P4 Stress and Strain Dr. A.B. Zavatsky
MT07

Lecture 1

Tension and Compression

Normal stress and strain of a prismatic bar
Mechanical properties of materials
Elasticity and plasticity
Hooke’s law
Strain energy and strain energy density
Poisson’s ratio

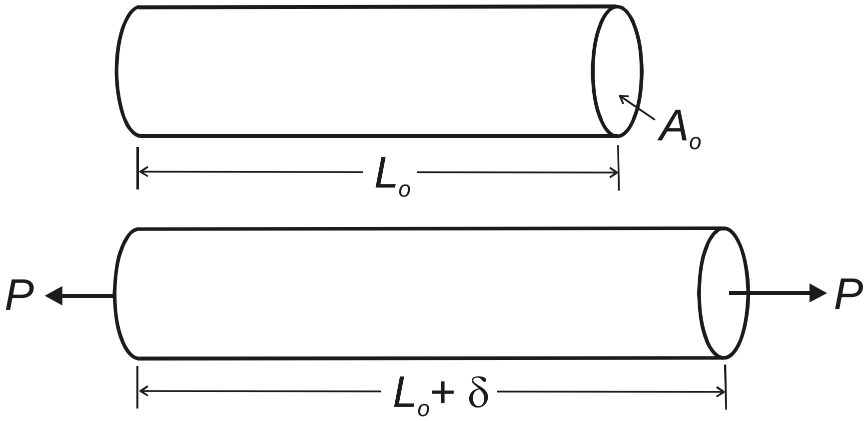
Normal <u>stress</u>

Prismatic bar: straight structural member having the same (arbitrary) cross-sectional area *A* throughout its length

Axial force: load *P* directed along the axis of the member

Free-body diagram disregarding weight of bar

Examples: members of bridge truss, spokes of bicycle wheels, columns in buildings, etc.



Define **normal stress** σ as the force P divided by the original area A_o perpendicular or normal to the force ($\sigma = P / A_o$).

Greek letters δ (delta) and σ (sigma)

When bar is stretched, stresses are **tensile** (taken to be positive)
If forces are reversed, stresses are **compressive** (negative)

Example: Prismatic bar has a circular cross-section with diameter d = 50 mm and an axial tensile load P = 10 kN. Find the normal stress.

$$\sigma = \frac{P}{A_o} = \frac{P}{(\pi d^2 / 4)} = \frac{4(10 \times 10^3)}{\pi (50 \times 10^{-3})^2} \frac{\text{N}}{\text{m}^2} = 5.0929 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

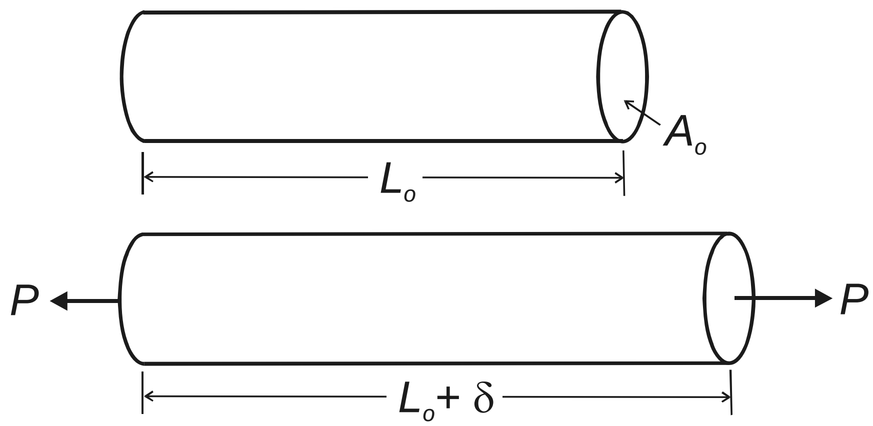
Units are force per unit area = N / m^2 = **Pa** (pascal). One Pa is very small, so we usually work in **MPa** (mega-pascal, Pa x 10^6).

