A Timeless Contract for Sharing Consumption Risk

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Abstract

We propose a social insurance design for sharing consumption risk that is time-consistent and preserves a meaningful distinction among generations of policymakers. Improved consumption risk-sharing under our policy design is attained through an endogenously-generated universal consumption safety net, which prevents long-run immiseration and decreases consumption inequality.

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1 Introduction

There has been a resurgence in inequality research following the publication of Thomas Piketty's "Capital in the Twenty-First Century" (Piketty and Goldhammer, 2014). However, the literature has been predominantly concerned with the issue of income or its components (Attanasio and Pistaferri, 2016). There is a strong case to be made for shifting the focus to the issue of consumption, not least because it is consumption that ultimately determines welfare of individuals. Moreover, consumption dynamics can diverge from income dynamics due to savings and insurance, which justifies an independent interest in the subject (Krueger and Perri, 2006; Heathcote, Perri and Violante, 2010; Meyer and Sullivan, 2017). Following this line of research, this paper investigates the impact of social insurance design on consumption dynamics and inequality in the long-run. The rapid expansion in government-provided social insurance as a risk-sharing mechanism across the world (Chetty and Finkelstein, 2013) suggests that the focus on consumption and welfare consequences of this policy is both warranted and justified.

Social insurance operates through redistribution of resources where government taxes today's fortunate to aid today's less fortunate, with the promise to do the same for the former if they were to face the reversal of fortune in the future. For it to remain sustainable, it is imperative that the incentives of the well-off are aligned to participate in the scheme, which in turn requires the belief that government will make good on its promises. This is the central theme of the paper to which we will return repeatedly.

The vast majority of the literature uses Ramsey policy to study risk-sharing under one-sided commitment of the principal (Phelan, 1995; Kocherlakota, 1996; Alvarez and Jermann, 2000; Ligon, Thomas and Worrall, 2002; Krueger and Uhlig, 2006; Broer, 2009; Ábrahám and Laczó, 2018; Ljunqvist and Sargent, 2018, Chapter 21) which is a suitable environment for considering the design of social insurance. With government as the principal or the social planner, Ramsey policy maximises the utilitarian social welfare function subject to the aggregate resource constraint and the participation constraints of individuals. The latter constraints, which ensue due to the lack of commitment to social insurance on the part of individuals, impede perfect risk-sharing where consumption is equalised among the participants.

The Ramsey insurance policy, even though optimal from the viewpoint of the initial period when government formulates the policy, has a number of undesirable features and implications. On the philosophical side, commitment to the Ramsey policy means preferences of the initial period policymaker prevail indefinitely in the operation of social insurance. In case of the Timeless Perspective (Woodford, 1999) version of the policy, it is preferences of the policymaker in the arbitrary distant past that dictate the working of social insurance. So, there is no meaningful distinction among generations of policymakers under the Ramsey policy.

As long as there is no intergenerational conflict among policymakers, say between the initial period policymaker and the policymaker in arbitrary future time, the commitment to the Ramsey policy is credible. However, due to the existence of the forward-looking participation constraints, the initial period Ramsey policy is generally not time-consistent: if a policymaker beyond the initial period were given the chance to redesign social insurance, the resulting optimal policy will deviate from the original optimal policy. Intuitively, the time-inconsistency issue arises because the later policymaker will not be obligated (hence will not have incentives) to keep promises made by the initial period policymaker. This failure of the principle of optimality in settings like ours has been well-known since the publication of Kydland and Prescott (1977). To the extent that the initial period policymaker is mindful of this intergenerational conflict and credibility of commitment is an important consideration for the design of social insurance, it is doubtful whether the Ramsey policy would be admissible.

Turning to the positive implications, the Ramsey insurance policy delivers very little risk-sharing in the long-run. To sustain the Ramsey policy, it is necessary to devote a considerable amount of available resource to keep promises made to individuals who were once well-to-do and contributed relatively more to the social insurance scheme to subsidise those who were less well-off. However, the decumulation of the earned privilege is very slow whose flip side is that unlucky individuals are driven down to autarkic outcomes in the absence of a good fortune. This issue emerges independently of the time-inconsistency problem: because the long-run outcome under the standard Ramsey policy is equivalent to the outcome under the Timeless Perspective policy which is time-consistent as long as it remains the modus operandi, the limited risk-sharing in the long-run does not go away. Naturally, this has an implication for cross-sectional consumption inequality in society which this paper is concerned with.

With the above results in mind, we propose an amendment to the Ramsey policy. The only change we make is that the policymaker in each time period, including the policymaker in the initial period, is restricted to make the same promise to individuals with the same history (which is well-defined in our setting; see Section 2). This policy framework guarantees intergenerational equity because it is not possible to make different promises to otherwise identical individuals whose only difference (in the eye of the policy) is that they arrive at the same history at different points in time. While this policy is no longer optimal from the viewpoint of the initial period, it is time-consistent by design. Moreover, it preserves a meaningful distinction among the policymakers at different time periods. This sets it apart from other popular time-consistent policy designs, for instance the Timeless Perspective policy (Woodford, 1999) or the unconditionally optimal policy (Damjanovic, Damjanovic and Nolan, 2008) where policymakers of all generations are effectively one and the same. Note also that our policy design is distinct from the discretionary policy where no promise about the future is made by the policymaker at any point in time because commitment is not possible.

One way to think about our proposal is that it is the timelessness in promises or contracts that we require as opposed to the timelessness in the cohort of policymakers as in the Timeless Perspective policy. For this reason, we refer to our policy framework as "Timeless Contract." The Timeless Contract policy is optimal in the class of policies where the promises are constrained to be time-invariant functionals of history. Brendon and Ellison (2018) provide a general theoretical treatment of this solution concept based on dominance ordering, which they refer to as "Time Consistently Undominated Policy."

We apply the Timeless Contract policy to the design of social insurance. In contrast to the Ramsey insurance policy where individuals, in the absence of good income draws, are driven down to their autarkic consumption levels in the long-run (or immediately under the Timeless Perspective policy), the Timeless Contract insurance policy endogenously generates the safety net level of consumption below which no individual falls. This minimum level of consumption is universal and above the bottom income levels. The flip side of this is that the earned privilege dissipates rather quickly, which currently well-to-do individuals accept because their worst-case consumption is sufficiently comfortable to compensate for this. Thus, the Timeless Contract policy breaks immiseration for unlucky individuals who experience persistent declines in their incomes. Note that the source of immiseration here is distinct from that of Thomas and Worrall (1990) which is due to asymmetric information. There is no information asymmetry in our setting and the immiseration in the long-run is a society's choice.

As a consequence of the enhanced consumption risk-sharing under the Timeless Contract policy, consumption inequality is lower and social welfare is higher than what would have been under the Ramsey policy. And as emphasised above, all of these require only a simple modification to the Ramsey policy which in turn is built on a rigorous theoretical foundation (Brendon and Ellison, 2018). Note that this improvement does not require social discounting to be higher than private discounting (Farhi and Werning, 2007).

Unlike the Ramsey insurance policy, the Timeless Contract insurance policy is not characterised by the existence at each income level of the mass of individuals for whom the participation constraints bind. In fact, the policy endogenously determines the income threshold below which individuals consume above their autarkic income levels as discussed above. For this reason, we cannot exploit the binding participation constraints to compute the equilibrium without evaluating a continuum of participation constraints first. We propose a solution method that endogenously and adaptively determines the number of binding participation constraints. The solution algorithm, which is based on the inspection of the Lagrange multipliers for the participation constraints, is intuitive, accurate, and flexible.

On the quantitative side, we use the maximum entropy method of Farmer and Toda (2017) to discretise the estimated non-Gaussian income process based on micro data. The method generalises Tauchen and Hussey's (1991) discretisation method and enables matching the skewness and the kurtosis in income data. The importance of these higher order moments in understanding income dynamics is studied by Guvenen et al. (2021). In Section 5, we apply this method to approximate the estimated income process with a high-dimensional Markov chain and use it to compute the equilibria under both Ramsey and Timeless Contract policy.

The paper is structured as follows: in Section 2, we introduce a simple model environment and discuss how our proposal differs from well-known solution concepts in the literature; we discuss the normative and positive implications of the Ramsey social insurance policy in Section 3 and of the Timeless Contract social insurance policy in Section 4; in Section 5, we generealise the simple model in Section 2 in several directions and carry out quantitative exercises that make use of micro data on consumption and income; Section 6 concludes.

