

**Further Empirical Evidence on the Forecasting of Volatility
with Smooth Transition Exponential Smoothing**

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Abstract

Smooth transition exponential smoothing (STES) uses a logistic function of a user-specified transition variable as adaptive time varying smoothing parameter. This paper empirically addresses three aspects of the use of STES for volatility forecasting. Previous empirical results showed the method performing well in comparison with fixed parameter exponential smoothing and a variety of GARCH models. However, those results related only to forecasting weekly volatility. In this paper, we address the use of STES for forecasting daily volatility. A second issue that we evaluate is the robustness of STES in the presence of extreme outlying observations. The third aspect that we consider is the use of trading volume within a transition variable in the STES method. Our simulation results suggest that STES performs well in terms of robustness, when compared with standard methods and several alternative robust methods. Analysis using stock return data shows that STES has the potential to outperform standard and robust forms of fixed parameter exponential smoothing and GARCH models. The results suggest the use of the sign and size of past shocks as STES transition variables, and provide no clear support for the incorporation of trading volume in a transition variable.

Key words: Smooth Transition Exponential Smoothing; Daily Volatility Forecasting; Robustness; Trading Volume.

1. Introduction

Volatility forecasts are important for a range of activities, including risk management, portfolio selection, and derivative pricing. A variety of time series methods have been proposed for volatility forecasting, including Generalised autoregressive conditional heteroskedasticity (GARCH) models, stochastic volatility models, and *ad hoc* time series methods, such as exponential smoothing (ES) (Poon and Granger, 2003). The literature contains a number of interesting nonlinear GARCH models, such as the smooth transition models of Hagerud (1997) and González-Rivera (1998), which allow a parameter to vary over time as a continuous function of a transition variable. This prompts Taylor (2004b) to develop smooth transition exponential smoothing (STES), which uses a logistic function of a user-specified transition variable as adaptive time varying smoothing parameter. STES is the focus of this paper.

Although the one previous empirical evaluation of STES for volatility prediction delivered encouraging results, consideration was only given to forecasting *weekly* volatility (see Taylor, 2004b). To estimate STES parameters, daily returns were used to construct realised weekly volatility, which was used as a proxy for actual volatility. In this paper, we provide empirical evidence on the use of STES for forecasting *daily* volatility based on daily returns, and without the use of realized volatility.

When dealing with volatility forecasting, an issue of concern is the existence of extreme observations or outliers. Such extremes in financial time series are commonly reported (see Franses and Ghijssels, 1999; Park, 2002; Poon and Granger, 2003), and they are evident in Fig. 1, which shows a plot of daily returns for the S&P 500 index. The robustness of STES in the presence of outliers has been considered in a study focusing only on the forecasting of the level of a time series (see Taylor 2004a). In this paper, we investigate the robustness of STES when it is used for the very different application of volatility forecasting.

----- Fig. 1 -----

Of crucial importance to the success of STES is the choice of the transition variable. The only variables that have previously been considered are the previous period's shock and the magnitude of this shock (see Taylor, 2004b). It has been suggested that trading volume has a potential role to play in forecasting volatility. For example, Lamoureux and Lastrapes (1990), Brooks (1998) and

Donaldson and Kamstra (2005), among others, consider trading volume as a regressor in a GARCH model. In this paper, we evaluate the usefulness of trading volume as a transition variable in STES.

In summary, the contribution of this paper is to provide further empirical evidence regarding the STES method for volatility prediction. More specifically, we address three main issues: the prediction of daily volatility; the robustness of STES for volatility forecasting in the presence of outliers; and the usefulness of trading volume as a STES transition variable.

In Section 2, we describe the STES method. In Section 3, we briefly review the literature on robust volatility forecasting, and propose several simple robust methods. Section 4 presents a simulation study to investigate the robustness of the STES method. Section 5 presents an empirical comparison of methods using stock index data. Section 6 summarises the paper.

2. Smooth Transition Exponential Smoothing (STES) for Volatility Forecasting

The GJRGARCH model of Glosten *et al.* (1993) provides a simple way to capture the leverage effect, whereby a negative shock has a greater impact on the next period's volatility than a positive shock of equal size. This popular GARCH model captures the leverage effect by switching between two different parameters, according to the sign of the previous period's shock. This switching is replaced by smoothing in the logistic smooth transition GARCH (LSTGARCH) model of Hagerud (1997) and González-Rivera (1998). Hagerud also introduces the exponential smooth transition GARCH (ESTGARCH) model, which uses the magnitude of the previous period's shock and an exponential function to dictate the smooth transition between parameters. A more detailed overview, including model expressions, for the GJRGARCH, LSTGARCH and ESTGARCH models is provided by Taylor (2004b).

Exponential smoothing (ES) is a simple and pragmatic approach to volatility forecasting. Its popularity has been due, at least partly, to its incorporation in the *RiskMetrics* methodology (*RiskMetrics*, 1996). Assuming that the conditional mean of the returns, r_t , is a constant, μ , we are interested in forecasting the variance, σ_t^2 , of errors, $\varepsilon_t = r_t - \mu$. The ES 1-step-ahead variance estimator is presented in expression (1) in recursive form with smoothing parameter, α , which is typically estimated by minimising the sum of squared in-sample prediction errors.

$$\hat{\sigma}_t^2 = \alpha \varepsilon_{t-1}^2 + (1-\alpha)\hat{\sigma}_{t-1}^2 \quad (1)$$

In the exponential smoothing literature, in the context of forecasting the level of a sales time series, it has been proposed that the smoothing parameter should be allowed to vary in order to capture the latest changing characteristics of the time series (e.g. Trigg and Leach, 1967). This, and the existence of smooth transition GARCH models, lead to the development of smooth transition exponential smoothing (STES) for predicting the level of a series (see Taylor, 2004a), and STES for volatility forecasting (see Taylor, 2004b), which we present in the following expressions:

$$\hat{\sigma}_t^2 = \alpha_{t-1} \varepsilon_{t-1}^2 + (1-\alpha_{t-1})\hat{\sigma}_{t-1}^2$$

where
$$\alpha_{t-1} = \frac{1}{1 + \exp(\beta + \gamma V_{t-1})}$$

V_{t-1} is the transition variable, and β and γ are constant parameters estimated in the standard way, by minimising the sum of squared in-sample prediction errors. This STES method uses a logistic function of a user-specified variable as adaptive time varying smoothing parameter. The logistic function restricts α_{t-1} to lie between 0 and 1. The method is called *STES-AE* when $|\varepsilon_{t-1}|$ is the transition variable; and *STES-SE* is the name given to the method when ε_{t-1}^2 is the transition variable.

