CL 117 Frege, Russell, Wittgenstein
Dr Hilary Greaves
Michaelmas Term 2010

Week 1: Frege's logic and foundations of arithmetic

1 Fregean logic

1. Aristotelian logic (SEP, "Aristotle's logic")
   (a) Aristotle held that all statements either assert or deny that some *predicate* holds of some *subject*. For example:
      i. Plato [S] was a philosopher [P].
      ii. All students [S] are lazy [P].
   (b) Aristotle explored the various two-premise arguments whose premises and conclusion are of such a form, giving names to the valid argument-forms.
      i. For example, the argument-form "All As are Bs. All Cs are As. Therefore all Cs are Bs" is valid, while "All As are Bs. All As are Cs. Therefore all Bs are Cs" is not.
2. Deficiencies in Aristotelian logic
   (a) Frege's key observation: *Not* all statements are of subject-predicate logical form.
      i. Some relations are *two-place*.
         A. "John loves Mary" entails "John loves someone". But if Aristotelian logic merely splits "John loves Mary" into the subject "John" and the predicate "loves Mary", then it cannot predict or explain this entailment.
      ii. Some statements are *quantified*.
         A. Aristotelian logic treats "All men are mortal" and "Socrates is mortal" as having the same logical structure – they are both treated as subject-predicate sentences, where the subject is "All men" or "Socrates" respectively. Frege notes that "All men are mortal" actually has a different logical form.
         B. Things get still more complicated when we have (as later logicians would say) *nested* quantifiers. For instance, "Every horse is an animal" entails "Every head of a horse is a head of an animal." Aristotelian logic would just treat "horse", "animal", "head of a horse" and "head of an animal" as four distinct and unrelated predicates, and hence it cannot predict or explain this entailment. (This is known as "the problem of multiple generality".)

3. The system of the *Begriffschrift* ("conceptual notation")

    (a) Frege's logical system in Begriffschrift is in many ways much like the logic you studied in the first year.

    (b) The main difference is that Frege's system is *second-order*: that is, it has quantifiers and variables ranging over concepts, as well as over individuals. (Your first-year logic was "first order".)

    (c) Frege's system also includes a 'Rule of substitution' and his 'Basic Law V' (on which more later).

2 Fregean logicism

1. Frege wanted to do for arithmetic what Euclid did for geometry: that is, to identify the *primitive* truths from which all of arithmetic can be *proved*.

2. The mathematician's motivations for Frege's project

    (a) One always prefers proof over appeal to "intuition", where this is possible.

    (b) Frege argues that the then-existing attempts to define e.g. the number 3, the number 10000, are hopelessly inadequate. For example:

        i. Abstractionism: (ideas/concepts of?) numbers are arrived at by abstraction from collections, just as concepts of colours and shapes are arrived at by abstraction from objects. (Schroeder, Cantor)

            A. Frege's reply: To *explain how we acquire an idea of* a given object is not to *define* that object.

            B. Diagnosis: Schroeder and Cantor conflate the *logical* with the *psychological* (the characteristic mistake of "psychologism").

        ii. Subjectivism: numbers are subjective ideas (in some sense).

            A. Frege's replies: this would mean that my number one and your number one were distinct; it would make mathematics part of psychology; and it would imply that there are only finitely many numbers.

        iii. The view that numbers are indefinable

            A. Frege's reply: This has no motivation, other than defeatism.

    (c) The provability of "numerical formulae" (e.g. "5+7=12")

        i. Kant says that such formulae are not provable (we are supposed to know each separately, by "intuition").

        ii. But clearly they *are* provable, because

            A. there are infinitely many of them, and "the assumption of infinitely many primitive truths conflicts with one of the the requirements of reason" (*Foundations*, §5);

B. formulae like 135664+37863=173527 are just *not* immediately obvious.

(d) If numerical formulae are provable, what is their proof?

i. Mill (an "empiricist") says that they are proved inductively, from observations of the behaviour of physical collections.

ii. Frege's objections:

A. If one object vanished every time 5 things and 7 things were juxtaposed, that wouldn't make "5+7=12" false.

B. Mill's account can't deal with adding numbers of events, since they cannot in the relevant sense be juxtaposed.

iii. Diagnosis: Mill's mistake is to conflate the *meaning* of an arithmetical proposition with its *applications*.

iv. A promising starting point: Leibniz's proof of "2+2=4" (Foundations, §6)

