Practical DSGE Modelling

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Overview

This is an introductory course for individuals with little or no previous knowledge of the practicalities involved in dynamic stochastic general equilibrium modelling. The intention of the course is not to train participants to become specialist model-builders but instead for them to be fully conversant in the main techniques involved in building and using dynamics models in their work. The aim is to make participants 'sophisticated consumers' of dynamic models and to provide a deeper framework and knowledge within which to frame their discussions. Although no prior knowledge is assumed it is a fast paced course and additional reading is provided to help participants to consolidate material.

Course objectives

By the end of the course participants should:

- Understand the economic foundations of dynamic models and how they relate to current policy issues
- Be able to write dynamic models in a general form suitable for solution and simulation by computer
- Implement basic solution and simulation techniques to analyse dynamic models, showing how the model economies behave and how they respond to different shocks
- Be aware of how to extend simple models to answer more complex questions associated with monetary policy
- Feel confident in discussing model-based analysis, evaluating the strength of models used and engaging in conversation with people building economic models

Organisation

The course runs over a series of five mornings. Each session will be a mixture of small-group teaching and practical exercises using MATLAB software to solve and simulate dynamic models. Afternoons are therefore free, allowing participants time to reflect and consolidate the morning work before the next session. The instructors will be available throughout each afternoon to answer any questions or queries that arise.

Pre-readings

Some pre-course readings will be assigned for those participants who wish to prepare in advance. The issues involved in the pre-reading will be dealt with in the course so there is no need to understand everything. The pre-reading is as much about getting participants to gauge what they do and don't understand as it is to get ideas and issues across.

Ouline

Session 1	Introduction to DSGE modelling.
	In this session we describe the key ingredients in a dynamic model, using the simplest available dynamic model with monetary policy.
	Exercises are designed to familiarise participants with the MATLAB computer language environment.
Session 2	Taking a model to the computer
	We show how to rewrite the model in a form suitable for solving and simulating on a computer. There are two parts to this: (i) approximating the nonlinear model by a linear model and (ii) collecting the equations together into the so-called state-space form.
	In exercises we practice the techniques of log-linearisation and define state- space forms.
Session 3	Solution techniques
	After a short discussion of the range of possibilities, we explain the standard Blanchard-Kahn technique for solving linear models. We also discuss the role of the Blanchard-Kahn conditions in ensuring determinacy of the solution.
	The computer exercises take participants from the state-space form to the solution to the model and show the relevance of determinacy conditions.
Session 4	Simulation techniques
	Having obtained the solution in the previous session, this session is devoted to developing a range of simulation techniques through which to analyse the model. Amongst others, we discuss stylised facts of volatilities and correlations at different leads and lags, impulse response analysis, forecast error variance decomposition.
	Computing exercises develop participants' skills at analysing dynamic models.
Session 5	Advanced dynamic models
	In the final session we survey some of the more advanced dynamic models that are useful for policy analysis. In particular, we focus on how to handle models with simple monetary policy rules such as those famously associated with Taylor.
	The exercises in the final session consolidate the work of the previous sessions. The participants will work through a series of exercises which simulate the behaviour of a dynamic economy under Taylor rules. Time permitting, participants will be able to analyse the response of the economy for different coefficients in the monetary policy rules.

Instructors

The course architects are Martin Ellison and Alina Barnett.

Martin Ellison, Assistant Professor, Warwick University

Martin has an MPhil from Oxford University and a PhD from the European University Institute, Florence. He is an affiliate of the Centre for Economic Policy Research and a consultant at the Bank of England. His research focuses on monetary theory and monetary policy and he has just received a three-year ESRC Research Fellowship for his project "Improving Monetary Policy for Macroeconomic Stability in the 21st Century". His research is regularly published in the top academic journals, as well as presented at leading conferences and central banks.

Alina Barnett, Postgraduate Research Fellow, Warwick University

Alina holds an MA in Economics from the Central European University in Budapest and a BA in Finance and Banking from the Academy of Economic Studies in Bucharest. Her research interests are in macroeconomics, time series econometrics and developing economies. Her professional experience includes financial analysis, banking and teaching economics at university and secondary school level.









Outline	
Day	Topics
1	Introduction to DSGE models / Introduction to MATLAB
2	Writing models in a form suitable for computer
3	Solution techniques
4	Simulation techniques
5	Advanced dynamic models







































Myopic price



Approximate myopic price with price that would prevail in flexible price equilibrium

$$p_t^* = k p_t m c_t$$

Price is constant mark-up *k* over marginal cost

In our hat (^) notation – to be explained later – the myopic price is given by

$$\hat{p}_t^* = \hat{p}_t + \hat{m}c_t$$



















Next steps



Introduction to MATLAB

How to write the DSGE model in a format suitable for solution

How to solve the DSGE model

Solving the DSGE model in MATLAB





Matrix



MATLAB works with essentially only one kind of object – a rectangular numerical matrix with possible complex entries.



























for ... end





Graphics



x = 0 : 0.01 : 100;y = x^2; plot(x,y)

Adding plots to an existing graph: hold on

Multiple plots in one figure: subplot









Log-linear approximation

Begin by taking logarithms of dynamic IS curve

$$-\sigma \ln Y_{t} = \ln \beta + \ln E_{t} \left[Y_{t+1}^{-\sigma} \frac{1+i_{t}}{1+\pi_{t+1}} \right]$$