Using stock market data, Taylor (2004b) obtains promising results using together the sign and size (ε_{t-1} and $|\varepsilon_{t-1}|$) of the previous period's shock as transition variables. In that study, the focus was on forecasting *weekly* volatility. In this paper, we instead evaluate *daily* volatility forecasting.

Taylor (2004a) performs a simulation study to address the robustness to outliers of the STES method for forecasting the level of a series. The results show that STES-AE and STES-SE perform well when the estimation sample and evaluation sample both contain an outlier. This can be attributed to the adaptive nature of the time-varying parameter, which decreases around the outlier in order to put a reduced weight on the outlier. In this paper, we use simulated data to evaluate the robustness to outliers of STES for volatility prediction.

3. Robust Volatility Forecasting Methods: A Review and Simple Proposals

3.1. The Impact of Outliers on Volatility Model Estimation

In time series analysis, two types of outliers have been considered: additive outliers and innovative outliers. An additive outlier gives an immediate and one-time effect on the observed time series, as only the current observation is affected. Hence, it has an additive impact. On the other hand, an occurrence of an innovative outlier at the present time also influences future observations.

The estimated standardized residuals of GARCH models tend to exhibit excess kurtosis, albeit less than the kurtosis in the raw returns (see, for example, Bollerslev, 1987). This is true even with the use of conditionally t -distributed errors. Franses and Ghijssels (1999) interpret this as evidence of additive outliers. Neglecting outliers can lead to biased parameter estimates in conditional mean equations, and also biased out-of-sample forecasts (Ledolter, 1989). This is supported by the empirical studies by Jorion (1995), and Andersen and Bollerslev (1998). They report that even though the GARCH parameters are highly significant in-sample, these models can produce poor out-of-sample forecasts due to the effects caused by outliers on parameter estimation. These findings lead to the perception that there are unavoidable limitations of GARCH models. To overcome these, Franses and Ghijssels (1999) advocate a method to detect additive outliers in GARCH models, and to reduce the impact of additive outliers on parameter estimates and forecasts. Carnero, *et al.* (2012) implement a few robust methods to estimating GARCH volatility in the presence of outliers, including Quasi Maximum Likelihood estimator (QML- t , proposed by Bollerslev, 1987) and Bounded-M (BM) estimator (proposed by Muler and Yohai, 2008). They find that these robust methods outperformed maximum likelihood procedures in estimate both parameters and volatilities. However, this procedure is computationally intensive, which is a practical concern. In the next subsection, we propose several simple robust methods that are easy to interpret and straightforward to implement.

3.2. Simple Proposals for Robust Volatility Prediction

Since ES for volatility forecasting is formulated in terms of variance forecasts, *RiskMetrics* (1996) suggests the minimisation of the sum of ‘squared’ in-sample one-step-ahead prediction errors between actual variance, using squared residual as the proxy, and predicted variance. However, this

criterion provides a quadratic loss function that can lead to spurious inference in the presence of outliers (Franses and Ghijssels, 1999). In order to give less weight to outliers, we include in our simulation study ES with parameter estimated using the minimisation of the sum of ‘absolute’ in-sample one-step-ahead prediction errors, between squared residuals and the variance forecasts. We refer to this method as *ES-Absolute*. (Note that it is unwise to use the absolute value of a residual as a proxy for standard deviation, because the absolute residual will almost certainly be a biased estimator of the conditional standard deviation. In view of this, we do not consider the robust GARCH method of Park (2002), which involves the absolute residual.)

Rousseeuw (1984) proposes the use of least median of squares regression to overcome estimation problems caused by outliers. This motivates us to consider, for ES parameter optimisation, the minimisation of the median absolute prediction error between squared residuals and the variance forecasts. We refer to this method as *ES-Median*. The analogous procedure for robust estimation of GARCH models is to use maximum median likelihood. We are not aware of the previous consideration of maximum median likelihood in GARCH modelling, or of the use of median absolute prediction error in the optimisation of ES parameters. We refer to this as *GARCH-MedianL*.

Winsorizing removes outliers by trimming a specified percentage, such as 1%, of the lowest and highest values in a dataset (see Hoaglin *et al.* 1983). Taylor’s (2005) winsorized GARCH approach involves first estimating the series of 1% conditional quantiles of the return series. For each period in the estimation sample, if a return exceeds the estimated quantile, it is replaced for the rest of the analysis by the quantile value. A similar procedure is used to reduce the magnitude of all returns that exceed the 99% conditional quantile. The winsorizing has the effect of removing outliers, and so, after it has been applied, standard maximum likelihood is used for GARCH parameter estimation, and the minimisation of squared prediction error is used for ES. Taylor uses CAViaR models to estimate the conditional quantiles. In this paper, we use the exponentially weighted moving averages method for quantiles of Boudoukh *et al.* (1998), which Taylor (2008) shows can be viewed as exponentially weighted quantile regression. Taylor (2005) finds that greater quantile forecast accuracy resulted when the decay parameter of this method is optimized using cross-validation with the quantile regression summation as cost function. We refer to the winsorizing with ES as *ES-Winsorized*, and we

use *GARCH-Winsorized* to denote winsorizing with GARCH(1,1) estimated using conditionally t -distributed errors.

Over the last decade, only limited literatures focusing on the applications of winsorizing in removing outliers. Among others are Lusk *et al.*(2011), Ghosh and Vogt (2012), Al-Khazaleh *et al.*(2015), Suleman *et al.* (2017), Kwak and Kim (2017).

4. Simulation Study to Evaluate the Robustness of STES to Outliers

4.1. The Simulated Data

In this section, we carry out Monte Carlo experiments to evaluate the robustness to outliers of STES compared to other volatility forecasting methods. We focus on additive outliers and one-step-ahead volatility forecasting. We use the contaminated GARCH(1,1) data-generating process of Park (2002). This process generates simulated returns, y_t , using the following data generating process:

$$\begin{aligned} y_t &= r_t + \eta o_t \\ r_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sigma_t u_t \\ \sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned}$$

where $u_t \sim N(0,1)$, $P(o_t = 1) = \wp$, $P(o_t = 0) = 1 - \wp$. The likelihood of the occurrence of an outlier in each period is controlled by the probability \wp . In all experiments, we set $\wp = 0.005$, as the occurrence of outliers is rare. The constant and GARCH parameters, μ , ω , α and β , take the values of 0, 0.02, 0.11 and 0.87, respectively. These values are taken by averaging the parameter estimates of GARCH(1,1) across the eight stock market indices considered in Section 5. In order to investigate the effect of different magnitudes of outliers, we set $\eta = \{0, 4, 6, 8\}$. $\eta = 0$ clearly corresponds to the case of no outlier. For simplicity, we did not try to incorporate a leverage effect in the simulated series, and so we do not consider asymmetric GARCH or STES methods in this study.

