb{E} \left[ (1 - \rho_j) L_j \right] \) and \( \Phi \equiv 1 + \frac{\text{Cov}\left[(1-\rho_j) L_j, dL_t\right]}{\mathbb{E}[(1-\rho_j) L_j] \mathbb{E}(dL_t)} \).

**A.7 Phillips curves under limit cases**

**Case \( \rho_j \to 1 \), non-tradable only**  When \( \rho_j \to 1 \), the non-tradable Phillips curve converges to:

$$\pi^{NT}_j = K^{NT}_j \left( \frac{1}{\varphi} dL_j + \frac{1}{\sigma} dC_j \right)$$  \hspace{1cm} (A.19)

Where:

$$K^{NT}_j \equiv \begin{bmatrix} \kappa^{NT}_j & \beta \kappa^{NT}_j & \beta^2 \kappa^{NT}_j & \ldots \\ 0 & \kappa^{NT}_j & \beta \kappa^{NT}_j & \ldots \\ 0 & 0 & \kappa^{NT}_j & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$  \hspace{1cm} (A.20)

where \( \kappa^{NT}_j = \frac{\xi}{\varphi} L_j v'(L_j) \). The National Phillips curve is undefined.

**Case \( \rho_j \to 0 \), tradable only**  When \( \rho_j \to 0 \), inflation is always zero for the non-tradable good, \( L^T_j = L_j \) and hence the national inflation is given by

$$\pi^T_t = \beta \pi^T_{t+1} + \kappa^T \left[ \frac{1}{\varphi} dL_t + \frac{1}{\sigma} dC_t \right]$$  \hspace{1cm} (A.21)

Where \( \kappa^T \equiv \frac{\xi}{\varphi} v'(L) L \).
B Empirical Appendix

B.1 Threats to Identification

Our main regressor of interest, the measure of monetary shocks, is relatively immune to standard endogeneity critiques as it is based on a fairly standard methodology that employs high-frequency changes in financial variables.\(^{38}\) A potential threat to our analysis could however come from omitted variable bias. In particular, it may be the case that we are not controlling for some other determinants of the local response to monetary policy. In so far as these unobserved determinants covary systematically with our two variables of interest –MPC and non-tradable employment– this would invalidate our previous claim that local MPCs and trade openness are some key drivers of the local response to monetary policy.

In Appendix B.2, we try to address these concerns by running a thorough battery of robustness checks. First, our main results do not change if we include an interaction of the monetary shock with the state fixed effect. This is isomorphic to allowing the regional response to change with any time invariant characteristic that varies across states. For example, since most of the fiscal response in the US takes place at the state and federal level, this exercise addresses concerns that our results are not driven by differential local fiscal responses. Second, we show that results are robust to controlling for the interaction between the state fixed effect, the time fixed effect, and the monetary shock. In this specification the slope of the response varies with any characteristic that is constant within a given state in a given month.\(^{39}\)

\(^{38}\)Recent studies question the exogeneity of monetary policy surprises based on high-frequency identification; see for example Miranda-Agrippino and Ricco (2021) or Bauer and Swanson (2022). In Appendix B.2, we address these concerns by showing that our results are robust to using a narratively identified instrument of monetary shocks –as in Romer and Romer (2000)– as well as measures that control for the information content that is embedded in policy announcements (Miranda-Agrippino and Ricco, 2021).

\(^{39}\)Appendix B.2 includes several other robustness checks, including different thresholds for the indicator variables \(D_{jt}^{NT}\) and \(D_{jt}^{M}\), using different measures of monetary shocks, and using different time and space samples.
B.2 Additional Results and Robustness Checks

Figure B.1: The Geography of Non-Tradable Employment
Table B.1: Estimates from Regression (21)

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<td></td>
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</tr>
<tr>
<td>45-64 y.o.</td>
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<td></td>
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<td>≥ 65 y.o.</td>
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<tr>
<td></td>
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<td>Y</td>
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</tbody>
</table>
Figure B.2: Regional Responses to Monetary Shocks – Different Horizons

(a) 30-month ahead  (b) 24-month ahead

Figure B.3: Accounting for Regional Heterogeneity

Note: we group the estimated county-specific coefficients $\hat{\beta}_{j,t}$ from (24) into 50 bins. Then, we regress within-bin population weighted averages of coefficients on within-bin population weighted averages of MPC. The blue bar represents the R-squared from this baseline regression. Each green bar represents the gain in R-squared when we add one extra variable to the baseline regression.
Figure B.4: Robustness – top 5%

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.

