INTRODUCTION TO LOGIC

2 Syntax and Semantics of Propositional Logic

Volker Halbach

Logic is the beginning of wisdom.
Thomas Aquinas

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In logic one abstracts from all stylistic variants etc of natural language and retains just the basic skeleton of the language in a regimented form.

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1. I specify the syntax or grammar of the language; in particular I define what the sentences of the language are.
2. I specify the semantics of the language; in particular, I say what it means for a sentence to be true under an interpretation (or in a 'structure'). Once the notion of an interpretation (or structure) is clear, I can define validity of arguments etc as for English.

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Examples of syntactic claims

- ‘Bertrand Russell’ is a proper noun.
- ‘likes logic’ is a verb phrase.
- ‘Bertrand Russell likes logic’ is a sentence.
- Combining a proper noun and a verb phrase in this way yields a sentence.

Semantics is all about *meanings* of expressions.

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Examples of semantic claims

- 'Bertrand Russell' refers to a British philosopher.
- 'Bertrand Russell' refers to Bertrand Russell.
- 'likes logic' expresses a property Russell has.
- 'Bertrand Russell likes logic' is true.

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Use

- The second occurrence of ‘Bertrand Russell’ is an example of use.
- This occurrence (without quotes) refers to a man.

Syntax: English vs. $\mathcal{L}_1$.

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Basic expressions of $\mathcal{L}_1$

1. *Sentence letters*: e.g. 'P', 'Q'.
2. *Connectives*: e.g. '$\neg$', '$\wedge$'. There are also brackets: '(' and ')'.

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Here's the full list of $\mathcal{L}_1$-connectives.

name	in English	symbol
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Greek letters: ϕ ('PHI') and ψ ('PSI'): not part of $\mathcal{L}_1$.

How to build a sentence of $\mathcal{L}_1$

Example

The following is a sentence of $\mathcal{L}_1$:

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ϕ and ψ are part of the metalanguage, not the object one.

Object language

The object language is the one we are theorising *about*.

- The object language is $\mathcal{L}_1$.

Metalanguage

The metalanguage is the one we are theorising *in*.

- The metalanguage is (augmented) English.

ϕ and ψ are used as variables in the metalanguage:
in order to generalise about sentences of the object language.

Bracketing conventions

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- $4 + 5 \times 3$

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Recall the characterisation of validity from week 1.

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An argument is *logically valid* if and only if there is *no* interpretation of subject-specific expressions under which:

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- Logical expressions: $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$.
- Subject-specific expressions: P, Q, R, …
- Interpretation: $\mathcal{L}_1$-*structure*.

$\mathcal{L}_1$-*structures*

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An $\mathcal{L}_1$-*structure* is an assignment of exactly one truth-value (T or F) to every sentence letter of $\mathcal{L}_1$.

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Examples

We can think of an $\mathcal{L}_1$-structure as an infinite list that provides a value T or F for every sentence letter.

	P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	$\cdots$
$\mathcal{A}$:	T	F	F	F	T	F	T	T	F	$\cdots$

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$\mathcal{B}$:	F	F	F	F	F	F	F	F	F	$\cdots$

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$\mathcal{B}$:	F	F	F	F	F	F	F	F	F	$\cdots$

We use $\mathcal{A}$, $\mathcal{B}$, etc. to stand for $\mathcal{L}_1$-structures.

Truth-values of complex sentences 1/3

$\mathcal{L}_1$-structures *only* directly specify truth-values for P, Q, R, …

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The meaning of ¬ is summarised in its *truth table*.

ϕ	$\neg\phi$
T	F
F	T

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Truth-conditions for ¬

The meaning of ¬ is summarised in its *truth table*.

ϕ	$\neg\phi$
T	F
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In words: $|\neg\phi|_{\mathcal{A}} = \mathrm{T}$ if and only if $|\phi|_{\mathcal{A}} = \mathrm{F}$.

Worked example 1

$|\phi|_{\mathcal{A}}$ is the truth-value of ϕ under $\mathcal{A}$.

ϕ	$\neg\phi$
T	F
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Compute the following truth-values.

Let the structure $\mathcal{A}$ be partially specified as follows.