We employed 1000 replications. For each, a time series of length 2500 observations is generated from the contaminated GARCH model. The first 500 observations are discarded to avoid initialisation effects, the next 1,500 observations are used for parameter estimation and the last 500 observations are reserved for post-sample evaluation. We could have also included an investigation of

the effect of outliers for different sample sizes. However, as pointed out by Park (2002, p. 387), the estimation sample size only slightly affects the performance of the forecasting models. The software Gauss was used for all computational work in this study. Figures 2 and 3 show two of the time series generated with the magnitude of outliers set by $\eta= 4$ and 8, respectively.

In the study, we included two versions of the STES method, three standard volatility forecasting methods, and five robust methods. We present the methods in the next three subsections.

----- Figs. 2 and 3 -----

4.2. Benchmark Methods

As a simple benchmark method, we estimated the variance using a moving average of the previous 30 squared simulated returns. We refer to this as *MA30*. We implemented standard fixed parameter ES. As suggested in *RiskMetrics* (1996), we optimised parameters by minimising the sum of squared in-sample one-step-ahead prediction errors between predicted and actual variance, using squared residual as the proxy for actual variance. We refer to this as *ES-Square*.

We included a standard GARCH(1,1) model. Even though the series was generated from a Gaussian distribution, the simulated series was contaminated by outliers, and so we opted to estimate the model assuming conditionally *t*-distributed errors. We refer to this simply as *GARCH*.

We implemented the following robust volatility forecasting methods, as described in Section 3: *ES-Absolute*, *ES-Median*, *GARCH-MedianL*, *ES-Winsorized* and *GARCH-Winsorized*. Both of these GARCH models are of order (1,1) with conditionally *t*-distributed errors.

4.3. STES Methods

We implemented the STES-AE and STES-SE methods as described in Section 2. Figures 4 and 5 display post-sample one-step-ahead volatility forecasts for STES-AE and GARCH, with the magnitude of the outliers set as $\eta = 4$ and 8, respectively. For clarity, we show only 100 out of 500 post-sample one-step-ahead forecasts. In the figures, both series of volatility forecasts react quickly to the outliers, but it is interesting to note that STES-AE is quicker to return to the previous volatility level. This feature is consistent across different magnitudes of outliers. The secondary axis in each

figure shows the plot of the adaptive parameter for STES-AE. This plot shows a decrease immediately after the two outliers in order to put a reduced weight on the outliers. It is important to appreciate that the reaction to the outliers are post-sample results.

----- Figs. 4 and 5 -----

Let us now consider briefly the adaptive smoothing parameter α_{t-1} for STES with $|\varepsilon_{t-1}|$ as transition variable. Expressions (2) and (3) show α_{t-1} for series with magnitude of outliers equal to 4 and 8, respectively. These expressions correspond to the series of STES-AE forecasts in Figures 4 and 5, respectively. The positive value for the coefficient of the transition variable implies that an increase in the size of the previous period's shock will result in a decrease in α_{t-1} . This reduces the forecast function's reaction to the outliers. The coefficient is larger for the series with larger outliers.

$$\alpha_{t-1} = \frac{1}{1 + \exp(0.71 + 0.11|\varepsilon_{t-1}|)} \quad (2)$$

$$\alpha_{t-1} = \frac{1}{1 + \exp(0.57 + 0.15|\varepsilon_{t-1}|)} \quad (3)$$

4.4. Post-Sample Forecasting Results

Mean square error (MSE) or root mean square error (RMSE) is the most preferred accuracy criterion for evaluating the performance of volatility forecasting models, as indicated by Brooks (1998), among others. However, Franses and Ghijssels (1999) write that this criterion provides a quadratic loss function that can lead to spurious inference in the presence of outliers. Hence, in the post-sample forecasting evaluation, we consider not only MSE but also mean absolute error (MAE), which is more robust to outliers. In addition, since the mean itself is a biased estimate in the presence of outliers, we also implement median absolute error (MedAE). These measures of accuracy used are RMSE, MAE and MedAE, given by

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (\varepsilon_t^2 - \hat{\sigma}_t^2)^2}$$

$$MAE = \frac{1}{N} \sum_{t=1}^N |\varepsilon_t^2 - \hat{\sigma}_t^2|,$$

and

$$MedAE = Median|\varepsilon_t^2 - \hat{\sigma}_t^2|,$$

where N is the post-sample size. The nature of the data, the fact that the volatility could be very close to zero, precludes the use of proportional accuracy measures, such as mean absolute percentage error (MAPE), advocated by Armstrong and Collopy (1992).

We calculated the mean square error (RMSE) for the 500 post-sample periods for each of the 1000 series simulated for each size of outlier. The average of the resulting 1000 RMSE values is shown, for each size of outlier, in the first four numerical columns of Table 1. In a similar way, we calculated the average MAE and average MedAE values, and these are also presented in Table 1.

----- Table 1 -----

For all three error measures, and all magnitudes of outliers, the robust GARCH method, *GARCH-MedianL*, performs worse than standard GARCH across all magnitudes of outliers. The results for *GARCH-Winsorized* are similarly to standard GARCH for all magnitudes of outliers.

Table 1 shows that, for all magnitudes of outliers, the robust ES method, *ES-Median*, performs poorly in terms of RMSE and MAE, but outperforms standard ES method, *ES-Square*, for the MedAE measure. The other two robust ES methods, *ES-Absolute* and *ES-Winsorized*, show more consistency, with both outperforming *ES-Square* for MedAE, and matching the performance of *ES-Square* in terms of RMSE and MAE.

Turning to the STES methods, we see that both STES-AE and STES-SE show the best results, in terms of all three measures, for the different magnitudes of outliers. Although the STES methods are matched by the performance of *ES-Median* for the MedAE, *ES-Median* is actually the poorest of all methods in terms of RMSE and MAE. The results for STES-SE and STES-AE are very encouraging. The feature of down-weighting the outlying observations is intuitively appealing, and is the key to the success of STES in terms of robustness.