$$\sigma = 5.093\,\text{MPa}$$

Note that N / mm^2 = MPa.

$$\frac{\text{N}}{\text{mm}^2} = \frac{\text{N}}{\text{mm}^2}\left(\frac{10^3\,\text{mm}}{\text{m}}\right)^2 = \frac{\text{N}}{\text{m}^2} \times 10^6 = \text{Pa} \times 10^6 = \text{MPa}$$

Normal strain



Define **normal strain** ε as the change in length δ divided by the original length L_o ($\varepsilon = \delta / L_o$).

Greek letter ε (epsilon)

When bar is elongated, strains are **tensile** (positive).
When bar shortens, strains are **compressive** (negative).

Example: Prismatic bar has length L_o = 2.0 m. A tensile load is applied which causes the bar to extend by δ = 1.4 mm. Find the normal strain.

$$\varepsilon = \frac{\delta}{L_o} = \frac{1.4 \times 10^{-3}}{2.0} \frac{\text{m}}{\text{m}} = 0.0007$$

Units: none, although sometimes quoted as $\mu\varepsilon$ (microstrain, $\varepsilon \times 10^{-6}$) or % strain

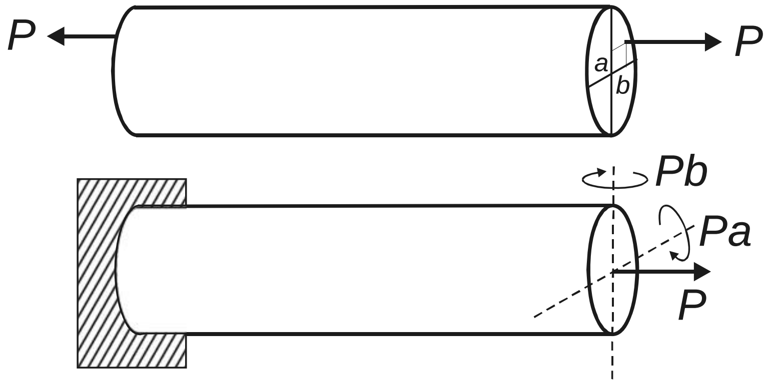
$$\varepsilon = 0.0007 = 7 \times 10^{-4} = \left(7 \times 10^{2}\right)\left(10^{-4} \times 10^{-2}\right) = 700 \times 10^{-6}$$

$$\varepsilon = 700\ \mu\varepsilon$$

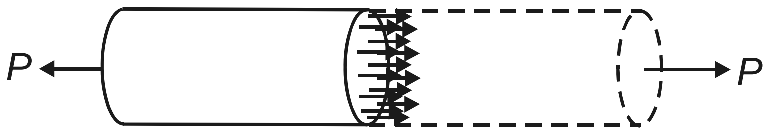
$$\varepsilon = 0.0007 = 0.07\%\ \text{strain}$$

Limitations of the theory for prismatic bars

Axial force P must act through the centroid of the cross-section. Otherwise, the bar will bend and a more complicated analysis is needed.



The stress must be uniformly distributed over the cross-section.



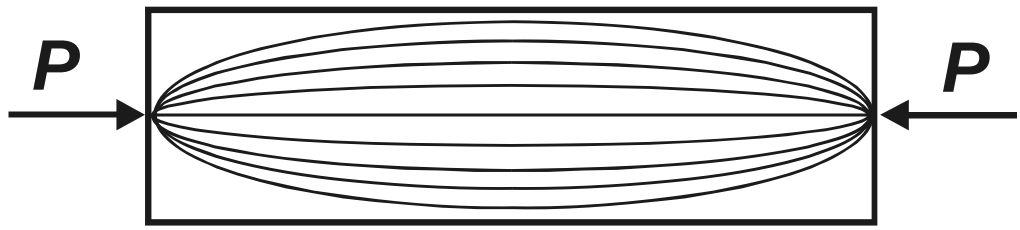
Material should be homogeneous (same throughout all parts of the bar).