P	Q	R	P_1	Q_1	R_1	P_2	Q_2	R_2	$\cdots$
T	F	F	F	T	F	T	T	F	$\cdots$

Compute:

$|P|_{\mathcal{A}} =$ $\quad$ $|Q|_{\mathcal{A}} =$ $\quad$ $|R_1|_{\mathcal{A}} =$

$|\neg P|_{\mathcal{A}} =$ $\quad$ $|\neg Q|_{\mathcal{A}} =$ $\quad$ $|\neg R_1|_{\mathcal{A}} =$

$|\neg\neg P|_{\mathcal{A}} =$ $\quad$ $|\neg\neg Q|_{\mathcal{A}} =$ $\quad$ $|\neg\neg R_1|_{\mathcal{A}} =$

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T	F	F	F	T	F	T	T	F	$\cdots$

Compute:

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Compute:

$$|P|_{\mathcal{A}} = \text{T} \qquad |Q|_{\mathcal{A}} = \text{F} \qquad |R_1|_{\mathcal{A}} =$$

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Truth-values of complex sentences 2/3

Truth-conditions for ∧ and ∨

The meanings of ∧ and ∨ are given by the truth tables:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-values of complex sentences 2/3

Truth-conditions for $\wedge$ and $\vee$

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ϕ	ψ	$(\phi \wedge \psi)$
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T	F	F
F	T	F
F	F	F

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T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \wedge \psi)|_{\mathcal{A}} = T$ if and only if $|\phi|_{\mathcal{A}} = T$ and $|\psi|_{\mathcal{A}} = T$.

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T	F	F
F	T	F
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T	F	T
F	T	T
F	F	F

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Truth-conditions for ∧ and ∨

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ϕ	ψ	$(\phi \land \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

ϕ	ψ	$(\phi \lor \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

$|(\phi \land \psi)|_{\mathcal{A}} = T$ if and only if $|\phi|_{\mathcal{A}} = T$ and $|\psi|_{\mathcal{A}} = T$.
$|(\phi \lor \psi)|_{\mathcal{A}} = T$ if and only if $|\phi|_{\mathcal{A}} = T$ or $|\psi|_{\mathcal{A}} = T$ (or both).

Truth-values of complex sentences 3/3

Truth-conditions for → and ↔

The meanings of → and ↔ are given by the truth tables:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

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T	F	F
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T	T	T
T	F	F
F	T	T
F	F	T

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T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \to \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.

Truth-values of complex sentences 3/3

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Truth-conditions for → and ↔

The meanings of → and ↔ are given by the truth tables:

ϕ	ψ	$(\phi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$(\phi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$|(\phi \to \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = \text{F}$ or $|\psi|_{\mathcal{A}} = \text{T}$.

$|(\phi \leftrightarrow \psi)|_{\mathcal{A}} = \text{T}$ if and only if $|\phi|_{\mathcal{A}} = |\psi|_{\mathcal{A}}$.

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}}$ = T and $|Q|_{\mathcal{B}}$ = F.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}}$ = F and $|(P \wedge Q)|_{\mathcal{B}}$ = F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \text{T}$
3. $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \text{F}$ and $|(P \land Q)|_{\mathcal{B}} = \text{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \text{T}$
3. $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \wedge Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}} = \mathrm{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Worked example 2

Let $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

What is the truth value of $\neg(P \to Q) \to (P \land Q)$ under $\mathcal{B}$?

1. $|(P \to Q)|_{\mathcal{B}} = \mathrm{F}$ and $|(P \land Q)|_{\mathcal{B}} = \mathrm{F}$
2. $|\neg(P \to Q)|_{\mathcal{B}} = \mathrm{T}$
3. $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}} = \mathrm{F}$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg\,(P \to Q) \to (P \wedge Q)$

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg\ (P \to Q) \to (P \wedge Q)$
T	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg\ (P \to Q) \to (P \land Q)$
T	F	T (under P)

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$($	P	$\to$	Q	$)$	$\to$	$($	P	$\land$	Q	$)$
T	F			T		F							