5. Evaluating STES using Stock Index Data

In this empirical study, we use stock index data to compare the forecast performance of the STES method against fixed parameter exponential smoothing and a variety of GARCH models. The study addresses the following issues:

- (i) In Taylor's (2004b) empirical work, the focus was on forecasting *weekly* volatility with models estimated using realized volatility constructed from daily data. In this section, we evaluate STES for forecasting *daily* volatility estimated using daily returns (and not realized volatility).
- (ii) For the STES method, we consider several potential transition variables that were not considered by Taylor (2004b). Two of these relate to trading volume.
- (iii) The presence of outliers in the stock index data means that the study also serves to investigate further the issue of robustness considered in Section 4.

5.1. The Stock Index Data

The data used in this study are the daily observed stock indices and their respective trading volume series. We used the same eight stock indices considered by Taylor (2004b). These are from the following major markets: Amsterdam (AEX), Frankfurt (DAX), Hong Kong (Hang Seng), London (FTSE100), New York (S&P500), Paris (CAC40), Singapore (STI) and Tokyo (Nikkei). Each time series consisted of 2000 log returns, implying a period of approximately eight years, with the sample period ending on 9 September 2010. Note that we opted to employ the data of this period which including the period of Global Financial Crisis, which is most probably providing us with outliers. The first 1500 observations are used for parameter estimation and the last 500 observations are reserved for post sample evaluation. We focused solely on one-step-ahead prediction.

5.2. Benchmark Methods

We implemented the standard and robust volatility forecasting methods that we included in the simulation study of Section 4. Since the leverage effect is often present in stock indices, we included in the study the following asymmetric GARCH models: GJRGARCH, LSTGARCH and ESTGARCH. For GJRGARCH, we also implemented the two robust forms of GARCH considered in

the simulation study of Section 4, and we refer to these as *GJRGARCH-MedianL* and *GJRGARCH-Winsorized*. The results for these methods were better than for the corresponding robust GARCH models, and so for simplicity, later in Section 5, we report the results for only the robust forms of GJRGARCH. All GARCH models are of order (1,1) with conditionally t -distributed errors.

5.3. STES Methods

We implemented the two STES methods considered in the simulation study of Section 4: STES-AE and STES-SE. Given the potential leverage effect in the stock index data, we also included asymmetric versions of the STES method. These involved the lagged shock, ε_{t-1} , as transition variable. More specifically, we considered STES with ε_{t-1} and $|\varepsilon_{t-1}|$ as transition variables, and STES with ε_{t-1} and ε_{t-1}^2 as transition variables. We term these STES-E&AE and STES-E&SE, respectively. STES-EAE was considered by Taylor (2004b), but STES-ESE was not.

We also used trading volume as a transition variable. Following the early work of Karpoff (1987), there have been many studies that have investigated the impact of volume on volatility, although few studies have considered the usefulness of volume for volatility prediction. Inclusion of a volume term in a GARCH model has been a common approach to investigating the relationship. The possible existence of a simultaneity problem is discussed by Lamoureux and Lastrapes (1990). Harvey (1989) addresses this and concludes that lagged volume should be used rather than contemporaneous volume. From a forecasting perspective, lagged volume is clearly more relevant. Unfortunately, Lamoureux and Lastrapes (1990) find that the lagged volume is a poor instrument for contemporaneous volume. Brooks (1998) concludes that lagged volume has little role to play in improving the out-of-sample forecasting performance of volatility models. By contrast, Donaldson and Kamtra (2005) conclude that lagged volume can significantly improve the accuracy of volatility forecasts when used with implied volatility within a GARCH model. Le and Zurbruegg (2010) combine the information from stock and option market to identify and confirm the forecast quality of volume.

Although Taylor (2008) emphasizes the importance of information flow in determining the significance of volatility forecasting model, by incorporating volatility index (VIX) and trading

volume as exogenous variable in six different GARCH models, Kambouroudis and McMillan (2016) discover that volume contributes a significant but small additional degree of predictive power which is in compliance with Brooks (1998).

However, by examining the predictive power of three information sets: daily trading volume, intraday returns and overnight returns, Fuertes, *et al.* (2015) find that volume is the most effective predictor.

In this paper, we carry out our own empirical analysis of the usefulness of lagged trading volume for volatility prediction. We consider the use of trading volume as STES transition variable to govern the weight on old versus new information. More specifically, we experimented with two alternative trading volume transition variables: the natural logarithm of lagged volume, and also the high/low volume indicator variable, $IndVol_{t-1}$, as implemented by Donaldson and Kamstra (2005):

$$IndVol_{t-1} = \begin{cases} 1 & \text{if } Volume_{t-1} \geq \frac{1}{n-1} \sum_{i=2}^n Volume_{t-i} \\ 0 & \text{otherwise} \end{cases}$$

We set $n = 5$ so that $IndVol_{t-1}=1$ if lagged volume is above its one-week lagged moving average. We also investigate a variety of other lag lengths, but the results were not affected qualitatively, which is consistent with the finding of Donaldson and Kamstra (2005). We considered STES with the individual transition variables, as well as pairs of transition variables. In Table 2, we list and define the ten different versions of STES that we included in our study.

----- Table 2 -----

5.4. Post-Sample Forecasting Results

Table 3 presents the RMSE calculated for the 500 post-sample forecasts produced by each method. The final column shows the ranking of each method averaged across the eight stock indices. The best five methods in each column are indicated by bold and underlining. For each method, the results are reasonably consistent across the eight stock indices. Of the standard methods, the best results correspond to GJRGARCH. Indeed, this was the best performing of all the methods in terms of RMSE. Of the five robust methods, the best RMSE results were produced by the GJRGARCH

model with winsorizing. For the STES method, the lowest RMSE results were achieved with ε_{t-1} and $|\varepsilon_{t-1}|$ as transition variables, or with ε_{t-1} and ε_{t-1}^2 as transition variables. These two versions of the STES method were only outperformed by GJRGARCH and by GJRGARCH with winsorizing. Although the results are reasonable for the versions of the STES method that feature trading volume in the transition variables, it would seem that better results can be achieved without trading volume.

----- Table 3 -----

Tables 4 and 5 present the MAE and MedAE results. In both tables, we see all of the STES methods outperforming all of the standard models, and all of the robust fixed parameter ES methods. In terms of MAE, the best STES methods used either $|\varepsilon_{t-1}|$ alone as transition variable, or ε_{t-1} and ε_{t-1}^2 together. In terms of MedAE, using $|\varepsilon_{t-1}|$ alone produced the best results for the STES method. As with the RMSE results, the MAE and MedAE results show that the trading volume transition variables did not seem to benefit the STES method. For MAE and MedAE, the best performing method was the GJRGARCH model estimated by maximising the median likelihood. Although the MAE and MedAE results for this method are impressive, it is interesting to note from Table 3 that it was one of the worst performing methods in terms of RMSE.