2 Motivation

This section presents a simple model environment to motivate the need for a risk-sharing mechanism among individuals subject to idiosyncratic risks. We focus on a social insurance scheme arranged by a Stackelberg leader such as government which is assumed to commit to it. However, we do not assume that private individuals necessarily hold up their end. This one-sided lack of commitment means perfect risk-sharing is not possible. The model environment, even though stylised and simplistic, is sufficient to illustrate how different social insurance designs work and develops the intuition for the generalised quantitative exercise in Section 5. We also briefly survey popular solution concepts in the related literature which correspond to different social insurance designs in our setting.

2.1 A Simple Model Environment

The model economy is occupied by a continuum of individuals with unit mass. The only source of risk in the economy is individual level income risk. There is no aggregate level risk which affects all individuals at once. For simplicity, there are only two income endowment levels $y^L < y^H$ where L stands for low and H high. For each individual, they evolve according to the first order Markov chain

$$\Pi = \begin{bmatrix} p_{LL} & 1 - p_{LL} \\ 1 - p_{HH} & p_{HH} \end{bmatrix} \tag{1}$$

whose stationary unconditional distribution is

$$\left(\frac{1 - p_{HH}}{2 - p_{LL} - p_{HH}}, \frac{1 - p_{LL}}{2 - p_{LL} - p_{HH}} \right). \tag{2}$$

Here, p_{ii} denotes the conditional probability of the subsequent period's income being y^i if the current period's income is also y^i where $i \in \{L, H\}$. Assuming a law of large numbers applies, (2) also gives the population sizes of low and high income individuals respectively at each point in time.

Because there are only two income levels, the distribution of types (i.e., income histories) is entirely characterised by the transition path from being high income individuals. Formally, the population size of each type, which is indexed by σ, is

$$\left(\frac{1 - p_{LL}}{2 - p_{LL} - p_{HH}} \right) \psi(\sigma) \tag{3}$$

where

$$\psi(0) = 1 \text{ and } \psi(\sigma) = (1 - p_{HH})(p_{LL})^{\sigma - 1} \text{ for } \sigma \geq 1.$$

$\sigma = 0$ corresponds to those whose income is y^H now, $\sigma = 1$ to those whose income was y^H one period ago but is y^L now, $\sigma = 2$ to those whose income was y^H two periods ago but y^L subsequently, and so on.

Each time period indexes the policymaker who is active at the time, which we take to be the generation s/he represents. The social planner's utilitarian welfare function at time period t is

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \sum_{\sigma=0}^{\infty} \left(\frac{1 - p_{LL}}{2 - p_{LL} - p_{HH}} \right) \psi(\sigma) u(c_s(\sigma)) \tag{4}$$

where $0 < \beta \leq 1$ is the discount factor and $u(.)$ is an instantaneous utility function of consumption $c(\sigma)$ which depends on the type σ. The inner summation is the type-population-weighted utility function and the outer summation aggregates this over time.[1] The instan-

[1] The social utility function in (4) can emerge in a dynastic overlapping generations setting with altruistic preferences (Barro, 1974).

taneous utility is given by the Constant Relative Risk Aversion function

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \tag{5}$$

where $\gamma \geq 0$ is the coefficient of relative risk aversion. The social planner also faces the aggregate resource constraint

$$\sum_{\sigma=0}^{\infty} \left(\frac{1-p_{LL}}{2-p_{LL}-p_{HH}} \right) \psi(\sigma) c_s(\sigma) \leq \left(\frac{1-p_{HH}}{2-p_{LL}-p_{HH}} \right) y^L + \left(\frac{1-p_{LL}}{2-p_{LL}-p_{HH}} \right) y^H \tag{6}$$

where the left-hand-side is the aggregate consumption expenditure (weighted by the population sizes of the types given in (3)) and the right-hand-side is the aggregate income endowment (weighted by the population sizes of low and high income individuals given in (2)). A particular social insurance design leads to a consumption allocation $\{c_t(\sigma)\}_{\sigma=0}^{\infty}$ for all t.

2.2 Commitment vs. Limited Commitment

It is assumed that participation in a social insurance scheme is voluntary. Whether individuals can commit to a social insurance scheme or not leads to a stark difference in the degree of consumption risk-sharing in the economy.

Suppose private individuals commit to the social insurance. Then, maximising the social welfare function in (4) subject to the aggregate resource constrain in (6) gives the first order condition

$$u^{'}(c_s(\sigma)) = \eta_s \text{ for all } \sigma \tag{7}$$

where η_s is the Lagrange multiplier on the aggregate resource constraint. This implies $c_s(\sigma) = c_s$ for all σ (i.e., constant across the types). Substituting this into (6), which is a binding constraint due to the monotonicity of the utility function in (5), leads to the optimal consumption at time period t

$$c_t = \left(\frac{1-p_{HH}}{2-p_{LL}-p_{HH}} \right) y^L + \left(\frac{1-p_{LL}}{2-p_{LL}-p_{HH}} \right) y^H. \tag{8}$$

This is equal to the unconditional expected income and constant over time. So, the optimal social insurance delivers perfect consumption risk-sharing across individuals and over time. However, this is rejected by data (Attanasio and Weber, 2010; Gervais and Klein, 2010).

As surveyed by Ljungqvist and Sargent (2018), the extensive literature has developed to account for imperfect consumption risk-sharing in data by dispensing with the assumption of commitment. Without commitment, individuals leave a social insurance scheme and revert to their outside options if the present discounted values of remaining are lower than the present discount values of exiting. These are the participation constraints that the social planner has to take into account in designing a social insurance policy. Suppose reneging on social insurance contributions excludes individuals from participating in the scheme at all future periods. In this case, the autarkic values become relevant outside options. With the simplifying assumption of no private savings, the outside options take the forms

$$V^H = \frac{\beta(1-p_{HH})}{(1-\beta)(1+\beta(1-p_{LL}-p_{HH}))}u(y^L) + \frac{1-\beta p_{LL}}{(1-\beta)(1+\beta(1-p_{LL}-p_{HH}))}u(y^H) \quad (9)$$

$$V^L = \frac{1-\beta p_{HH}}{(1-\beta)(1+\beta(1-p_{LL}-p_{HH}))}u(y^L) + \frac{\beta(1-p_{LL})}{(1-\beta)(1+\beta(1-p_{LL}-p_{HH}))}u(y^H) \quad (10)$$

where V^L and V^H denote the outside options of low and high income individuals respectively. It follows that the participation constraints are of the forms

$$E_t \sum_{s=t} \beta^{s-t} u(c_s(0)) \geq V^H \ (= V^0) \quad (11)$$

$$E_t \sum_{s=t} \beta^{s-t} u(c_s(\sigma)) \geq V^L \ (= V^\sigma \text{ for } \sigma > 0) \quad (12)$$

where E_t is a mathematical expectation conditional on information up to time period t. (11) is the participation constraint for high income individuals whose type is $\sigma = 0$ and (12) is the participation constraint for low income individuals whose types correspond to $\sigma \geq 1$.

Under the standard assumptions[2], the lack of commitment to a social insurance scheme, which works through the participation constraints above, becomes the source of limited risk-sharing among individuals. Using the simple model environment here, we will study the social insurance under the Ramsey policy in Section 3 and under the Timeless Contract policy in Section 4. In Section 5, we will extend the setting here to allow for $N \geq 2$ income endowment levels and the outside options with private savings.

2.3 A Review of Solution Concepts

Before proceeding to the illustration of different social insurance policies using the simple model environment above, we take a short digression and discuss popular solution concepts in

[2]These assumptions guarantee that perfect risk-sharing is not possible without commitment and rule out autarky as an equilibrium. See Broer (2009).

the related literature to (a) distinguish our proposed policy design from those that correspond to these solution concepts on an abstract level and (b) also justify focusing on the Ramsey policy as the benchmark against which our proposal is compared. In what follows, we assume a stationary environment.

1. **Ramsey Policy**: As discussed in the introduction, Ramsey policy is the most popular solution concept in the literature. It optimises the conditional expectation of the objective function of a policymaker at the designated initial period or time-0 subject to optimising behaviour of economic agents and other relevant constraints (e.g., resource constraints). It is assumed that future policymakers will commit to this policy indefinitely. So, there is no meaningful distinction among policymakers at different points in time: the time-0 policymaker's preferences prevail indefinitely. It is well-known that this policy is generally not time-consistent when the economic agents' behaviours introduce forward-looking constraints (Kydland and Prescott, 1977) because a Ramsey policymaker is not bound to keep promises that were made for the current and future periods prior to the commencement of her/his policy regime. This is true whether the policy is being set at time-0 or later if a future policymaker were in a position to reset the policy, even if all policymakers have the same intrinsic preferences. Therefore, policy continuity is generally not attainable across generations of policymakers (i.e., the principle of optimality does not hold) without imposing an additional constraint on the policy-setting.[3] This is what we turn to next.