Figure B.5: Robustness – top 50%

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.
Figure B.6: Robustness – state×date FE

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.

Figure B.7: Robustness – seasonally adjusted employment

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.
Figure B.8: Robustness – Romer and Romer (2000) shock

![Graph](image)

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.

Figure B.9: Robustness – ending the sample in 2006m12

![Graph](image)

Note: errors are two-way clustered at the time and county level. Lightly shaded areas represent 95% confidence intervals. Darkly shaded areas are 90% confidence intervals. The y-axis represents the cumulative percentage change in employment. The x-axis represents months elapsed since the shock.

### B.3 Data for Figure 2

In this section, we briefly describe the data used to perform our gain in R-squared exercise presented in Figure 4.

**Bank Deposits** First, we obtain county-level deposit HHI data from the Federal Deposit Insurance Corporation (FDIC) Summary of Deposits database. This dataset includes
annual information on branch-level deposits for the universe of FDIC insured US institutions. Our sample covers the years 1994-2015. For each year \( t \) and each county \( j \) in the data, we compute the deposit HHI \( H_{jt} \) according to the standard formula \( H_{jt} = \sum_{i}^{N_t} s_{ijt}^2 \), where \( s_{ijt} \) represents the share of deposits held by bank \( i \) in county \( j \), year \( t \). We then average \( H_{jt} \) for each county over the time period 1994-2015.

**Housing Cost, Share of Homeowners, Population Density & Temperature** We rely on the Social Capital Project to obtain county-level measures of housing costs, the share of homeowners, population density, and temperature. The Social Capital Project is an initiative conducted by the US Congress Joint Economic Committee to collect state and county-level data on a variety of social, economic, health, and religious indicators from different sources.\(^{40}\) We define housing costs as the share of households for which annual housing costs exceed 35\% of yearly household income. Similarly, we define the share of homeowners as the share of houses which are owner-occupied. Both of these measures are based on data from the American Community Survey for the period 2011-2015. Population density is simply defined as the ratio between county population and county size (in square miles). This measure is obtained from the 2010 US census. As for temperature, we consider the mean temperature recorded in the county in the year 2011. This measure is obtained from the North America Land Data Assimilation System.

**Firm Size Distribution** Data on the distribution of firm size come from the County Business Patterns (CBP) dataset published by the US Census. In particular, for each county and each year, we compute the average number of employees per establishment. We then average this measure for each county over our annual sample 1990-2015.

**Participation Rate, Reallocation Rate & Firm Entry Rate** We compute the participation rate directly from our main dataset, i.e., the Local Area Unemployment Statistics published by the BLS. In particular, for each year in our data we define the participation rate as the share of people in the labor force over total county population. We then average this measure for each county across 1990-2015. For the reallocation rate and the firm entry rate, instead, we rely on the Business Dynamism Statistics published by the US Census. The reallocation rate is obtained as the sum of the jobs created and destroyed in a given county and year, as a share of total jobs. Similarly, we define the firm entry rate as the ratio of the number of new establishments opened in a given county-year over the total number of the establishments operating in that county. The data spans 1990-2015.

\(^{40}\)For more details, see jec.senate.gov/public/index.cfm/republicans/socialcapitalproject.
Land Availability  Our data on land availability come from Lutz and Sand (2022). The authors build upon the seminal work by Saiz (2010) and develop a time varying and geographically disaggregated measure of land unavailability using satellite imagery data. We refer the reader to Lutz and Sand (2022) for a thorough description of the construction of land unavailability. The data start in 2002m1. We generate county-level averages of the measure of land unavailability over our time sample 2002m1-2015m12.

Age Structure, Race Structure & Gender Structure  Our data on the age, race, and gender structure come from the Census Bureau’s Population Estimates Program. This database includes annual county-level estimates of population by age, race, and gender. To analyze the age structure, we follow Leahy and Thapar (2022) and focus on the share of population within a county which is less than 35 years old and in between 40 and 65 years old. For the race structure, we compute the share of population within a county which is black and the share of population which is hispanic. For the gender structure, we compute the share of women within a county. We then average each of these variables for each county over our annual sample 1990-2015.

Voting Rate  Data on the voting rate come from the County Presidential Election Returns published by the MIT Election Data and Science Lab. In particular, for each presidential election from 2000 to 2020, we compute the total number of votes in each county as a share of county-level population. For each county, we then average the participation rate across the 6 presidential elections that took place between 2000 and 2020.