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F		T		F		T		

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	F		T		F		T		F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	F	T	F	T	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = T$ and $|Q|_{\mathcal{B}} = F$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	F	T	F	T	F F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \text{T}$ and $|Q|_{\mathcal{B}} = \text{F}$.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P$	$\land$	$Q)$
T	F	T	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F		T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$($	P	$\to$	Q	$)$	$\to$	$($	P	$\wedge$	Q	$)$
T	F			T	F	F				T	F	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = T$ and $|Q|_{\mathcal{B}} = F$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	F	T	T	F	F		T	F	F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}} = \mathrm{T}$ and $|Q|_{\mathcal{B}} = \mathrm{F}$.

Compute $|\neg(P \to Q) \to (P \wedge Q)|_{\mathcal{B}}$

P	Q	$\neg$	$($	P	$\to$	Q	$)$	$\to$	$($	P	$\wedge$	Q	$)$
T	F	T		T	F	F				T	F	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

For actual calculations it's usually better to use tables.

Suppose $|P|_{\mathcal{B}}$ = T and $|Q|_{\mathcal{B}}$ = F.

Compute $|\neg(P \to Q) \to (P \land Q)|_{\mathcal{B}}$

P	Q	$\neg$	$($	P	$\to$	Q	$)$	$\to$	$($	P	$\land$	Q	$)$
T	F	T		T	F	F		**F**		T	F	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg\,(P \to Q) \to (P \wedge Q)$
T	T	
T	F	
F	T	
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T						
T	F								
F	T								
F	F								

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg\,(P \to Q) \to (P \land Q)$
T	T	T
T	F	T
F	T	
F	F	

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T						
T	F		T						
F	T		F						
F	F								

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg\ (P \to Q) \to (P \land Q)$
T	T	T
T	F	T
F	T	F
F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T				T		
T	F		T						
F	T		F						
F	F		F						

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T				T		
T	F		T				T		
F	T		F						
F	F		F						

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T				T		
T	F		T				T		
F	T		F				F		
F	F		F						

Using the same technique we can fill in the full truth table for
$\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	T		T		T
T	F		T		T
F	T		F		F
F	F		F		F

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \land Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T		T		T		T		
T	F		T				T		
F	T		F				F		
F	F		F				F		

Using the same technique we can fill in the full truth table for
$\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T		T		T		
T	F		T		F		T		
F	T		F				F		
F	F		F				F		

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T			T		
T	F		T	F			T		
F	T		F	T			F		
F	F		F				F		

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg$	$(P$	$\rightarrow$	$Q)$	$\rightarrow$	$(P$	$\wedge$	$Q)$
T	T		T		T		T		
T	F		T		F		T		
F	T		F		T		F		
F	F		F		F		F		

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T	T		T		T		T		T
T	F		T		F		T		
F	T		F		T		F		
F	F		F		F		F		

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T	T		T		T		T		T
T	F		T		F		T		F
F	T		F		T		F		
F	F		F		F		F		

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg\,(P \to Q) \to (P \wedge Q)$			
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P \to Q)$	$\to$	$(P \wedge Q)$
T	T	T	T	T	T
T	F	T	F	T	F
F	T	F	T	F	T
F	F	F	F	F	F

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T	T		T		T
T	F		T		F		T		F
F	T		F		T		F		T
F	F		F		F		F		F

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T	T		T		T
T	F		T	F	F		T		F
F	T		F		T		F		T
F	F		F		F		F		F

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T	T		T	T	T		T		T
T	F		T	F	F		T		F
F	T		F	T	T		F		T
F	F		F		F		F		F

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F	T		F	T	T		F		T
F	F		F	T	F		F		F

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T	T		T	T	T
T	F		T	F	F		T		F
F	T		F	T	T		F		T
F	F		F	T	F		F		F

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T	T		T	T	T		T	T	T
T	F		T	F	F		T	F	F
F	T		F	T	T		F		T
F	F		F	T	F		F		F

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T		T	T	T		T	T	T
T	F		T	F	F		T	F	F
F	T		F	T	T		F	F	T
F	F		F	T	F		F		F

ϕ	$\neg\phi$
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F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \rightarrow Q) \rightarrow (P \wedge Q)$