2. **Timeless Perspective Policy (Woodford, 1999)**: Timeless Perspective policy optimises the conditional expectation of the objective function of a fictitious policymaker in the arbitrarily distant past (i.e., $t \to -\infty$) subject to optimising behaviour of economic agents and other relevant constraints. The policymaker at any point in time is constrained to solve this fictitious policymaker's problem if s/he were in a position to reset the policy. This constrains the policymaker to keep promises that were made prior to the beginning of her/his policy regime as if the fictitious policymaker would have made and kept them. It follows that there is no meaningful distinction among generations of policymakers: the preferences of the fictitious policymaker from the distant past prevail at all times. In this respect, the Timeless Perspective policy is similar to the Ramsey policy with their difference being the time period that the prevailing policymaker belongs to. Woodford (1999) proposed this solution concept to eliminate the time-inconsistency issue from the Ramsey policy.[4] Because the policy framework

[3]Chari and Kehoe (1990) show that it is possible to sustain the Ramsey outcome using a trigger strategy under discretion.

[4]See Woodford (2010) for a survey.

forces all policymakers to be the agents of the fictitious policymaker, time-inconsistency cannot arise. In practice, the Timeless Perspective policy is equivalent to applying the long-run standard Ramsey policy at all times if all policymakers, including the fictitious policymaker, have the same intrinsic preferences.[5] This implies that the Timeless Perspective policy is generally not welfare-maximising from the perspective of time-0.[6]

3. **Unconditionally Optimal Policy (Damjanovic et al., 2008)**: Unconditionally Optimal policy, as the name suggests, optimises the unconditional expectation of the objective function of a policymaker subject to optimising behaviour of economic agents and other relevant constraints. Suppose all policymakers have the same intrinsic preferences. At each time period, the objective function of the policymaker is averaged over all possible initial conditions (i.e., shock histories up until that time) to arrive at the final objective function. This objective function is invariant over time under the assumption that there is an indefinitely long past behind each policymaker which is frequently invoked (and invariant in the long-run even without this assumption). In this case, the policymaker at each point in time ends up having exactly the same objective function. This makes policymakers of all generations effectively one and the same and from this it follows that the unconditionally optimal policy is necessarily time-consistent. In this respect, the Unconditionally Optimal policy is similar to the Timeless Perspective policy. Damjanovic et al. (2015) formalise this connection and show that the two policies are equivalent when the time discount factor is equal to one.

4. **Timeless Contract Policy**: Timeless Contract Policy, which is our proposed policy framework, sits somewhere between the alternatives above. Again, assume that all policymakers have the same intrinsic preferences. Our proposal optimises the conditional expectation of the objective function of a policymaker subject to one additional constraint that the policymaker in each time period, including the policymaker in the initial period, is confined to make the same promise to individuals with the same history. This dynamic symmetry constraint, which ensures that the policymaker at a particular time period is not privileged over others (e.g., the time-0 policymaker in the Ramsey case), guarantees time-consistency by eliminating time-variations in promises which constitute recursive contracts. The trade-off is that it is generally not time-0 optimal. Because the policy is otherwise unconstrained (for instance, there is no restriction on the form of the objective function unlike 1 to 3 above) and the prefer-

[5] Svensson (2010) shows that a Timeless Perspective policy can be implemented as a discretionary policy as long as the policy objective is suitably modified to include time -1 Lagrange multipliers.

[6] Dennis (2010) provides examples where a Timeless Perspective policy is welfare-dominated even by a discretionary policy.

ences of the policymakers at different points in time are reflected in determining the time-invariant contracts, a meaningful distinction among generations of policymakers is preserved in our proposal. As discussed in the introduction, this timelessness in promises or contracts inspires us to name our proposal Timeless Contract policy. The policy is optimal in the class of policies where the promises are constrained to be time-invariant functionals of history. Brendon and Ellison (2018) provide general principles behind this policy design under the name of "Time Consistently Undominated Policy" and apply them to a variety of policy contexts. The connection to this solution concept will be discussed in Section 4. There, it will also be demonstrated that the Timeless Contract Policy requires only a minor amendment to the Ramsey policy, which is another advantage as the latter is well-understood by researchers and practitioners due to its popularity.

Both Timeless Contract policy and Unconditionally Optimal policy exhibit a lot of concern for future time periods by construction. But, this comes quite differently: whereas the former constrains the form of promises to be time-invariant, the latter imposes the form of objective function to be state-invariant. To the extent that it is more straightforward or practicable for a society to consider restricting the space of social contracts as opposed to the preferences of policymakers in designing a policy, our proposal is more attractive.[7]

Finally, as discussed above, both Timeless Perspective policy and Unconditionally Optimal policy are closely connected to the long-run Ramsey policy. For this reason as well as its popularity in the related literature, we adopt this particular solution concept, whether interpreted as the long-run Ramsey policy or the Timeless Perspective policy, as the benchmark against which our policy proposal is compared below.

3 Long-run Ramsey or Timeless Perspective Policy

We first characterise the social insurance that is associated with the long-run Ramsey policy, which is equivalent to the Timeless Perspective policy as discussed in Subsection 2.3. For the purpose of illustration, we consider the simple environment in Subsection 2.1 here. Because the key features of the policy carry over to a more general environment, this section serves as the springboard for the quantitative exercise in Section 5.

[7]An analogy from the monetary policy literature is the society's choice between an appointment of a conservative central bank chair/governor (Rogoff, 1985) and the use of incentive contracts for central bankers (Walsh, 1995; Svensson, 1997) to attain a socially desirable outcome.

3.1 Ramsey Social Insurance Policy

The initial period policymaker maximises the social welfare function in (4) subject to the aggregate resource constraint in (6) and the participation constraints in (11) and (12) due to limited commitment of private individuals. The policymakers in subsequent time periods commit to the optimal policy of the initial period policymaker. Because the participation constraints are forward-looking, standard dynamic programming cannot be used to solve the problem. For this reason, we adopt the dynamic programming squared methodology (Ljungqvist and Sargent, 2018) to set up and solve the problem. To be specific, we combine promised value approach of Abreu, Pearce and Stacchetti (1990) and saddle-point dynamic programming of Marcet and Marimon (2019) and use promised value as a choice variable in the social planner's problem. This is the case for the later sections of the paper as well.

First, we separate the participation constraints in (11) and (12) into the promise-making constraint

$$V^{\sigma} \leq u(c_s(\sigma)) + \beta E_s \omega_{s+1}(\sigma^{'}) \tag{13}$$

and the promise-keeping constraint

$$\omega_s(\sigma) \leq u(c_s(\sigma)) + \beta E_s \omega_{s+1}(\sigma^{'}) \tag{14}$$

where $\sigma^{'} = \sigma + 1$ (i.e., if low income is realised in the next period).[8] The promised value $\omega_s(\sigma)$ summarises relevant income history up to time period s (Abreu, Pearce and Stacchetti, 1990) thus simplifies the problem.

Now, maximising the social welfare function in (4) subject to the aggregate resource constraint in (6) and the reformulated participation constraints in (13) and (14) gives the two first order conditions for currently low income individuals with income history or type $\sigma > 0$:

$$c_s(\sigma): \ (1 + \lambda_s^m(\sigma) + \lambda_s^k(\sigma))u^{'}(c_s(\sigma)) = \eta_s \tag{15}$$

$$\omega_s(\sigma): \ \lambda_s^k(\sigma) = \lambda_{s-1}^m(\sigma - 1) + \lambda_{s-1}^k(\sigma - 1) \tag{16}$$

where η_s is the Lagrange multiplier on the aggregate resource constraint, $\lambda_s^m(\sigma)$ on the promise-making constraint, and $\lambda_s^k(\sigma)$ on the promise-keeping constraint in time period s. The solution for currently high income individuals with $\sigma = 0$ is characterised by $\lambda_s^m(0) > 0$ and $\lambda_s^k(0) = 0$. This means that the solution for these individuals is independent of their income histories. What is the intuition behind this? First, the participation constraint binds for high income individuals because the social insurance is sustained by transferring

[8] If a plan satisfies (13) and (14), it also satisfies (11) and (12) by construction.

resources from high to low income individuals. And V^0 is a function only of current income as shown in (9) and (11). These together imply that the solution is a function only of the current income. Kocherlakota (1996) refers to this condition as the "amnesia property." This property also applies to a more general setting with more than two income levels.