----- Tables 4 to 5 -----

In Table 6, we summarise the performance of the methods for the three error measures. The table shows the mean ranks taken from the final column of each of Tables 3, 4 and 5. The final column of Table 6 shows the average of the three mean ranks for each method. This final column shows good overall performance from GJRGARCH in its standard form, and when estimated by maximising the median likelihood. It is also clear from Table 6 that several of the STES methods performed very competitively, with the best overall results of all methods across all three measures coming from STES with either $|\varepsilon_{t-1}|$ alone as transition variable, or ε_{t-1} and ε_{t-1}^2 together. Note that ε_{t-1} and ε_{t-1}^2 were not considered together as STES transition variables by Taylor (2004b). The results for all the STES methods are better than those for all the fixed parameter ES methods. The poorest overall results in Table 6 correspond to the moving average, MA30, and the two smooth transition

GARCH models, LSTGARCH and ESTGARCH. The poor performance of these two GARCH models is consistent with the results of Taylor (2004b) for weekly volatility prediction.

----- Table 6 -----

We also applied to the model confidence set testing (MCS test) of Hansen et al. (2011). In Tables 7 and 8, present the compositions of the Superior Set of Models (SSM) discriminating by model. The different entries in each column represent the number of models that belong to the SSM at the end of the MCS procedure discriminated by model. Using the absolute forecast errors, we found that STES methods are least eliminated model among all the models used in this study. However, the results from squared forecast errors show that the SSM is quite homogeneous with respect to the type of the methods.

----- Tables 7 and 8 -----

6. Summary

The aim of this paper was to provide further empirical evidence on the accuracy of the STES method for volatility prediction. By contrast with the only previous study of this method, which considered weekly volatility, we focused in this paper on the prediction of daily volatility, which is likely to be of more practical use. Our analysis of eight stock indices showed the STES method performing well in comparison with a range of standard and robust ES and GARCH models. With regard to the choice of STES transition variables, our results suggest that the size of the previous period's shock is the most important variable to use, and that it may be beneficial to also include the sign of the previous period's shock as a second transition variable. Our results do not support the incorporation of trading volume in a STES transition variable. Of the other methods, GJRGARCH performed particularly well in terms of RMSE, while the best MAE and MedAE results came from a robust form of this model that had parameters optimised by maximising the median likelihood.

STES has been reported to have robust feature in the presence of outliers when used to forecast the level of a time series. In this paper, we extended consideration of the robustness of STES to volatility forecasting. The results from the simulation study show that, regardless of the magnitude of the outliers and post-sample forecast evaluation criterion used, the STES methods outperformed

standard methods, including standard ES, GARCH and several robust volatility forecasting methods. Hence, we can conclude that in the presence of outliers, STES with suitable transition variables is a robust volatility forecasting method.

In terms of future research, it would be interesting to consider alternative transition variables, such as implied volatility. The evaluation of multi-step-ahead prediction from STES is another potential area of research. It would also be interesting to investigate the use of STES with intraday returns data. Another potential research area would be the development of STES for conditional quantile forecasting, with a view to contributing to the literature on value at risk estimation.

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Fig. 1. S&P500 (NewYork), 2000 daily log return

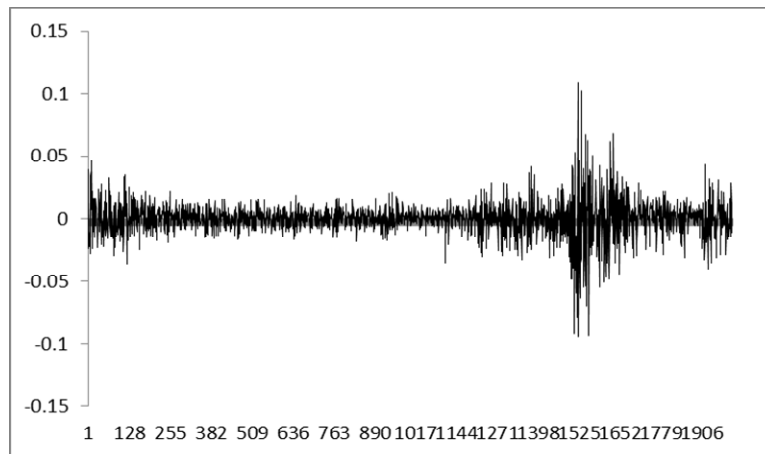


Fig. 2. Simulated time series generated from the GARCH (1,1) with outlier magnitude $\eta=4$.

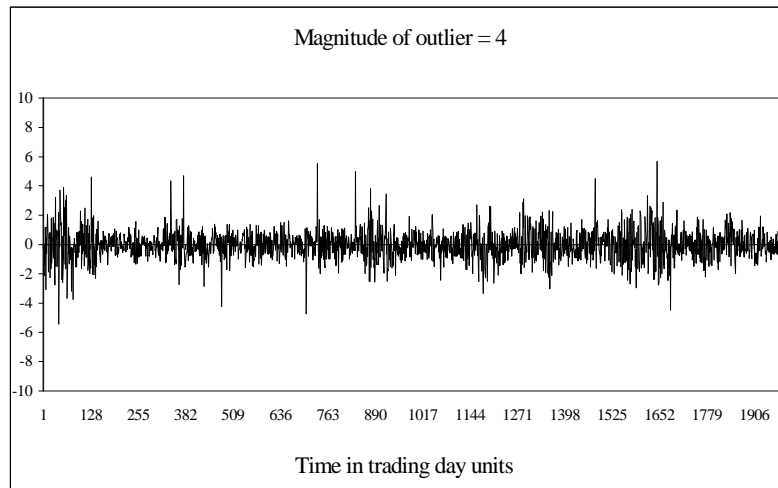


Fig. 3. Simulated time series generated from the GARCH (1,1) with outlier magnitude $\eta=8$.