A. Suppose we define: 2:=1+1, 3:=2+1, 4:= 3+1.

B. Add a single axiom: "when equals are substituted for equals, equals remain."

C. Then, we can reason as follows: 2+2=2+1+1 (by definition of '2')=3+1 (by definition of '3')=4 (by definition of '4').

v. Frege's verdict: This basic idea is right. But it is not a *complete* foundation of arithmetic, because

A. Leibniz needs an extra axiom, viz. the associativity of addition;

B. Leibniz has no definition of zero or one;

C. Leibniz has no definition of the "+1" (successor) operation.

3. The empiricists' motivation for logicism

(a) A priori vs a posteriori knowledge

i. Kant: a true proposition is *a priori* iff no experience is required to judge its truth beyond the experience that is required to understand the requisite concepts (e.g. 2+2=4); otherwise it is *a posteriori* (e.g. Cameron won the election).

(b) The analytic/synthetic distinction

i. Intuitive idea: a true proposition is analytic iff it is "true in virtue of meaning" (e.g. "All bachelors are unmarried", "All men are mortal").

ii. Kant: A true proposition is *analytic* iff the concept of the predicate is contained in the concept of the subject.

iii. Frege: A true proposition is *analytic* iff its proof depends only on general logical laws and on definitions.

(c) The available combinations:

   i. Analytic a posteriori: not possible.
   ii. Analytic a priori: e.g. "all men are mortal", "all bachelors are unmarried".
   iii. Synthetic a posteriori: e.g. "Cameron won the election", "Jones worked as a bricklayer before his accident."
   iv. Synthetic a priori: controversial! *Kant* thought that both arithmetic and (Euclidean) geometry were synthetic *a priori*.

(d) The empiricists' puzzle: *How can there be* any synthetic a priori propositions?
   i. Knowledge via a "faculty of rational intuition" is unacceptably mysterious.
   ii. Frege and relativity to the rescue of the empiricists: there aren't any. Arithmetic is *analytic* a priori (Frege), while relativity is *synthetic* a posteriori (relativity).

4. Frege's foundations for arithmetic

(a) Some Fregean background: Concepts and objects
   i. Frege draws a sharp distinction between concepts and objects. [First-level] *Concepts* include things like *student at Oxford, murderer*, and *green.*
   ii. There is a hierarchy of concepts. *First-level* concepts are concepts under which objects fall. *Second-level* concepts are concepts under which first-level concepts fall. One further has third-, fourth- etc level concepts.
      A. The concept *student at Oxford* falls under the second-level concept *used as an example in Dr Greaves' lectures.*
   iii. Frege holds to every concept F is associated an object – the *extension of the concept F.* (Unofficially: This extension seems to be something like the class containing all and only Fs.)

(b) Frege's definition of number
   i. Frege first defines "the number which belongs to the concept F" (we will write "#F", and say "the number of Fs"):
      A. #F := the extension of the concept *equinumerous to the concept F.*
   ii. This facilitates his definitions of the key notions for basic arithmetic:
      A. *Zero* is defined to be: the number of the concept *not identical with itself.*
      B. "n is the successor of m" is defined to mean: there is a concept F and an object x falling under F such that the number that belongs to the concept F is n and the number that belongs to the concept "falling under F but not identical with x" is m.

C. “n is a natural number” is defined to mean: n stands in the ancestral of the successor relation to zero.

(c) The fruits: From these definitions and Frege's logic, we will be able to *prove* the various general laws that arithmeticians need to appeal to. E.g.

i. the fact that there are infinitely many natural numbers;

ii. the fact that every number has a successor;

iii. the fact that zero is not the successor of any number;

iv. the principle of mathematical induction (i.e. the fact that for any property P, if (i) P holds of zero, and (ii) for all n, if P holds of n then P holds of the successor of n, then P holds of all natural numbers).

Exercises

1. Frege holds that the notion of the extension of a concept is itself a notion of pure logic. But other key notions that are required for his definitions of ‘zero’, ‘successor’ and ‘natural number’ are *not obviously* purely logical. These include the notion of equinumerosity, and the notion of the ancestral of a given relation. Write down definitions of ‘F is equinumerous to G’ and ‘x stands in the ancestral of the relation R to y’ (where R is an arbitrary 2-place relation), in the language of second order logic.