Problem is last term on right hand side


































































Solution of unstable equation

Solve unstable equation forward to time t+j

$$E_t \widetilde{y}_{t+j} = (\Lambda_2)^j \widetilde{y}_t$$

As $|\Lambda_2| > 1$, only stable solution is $\tilde{y}_t = 0 \ \forall t$

$$\tilde{y}_t = P_{21}^* w_t + P_{22}^* y_t = 0$$
 $y_t = -P_{22}^{*-1} P_{21}^* w_t$

Forward-looking (control) variables are function of backward-looking (predetermined) variables

Solution of stable equation Solve stable equation forward to time t+j $E_t \widetilde{w}_{t+j} = (\Lambda_1)^j \widetilde{w}_t$ As $|\Lambda_1| < 1$, no problems with instability $\widetilde{w}_t = P_{11}^* w_t + P_{12}^* y_t$ $y_t = -P_{22}^{*-1} P_{21}^* w_t$ $\widetilde{w}_t = (P_{11}^* - P_{12}^* P_{22}^{*-1} P_{21}^*) w_t$

























Variance	S		
		Standard deviation	
	Interest rate	0.46	
	Output gap	1.39	
	Inflation	0.46	

Со	rrelations	5			
		Interest	Output	Inflation	
		rate	gap		
	Interest rate	1	-1	-1	
	Output gap	-1	1	1	
	Inflation	-1	1	1	
		1	1		

tocorrela	tions			
	t,t-1	t,t-2	t,t-3	t,t-4
Interest rate	0.50	0.25	0.12	0.06
Output gap	0.50	0.25	0.12	0.06
Inflation	0.50	0.25	0.12	0.06

Cross-correlations								
Correla	tion wi	th outp	ut gap	at time	t			
				1	1			
	t-2	t-1	t	t+1	<i>t</i> +2			
Output gap	<i>t-2</i> 0.25	<i>t-1</i> 0.50	<i>t</i> 1	<i>t</i> +1 0.50	<i>t</i> +2 0.25			
Output gap Inflation	<i>t-2</i> 0.25 0.25	<i>t-1</i> 0.50 0.50	t 1 1	<i>t</i> +1 0.50 0.50	t+2 0.25 0.25			











Shock v_t^1 v_t^2 Impulse v_1^1 Ψ_1^1 response at Ψ_1^1 Ψ_1^2 horizon 1 v_1^2	FEVD At horizo	at horizon on <i>h</i> = 1, two s	h = 1 ources o	of foreca	st errors
Impulse response at Ψ_1^1 Ψ_1^2 horizon 1		Shock	v_t^1	v_t^2	
		Impulse response at horizon 1	Ψ_1^1	Ψ_1^2	
Contribution to variance at horizon 1 $(\Psi_1^1)^2 \sigma_{\nu^1}^2 (\Psi_1^2)^2 \sigma_{\nu^2}^2$		Contribution to variance at horizon 1	$\left(\Psi_1^1 ight)^2\sigma_{_{V^1}}^2$	$\left(\Psi_1^2\right)^2 \sigma_{_{\mathcal{V}^2}}^2$	



EVD at ho	rizon /	h = 2		
horizon $h = 2$,	, four so	urces of	forecas	t errors
Shock	v_t^1	v_t^2	v_{t+1}^1	v_{t+1}^2
Impulse response at horizon 2	Ψ_2^1	Ψ_2^2	Ψ_1^1	Ψ_1^2
Contribution to variance at horizon 2	$\left(\Psi_2^1 ight)^2\sigma_{_{\mathcal{V}^1}}^2$	$\left(\Psi_2^2\right)^2 \sigma_{_{\mathcal{V}^2}}^2$	$\left(\Psi_1^1 ight)^2\sigma_{_{V^1}}^2$	$\left(\Psi_1^2 ight)^2\sigma_{_{arsigma^2}}^2$

















Impulses



Can add extra shocks to the model

$$\hat{x}_{t} = E_{t}\hat{x}_{t+1} - \sigma^{-1}(\hat{i}_{t} - E_{t}\hat{\pi}_{t+1}) + g_{t}$$
$$\hat{\pi}_{t} = \beta E_{t}\hat{\pi}_{t+1} + \kappa \hat{x}_{t} + u_{t}$$
$$\hat{i}_{t} = \delta \hat{\pi}_{t} + v_{t}$$

Shocks may be correlated

$\left(v_{t+1} \right)$	$\left(\rho_{11} \right)$	$ ho_{\!12}$	ρ_{13}	$\left(v_{t}\right)$		(v_{11})	v_{12}	v_{13}	$\left(\mathcal{E}_{t+1}^{v} \right)$
u_{t+1}	$= \rho_{21}$	$ ho_{\scriptscriptstyle 22}$	$ ho_{ m 23}$	<i>u</i> _t	+	v_{21}	υ_{22}	v_{23}	\mathcal{E}_{t+1}^{u}
$\left(g_{t+1}\right)$	$\left(\rho_{31} \right)$	$ ho_{ m 32}$	$ ho_{33})$	(g_t)		v_{31}	v_{32}	v_{33})	$\left(\mathcal{E}_{t+1}^{g} \right)$



Solution of complex models



Blanchard-Kahn technique relies on invertibility of A_0 in state-space form.

$$A_{0}E_{t}X_{t+1} = A_{1}X_{t} + B_{0}v_{t+1}$$

$$E_{t}X_{t+1} = A_{0}^{-1}A_{1}X_{t} + A_{0}^{-1}B_{0}v_{t+1}$$

$$A \qquad B$$

$$E_{t}X_{t+1} = AX_{t} + Bv_{t+1}$$
