The evolution of the Pareto weights under the Ramsey social insurance policy as given in (15) is brought out by (16). It shows that $\lambda_s^k(\sigma)$ contains a unit root, which implies that the Ramsey policy permanently increases the "absolute" Pareto weights of individuals whenever their participation constraints bind (because $\lambda_s^m(0) > 0$). The "relative" Pareto weights eventually decline, but only due to the increases in the absolute Pareto weights of other individuals. This suggests that the privilege of having high income today is long-lasting under the Ramsey policy.

Solving (15) and (16) with the utility function in (5) gives

$$c_t^{Ramsey}(\sigma) = \left[\frac{(1+\lambda_{t-\sigma}^m(0))}{\eta_t}\right]^{1/\gamma} \tag{17}$$

which describes consumption at time period t for individuals whose income realisation was y^H (high income) σ time periods ago. The equilibrium is computed by exploiting the fact that at each income level there are individuals whose participation constraints bind (more on this in Section 5). Figure 1 plots (17) with $c_t^{Ramsey}(\sigma)$ on the vertical axis and σ on the horizontal axis.[9] As discussed above, the unit root for the Pareto weight in (16) leads to the slow decline of consumption level along the unconstrained downward path. In the limit, consumption converges to the worst autarkic income level y^L as predicted by the theory. In what follows, we will examine these features of the Ramsey insurance policy from both normative and positive perspectives.

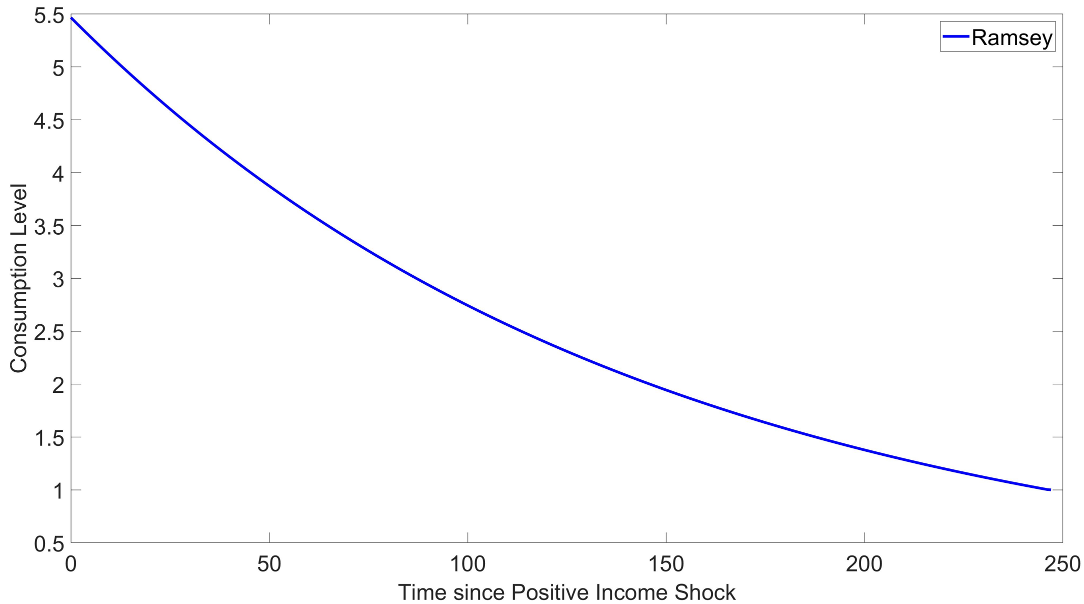
3.2 Is Ramsey Policy Appropriate?

The discussion so far raises two principal objections to Ramsey policy. On the normative side, the objection is whether commitment to a policy that reflects only the preferences of the initial period policymaker can be justified from the viewpoint of intergenerational equity. For instance, this issue features prominently in the exchange between Thomas Jefferson and James Madison on the eve of the creation of the US Bill of Rights. On September 6, 1789, Jefferson wrote to Madison[10]

> The question whether one generation of men has a right to bind another, seems

[9] The parameter values used are $\beta = 0.96$, $\gamma = 1$, $p_{LL} = 0.99$, $p_{HH} = 0.98$, $y^L = 1$, and $y^H = 10$.
[10] https://founders.archives.gov/documents/Madison/01-12-02-0248

Figure 1: Evolution of Consumption under Ramsey: An Illustration with a Two State Income Process



> never to have been started on this or our side of the water. Yet it is a question of such a consequence as not only to merit decision, but place also, among the fundamental principles of every government ... I set out on this ground which I suppose to be self-evident, that the earth belongs in usufruct to the living; that *the dead have neither powers nor rights over it* (our own italics).

This raises concern for the current generation binding future generations to a policy that is not necessarily optimal from their perspectives. More generally, the asymmetry between present and future people is a central issue in intergenerational justice and its resolution has important implications for policy design. On this issue, Meyer (2015) argues that "present generations may be obligated by considerations of justice not to pursue policies that impose an unfair intergenerational distribution of costs and benefits." As the discussion in Subsection 2.3 reveals, Ramsey policy cannot be taken to be acceptable in this regard. We will return to this point in Section 4.

Narrowing down the scope to social insurance, there is an objection to Ramsey policy grounded in its positive implications as well. As shown in Subsection 3.1, while an unlucky individual's consumption is driven down to the worst autarkic income level, a lot of resources are devoted to keep promises made to individuals for whom the participation constraints are no longer binding (because they no longer receive high income realisations). And this waste of resources is very persistent. This casts doubt upon the desirability of the Ramsey social insurance policy as a consumption risk-sharing mechanism. The case against it is further reinforced by taking note of its time-inconsistency (under the standard version) which turns up even in settings without aggregate uncertainty like ours.

The normative and positive objections raised above suggest that Ramsey policy is undesirable at least as a solution concept for designing a social insurance policy. In what follows, we propose an amendment to this that delivers starkly different outcomes.

4 Timeless Contract

This section introduces a simple modification to the Ramsey social insurance policy that brings about improved consumption risk-sharing in the long-run. This policy is feasible in that it satisfies the same participation constraints and resource constraint as the Ramsey insurance policy. Moreover, it is optimal in the class of policies to be defined below. We continue to use the simple model environment in Subsection 2.1 so that we can compare the outcome under our proposed policy to that under the Ramsey policy in Section 3.

4.1 Our Proposal

Is there an alternative risk-sharing scheme that prevents the long-run immiseration of unlucky individuals that is characteristic of the Ramsey social insurance policy? If there is such a policy, is it also time-consistent? The answers to these questions are yes and yes as alluded in Subsection 2.3. Moreover, it can be implemented by simply adding one more constraint

$$\omega_{s+1}(\sigma) = \omega_s(\sigma) \tag{18}$$

for all s to the Ramsey problem in Section 3. This dynamic symmetry constraint requires all individuals of the type σ (i.e., income history) are promised the same utility level at all times (i.e., no transitional dynamics) *ex-ante*. This is not true of either the standard Ramsey policy or the Timeless Perspective policy because it is essentially the long-run version of the standard Ramsey policy (where the transitional dynamics in promises disappear *ex-post*).

The intuitive interpretation of (18) as discussed in the above sections is that the policymaker at a particular time period is not privileged over those belonging to other time periods. And this eliminates the time-inconsistency issue by going directly to the source and performing a surgical intervention of restricting time-variations in promises or utility contracts rather than casting a wide net that may bring about unintended consequences. Moreover, there is a general theoretical foundation behind this procedure which will be discussed in the next subsection. With this rather minimal intergenerational equity requirement in place, the preferences of the policymakers at different points in time are taken into account in determining time-invariant utility contracts, which culminates to our Timeless Contract social insurance policy. As discussed in Subsection 2.3, what we bring to the table is the timelessness in promises or contracts as opposed to the timelessness of policymakers that is the hallmark of the Timeless Perspective policy.