Deformation is uniform. That is, we assume that we can choose any part of the bar to calculate the strain.

$$\varepsilon = \frac{\delta}{L_o} = \frac{(\delta/2)}{(L_o/2)} = \frac{(\delta/3)}{(L_o/3)} \text{ etc.}$$

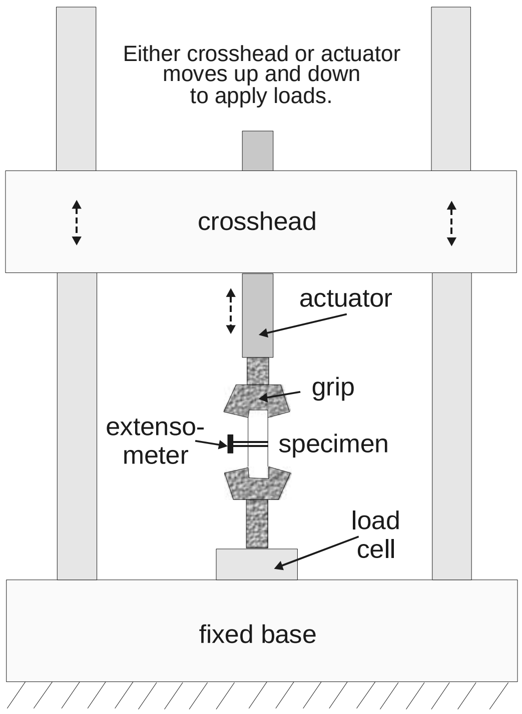
Stress Concentrations

If the stress is not uniform where the load is applied (say a point load or a force applied through a pin or bolt), then there will be a complicated stress distribution at the ends of the bar (known as a “stress concentration”).



If we move away from the ends of the bar, the stresses become more uniform and $\sigma = P/A$ can be used (usually try to be **at least** as far away as the largest lateral dimension of the bar, say one diameter).

Mechanical properties of materials: tensile tests



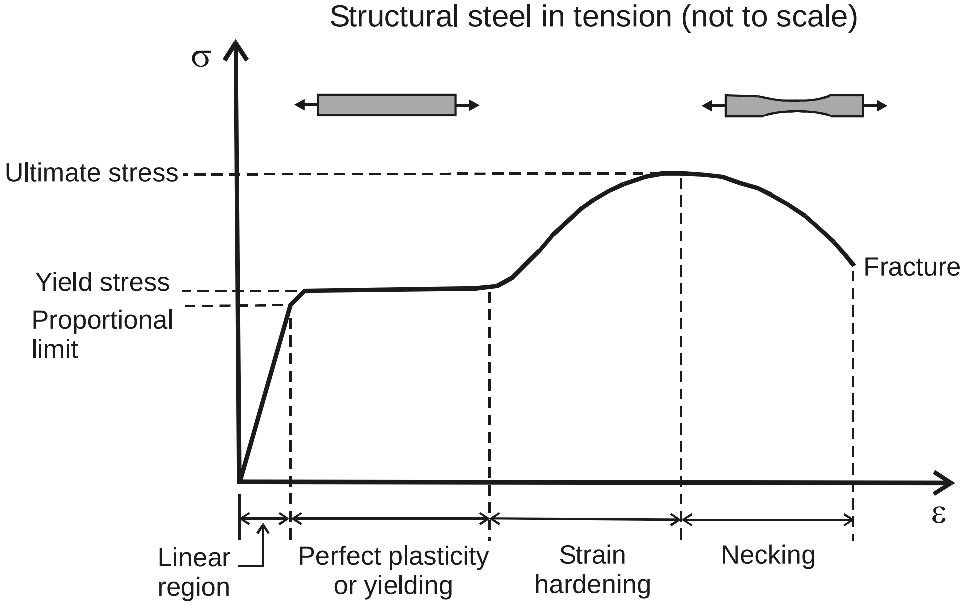
Apply loads under computer control

Log voltage readings from load cell and extensometer (or crosshead/actuator) to computer

Standardization of specimen size and shape and of test procedure (ASTM, BSI, ISO)

Output plots of force versus extension, but slope of curve, maximum values, etc depend on specimen size

Stress-strain diagram for tension



Structural steel (also called mild steel or low-carbon steel; an iron alloy containing about 0.2% carbon). Static (slow) loading.

