INTRODUCTION TO LOGIC

Lecture 3
Formalisation in
Propositional Logic

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There is no other way to learn the truth
than through logic
Averroes

Outline

English connectives

Recall that connectives join one or more sentences together to make compound sentences.

English connectives

- 'It is not the case that'
- 'and'
- 'or'
- 'if, ... then'
- 'if and only if'

'It is not the case that' and 'Bertrand Russell likes logic' make 'It is not the case that Bertrand Russell likes logic'

These correspond to the connectives of $\mathcal{L}_1$: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

But English also contains other connectives.

More English connectives

- 'It could be the case that'
- 'It must be the case that'
- 'Pope Benedict XVI thought that'
- 'because'
- 'logically entails that'

'Pope Benedict XVI thought that' and 'Bertrand Russell likes logic' make 'Pope Benedict XVI thought that Bertrand Russell likes logic'

Only some English connectives can be captured in $\mathcal{L}_1$.
None of these connectives can be.

Truth functionality

Only truth-functional connectives can be captured in $\mathcal{L}_1$.

Example: a truth-functional connective

The truth-value of 'It is not the case that A' is fully determined by the truth-value of A

A	It is not the case that A
T	F
F	T

Example: a non-truth-functional connective

The truth-value of 'It is possibly the case that A' is **not** fully determined by the truth-value of A

A	It is possibly the case that A
T	T
F	?

Consider the false sentences A_1 and A_2

A_1	V. Halbach is giving this lecture.	F
A_2	Two plus two equals five.	F

It is possibly the case that A_1.	T
It is possibly the case that A_2.	F

Characterisation: truth-functional (p. 54)

A connective is **truth-functional** if and only if the truth-value of the **compound sentence** cannot be changed by replacing a **direct subsentence** with another having the same truth-value.

$$\underbrace{\overbrace{\text{It is not the case that}}^{\text{Connective}}\ \overbrace{\text{it is possibly the case that } \underbrace{2+2=5}_{\text{Subsentence}}}^{\text{Direct subsentence}}}_{\text{Compound Sentence}}$$

$$\underbrace{\overbrace{\text{It rains}}^{\text{Direct subsentence}}\ \overbrace{\text{and}}^{\text{Connective}}\ \overbrace{\text{sometimes } \underbrace{\text{it snows}}_{\text{Subsentence}}}^{\text{Direct subsentence}}}_{\text{Compound Sentence}}$$

NB: replacing non-direct subsentences may change the truth-value.

Formalisation

This is the process of translating English into $\mathcal{L}_1$.

Formalise:

It is not the case that Russell likes logic.

$\neg$ corresponds to 'It is not the case that'.
Let R correspond to 'Russell likes logic'.

Formalisation

$\neg R$

Dictionary

R: Russell likes logic.

Formalise:

Russell likes logic and philosophers like conceptual analysis.

Formalisation
$(\boldsymbol{R} \wedge \boldsymbol{P})$

Dictionary
R: Russell likes logic.
P: Philosophers like conceptual analysis.

Only truth-functional connectives can be formalised in $\mathcal{L}_1$.

Formalise:

It could be the case that Russell likes logic

Formalisation: $\boldsymbol{C}$
Dictionary: C: It could be the case that Russell likes logic.

Formalise:

It is not the case that it could be the case that Russell likes logic.

Formalisation: $\neg \boldsymbol{C}$
Dictionary: C: It could be the case that Russell likes logic.

Note: it's fine to use letters other than P, Q, R when formalising English sentences.

Sometimes we need to paraphrase first.

Formalise:

Russell doesn't like logic

Paraphrase: It is not the case that Russell likes logic.
Formalisation: $\neg \boldsymbol{R}$
Dictionary: R: Russell likes logic.

Formalise:

Neither Russell nor Whitehead likes logic.

Paraphrase: It is not the case that Russell likes logic and it is not the case that Whitehead likes logic.
Formalisation: $\neg \boldsymbol{R} \wedge \neg \boldsymbol{W}$
Dictionary: R: Russell likes logic. W: Whitehead likes logic.

Common variants

Here are some of the most common variants of the standard connectives.

$\mathcal{L}_1$	standard connective	some other formulations
$\wedge$	and	but, although
$\vee$	or	unless
$\neg$	it is not the case that	not, none, never
$\rightarrow$	if ... then	provided that, only if
$\leftrightarrow$	if and only if	precisely if, just in case

Rules of thumb for $\to$

Formalise:

(1) If John revised, [then] he passed. $R \to P$

(2) John passed if he revised. '$P \leftarrow R$' i.e. $R \to P$

(3) John passed only if he revised. $P \to R$

(4) John only passed if he revised. $P \to R$

Dictionary: R: John revised. P: John passed.

(1) **Formalisation**: $\boldsymbol{R \to P}$

(2) Paraphrase: (1). **Formalisation**: $\boldsymbol{R \to P}$

(3) Paraphrase: If John passed, John revised.
Formalisation: $\boldsymbol{P \to R}$

(4) Paraphrase: (3). **Formalisation**: $\boldsymbol{P \to R}$

'If' **mid**-sentence corresponds to '$\leftarrow$'; 'only if' to $\to$.