Formally, the social planning problem under our proposal is to maximise the social welfare function in (4) subject to the aggregate resource constraint in (6), the participation constraints in (13) and (14), and the dynamic symmetry constraint in (18). From this, we obtain the first order conditions (15) and

$$\omega_s(\sigma): \ \lambda_s^k(\sigma) = \beta[\lambda_s^m(\sigma-1) + \lambda_s^k(\sigma-1)] \text{ for } \sigma > 0 \tag{19}$$

where the Lagrange multipliers have the same definitions as those in Section 3. As was the case with the Ramsey social insurance policy, the solution for currently high income individuals with $\sigma = 0$ comes with $\lambda_s^m(0) > 0$ and $\lambda_s^k(0) = 0$ because any social insurance is sustained by contributions from the better-off and the amnesia property continues to hold here. However, (19) shows that unlike the Ramsey insurance policy, the Timeless Contract insurance policy is entirely cross-sectional or intratemporal (i.e., the current multipliers do not depend on the past multipliers). Moreover, (19) also indicates that the absolute Pareto weights decay cross-sectionally for $\beta < 1$, which suggests that the relative Pareto weights should decline more quickly compared to the Ramsey insurance policy where the decrease in the relative Pareto weights of currently low income individuals is driven only by the increase of the absolute Pareto weights of currently high income individuals. Finally, the Timeless Contract insurance policy, by construction, is optimal in the class of policies where the utility contracts are constrained to be time-invariant functionals of the types.

Because it is socially optimal to lift as many as possible above their autarkic values without violating the constraints, we naturally conjecture that the solution entails currently low income individuals permanently consuming above their autarkic income level y^L. The corresponding minimum consumption level, which is endogenously generated by this policy, is the social safety net level of consumption below which no individual falls at all times. This distinguishes the Timeless Contract policy from the Ramsey policy where the consumption level of persistently unlucky individuals converges down to the worst autarkic income level y^L over time. Formally, the conjecture is $\lambda_s^m(\sigma) = 0$ for all $\sigma > 0$. Solving (15) and (19) with the utility function in (5) gives

$$c_t^{TC}(\sigma) = \left[\frac{(1 + \beta^\sigma \lambda_t^m(0))}{\eta_t}\right]^{1/\gamma} \tag{20}$$

where the superscript "TC" stands for Timeless Contract. This gives consumption at time period t for individuals whose income realisation was y^H σ time periods ago. The expression in (20) brings out all features of the Timeless Contract insurance policy discussed so far, especially the cross-sectional decaying of the Pareto weights. We now verify this conjecture numerically using the same parameter values as the Ramsey example in Section 3. Note that the resulting equilibrium is immediately at steady state because our model environment is stationary and without aggregate uncertainty.

Figure 2 plots the evolution of consumption under both the Ramsey social insurance policy in (17) and the Timeless Contract social insurance policy in (20) for comparison. The vertical axis gives consumption level and the horizontal axis time since high income realisation (i.e., σ). The figure confirms our intuitions above. First, the privilege of having high income once as embodied in the Pareto weights declines more quickly under the Timeless Contract policy than the Ramsey policy, which implies that the consumption level also comes down more rapidly under the former. Second, the consumption level is higher both initially ($\sigma = 0$) and in the limit ($\sigma \to +\infty$) under the Timeless Contract policy to compensate for the aforementioned faster decline of consumption. These ensure that the participation constraints continue to be satisfied. The consumption level in the limit, which is the endogenously-generated social safety net level of consumption, is more comfortable than that under the Ramsey policy as conjectured above. This shows that the Timeless Contract policy delivers improved downside risk-sharing in the long-run relative to the Ramsey policy. Moreover, it is also credible because it is time-consistent.

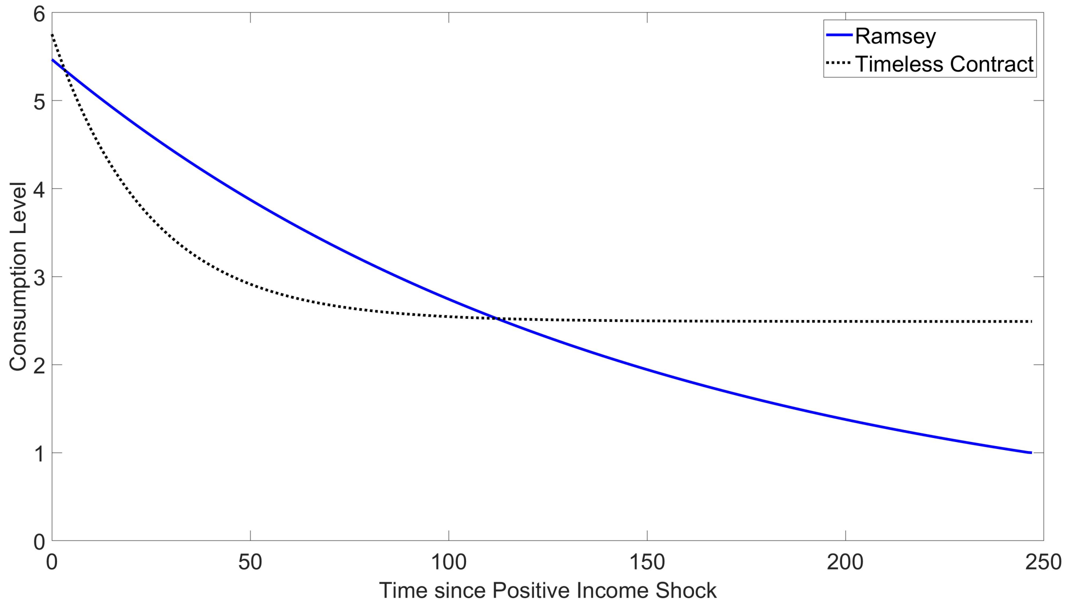
In the quantitative exercise in Section 5 where there are more than two income levels, determining the threshold income level below which the consumption safety net becomes applicable is no longer trivial. We defer the description of the solution algorithm as well as welfare evaluation and measurements of inequality to that section where these become more relevant.

4.2 Discussion

The results so far warrant a further discussion regarding normative and positive aspects of Timeless Contract policy. We briefly touch on these in turn.

On the normative front, the imposition of the dynamic symmetry constraint in (18) is compatible with Harsanyi's utilitarian model of an original position behind a veil of ignorance (Harsanyi, 1953; 1955; 1978). Harsanyi emphasised impartiality and impersonality in making a decision in the original position. Building on von Neumann and Morgenstern's expected utility hypothesis, these principles led him to propose the equiprobability model of moral

Figure 2: Evolution of Consumption under Ramsey and Timeless Contract: An Illustration with a Two State Income Process



value judgments where individuals make a decision as if they face an equal chance of being subject to any of the social positions existing in the situation. This can be taken to be an application of Jaynes' (1968) principle of maximum entropy to the prior determination which formalises the Bayes-Bernoulli-Laplace principle of insufficient reason. In the context of our problem, the question is how a group of policymakers will design a social insurance scheme without knowledge of which time period they will be residing over. Whereas the Ramsey insurance policy does not survive the Harsanyi criteria as this amounts to assigning the entire probability mass of one to a particular time period's policymaker, the Timeless Contract insurance policy does because it treats the policymakers uniformly in all respects (which makes it both impartial and impersonal). And as a consequence of this, there is more risk-sharing in the long-run as illustrated in Figure 2. Binmore (1989) demonstrates that the Harsanyi argument is at its most powerful in a contractual setting like ours even though he originally set out the argument to defend utilitarianism against Rawl's theory of social contract (Rawls, 1971).[11] In further support of this result, we demonstrate that the Timeless Contract insurance policy lives up to Rawl's difference principle by lifting as many less fortunate as possible above their outside options through the endogenously-generated

[11] Roemer (2002) agrees with Harsanyi's decision theory but objects to his model being labelled utilitarian because it does not provide sufficient information to define utilitarianism, for instance cardinality of utility and interpersonal comparability. Because our model satisfies at least difference-comparability of utility, it is utilitarian (see Roberts, 1980).

consumption safety net, for good. In light of this, our proposed policy could be seen as an instance where Harsanyi's and Rawl's visions of an original position behind a veil of ignorance do not necessarily conflict.

As mentioned in several places above, Timeless Contract policy is an instance of Brendon and Ellison's (2018) "Time Consistently Undominated Policy." They advocate replacing time-0 optimality of Ramsey policy with Pareto undominance to overcome the credibility problem associated with it. Under Time Consistently Undominated Policy, (19) holds generally at steady state without the symmetry constraint in (18). However, this generality comes at the cost of potential multiplicity. The imposition of the symmetry constraint, which removes transitional dynamics in policy that are difficult to justify in a stationary setting like ours, delivers a unique time-consistent policy.[12] Moreover, the policy is optimal in the class of policies where the utility contracts are constrained to be time-invariant functionals.