2. Which of the ‘general laws’ quoted above are we already in a position to prove, from the definitions given above and what we know of Frege's logic?

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Week 2: Russell's Paradox and Neo-Fregean Logicism

Some useful references: The articles 'Gottlob Frege' and 'Frege's Logic, Theorem, and Foundations for Arithmetic', both by Edward N Zalta, in the Stanford Encyclopaedia of Philosophy (http://plato.stanford.edu).

1 Russell's Paradox

1. In a letter to Frege just as the Grundgesetze was going to press, Russell pointed out that Frege's system (second-order logic augmented by Frege's "Basic Law V") is actually *inconsistent*.

2. The basic idea of Russell's paradox: Let C be the concept 'being an object x that is the extension of some concept that x does not fall under'. Then, we are forced to admit that the extension of C falls under C iff it does not.

3. Deriving Russell's Paradox *within Frege's logical system*

   (a) Outline of a formal derivation of the paradox within Frege's system:

      i. Let Ψ be the open formula '$\exists\phi(x = \epsilon\phi \wedge \neg\phi x)$'; [by (CC),] let C be the corresponding concept.

      ii. Suppose first that ϵC [whose existence is established by (EE)] falls under C, i.e. that $C(\epsilon C)$.

         A. Since C was defined to correspond to the above formula Ψ, $C(\epsilon C)$ entails $\exists\phi(\epsilon C = \epsilon\phi \wedge \neg\phi(\epsilon C))$. Instantiating the variable ϕ to the concept-constant F, this in turn gives $(\epsilon C = \epsilon F \wedge \neg F(\epsilon C))$.

         B. By Basic Law V (left-to-right), from $\epsilon C = \epsilon F$ we deduce $\forall x(Cx \leftrightarrow Fx)$.

         C. But we also have, from the above, $\neg F(\epsilon C)$. Hence $\neg C(\epsilon C)$; contradiction.

      iii. So suppose instead that ϵC does not fall under C, i.e. that $\neg C(\epsilon C)$, i.e. that $\neg\exists\phi(\epsilon C = \epsilon\phi \wedge \neg\phi(\epsilon\phi))$. In this case ordinary (second-order) logic alone suffices to derive the negation of this supposition:

         A. By (first-order) predicate logic, $\epsilon C = \epsilon C$. Hence we have $\epsilon C = \epsilon C \wedge \neg C(\epsilon C)$.

         B. By (second-order) existential generalisation, $\exists\phi(\epsilon C = \epsilon\phi \wedge \neg\phi(\epsilon C))$.

C. But this just is the condition of application of the concept C to the object ϵC; hence we have $C(\epsilon C)$.

iv. Hence, in Frege's system one can prove both $C(\epsilon C)$ and $\neg C(\epsilon C)$, i.e. the system is formally inconsistent.

(b) Some now-important Fregean background: In addition to (what we now think of as) the standard machinery of second-order logic, Frege's system contained a 'Rule of substitution' and Frege's 'Basic Law V':

Rule of substitution (RS), simplified version: In any statement of the form $\ldots \phi x \ldots$ in which the variable ϕ is free that is derivable as a theorem of logic, we may substitute any open formula $\Psi(x)$ (with the free variable x) for all the occurrences of the atomic formula ϕx in $\ldots \phi x \ldots$.

i. In the context of (the remainder of) second-order logic, (RS) entails a *comprehension principle for concepts*:

Comprehension principle for concepts (CC): For any formula Ψ that has x free and no free ϕs, $\exists \phi \forall x\, (\phi x \leftrightarrow \Psi(x))$. ('For any open formula there exists a corresponding concept'.)

Basic Law V (BLV), concept version: the extension of the concept F is equal to the extension of the concept G iff the same objects fall under F as fall under G, i.e. $\epsilon F = \epsilon G \leftrightarrow \forall x(Fx \leftrightarrow Gx)$.

i. (BLV) entails a principle of *existence of extensions*:

Existence of extensions principle (EE): Every concept has an extension, i.e. $\forall \phi \exists x(x = \epsilon \phi)$.

(c) In the context of (the remainder of) second-order logic, (RS) and (BLV) entail respectively a *Comprehension principle for concepts* and an *Existence of extensions principle*:

(d) A natural first reaction to Russell's paradox is the thought is that the paradox can be avoided by saying either that the offending concept does not exist, or that it has no extension. But (CC) and (EE) rule out these moves.

i. Frege's reduction of arithmetic does need (something like) these principles: they are involved in e.g. the guarantee that *there exists* any object answering to Frege's definition of the number zero, or of the successor of any given natural number.