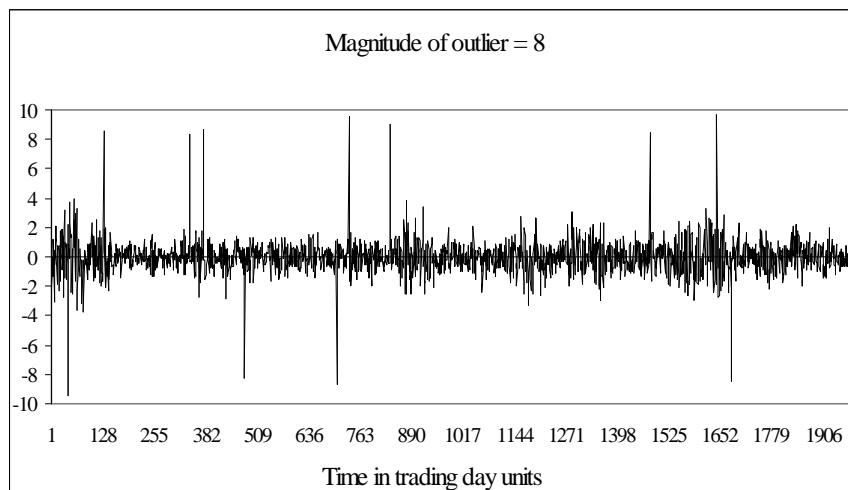


Fig. 4. Volatility forecasts for the STES-AE and GARCH models for simulated time series with outlier magnitude $\eta=4$.

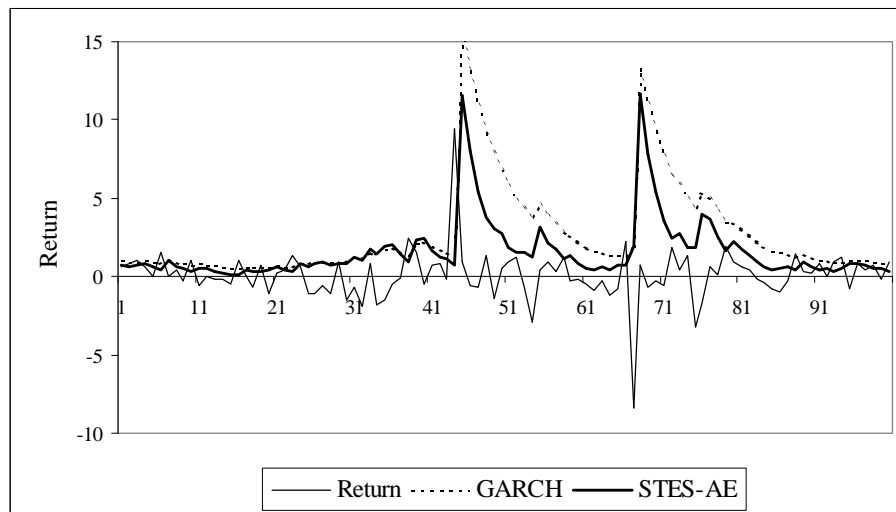


Fig. 5. Volatility forecasts for the STES-AE and GARCH models for simulated time series with outlier magnitude $\eta=8$.

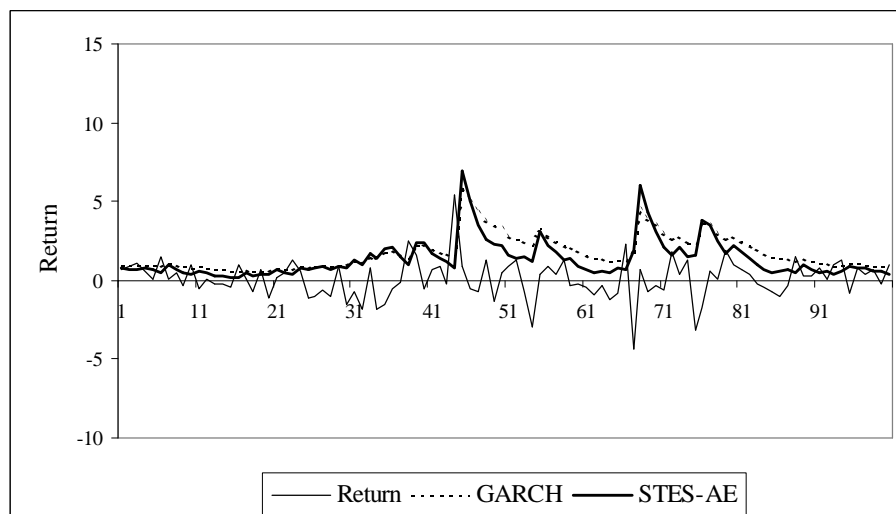


Table 1

Evaluation of 500 post-sample one-step-ahead forecasts for 1000 simulated series from the contaminated GARCH (1,1) process. η is the outlier magnitude.

	Mean RMSE				Mean MAE				Mean MedAE			
	η				η				η			
	0	4	6	8	0	4	6	8	0	4	6	8
Standard Methods												
MA30	1.78	2.46	3.71	5.61	1.02	1.18	1.38	1.67	0.59	0.66	0.70	0.72
ES-Square	<u>1.75</u>	2.45	3.72	5.60	1.00	1.17	1.36	1.64	0.57	0.67	0.81	1.03
GARCH	<u>1.75</u>	<u>2.44</u>	3.71	5.63	1.02	1.16	1.34	1.58	0.62	0.66	0.69	0.72
Robust Methods												
ES-Absolute	<u>1.75</u>	2.45	3.71	5.59	1.00	1.17	1.38	1.64	0.57	0.66	0.77	0.91
ES-Median	1.88	2.76	4.22	6.42	1.05	1.27	1.48	1.78	0.55	0.58	<u>0.60</u>	<u>0.61</u>
ES-Winsorized	<u>1.75</u>	<u>2.44</u>	3.71	5.61	0.99	1.16	1.36	1.61	0.57	0.64	0.68	0.73
GARCH-MedianL	2.11	2.69	3.94	5.76	1.30	1.31	1.44	1.60	0.93	0.80	0.79	0.78
GARCH-Winsorized	<u>1.75</u>	<u>2.44</u>	3.71	5.63	1.03	1.18	1.37	1.64	0.63	0.67	0.69	0.70
STES Method												
STES-AE	<u>1.74</u>	<u>2.43</u>	<u>3.68</u>	<u>5.56</u>	<u>0.96</u>	<u>1.09</u>	<u>1.22</u>	<u>1.42</u>	<u>0.53</u>	<u>0.57</u>	<u>0.60</u>	0.66
STES-SE	<u>1.75</u>	<u>2.44</u>	<u>3.67</u>	<u>5.54</u>	<u>0.96</u>	<u>1.08</u>	<u>1.21</u>	<u>1.40</u>	<u>0.54</u>	<u>0.57</u>	<u>0.59</u>	<u>0.63</u>

Note: Bold and underline indicates best two values in each column.