The normative and positive arguments above provide justifications for our policy proposal. Armed with these and the intuitions developed so far, we proceed to consider a more general setting where more elaborate policy comparisons are permitted.

5 A Quantitative Exercise

This section conducts quantitative evaluations of our policy proposal. Doing so requires extending the simple model environment in Section 2 in several dimensions which in turn necessitates the introduction of many additional technicalities. We start with these.

5.1 A Model Environment

The economy is again occupied by a continuum of individuals with unit mass. They are subject to idiosyncratic or individual level income risk but not aggregate level risk. There are $N \geq 2$ income endowment levels denoted by Z^j and arranged in increasing order of the outside option V^j where $j = 1, 2, ..., N$. Z^j follows a first order Markov chain that approximates an income process fitted to actual data (more on this in the next subsection). For $j \geq 2$, σ^j indexes individuals whose income realisations have been below Z^j since it was last realised σ^j time periods ago and made their promise-making constraints bind (i.e., type). $\sigma^j = 0$ for individuals whose current income realisation is Z^j and promise-making constraint binds. The outside option V^j is given by the corresponding autarkic value as before, but this time allowing for private savings (but not borrowing) in the form of riskless

[12]For a general discussion on multiplicity of subgame perfect equilibria in a related setting, see Sargent (1999).

storage technology. This makes the autarky a self-insurance sphere à la Aiyagari (1994). For simplicity, we assume that private savings do not depreciate over time.[13] The autarkic values at different income levels are obtained by solving the associated dynamic programming problem with value function iteration on 20,000 grid points.

The utilitarian social welfare function at time period t takes the form

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \sum_{j=2}^{N} \sum_{\sigma^j=0}^{\infty} \Phi(j, \sigma^j) u(c_s(\sigma^j)) \tag{21}$$

where $\Phi(j, \sigma^j)$ denotes the population size of individuals who were last constrained at the income level Z^j σ^j time periods ago and $u(.)$ is the instantaneous utility function in (5) according to which consumption $c_s(\sigma^j)$ is evaluated. σ^j is well-defined for all j as in the previous sections. The set of constraints includes the aggregate resource constraint

$$\sum_{j=1}^{N} \Psi(j) Z^j \geq \sum_{j=2}^{N} \sum_{\sigma^j=0}^{\infty} \Phi(j, \sigma^j) c_s(\sigma^j) \tag{22}$$

where $\Psi(j)$ gives the population size of individuals whose current income level is Z^j, the promise-making constraint

$$V^{\sigma^j} \leq u(c_s(\sigma^j)) + \beta E_s \omega_{s+1}((\sigma^j)'), \tag{23}$$

the promise-keeping constraint

$$\omega_s(\sigma^j) \leq u(c_s(\sigma^j)) + \beta E_s \omega_{s+1}((\sigma^j)'), \tag{24}$$

and the dynamic symmetry constraint

$$\omega_{s+1}(\sigma^j) = \omega_s(\sigma^j). \tag{25}$$

The Ramsey social insurance policy is derived by maximising the welfare function in (21) subject to (22), (23), and (24). The Timeless Contract social insurance policy is obtained by including (25) as an additional constraint to the Ramsey problem. These give the first order conditions that parallel (15) and (16) for the Ramsey problem and (15) and (19) for the Timeless Contract problem, which are now indexed by σ^j. The Lagrange multipliers $\lambda_s^m(j, \sigma^j)$, $\lambda_s^k(j, \sigma^j)$, and η_s have the same properties as those in Sections 3 and 4, for instance

[13] This makes commitment as limited as possible in this setting. The results are robust to using constant positive depreciation rates instead.

$\lambda_s^m(j,0) > 0$ and $\lambda_s^k(j,0) = 0$ (i.e., the amnesia property). For the Timeless Contract policy, there is an additional procedure of determining the income threshold below which the universal consumption safety net becomes applicable. This will be explained below.

5.2 Estimation and Discretisation of an Income Process

We consider a stochastic income process

$$\log(y_{ijt}) = \alpha_{jt} + x_{it} \tag{26}$$

where y_{it} is real income of individual i belonging to group j at time period t, α_{jt} is a component of income specific to group j at time period t (between-group component) determined by observed characteristics such as age, education, gender, occupation, and race, and x_{it} is an idiosyncratic income component of individual i at time period t (within-group component). As the models above indicate, we focus on the idiosyncratic part of income which captures the risk that income (therefore consumption) varies through no fault of individuals.[14] To isolate the idiosyncratic component, we regress real income on the aforementioned observed characteristics and obtain the residual (Katz and Autor, 1999; Krueger and Perri, 2006; Gervais and Klein, 2010; Broer, 2013). It is important to note that our theoretical point holds generally and does not depend on what income process is used or how income is decomposed.

For comparability to the recent literature, we follow the procedure outlined in Broer (2013) and use the US Consumer Expenditure Survey (CEX) data to obtain the idiosyncratic income series.[15] The sample period is from 1999 to 2003 as recommended by Broer (2013). This is a compromise between a sufficient number of observations and small changes to the income process.

Figure 3 gives the empirical distribution of idiosyncratic real income both in level (in the left panel) and in logarithm (in the right panel). It displays non-normality in the form of non-zero skewness and excess kurtosis whether in level or in logarithm which the recent literature considers to be important for understanding income dynamics (see Guvenen et al., 2021). In light of this, we model the idiosyncratic component x_{it} as the sum of the persistent component m_{it} and the transitory component ε_{it}

$$x_{it} = m_{it} + \varepsilon_{it} \tag{27}$$

[14] It is unlikely that individuals can insure against the changes in the group specific component of income. See Krueger and Perri (2006).

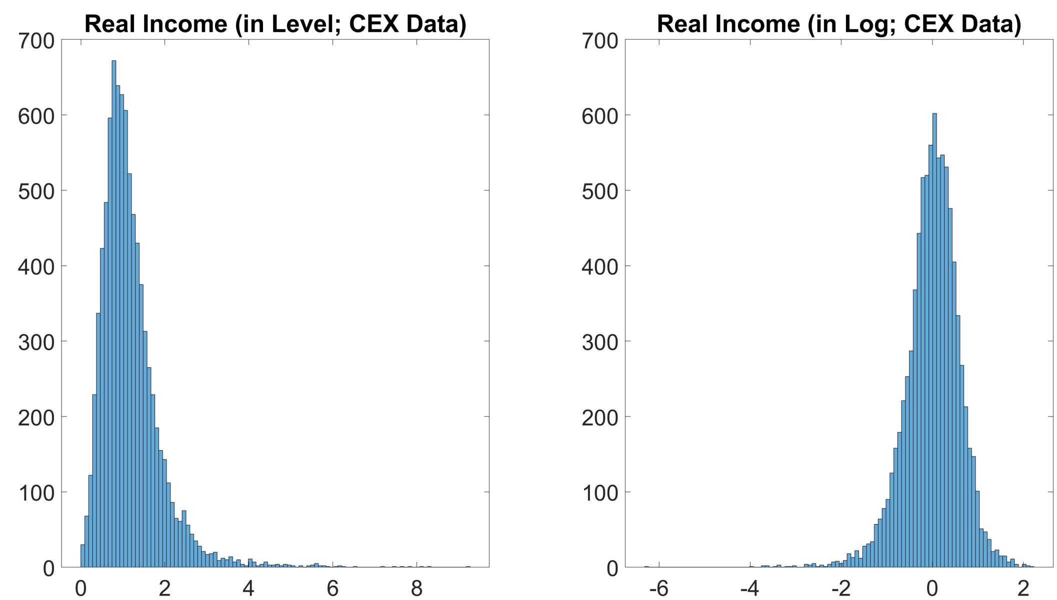
[15] We use the LEA+ series as the raw data which is the sum of after-tax labour income and transfers. See Broer (2013) for the detail.

where the persistent component is given by the autoregressive process

$$m_{it} = \rho m_{it-1} + v_{it} \tag{28}$$

featuring the iid shock term v_{it} that follows a normal mixture distribution $F(v) = p_1 N(\mu_1, \sigma_1^2) + p_2 N(\mu_2, \sigma_2^2)$ with $p_1, p_2 \geq 0$ and $p_1 + p_2 = 1$. The use of the mixture distribution allows us to capture the non-zero skewness and the excess kurtosis observed in the data. Given that x_{it} is a mean zero series by construction (i.e., it is a regression residual), we constrain $F(v)$ to be zero-mean. The iid transitory component ε_{it} follows a usual zero-mean normal distribution $N(0, \sigma_\varepsilon^2)$. Following Civale, Díez-Catalán and Fazilet (2017), we impose $\sigma_\varepsilon^2 = \alpha Var(v_{it})$ for the purpose of parametric parsimony.