2 Frege's Way Out

1. In an appendix to the second volume of *Basic Laws*, Frege suggests that a minor modification of Basic Law V may block the paradox. His suggested revised Law is:

2. BLV$'$: $(\forall x(Fx \leftrightarrow Gx) \rightarrow \epsilon F = \epsilon G) \wedge (\epsilon F = \epsilon G \rightarrow ((x \neq \epsilon F \wedge x \neq \epsilon G) \rightarrow (Fx \leftrightarrow Gx)))$.

3. Frege realises that it is not *a priori* obvious that his foundation for arithmetic will still function, once BLV is weakened in this way. But he is hopeful.

4. It is now known that Frege's modified law BLV$'$ is consistent with the remainder of his system, but is not consistent with the remainder of his system *together with* the assertion that there exist more than two objects.

    (a) In particular, Frege would no longer be able to prove the existence of infinitely many natural numbers, in his revised system.

3 Neo-Fregeanism

1. In Frege's reduction of arithmetic to 2OL+RS+BLV, Frege uses BLV to derive "Hume's Principle", *but does not make any further essential appeal to BLV.*

    Hume's Principle (HP): The number of Fs is identical to the number of Gs iff the concepts F and G are equinumerous.

2. "Neo-Humeans" propose jettisoning BLV, adopting HP as a primitive, and keeping the remainder of Frege's reduction ('Frege's Theorem') intact.

    (a) Unlike BLV, the conjunction of HP with 2OL is consistent. (Bibliographic note: George Boolos gives a 'relative consistency' proof in his article 'The consistency of Frege's foundations of arithmetic', cited in Hale and Wright (below).)

3. Why Frege didn't take this line: the "Julius Caesar" problem

    (a) Contextual vs explicit definitions

        i. An *explicit definition* takes the form of an equality, with the term to be defined on the LHS, and (for a non-circular definition) that term not appearing on the RHS. Example: 'A mother is a female parent'.

        ii. A *contextual definition* provides one or more equivalences between phrases/sentences containing the term to be defined and phrases/sentences not containing that term. Example: a contextual definition of 'legal duty' might be 'X has a legal duty to do Y means that X is required to do Y by a contract relationship that would be upheld in a court of law'".

    (b) Abstraction principles

        i. An *abstraction principle* is a biconditional giving the equality of 'two' abstract objects on its LHS, and the holding of an equivalence relation between two different objects on its RHS. For example:

A. Directions from parallelism: The direction of line a is equal to the direction of line b iff a and b are parallel.

B. Ages from birthday-sharing: the age of object a is equal to the age of object b iff a and b came into existence at the same time as one another.

C. Colours from light-reflectance: the colour of object a is equal to the colour of object b iff a and b reflect and transmit the same wavelengths of light.

ii. Abstraction principles entail the existence of the abstract objects referred to on their LHSs.

iii. Frege complained about the use of abstraction principles to *define* the objects referred to on their LHSs, on the grounds that these are contextual definitions and, as such, *do not enable us to eliminate the defined term from* every *context in which it might appear.* It follows that they fail to capture all that we know about the objects being defined. ('One cannot [on the basis of the above definition of direction] decide whether England is the same as the direction of the Earth's axis. ... Naturally no-one is ever going to confuse England with the direction of the Earth's axis; but that is not owing to our definition.' (*Foundations*, §66)) *Frege* infers from this that they are not admissible as definitions.

iv. Hume's Principle is an abstraction principle: thus Frege would similarly complain that it fails to capture all that we know about numbers. ('One cannot [on the basis of e.g. HP] decide whether Julius Caesar belongs to a number concept or not, whether this same well-known conqueror of Gaul is a number or not.' (*Foundations*, §56.))

(c) The neo-Fregean response: *Tu quoque*

i. BLV is an abstraction principle too! So *if* the only thing wrong with BLV is its inconsistency with the remainder of Frege's system, then HP is an acceptable substitute.

4. Objections to (both Fregeanism and) neo-Fregeanism

(a) The 'Bad Company' objection: abstraction principles cannot be analytically true, and therefore (neo-)Fregeanism is 'not really logicism'

i. Background principle: If a given definition is analytically true, then any other definition with the same logical structure and no relevant difference must be analytically true also: e.g.