Table 2

STES methods and their choices of transition variables

STES Method	Transition variables
STES-AE	$ \varepsilon_{t-1} $
STES-SE	ε_{t-1}^2
STES-E&AE	ε_{t-1} and $ \varepsilon_{t-1} $
STES-E&SE	ε_{t-1} and ε_{t-1}^2
STES-IndVol	Indicator for high/low volume
STES-IndVol&AE	Indicator for high/low volume and $ \varepsilon_{t-1} $
STES-IndVol&SE	Indicator for high/low volume and ε_{t-1}^2
STES-LnVol	Log volume
STES-LnVol&AE	Log volume and $ \varepsilon_{t-1} $
STES-LnVol&SE	Log volume and ε_{t-1}^2

Table 3
 RMSE ($\times 10^6$) for 500 post-sample one-step-ahead variance forecasts for eight stock indices.

	Amsterdam	Frankfurt	Hong Kong	London	New York	Paris	Singapore	Tokyo	Mean Rank
Standard Methods									
MA30	1144	1038	1544	895	1111	1092	272	1412	19.9
ES-Square	1115	1029	1466	875	1094	1072	268	1345	10.3
GARCH	<u>1111</u>	1025	1513	872	1099	1068	<u>265</u>	<u>1326</u>	7.8
GJRGARCH	<u>1099</u>	<u>985</u>	1463	<u>849</u>	<u>1074</u>	<u>1026</u>	<u>262</u>	<u>1285</u>	<u>2.3</u>
LSTGARCH	1115	1029	1543	875	1099	1073	267	1337	11.6
ESTGARCH	1118	1036	1530	876	1106	1070	<u>265</u>	1344	12.6
Robust Methods									
ES-Absolute	1115	1029	1512	876	1135	1076	267	1366	14.1
ES-Median	1181	1055	1466	902	1144	1076	267	<u>1286</u>	15.3
ES-Winsorized	1124	1033	1483	876	1098	1080	266	1348	14.6
GJRGARCH-MedianL	1123	1027	1470	932	1098	1149	280	<u>1297</u>	14.1
GJRGARCH-Winsorized	<u>1099</u>	<u>987</u>	1473	<u>852</u>	<u>1082</u>	<u>1034</u>	<u>264</u>	<u>1302</u>	<u>3.9</u>
STES Method									
STES-AE	1117	<u>1024</u>	<u>1441</u>	876	1096	1070	<u>265</u>	1361	9.3
STES-SE	1114	<u>1023</u>	<u>1439</u>	873	1094	<u>1067</u>	266	1374	<u>7.6</u>
STES-E&AE	<u>1094</u>	1029	<u>1370</u>	<u>857</u>	<u>1087</u>	<u>1046</u>	268	1345	<u>5.8</u>
STES-E&SE	<u>1101</u>	<u>1009</u>	<u>1391</u>	<u>853</u>	<u>1082</u>	<u>1053</u>	<u>264</u>	1389	<u>5.0</u>
STES-IndVol	1119	1029	<u>1441</u>	874	1099	1071	266	1345	9.5
STES-IndVol&AE	1119	1029	1467	873	1099	1072	<u>265</u>	1345	9.6
STES-IndVol&SE	1132	1029	<u>1440</u>	874	1099	1081	<u>265</u>	1346	11.4
STES-LnVol	1123	1029	1459	875	1091	1099	268	1345	12.0
STES-LnVol&AE	1114	1027	1444	873	1088	1080	266	1345	8.1
STES-LnVol&SE	1124	1029	1447	<u>872</u>	<u>1087</u>	1081	268	1345	10.5

Note: Bold and underline indicates best five values in each column.

Table 4
MAE ($\times 10^6$) for 500 post-sample one-step-ahead variance forecasts for eight stock indices.

	Amsterdam	Frankfurt	Hong Kong	London	New York	Paris	Singapore	Tokyo	Mean Rank
Standard Methods									
MA30	543	471	621	399	502	512	139	556	18.6
ES-Square	526	466	596	385	499	499	137	542	12.8
GARCH	518	455	599	377	500	483	138	526	10.8
GJRGARCH	538	435	576	388	490	464	142	509	11.0
LSTGARCH	535	477	626	393	506	510	145	552	18.5
ESTGARCH	543	494	652	394	528	496	135	566	18.1
Robust Methods									
ES-Absolute	528	466	606	385	518	502	137	549	14.8
ES-Median	549	474	593	395	522	500	139	533	16.9
ES-Winsorized	528	466	597	385	500	498	137	543	13.4
GJRGARCH-MedianL	542	416	546	324	453	444	120	485	3.1
GJRGARCH-Winsorized	539	468	624	387	517	505	153	569	18.5
STES Method									
STES-AE	491	437	563	357	465	459	132	515	4.3
STES-SE	502	439	566	369	473	472	135	519	6.8
STES-E&AE	496	466	567	356	489	460	137	540	8.5
STES-E&SE	489	428	555	359	481	457	134	512	4.5
STES-IndVol	521	464	557	377	474	492	129	534	7.0
STES-IndVol&AE	485	466	576	367	474	457	131	532	5.6
STES-IndVol&SE	487	466	561	377	474	455	133	534	5.9
STES-LnVol	521	465	591	385	487	479	132	542	10.3
STES-LnVol&AE	496	461	566	378	475	461	130	542	7.9
STES-LnVol&SE	489	466	563	377	474	452	132	541	6.0

Note: Bold and underline indicates best five values in each column.

Table 5
MedAE ($\times 10^6$) for 500 post-sample one-step-ahead variance forecasts for eight stock indices.