Linear elasticity & Hooke's law

When a material behaves elastically and also exhibits a linear relationship between stress and strain, it is said to be "linearly elastic".

Hooke's Law (one dimension) $\boldsymbol{\sigma = E\,\varepsilon}$

where ***E* = modulus of elasticity**, units Pa

E is the slope of the stress-strain curve in the linear region.

For a prismatic bar made of linearly elastic material,

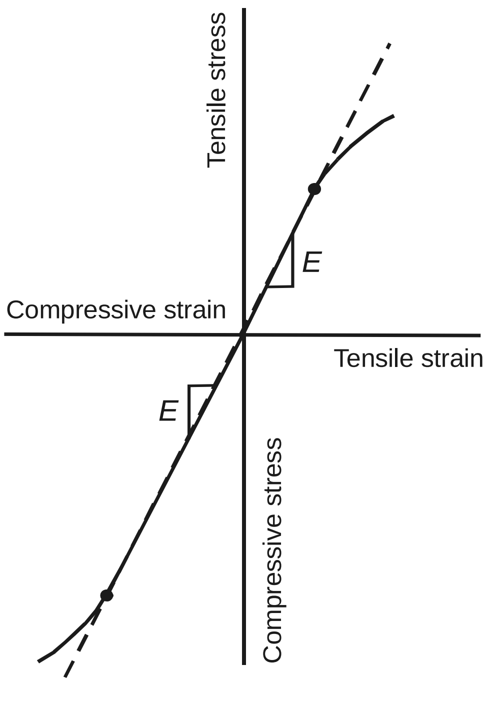
$$\sigma = E\varepsilon \qquad \left(\frac{P}{A_o}\right) = E\left(\frac{\delta}{L_o}\right) \qquad \delta = \frac{PL_o}{EA_o}$$

Tables of mechanical properties (Howatson, Lund, Todd – HLT)

E Young modulus (GPa)
G Shear modulus (GPa)
K Bulk modulus (GPa)
ν Poisson ratio
σ_y Proof or yield stress (MPa)
σ_f Ultimate (failure) stress or tensile strength (MPa)
ϵ_f Tensile strain to failure (%)
K_{Ic} Fracture toughness (MPa m$^{1/2}$)

	E	G	K	ν	σ_y	σ_f	ϵ_f
Alloys							
Aluminium 2024							
(age-hardened)	72	28	75	0·33	395	475	10
Brass (70/30) (annealed)	101	37	112	0·35	115	320	67
(rolled)	101	37	112	0·35	390	460	20
Cast iron (grey)	100–145	40–58		0·26	100–260	150–400	
(nodular)	169–172	66		0·28	230–460	370–800	2–17
Constantan (60% Cu)	163	61	157	0·33	200–440	400–570	
Manganin (84% Cu)	124	47				465	
Mumetal (77% Ni)	220					500–900	
Nimonic 80A							
(superalloy)	214			0·35	800	1300	20
Nichrome (80/20)	186				100–400	170–900	
Phosphor-bronze (5% Sn)	100			0·38	110–670	330–750	2–50
Solder (soft) (50% Sn)	40				33	42	60
Steel: mild	210	81	160–170	0·27–0·30	240	400–500	10–20
Steel: high-yield structural	210	81	170	0·30	400	600	20
Steel: ultra high strength					1600	2000	10
Steel: austenitic stainless	190–200	74–86	–	0·25–0·29	255	660	45
Titanium-6Al-4V	115				800–900	900–1000	10–20