Figure 3: Empirical Distributions of Idiosyncratic Real Income



We estimate and discretise the income process in (27) and (28) to arrive at the Markov chain for the income endowment Z^j mentioned in the previous subsection. For estimation, we use the Generalised Method of Moments (GMM; Hansen, 1982) where the variance, the skewness, and the kurtosis of log income and log income change (which have closed-form expressions) serve as the target moments. Given that we do not prioritise matching certain moments over others, we use equal weights here.[16] Because we estimate six parameters $\theta = (\mu_1, \sigma_1, \sigma_2, p_1, \rho, \alpha)$ using six target moments, the system is exactly identified.[17] The

[16] The estimation results are robust to using non-uniform weights.

[17] The remaining parameters are recovered using the parametric restrictions $p_2 = 1 - p_1$ and $\mu_2 = -p_1\mu_1/p_2$.

parameter estimates are provided in Table 1.

Table 1: Parameter Estimates

Parameter	Value
μ_1	0.0038
σ_1	0.0207
σ_2	2.7982
p_1	0.9926
ρ	0.9274
α	1.8207

The next step is to discretise the estimated persistent component of the idiosyncratic income in (28). Even though Tauchen and Hussey's (1991) method, which is the standard choice in the literature, is useful in many contexts, it does not allow directly targeting the moments beyond the first two that we deem to be necessary for the reason stated above. To this end, we adopt the maximum entropy method of Farmer and Toda (2017) which enables us to target the skewnesss and the kurtosis additionally. The resulting discrete Markov chain for the estimated persistent component is combined with a binary discretisation for the estimated transitory component to parallel the expression in (27). This step follows the procedure in Broer (2013) where one standard positive and negative transitory shocks occur with equal probabilities. At the end of these steps, we arrive at the discrete approximation of the estimated income process in (27) and (28). Finally, we obtain Z^j in the previous section by taking the antilog of the discretised idiosyncratic income values.

We use a 30-state discretisation for the estimated persistent component, which together with a binary discretisation for the estimated transitory component leads to a 60-state discrete approximation for the idiosyncratic income. This Markov chain gives a good trade-off between accuracy and speed. Table 2 presents the first four moments of the idiosyncratic income based on the CEX data, the GMM-estimated income process, and the discrete approximation. It shows that the estimated income process and its discrete approximation broadly capture the negative skewness and the excess kurtosis in the data even though the income process in (27) and (28) does not represent a major departure from that frequently used in the literature.[18]

Finally, Figure 4 illustrates the heat map of the transition probability matrix for the discretised income process. The horizontal axis gives the present income and the vertical axis the subsequent income. For the ease of comparison with the figures to follow, income

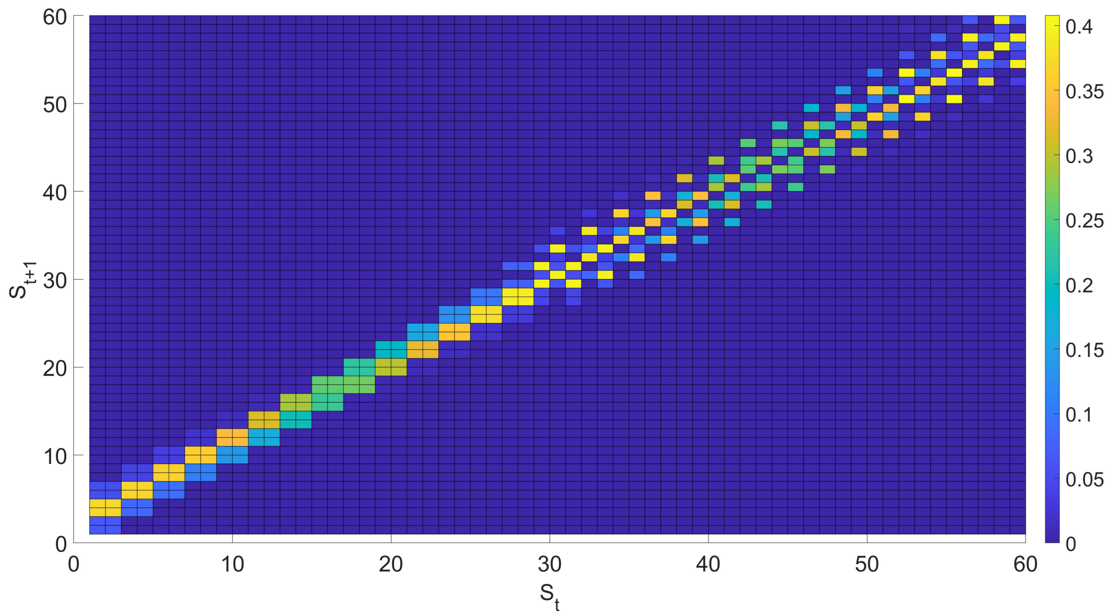
[18]It is possible to improve the fit further by adding more elements to the normal mixture random variable v_{it} in (28). However, this comes at the cost of having to include in estimation moments beyond the first four. We refrain from doing this to preserve the interpretability of the target moments.

Table 2: First Four Moments of Idiosyncratic Income (in Logarithm)

	CEX Data	Estimated	Discretised
Mean	0	0	0
Variance	0.362	0.388	0.388
Skewness	-0.806	-0.661	-0.658
Kurtosis	6.833	8.172	8.002

is arranged in descending order of the outside option (to be computed below) here.[19] As the sidebar of the figure designates, the brighter the color the higher the probability. It indicates that an income transition happens at most a few steps at a time, with the upper half of the distribution being more compressed. This should translate into the persistence of consumption in autarky which in turn poses a challenge in inducing the currently better-off individuals to participate in a social insurance scheme.

Figure 4: 60-State Transition Probability Matrix (in Descending Order of Outside Option)



[19] Due to the existence of the transitory component of the idiosyncratic income, the value of the outside option is not monotonic increasing in income level (see Broer, 2009). This is why ordering is done in terms of the outside option (or equivalently the consumption level associated with the binding participation constraint).

5.3 Solution Methods

Computing the equilibrium under the Timeless Contract social insurance policy comes with an extra challenge of determining the income threshold below which the universal consumption safety net applies. This issue does not arise in computing the equilibrium under the Ramsey social insurance policy because it is characterised by the existence at each income level of the mass of individuals whose participation constraints bind. Exploiting this feature, the equilibrium is computed by determining for each income level the consumption cutoff and the transition path therefrom (a) whose resulting utility is equal to the outside option at the same income level (due to the binding participation constraint) and (b) whose resulting joint consumption-income distribution satisfies the constraints (22) to (24). This can be implemented using a standard method, for instance the bisection method (see Broer, 2009).

To deal with the aforementioned extra complication for the Timeless Contract insurance policy, we modify the bisection method to be able to check whether the participation constraint at a particular income level binds. Moving up from the income level associated with the lowest outside option, we solve for the Lagrange multipliers for the participation constraints $\lambda_t^m(j, \sigma^j)$ for a given value of the Lagrange multiplier for the aggregate resource constraint η_t, and adjust the latter depending on the sign of the aggregate excess demand (increase if positive and decrease otherwise). We iterate on these steps, dropping the participation constraints associated with zero λ_t^ms (hence not binding) along the way. The bisection on η_t makes use of a lower limit that decreases more slowly when the participation constraint being evaluated is potentially not binding to avoid dropping it erroneously (i.e., adaptive bisection). Even though this procedure is equivalent to determining the consumption cutoffs through (20), working with the Lagrange multipliers is more convenient for the problem at hand as it enables direct evaluations of the constraints. The solution consists of the joint consumption-income distribution and the income threshold for implementing the consumption safety net that satisfy the constraints (22) to (25). The proposed solution algorithm is intuitive (as it is based on the Lagrange multipliers), accurate, and flexible (for instance, it can be used for computing the equilibrium under the Ramsey policy).

5.4 Results

Using the solution method introduced above, we now compute the equilibrium under the Timeless Contract social insurance policy and compare it to the equilibrium under the Ramsey social insurance policy. For this, we continue to use the parameter values that are standard for annual calibration: $\beta = 0.96$ and $\gamma = 1$.[20] The outside options are given by the autarkic values with savings as described in Subsection 5.1 and the income distribution is generated by the 60-state Markov chain in Subsection 5.2.

