INTRODUCTION TO LOGIC

Lecture 7

Formalisation in Predicate Logic

Dr. James Studd

‘Contrariwise,’ continued Tweedledee,
‘if it was so, it might be;
and if it were so, it would be;
but as it isn’t, it ain’t.
That’s logic’
Lewis Carroll
Through the Looking-Glass

Outline

(1) Review of adequacy.

(2) Logical properties of English sentences.

(3) Further issues in predicate formalisation.

Recap: Adequacy

Two notions of consequence coincide.

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Let Γ be a set of $\mathcal{L}_2$-sentences and ϕ a $\mathcal{L}_2$-sentence

Definition: provable (syntactic)

$\Gamma \vdash \phi$ iff there is a proof of ϕ with only sentences in Γ as non-discharged assumptions.

Recap: Adequacy

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Definition: valid (semantic)

$\Gamma \models \phi$ iff there is no $\mathcal{L}_2$-structure in which all sentences in Γ are true and ϕ is false.

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Adequacy theorem (Soundness and Completeness)

$\Gamma \vdash \phi$ iff $\Gamma \models \phi$

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Step (i) Formalise the argument in $\mathcal{L}_2$.

Step (ii) Prove the formalised argument in Natural Deduction.

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Method to establish non-validity

Step (i) Formalise the argument in $\mathcal{L}_2$.

Step (ii) Construct a counterexample. (An $\mathcal{L}_2$-structure in which the premisses are true and the conclusion is false.)

Unfinished business

Show that argument 2 is valid

(1) Zeno is a tortoise.
(2) All tortoises are toothless.
Therefore, (C) Zeno is toothless.

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Step (i): formalise

(1) Ta
(2) $\forall x(Tx \to Lx)$
(C) La

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Need to show: $Ta, \forall x(Tx \to Lx) \models La$.

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Sufficient to show: $Ta, \forall x(Tx \to Lx) \vdash La$.

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That completes the proof
Consequently: $Ta, \forall x(Tx \to Lx) \models La$ (by adequacy)
The English argument about Zeno is valid in predicate logic.

Exercise: show that the following argument is valid

All concrete objects are located in space. The number 5 isn't located in space. So the number 5 isn't a concrete object.

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So the argument is valid in predicate logic.

Note on partial formalisation

To establish validity: partial formalisation may suffice.

	Less detailed (I)
Premiss 1	$\forall x\,(Cx \to Lx)$
Premiss 2	$\neg La$
Conclusion	$\neg Ca$

Note on partial formalisation

To establish validity: partial formalisation may suffice.

	Less detailed (I)	More detailed (II)
Premiss 1	$\forall x\,(Cx \to Lx)$	$\forall x\,(Cx \to L^2xb)$
Premiss 2	$\neg La$	$\neg L^2ab$
Conclusion	$\neg Ca$	$\neg Ca$

Dictionary: b: Space. L^2: … is located in ….

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Both (I) and (II) are fine.

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To establish validity: partial formalisation may suffice.

	Less detailed (I)	More detailed (II)	Not detailed enough (III)
Premiss 1	$\forall x\,(Cx \to Lx)$	$\forall x\,(Cx \to L^2xb)$	A
Premiss 2	$\neg La$	$\neg L^2ab$	S
Conclusion	$\neg Ca$	$\neg Ca$	C

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Both (I) and (II) are fine. (III) is not.

A: All concrete objects are located in space.
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Both (I) and (II) are fine. (III) is not.

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C The number 5 isn't a concrete object.

NB: to show non-validity: full formalisation required.

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Conclusion: Ka.

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Step (ii): construct a counterexample.

$\forall x\,(Bx \to (Kx \to (Tx \wedge Jx))), Ba \wedge Ta \wedge Ja \nvDash Ka$

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$$|a|_{\mathcal{A}} = 1$$

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The premisses are true, and the conclusion is false in $\mathcal{A}$.
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Step (i) formalise the sentence as a sentence ϕ of $\mathcal{L}_2$.
Step (ii) prove that $\vdash \phi$.

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Step (i) formalise the sentence as a sentence ϕ of $\mathcal{L}_2$.
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Example: show the sentence is a contradiction.

Something is bigger than everything but not bigger than itself.

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Step (i): formalise

Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

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Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

- x is bigger than everything: $\forall y Bxy$

Dictionary: B: …is bigger than ….

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Paraphrase: Some x is such that (x is bigger than everything and x is not bigger than itself)

- x is bigger than everything: $\forall y Bxy$
- x is not bigger than itself: $\neg Bxx$

Dictionary: B: …is bigger than ….

Example: show the sentence is a contradiction.

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Further issues: scope ambiguity

Every philosopher knows a metaphysician. 40

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Paraphrases

(1) Every philosopher is such that they know some metaphysician.

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Issue 2: variable arity.

Example: formalise in $\mathcal{L}_2$ as a valid argument.

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Manny is eating the scrambled egg out of his shoe with his comb. So, Manny is eating the scrambled egg and Manny is eating out of his shoe.

The trick is to formalise the argument just using E_1.

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Issue 3: adverbs

Example: formalise in $\mathcal{L}_2$ as a valid argument.

Usain ran quickly; so Usain ran. 45

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Example: formalise in $\mathcal{L}_2$ as a valid argument.

Usain ran quickly; so Usain ran. 45

The following formalisation is clearly not valid.

Premiss: Qb. Conclusion: Rb.

Dictionary: b: Usain. Q: …ran quickly. R: …ran.

Issue 3: adverbs

Example: formalise in $\mathcal{L}_2$ as a valid argument.

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The following formalisation is clearly not valid.

Premiss: Qb. Conclusion: Rb.

Dictionary: b: Usain. Q: …ran quickly. R: …ran.

But there is a (somewhat contrived) way to formalise it.

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Issue 3: adverbs

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The following is valid:

Premiss: $\exists x(R_1 x \land Pbx \land Q_1 x)$. Conclusion: $\exists x(R_1 x \land Pbx)$.

Issue 4: non-extensionality

Example

Miles wants to live in Oxford. Oxford is a city with high levels of air pollution. Therefore Miles wants to live in a city with high levels of air pollution.

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Yet the obvious $\mathcal{L}_2$-formalisation is valid.

Premiss 1: Lmo.
Premiss 2: Po.
Conclusion: $\exists x\,(Lmx \land Px)$.

L:	… wants to live in …
P:	… is a city with high levels of air pollution
m:	Miles
o:	Oxford

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What has gone wrong?

Extensionality of $\mathcal{L}_2$

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$\mathcal{L}_2$-structures assign extensions to expressions.

$\mathcal{L}_2$-expression	*extension*
constant	object
sentence	truth-value
unary predicate	set
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They have the following feature.

Extensionality

In a $\mathcal{L}_2$-structure, the extension of a sentence depends only on the extensions of its constituent expressions.

Let $\mathcal{A}$ be an $\mathcal{L}_2$-structure, ϕ an $\mathcal{L}_2$-sentence.

Extensionality in $\mathcal{L}_2$

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'... knows that Superman wears a cape' *is* extensional.

http://logicmanual.philosophy.ox.ac.uk