Stress-strain diagram for compression

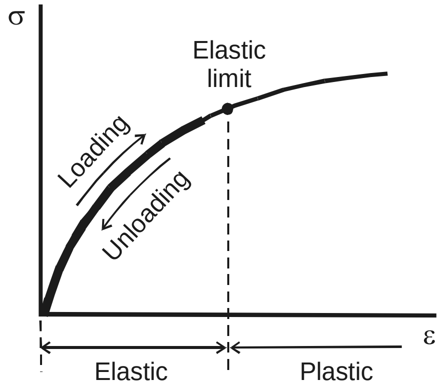


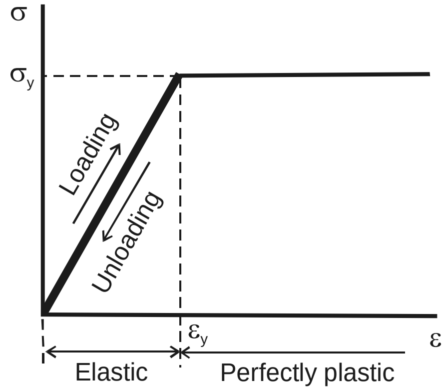
If we load a crystalline material sample in **compression**, the force-displacement curve (and hence the stress-strain curve) is simply the reverse of that for loading in tension **at small strains** (in the elastic region).

The tension and compression curves are different at larger strains (the compression specimen is squashed; the tension specimen enters the plastic region).

Elasticity and Plasticity

Static loading (gradually increases from zero, with no dynamic or inertial effects due to motion) and slow unloading. Within the elastic region, the curves for loading and unloading are the same.



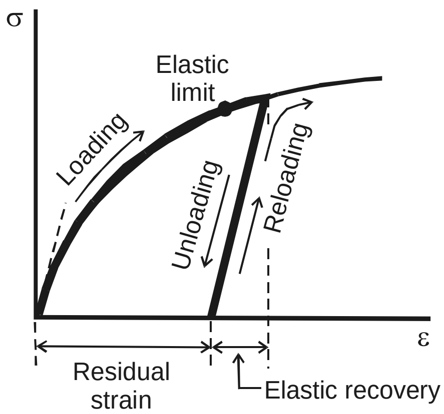


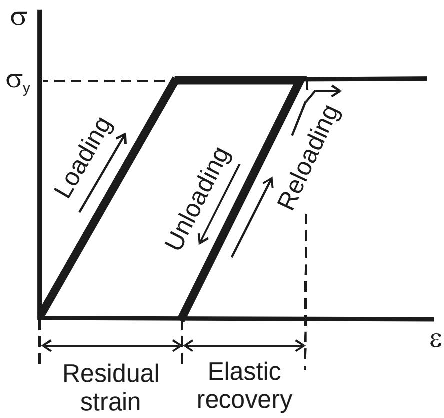
The stress-strain curve need not be linear in the elastic region.

The stress-strain curve for structual steel (and some other metal alloys) can be idealized as having a **linear elastic** region and a **perfectly plastic** region.

Elasticity and Plasticity

Static loading and slow unloading. Past the elastic limit, the curves for loading and unloading are different. The un-loading curve is parallel to the (initial) elastic loading curve.

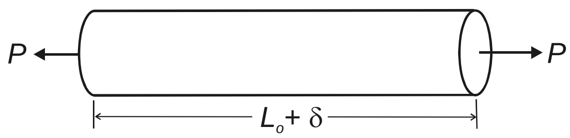


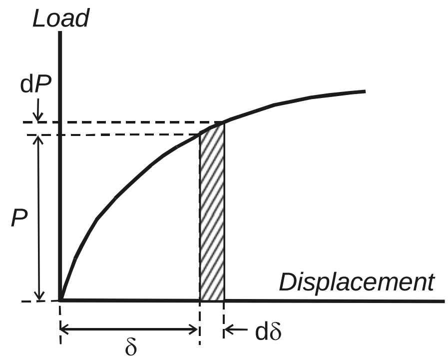


After unloading, there is a certain amount of **elastic recovery** and some **residual strain**, that is, a permanent elongation of the specimen. Upon reloading, the unloading curve is followed.