5.4.1 Evolution of Consumption

We first compare the evolution of consumption under the Timeless Contract social insurance policy and the Ramsey social insurance policy in the long-run. For the 60-state income process estimated on the CEX data, the median income level turns out to be the income threshold for the application of the consumption safety net. The Timeless Contract social planner essentially treats those who have been below this income level for sufficiently long time periods as one identical group. Figure 5 plots the evolution of consumption under the Timeless Contract insurance policy (black dotted) and the Ramsey insurance policy (blue solid) for all income levels above this threshold. Income is in descending order of the outside option.[21] The vertical axis gives consumption level and the horizontal axis time since income realisation that made the participation constraint bind (i.e., σ^j). So, each plot gives the unconstrained downward path of consumption in the absence of a positive income shock. The figure is identical to Figure 2 above except for the number of income levels. It shows that the properties of the Timeless Contract insurance policy documented in Section 4 continue to hold in this quantitative setting. In particular, the Timeless Contract policy improves downside consumption risk-sharing in the long-run relative to the Ramsey policy through the universal consumption safety net[22] that prevents the immiseration towards the worst autarkic consumption level under the latter. The trade-off is the relatively quicker dissipation of the social privilege due to income history under the former as reflected in the speed at which consumption comes down, which the better-off accept given the compensation through the improved consumption risk-sharing.

5.4.2 Inequality and Welfare

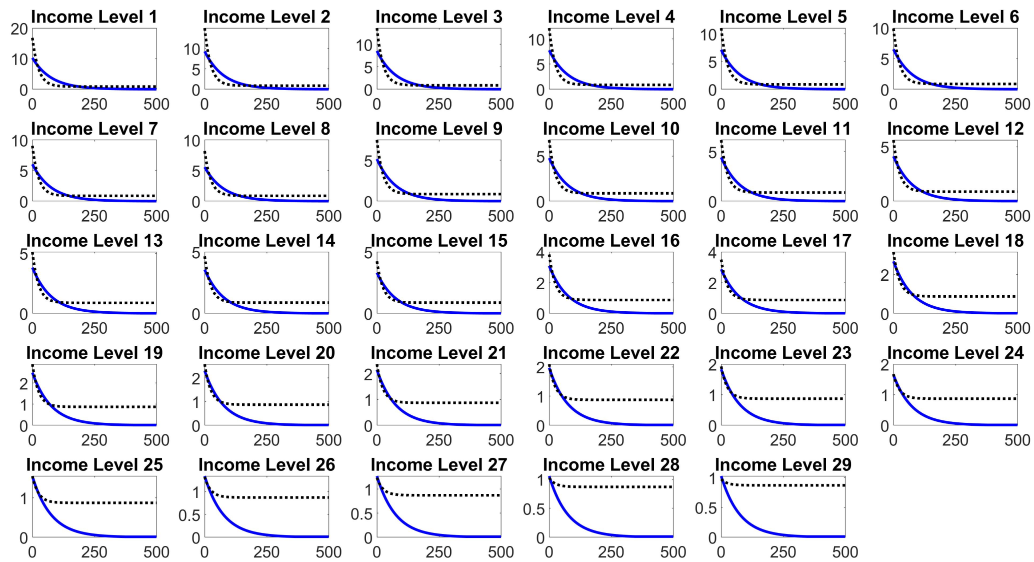
We now turn to the distributional and welfare implications of the Timeless Contract social insurance policy. We simulate both the Timeless Contract economy and the Ramsey economy for 200,000 time periods, discarding the first 1,000 periods as the burn-in sample. Figure 6 plots the distribution of consumption under the Ramsey social insurance policy (in the

[20] In Sections 3 and 4 above, we use the same parameter values to demonstrate how these policies work in a simple model environment there.

[21] For the Ramsey social insurance policy, the consumption transition paths exist up to the 59th income level. They are not plotted here in the interest of space.

[22] As the figure illustrates, this applies equally no matter what income level individuals descend down from.

Figure 5: Ramsey (Blue Solid) and Timeless Contract (Black Dotted) (Income in Descending Order of Outside Option)



upper panel) and the Timeless Contract social insurance policy (in the lower panel). The red vertical dotted line gives the worst consumption level under the Ramsey policy, and the green vertical dash-dotted line the safety net level of consumption under the Timeless Contract policy. The histograms indicate a substantial improvement on the left side of the distribution under the Timeless Contract policy relative to that under the Ramsey policy as the consumption safety net is above the lowest quintile under the latter. In the consumption-equivalent term, the Timeless Contract policy delivers the welfare improvement of 1.39% per person per annum in the long-run.

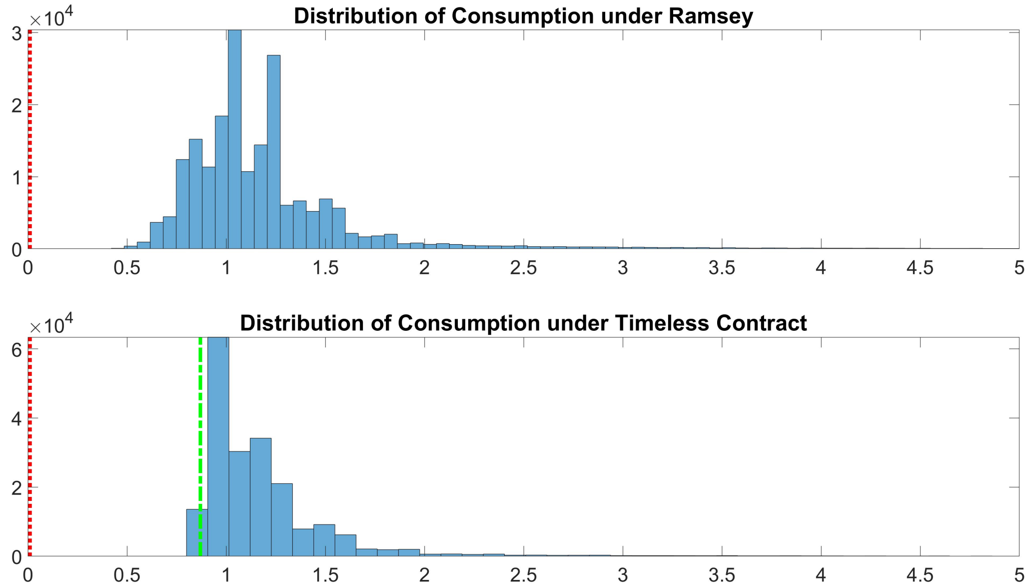
Table 3 computes standard inequality measures[23] based on the CEX data[24] as well as the simulated data above. As well-known in the literature, the consumption distribution in the actual data is closely approximated by that under the Ramsey social insurance policy (see Krueger and Perri, 2006; Broer, 2013). To the extent that we take the Ramsey policy to be a description of reality, switching to the Timeless Contract policy reduces consumption inequality in the long-run, for instance improving the Gini coefficient by 0.036.

The quantitative exercise in this section is useful in many respects, especially for illustrating the mechanics of the Timeless Contract social insurance policy and establishing the

[23] See Cowell (2000).

[24] These are idiosyncratic components of consumption that are obtained following the procedure outlined in Subsection 5.2. We use the ND+ series which includes service flows from durables as the raw data. See Broer (2013) for the detail.

Figure 6: Simulated Distributions of Consumption



connection to the literature. However, it is worth reemphasising that our theoretical point is not dependent on a specific data set or income process and holds generally.

Table 3: Consumption Inequality

	CEX Data	Ramsey	Timeless Contract
Gini	0.209	0.197	0.161
Theil L	0.071	0.071	0.056
Theil T	0.073	0.090	0.081
Atkinson 1	0.069	0.068	0.055
Atkinson 2	0.132	0.112	0.083
90-10 Ratio	2.552	1.959	1.664
50-10 Ratio	1.593	1.345	1.180

The numbers next to the Atkinson indices are the values of the inequality aversion parameter.

6 Conclusion

If the social planner were to commit to a social insurance policy today, how would s/he make the choice? The answer depends not only on the assumptions on the preferences and the technologies of a society but also on the decision protocol. To this end, we apply a decision protocol that is mindful of intergenerational equity and continuity in devising a consumption risk-sharing mechanism. We show that the resulting policy prevents immiseration in the long-run through an endogenously-generated consumption safety net, and this reduces consumption inequality and improves the social welfare relative to the standard policy. Moreover, this does not require a major departure from the standard policy in formulation and implementation. The proposed policy framework is broadly applicable, having potential applications in other fields of economics such as fiscal, international, and monetary economics where optimal policy questions are frequently invoked.

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