A. ζ is the limit of the series $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$.

B. ξ is the limit of the series $9, 9\frac{1}{2}, 9\frac{2}{3}, 9\frac{3}{4}, \ldots$.

ii. Objection 1: Clearly not all abstraction principles are analytically true, since BLV is not true at all. In the absence of any relevant characteristic distinguishing HP from BLV, HP cannot be analytically true either. Hence 'neo-Fregean logicism' is not logicism.

iii. Reply to Objection 1: There is a relevant difference, viz. the fact that HP is consistent with 2OL whereas BLV is not.

iv. Responses to this reply:

A. It is debatable whether this difference is *relevant*.

B. Even if it is relevant: There are other abstraction principles that are consistent with 2OL, but inconsistent with 2OL + HP (e.g. Boolos' 'Nuisance Principle'; see p.181 of Hale and Wright, 'Logicism in the twenty-first century', in Shapiro (ed.), *The Oxford Handbook of philosophy of mathematics and logic*, OUP (2005)).

(b) HP cannot be a 'truth of logic', since it entails the existence of infinitely many objects (and hence (neo-)Fregeanism is 'not really logicism')

i. The background thought here is that *logic alone* cannot dictate anything about *what exists*.

ii. It's not clear that this thought is correct where *abstract* objects are concerned.

iii. In any case: if this background principle is accepted, then any logicist reduction of arithmetic is obviously doomed from the outset!

(c) Second-order logic 'is not really logic' (and hence (neo-)Fregeanism is 'not really logicism')

i. Theorem: There is no finite axiomatization of second-order logic that is both sound and complete (unlike first-order logic).

ii. Some people infer from this that second-order logic is not *epistemologically unproblematic*, in the way that first-order logic is supposed to be unproblematic: if we cannot recover all its 'logical truths' by mechanical application of the axioms and inference rules, then we may have to appeal to precisely the sort of 'intuition' that Frege sought to banish from the foundations of mathematics.

(d) Benacerraf's objection: Numbers cannot be the objects they are claimed to be by *any* of various reduction programmes (neo-Fregean and otherwise), precisely because *there is more than one equally viable such program.*

i. E.g. set theorists are fond of 'identifying' the number 0 with the empty set. This too enables them to recover the basic truths of arithmetic.

ii. There do not seem to be any considerations available to make true one of these reductionist proposals in preference to the others.

iii. This is the motivation for 'structuralism' in the philosophy of mathematics.

5. A Hard Question: What exactly *is* it for a given truth to be (or not to be) a 'truth of logic'?

Exercises

1. Show how the Comprehension Principle for Concepts and the Existence of Extensions principle follow from (RS) and (BLV) respectively. (Hint: The SEP is your friend!)

2. Convince yourself that Frege can prove the existence of objects answering to his definitions of zero and the successor of zero *given* the Comprehension Principle for Concepts and the Existence of Extensions principle, but that these proofs do not go through without those principles.

3. Convince yourself

   (a) that Frege can prove, using his *original* BLV and his definitions of zero and of the successor operation, that zero is not identical to the successor of zero (i.e. that $0 \neq 1$);

   (b) that this proof no longer goes through, once BLV is weakened to BLV$'$ following Frege's suggestion.

4. Consider three wildly different concepts, and attempt to use the Existence of Extensions principle to prove the existence of three *distinct* concept-extensions, first in Frege's original system and then in Frege's modified system. Where does your attempt get blocked in the latter case?

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Week 3: Frege on functions, concepts and objects

1 Function and Concept

1. The mathematician's notion of a function (informally): E.g. $2x + 1$, $3x^2$ are functions of x.
   (a) Functions map *arguments* to *values*. E.g. the value of the function $2x + 1$ for the argument 3 is $2 \times 3 + 1$, i.e. the number 7.
2. What *is* a function?
   (a) Frege criticises *formalism*: the claim that a function is an *expression* containing a symbol that can be replaced with names. This is to conflate the *sign* with the *thing signified*.
   (b) Frege's own answer is that a function is something 'incomplete' or 'unsaturated', which when *combined with* an object yields another object. (??)
   (c) Note that Frege's claim (whatever it is!) is *not* that expressions for functions do not themselves refer to anything: such expressions refer to functions. (See 'Letter to Husserl, 24.5.1891')
3. Frege's generalisations of the notion of function
   (a) First generalisation: allow natural-language expressions, as well as mathematical expressions, to designate functions.
      i. Expressions designating functions are arrived at by replacing one or more (object-)names in a sentence or other complex name with variables. E.g. from "the father of Hilary", we can obtain the function-expression "the father of x": this names a function that maps every object to its father ...
   (b) Second generalisation: treat *sentences* with gaps/placeholders as designating functions.
      i. Mathematical example: '$x^2 = 4$'.
      ii. The *values* of these functions are *truth-values*: *The True* for arguments (e.g. 2) that make the sentence true, *The False* for those (e.g. 3) that make the sentence false.
         A. The True and The False are *objects*, according to Frege.
   (c) Both generalisations simultaneously: E.g. from 'Caesar conquered Gaul', we can obtain the function-expressions