Strain energy

Prismatic bar subjected to a static load P .
P moves through a distance δ and hence does work.





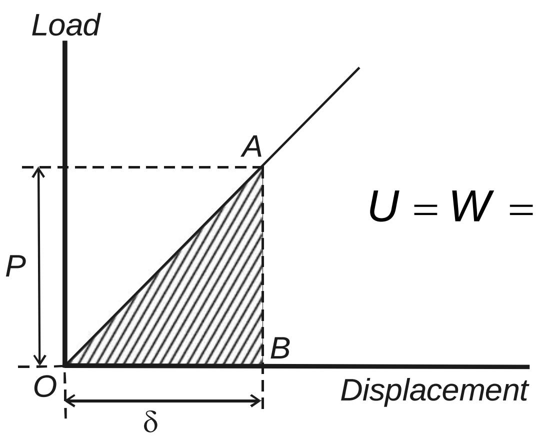
The work done by the load is equal to the area below the load-displacement diagram:

$$dW = (dP)(d\delta)$$

$$W = \int_0^{\Delta} P \, d\delta$$

This work W produces strains, which increase the energy of the bar itself. The **strain energy U** (= W) is defined as the energy absorbed by the bar during the loading process. Units are N m or J (joules).

Linearly elastic behaviour

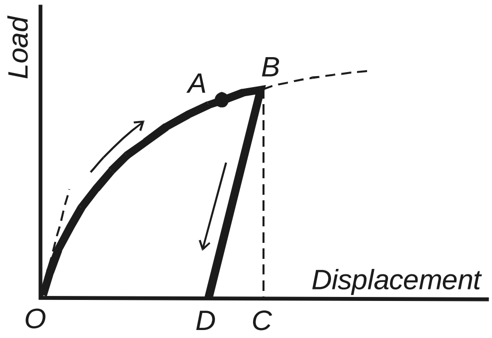


$$U = W = \text{area OAB} = \frac{1}{2} P\delta$$

For a linearly elastic bar, $\delta = \dfrac{PL_o}{EA_o}$, so $U = \dfrac{P^2 L_o}{2EA_o} = \dfrac{EA_o \delta^2}{2L_o}$

For a linearly elastic spring, $P = k\delta$, so $U = \dfrac{k\delta^2}{2} = \dfrac{P^2}{2k}$

Elastic and inelastic strain energy



During loading along curve *OAB*, the work done is the area under the curve (*OABCDO*).

If loading is past the elastic limit *A*, the bar will unload along line *BD*, with permanent elongation *OD* remaining.

The **elastic strain energy** (area *BCD*) is recovered during unloading.

Inelastic strain energy (area *OABDO*) is lost in the process of permanently deforming the bar.

Strain energy density

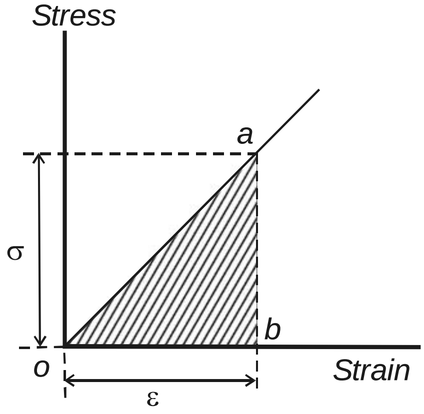
Strain energy density *u* is the strain energy per unit volume of material. The units are J / m^3 = N m / m^3 = N / m^2 = Pa