Differences between $\to$ and 'if'

Formalise

If the lecturer hadn't shown up last week, Plato would have given the lecture.

Consider: $\neg S \to P$.
Dictionary: S: The lecturer showed up last week.
P: Plato gave the lecture.

The English sentence appears to be false.
But when $|S|_{\mathcal{A}} = \mathrm{T}$, $|\neg S \to P|_{\mathcal{A}} = \mathrm{T}$.

See Sainsbury, *Logical Forms*, ch. 2 for further discussion.

Complex Sentences

Formalise

If David folded or David didn't have the ace, Victoria won.

Logical Form

(If ((David folded) or it is not the case that (David had the ace)), (Victoria won))

This is in (propositional) **logical form**.

- All connectives are standard connectives
- No sentence can be further formalised in $\mathcal{L}_1$.

Formalisation: $\boldsymbol{((F \lor \neg A) \to W)}$
Dictionary: F: David folded.
A: David had the ace.
W: Victoria won.

Sometimes the paraphrase may need to be quite loose.

Formalise

(1) Exactly one of the following happened: David won or Victoria won.

(2) Exactly one of the following happened: David won or Victoria won or it was a tie.

(1) Paraphrase: ((David won and Victoria did not win) or (Victoria won and David did not win))
Formalisation: $\boldsymbol{(D \land \neg V) \lor (V \land \neg D)}$
Dictionary: D: David won. V: Victoria won.

(2) **Formalisation**:
$\boldsymbol{(D \land \neg V \land \neg T) \lor (V \land \neg D \land \neg T) \lor (T \land \neg D \land \neg V)}$
Dictionary: T: It was a tie.
(and as before)

Scope ambiguity

Example

David's hand was weak and Victoria was bound to win unless the Jack came up on the turn.

Logical forms

(1) (((David's hand was weak) and (Victoria was bound to win)) **or** (the Jack came up on the turn))

(2) ((David's hand was weak) **and** ((Victoria was bound to win) or (the Jack came up on the turn)))

Formalisation

(1) $(\boldsymbol{D} \wedge \boldsymbol{V}) \vee \boldsymbol{J}$

(2) $\boldsymbol{D} \wedge (\boldsymbol{V} \vee \boldsymbol{J})$

Dictionary

D : David's hand was weak.

V: Victoria was bound to win.

J: The Jack came up on the turn.

This is a case of **scope ambiguity**.

Definition (p. 65)

The **scope** of an occurrence of a connective in a sentence ϕ is the occurrence of the smallest subsentence of ϕ that contains this occurrence of the connective.

A **subsentence** of ϕ is any sentence occurring as part of ϕ (including ϕ itself).

$$(1)\ \overbrace{\underbrace{(D \wedge V)}_{\text{Scope of } \wedge} \vee J}^{\textbf{Scope of } \vee} \qquad (2)\ \underbrace{D \wedge \overbrace{(V \vee J)}^{\textbf{Scope of } \vee}}_{\text{Scope of } \wedge}$$

In (1): $\vee$ has wider scope.

In (2): $\wedge$ has wider scope.

More on paraphrase

(1) Tom and Jerry are animals.

(2) Tom and Jerry are apart.

(3) Jerry is a white mouse.

(4) Jerry is a large mouse.

Worked example: are these acceptable paraphrases?

(1) Tom is an animal and Jerry is an animal.

(2) Tom is apart and Jerry is apart.

(3) Jerry is white and Jerry is a mouse.

(4) Jerry is large and Jerry is a mouse.

Logical notions in English

Recall last week's definitions of tautology, contradiction and validity for $\mathcal{L}_1$ sentences and arguments.
These properties of $\mathcal{L}_1$ can be transposed to English.

Definition

(1) An English sentence is a **tautology** if and only if its formalisation in propositional logic is a tautology.

(2) An English sentence is a **propositional contradiction** if and only if its formalisation in propositional logic is a contradiction.

(3) An argument in English is **propositionally valid** if and only if its formalisation in $\mathcal{L}_1$ is valid.

Worked Example

Show that the following argument is propositionally valid.

Unless CO_2-emissions are cut, there will be more floods. CO_2-emissions won't be cut. Therefore there will be more floods.

- Start by identifying the premisses and conclusion.
- Next, specify a dictionary.

  Dictionary.
  C: CO_2 emissions will be cut.
  M: There will be more floods.
- Next, formalise the premisses and conclusion.
- Finally, check the formalised argument is valid.

Worked example (cont.)

P1 Unless CO_2-emissions are cut, there will be more floods.
P2 CO_2-emissions won't be cut.
C There will be more floods

Formalise the argument:

It remains to show.

$(C \vee M), \neg C \models M$

You know two ways to do this.

- Method 1: Forwards truth table.
- Method 2: Backwards truth table.

Backwards truth-table

C	M	$(C \vee M)$	$\neg C$	M

This shows there cannot be a line in the truth-table in which both premisses are true and the conclusion is false.
So, the English argument is propositionally valid.