i. 'x conquered Gaul': this designates a function that maps an object to The True if that object conquered Gaul, otherwise maps the object to The False.

ii. 'Caesar conquered x': similarly, *mutatis mutandis*.

iii. 'x conquered y': this designates a function that maps an ordered pair of objects to The True if the first object conquered the second; otherwise maps that pair of objects to The False.

(This is one of the keys to Frege's observation that sentences need not be of subject—predicate form.)

(d) Concepts as functions: A *concept* is just a function whose values are always truth-values. (Thus 'x conquered Gaul' and 'Caesar conquered x' both name concepts.)

4. Fregean "grammar"

(a) Frege categorises every meaningful expression as falling into exactly one of the categories object, 1^{st}-level concept, 2^{nd}-level concept, 1^{st}-level function whose values are objects, etc.

(b) Frege insists that whether or not a given string of expressions designates an object, a truth-value, etc, depends *only* on these categories. For example:

i. Any object-name can be paired with any 1^{st}-level predicate to yield the name of a truth-value.

A. Pairing the name "2" and the predicate "$x^2=4$" gives "$2^2=4$"; this names a truth-value (specifically, The True).

B. Pairing the name "Hilary" and the predicate "$x^2=4$" gives "$\text{Hilary}^2=4$"; this looks like nonsense, but, according to Frege, it too is a sentence and names a truth-value (presumably, The False).

C. Pairing the name "2" and the predicate "x is the best university in the world" gives "2 is the best university in the world"; this too looks like nonsense, and here too Frege would insist that it is a sentence and names a truth-value (specifically, The False).

ii. A possible argument against Frege's theory: "$\text{Hilary}^2=4$" is not a meaningful sentence; it is nonsense. But Frege's theory entails that it is a meaningful sentence. Therefore Frege's theory is false.

A. Possible Fregean reply: Frege's theory is a theory of "meaningfulness" in a very particular sense: it is a theory of which expressions have reference. It is not at all obvious that the intuitive sense in which "$\text{Hilary}^2=4$" is "nonsense" corresponds to the theoretical notion of *reference failure* (as opposed to: some other normatively negative theoretical notion).

iii. Any name can be paired with any first-level function-expression to yield another name.

A. Pairing the name "2" and the function-expression "x^2" gives "2^2"; this names an object (specifically, 4).

B. Pairing the name "Hilary" and the function-expression "x^2" gives "$(\text{Hilary})^2$". According to Frege this must be the name of some object, but it is not clear which object!

C. Pairing the name "2" and the function-expression "the father of x" gives "the father of 2"; here too Frege is committed to the claim that this is the name of some object.

iv. Another possible argument against Frege's theory: As observed above, according to Frege's theory 'Hilary^2" must name some object. Whatever this object is, there will be some sentences that Frege's theory deems *true* that (intuitively!) are either false or meaningless. For example, if we arbitrarily stipulate that "Hilary^2" is to designate the moon, the sentence "Hilary^2 orbits the Earth" will have to name The True.

A. This objection is more difficult for the Fregean to answer than the previous one.

B. Possible reply: Frege's theory is not supposed to be a theory of natural language. He is *merely* constructing a formal theory to provide foundations for arithmetic; his discussions of natural language are for purely pedagogical purposes. (But if it is the case, the Fregean should consistently come clean about it!)

(c) Did Frege *have* to back himself into this corner?

i. Frege says that, if not every function can take every object as argument, then "scientific rigour" will be threatened. He seems to have in mind fallacies that mathematicians had fallen into by unwittingly theorising with empty names (e.g., "the limit of the following series...").

ii. At first sight, it seems very odd to suggest that *grammar* is the right way to perform this "guarding" task.