For a prismatic bar of initial length L_o and initial cross-sectional area A_o:

$$u = \frac{U}{volume} = \frac{\left(P^2 L_o / 2EA_o\right)}{\left(A_o L_o\right)} = \frac{P^2}{2EA_o^{\;2}} = \frac{\left(EA_o\delta^2 / 2L_o\right)}{\left(A_o L_o\right)} = \frac{E\delta^2}{2L_o^{\;2}}$$

Using $\sigma = P / A_o$ and $\varepsilon = \delta / L_o$ gives: $u = \dfrac{\sigma^2}{2E} = \dfrac{E\varepsilon^2}{2}$

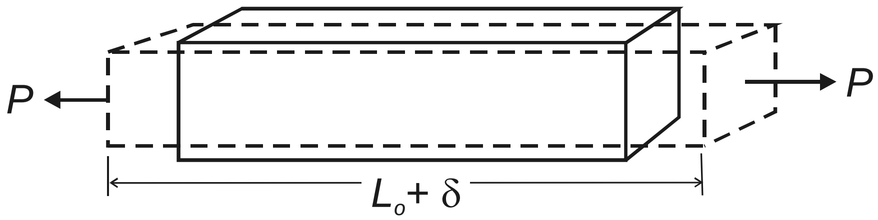
If the material follows Hooke's Law ($\sigma = E\,\varepsilon$), then u is the area under the stress-strain diagram.



$$u = \text{area oab} = \frac{1}{2}\sigma\varepsilon = \frac{\sigma^2}{2E} = \frac{E\varepsilon^2}{2}$$

Poisson's ratio

When a prismatic bar is stretched, it not only gets longer, it gets thinner.



So there is a tensile strain in the axial direction and a compressive strain in the other two (lateral) directions.

Define **Poisson's ratio** as: $\nu = \dfrac{-\text{lateral strain}}{\text{axial strain}} = \dfrac{-\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}}$

Greek letter ν (nu)

If axial strain is tensile (+), lateral strain is compressive (-).
If axial strain is compressive (-), lateral strain is tensile (+).
So Poisson's ratio is a positive number.

For most metals and many other materials, ν ranges from 0.25 – 0.35. The theoretical upper limit is 0.5 (rubber comes close to this).

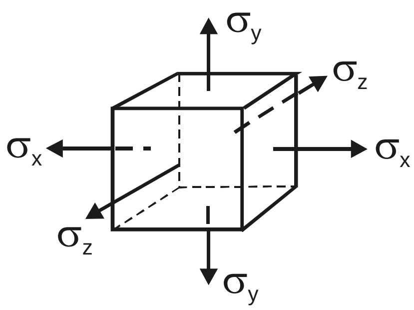
Poisson's ratio holds for the linearly elastic range in both tension and compression. When behaviour is non-linear, Poisson's ratio is not constant.

Limitations

For the lateral strains to be the same throughout the entire bar, the material must be homogeneous (same composition at every point).

The elastic properties must be the same in all directions perpendicular to the longitudinal axis (otherwise we need more than one Poisson's ratio).

Generalized Hooke's Law



Apply σ_x, get ε_x, $\varepsilon_y = -\nu\varepsilon_x$, $\varepsilon_z = -\nu\varepsilon_x$
Apply σ_y, get ε_y, $\varepsilon_x = -\nu\varepsilon_y$, $\varepsilon_z = -\nu\varepsilon_y$
Apply σ_z, get ε_z, $\varepsilon_x = -\nu\varepsilon_z$, $\varepsilon_y = -\nu\,\varepsilon_z$

For an isotropic linearly elastic material, $\varepsilon = \sigma / E$ in the x, y, and z directions.

Use superposition to get the overall strains:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\varepsilon_y - \nu\varepsilon_z$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\left(\frac{\sigma_y}{E}\right) - \nu\left(\frac{\sigma_z}{E}\right)$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right)$$

… and similarly for ε_y and ε_z.