P	Q	$\neg$	$($	P	$\rightarrow$	Q	$)$	$\rightarrow$	$($	P	$\wedge$	Q	$)$
T	T			T	T	T				T	T	T	
T	F			T	F	F				T	F	F	
F	T			F	T	T				F	F	T	
F	F			F	T	F				F	F	F	

ϕ	$\neg\phi$
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T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

Using the same technique we can fill in the full truth table for $\neg(P \to Q) \to (P \wedge Q)$

P	Q	$\neg$	$($	P	$\to$	Q	$)$	$\to$	$($	P	$\wedge$	Q	$)$
T	T	F		T	T	T				T	T	T	
T	F			T	F	F				T	F	F	
F	T			F	T	T				F	F	T	
F	F			F	T	F				F	F	F	

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\land$	$Q)$
T	T	F	T	T	T		T	T	T
T	F	T	T	F	F		T	F	F
F	T		F	T	T		F	F	T
F	F		F	T	F		F	F	F

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P	Q	$\neg$	$(P$	$\to$	$Q)$	$\to$	$(P$	$\wedge$	$Q)$
T	T	F	T	T	T		T	T	T
T	F	T	T	F	F		T	F	F
F	T	F	F	T	T		F	F	T
F	F		F	T	F		F	F	F

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T	T	F	T	T	T		T	T	T
T	F	T	T	F	F		T	F	F
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T	T	F	T	T	T	**T**	T	T	T
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F	T	F	F	T	T	**T**	F	F	T
F	F	F	F	T	F		F	F	F

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F	T	F	F	T	T	**T**	F	F	T
F	F	F	F	T	F	**T**	F	F	F

The main column (in boldface) gives the truth-value of the whole sentence.

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \to \psi)$
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Validity

Let Γ be a set of sentences of $\mathcal{L}_1$ and ϕ a sentence of $\mathcal{L}_1$.

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Definition

The argument with all sentences in Γ as premisses and ϕ as conclusion is *valid* if and only if there is no $\mathcal{L}_1$-structure under which:

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Notation: when this argument is valid we write $\Gamma \vDash \phi$.

$\{P \to \neg Q, Q\} \vDash \neg P$ means that the argument whose premises are $P \to \neg Q$ and Q, and whose conclusion is $\neg P$ is valid.
Also written: $P \to \neg Q, Q \vDash \neg P$

Worked example 3

We can use truth-tables to show that $\mathcal{L}_1$-arguments are valid.

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

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P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

Rows correspond to interpretations.

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
T	T	T	**F**	F	T	**T**	**F**	T
T	F	T	**T**	T	F	**F**	**F**	T
F	T	F	**T**	F	T	**T**	**T**	F
F	F	F	**T**	T	F	**F**	**T**	F

Rows correspond to interpretations.
One needs to check that there is no row in which all the premisses are assigned T and the conclusion is assigned F.

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
►	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
	F	T	F	**T**	F	T	**T**	**T**	F
	F	F	F	**T**	T	F	**F**	**T**	F

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
	T	T	T	**F**	F	T	**T**	**F**	T
▶	T	F	T	**T**	T	F	**F**	**F**	T
	F	T	F	**T**	F	T	**T**	**T**	F
	F	F	F	**T**	T	F	**F**	**T**	F

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Show that $\{P \to \neg Q, Q\} \vDash \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
▶	F	T	F	**T**	F	T	**T**	**T**	F
	F	F	F	**T**	T	F	**F**	**T**	F

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We can use truth-tables to show that $\mathcal{L}_1$-arguments are valid.

Example

Show that $\{P \to \neg Q, Q\} \models \neg P$.

	P	Q	P	$\to$	$\neg$	Q	Q	$\neg$	P
	T	T	T	**F**	F	T	**T**	**F**	T
	T	F	T	**T**	T	F	**F**	**F**	T
	F	T	F	**T**	F	T	**T**	**T**	F
▶	F	F	F	**T**	T	F	**F**	**T**	F

Rows correspond to interpretations.
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Other logical notions

Definition

A sentence ϕ of $\mathcal{L}_1$ is *logically true* (a *tautology*) iff:

- ϕ is true under all $\mathcal{L}_1$-structures.