	Amsterdam	Frankfurt	Hong Kong	London	New York	Paris	Singapore	Tokyo	Mean Rank
Standard Methods									
MA30	202	187	213	153	180	214	74	<u>201</u>	13.0
ES-Square	197	191	214	149	182	207	77	202	14.5
GARCH	196	183	203	146	187	200	74	205	11.6
GJRARCH	209	<u>171</u>	<u>197</u>	161	179	194	76	<u>201</u>	10.6
LSTGARCH	212	205	236	162	189	224	83	229	19.1
ESTGARCH	218	211	247	162	208	208	72	235	18.5
Robust Methods									
ES-Absolute	201	190	206	148	207	220	77	209	14.9
ES-Median	194	214	215	143	204	202	72	211	14.5
ES-Winsorized	204	192	205	151	184	212	72	202	14.5
GJRARCH-MedianL	226	<u>147</u>	<u>189</u>	<u>72</u>	<u>142</u>	<u>144</u>	<u>44</u>	<u>165</u>	<u>3.5</u>
GJRARCH-Winsorized	213	199	249	162	197	227	93	257	19.8
STES Method									
STES-AE	<u>185</u>	<u>174</u>	209	<u>134</u>	<u>164</u>	<u>187</u>	71	<u>194</u>	<u>4.9</u>
STES-SE	191	<u>178</u>	215	142	175	196	72	202	9.1
STES-E&AE	<u>181</u>	191	211	<u>134</u>	175	<u>186</u>	77	<u>201</u>	8.0
STES-E&SE	<u>182</u>	<u>177</u>	212	<u>140</u>	182	196	71	<u>196</u>	<u>6.9</u>
STES-IndVol	194	190	<u>190</u>	144	<u>162</u>	203	<u>69</u>	<u>201</u>	<u>7.1</u>
STES-IndVol&AE	<u>188</u>	191	<u>201</u>	<u>137</u>	<u>162</u>	193	<u>67</u>	<u>201</u>	<u>5.9</u>
STES-IndVol&SE	193	191	<u>189</u>	144	<u>162</u>	205	<u>70</u>	<u>201</u>	7.5
STES-LnVol	190	189	211	149	171	<u>176</u>	71	202	8.8
STES-LnVol&AE	<u>188</u>	188	216	144	167	<u>180</u>	<u>67</u>	202	7.8
STES-LnVol&SE	189	191	218	145	168	188	71	202	10.0

Note: Bold and underline indicates best five values in each column.

Table 6
Summary of the ranking of methods in Tables 3, 4 and 5 for the eight stock indices.

	Mean Rank for RMSE from Table 3	Mean Rank for MAE from Table 4	Mean Rank for MedAE from Table 5	Mean of Mean Ranks
Standard Methods				
MA30	19.9	18.6	13.0	17.2
ES-Square	10.3	12.8	14.5	12.5
GARCH	7.8	10.8	11.6	10.0
GJRGARCH	<u>2.3</u>	11.0	10.6	8.0
LSTGARCH	11.6	18.5	19.1	16.4
ESTGARCH	12.6	18.1	18.5	16.4
Robust Methods				
ES-Absolute	14.1	14.8	14.9	14.6
ES-Median	15.3	16.9	14.5	15.5
ES-Winsorized	14.6	13.4	14.5	14.2
GJRGARCH-MedianL	14.1	<u>3.1</u>	<u>3.5</u>	<u>6.9</u>
GJRGARCH-Winsorized	<u>3.9</u>	18.5	19.8	14.0
STES Method				
STES-AE	9.3	<u>4.3</u>	<u>4.9</u>	<u>6.1</u>
STES-SE	<u>7.6</u>	6.8	9.1	7.8
STES-E&AE	<u>5.8</u>	8.5	8.0	<u>7.4</u>
STES-E&SE	<u>5.0</u>	<u>4.5</u>	<u>6.9</u>	<u>5.5</u>
STES-IndVol	9.5	7.0	<u>7.1</u>	7.9
STES-IndVol&AE	9.6	<u>5.6</u>	<u>5.9</u>	<u>7.0</u>
STES-IndVol&SE	11.4	<u>5.9</u>	7.5	8.3
STES-LnVol	12.0	10.3	8.8	10.3
STES-LnVol&AE	8.1	7.9	7.8	7.9
STES-LnVol&SE	10.5	6.0	10.0	8.8

Note: Bold and underline indicates best five values in each column.

Table 7

Composition of remaining models in the Superior Set for eight stock indices using squared forecast errors

	Amsterdam	Frankfurt	Hong Kong	London	New York	Paris	Singapore	Tokyo	Total Count
Standard Methods									
MA30	1	1	1	1	1	1	1	1	8
ES-Square	1	1	1	1	1	1	1	1	8
GARCH	1	1	1	1	1	1	1	1	8
GJRGARCH	1	1	1	1	1	1	1	1	8
LSTGARCH	1	1	1	1	1	1	1	1	8
ESTGARCH	1	1	1	1	1	1	1	1	8
Robust Methods									
ES-Absolute	1	1	1	1	1	1	0	1	7
ES-Median	1	1	1	1	1	1	0	1	7
ES-Winsorized	1	1	1	1	1	1	1	1	8
GJRGARCH-MedianL	1	1	1	1	1	1	1	0	7
GJRGARCH-Winsorized	1	1	1	1	1	1	0	1	7
STES Method									
STES-AE	1	1	1	1	1	1	1	1	8
STES-SE	1	1	1	1	1	1	1	1	8
STES-E&AE	1	1	1	1	1	1	1	1	8
STES-E&SE	1	1	1	1	1	1	1	1	8
STES-IndVol	1	1	1	1	1	1	1	1	8
STES-IndVol&AE	1	1	1	1	1	1	1	1	8
STES-IndVol&SE	1	1	1	1	1	1	1	1	8
STES-LnVol	1	1	1	1	1	1	1	1	8
STES-LnVol&AE	1	1	1	1	1	1	1	1	8
STES-LnVol&SE	1	1	1	1	1	1	1	1	8

Table 8:
Composition of remaining models in the Superior Set for eight stock indices using absolute forecast errors

	Amsterdam	Frankfurt	Hong Kong	London	New York	Paris	Singapore	Tokyo	Total Count
Standard Methods									
MA30	0	0	0	0	0	0	1	0	1
ES-Square	0	0	0	0	1	0	1	0	2
GARCH	0	0	0	1	1	0	1	0	3
GJRGARCH	0	1	1	1	1	0	1	0	5
LSTGARCH	0	0	0	0	0	0	1	0	1
ESTGARCH	0	0	0	0	0	0	0	0	0
Robust Methods									
ES-Absolute	0	0	0	0	1	0	1	0	2
ES-Median	0	0	0	1	1	0	1	0	3
ES-Winsorized	0	0	0	0	1	0	1	0	2
GJRGARCH-MedianL	0	1	1	1	1	1	1	1	7
GJRGARCH-Winsorized	0	0	0	0	1	0	0	0	1
STES Method									
STES-AE	1	1	1	1	1	0	1	0	6
STES-SE	0	0	0	1	1	0	1	0	3
STES-E&AE	1	1	0	0	1	0	1	0	4
STES-E&SE	1	1	1	1	1	0	1	0	6
STES-IndVol	0	0	0	1	1	0	1	0	3
STES-IndVol&AE	1	1	0	1	1	0	1	0	5
STES-IndVol&SE	1	1	1	1	1	0	1	0	6
STES-LnVol	0	1	0	0	1	0	1	0	3
STES-LnVol&AE	1	1	0	0	1	0	1	0	4
STES-LnVol&SE	1	1	1	0	1	0	1	0	5