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e.g. $P \vee \neg P$, and $P \to P$ are tautologies.

Truth tables of tautologies

Every row in the main column is a T.

P	P	$\vee$	$\neg$	P	P	$\to$	P
T	T	**T**	F	T	T	**T**	T
F	F	**T**	T	F	F	**T**	F

Definition

A sentence ϕ of $\mathcal{L}_1$ is a *contradiction* iff:

- ϕ is not true under any $\mathcal{L}_1$-structure.

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A sentence ϕ of $\mathcal{L}_1$ is a *contradiction* iff:

- ϕ is not true under any $\mathcal{L}_1$-structure.

e.g. $P \wedge \neg P$, and $\neg(P \rightarrow P)$ are contradictions.

Truth tables of contradictions

Every row in the main column is an F.

P	P	$\wedge$	$\neg$	P	$\neg$	$(P$	$\rightarrow$	$P)$
T	T	**F**	F	T	**F**	T	T	T
F	F	**F**	T	F	**F**	F	T	F

Definition

Sentences ϕ and ψ are *logically equivalent* iff:

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Definition

Sentences ϕ and ψ are *logically equivalent* iff:

- ϕ and ψ are true in exactly the same $\mathcal{L}_1$-structures.

- P and $\neg\neg P$ are logically equivalent.
- $P \wedge Q$ and $\neg(\neg P \vee \neg Q)$ are logically equivalent.

Truth tables of logical equivalents

The truth-values in the main columns agree.

P	Q	P	$\wedge$	Q	$\neg$	$(\neg$	P	$\vee$	$\neg$	$Q)$
T	T	T	**T**	T	**T**	F	T	F	F	T
T	F	T	**F**	F	**F**	F	T	T	T	F
F	T	F	**F**	T	**F**	T	F	T	F	T
F	F	F	**F**	F	**F**	T	F	T	T	F

Worked example 4

Example

Show that the sentence $(P \rightarrow (\neg Q \wedge R)) \vee P$ is a tautology.

Worked example 4

Example

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 1: Full truth table

- Write out the truth table for $(P \to (\neg Q \wedge R)) \vee P$.
- Check there's a T in every row of the main column.

Worked example 4

Example

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 1: Full truth table

- Write out the truth table for $(P \to (\neg Q \land R)) \lor P$.
- Check there's a T in every row of the main column.

P	Q	R	(P	$\to$	($\neg$	Q	$\land$	R))	$\lor$	P
T	T	T	T	F	F	T	F	T	**T**	T
T	T	F	T	F	F	T	F	F	**T**	T
T	F	T	T	T	T	F	T	T	**T**	T
T	F	F	T	F	T	F	F	F	**T**	T
F	T	T	F	T	F	T	F	T	**T**	F
F	T	F	F	T	F	T	F	F	**T**	F
F	F	T	F	T	T	F	T	T	**T**	F
F	F	F	F	T	T	F	F	F	**T**	F

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \to (\neg\, Q \land R)) \lor\; P$

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
									F	

Worked example 4 (cont.)

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P	Q	R	$(P \to (\neg\ Q \wedge R))$	$\vee$	P
				F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \to (\neg Q \land R))$	$\lor$	P
			F_1	F	

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \to (\neg\, Q \wedge R)) \vee P$
			F_1 F F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P \to (\neg Q \land R))$	$\lor$	P
			F_1	F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$($	P	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
				T_3	F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \land R)) \lor P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\land$	$R))$	$\lor$	P
			T_3	F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \land \psi)$	$(\phi \lor \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Worked example 4 (cont.)

Show that the sentence $(P \to (\neg Q \wedge R)) \vee P$ is a tautology.

Method 2: Backwards truth table.

- Put an F in the main column.
- Work backwards to show this leads to a contradiction.

P	Q	R	$(P$	$\to$	$(\neg$	Q	$\wedge$	$R))$	$\vee$	P
			?	F_1					F	F_2

ϕ	$\neg\phi$
T	F
F	T

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$	$(\phi \to \psi)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T