A. "Partial functions": functions that are undefined for some arguments.

B. Frege could have said that the function designated by "the father of x" is *undefined* for arguments that do not have fathers; thus that "the father of 2" fails to name anything *even though* 2 is an object and the arguments of *the father of x* are also objects.

C. Usual practice: Rigor is recovered by *proving* (somehow) that an object must have a father/a square/etc (and only one), before permitting the use of expressions like "the father of Hilary"/"the square of 2".

iii. Frege's remarks are probably motivated by the fact that *in formal logic*, if not every well-formed (i.e. grammatically correct) closed term has a referent, one requires a more complicated axiom system...

2 Concept and Object: The paradox of the concept *horse*

1. How the 'paradox' arises
   (a) Frege insists on a *linguistic* – specifically, a grammatical – criterion for identifying some phrases as names of objects and others as predicates (i.e. as names of concepts):
      i. Concepts are *essentially predicative*, *unsaturated* entities.
      ii. Any expression denoting a concept must be similarly unsaturated, i.e. must contain a gap.
      iii. In consequence, any expression consisting of a *definite* article followed by a common noun can refer only to an object: it cannot refer to a concept.

   (b) This has the prima facie odd result that the reference of the phrase "the concept *horse*" must be an *object*, not a concept.
      i. Frege insists that nothing can be both a concept *and* an object.
      ii. Which object is it? Presumably, the <u>extension</u> of the concept *horse*.

   (c) As a result, Frege is committed to the apparently bizarre claim that the sentence "The concept *horse* is not a concept" is true.
      i. This is "the paradox of the concept *horse*".

   (d) Frege agrees that this is counterintuitive, but he doesn't take it to ground any objection to his system. He regards it as an unavoidable consequence of an "awkwardness of [natural] language".

2. Setting this out as an argument:

   P1 No saturated expression names a concept.

   P2 'The concept *horse*' is saturated.

   C1 'The concept *horse*' does not name a concept. (From (P1) and (P2).)

   P3 'ζ is a concept' is true iff 'ζ' names a [first-level] concept; otherwise it is false.

   C2 'The concept *horse* is a [first-level] concept' is false. (From (C1) and (P3).)

   P4 'ζ is not a [first-level] concept' is true iff 'ζ' does not name a [first-level] concept; otherwise it is false.

C3 'The concept *horse* is not a [first-level] concept' is true. (From (C1) and (P4).)

3. Possible responses to the "paradox"

   (a) Agree with Frege that "the concept horse" names an object and that hence "the concept horse is not a concept" is true, but argue that the resulting "paradox" is in fact unproblematic.

      i. Dummett's suggestion: The [only] reason the "paradox" threatens to be problematic is that it suggests that "it is not possible, *by any means whatever*, to state, for any predicate, which particular concept it stands for". But in fact we *can* do this (albeit in a somewhat roundabout way): we can use the locution "what the predicate 'x is a horse' stands for."

      ii. Objection: The grammatical test will decide that this locution, too, must name an object ...

   (b) Say that Frege should have said that "the concept *horse*" fails to name anything at all, not that it names an object.

      i. Kenny's suggestion: "The concept *horse*" is ungrammatical in the same way that "the verb ' "swims" ' " is ungrammatical [note the two sets of quotes enclosing 'swims'!].

      ii. In that case, "The concept *horse* is not a concept" would be *meaningless*, not true (or false), on Frege's theory.

      iii. Objection: It's not clear that these cases *are* relevantly analogous: more needs to be said about how the context 'the concept ... ' functions.

   (c) Abandon the definite-article test for names of objects, and hold that "the concept *horse*" names a concept after all.

      i. In which case "The concept *horse* is not a concept" is false after all.

      ii. Parsons' suggestion: This is what Frege should have done. Frege had already abandoned the definite-article completeness test for examples like "The whale is not a mammal", hence had already admitted that that test was violable, hence it would be no great cost to admit another violation.

      iii. Objection: It is not clear that we can get any handle on the distinction between names and predicates, and the related distinction between objects and concepts, if not via the grammar of natural languages.

   (d) Agree with Frege that "the concept horse" names an object, but argue that in that case, "is not a concept" must in this sentence name a *first*-level concept: the concept that all and only *proxies for* concepts fall under.

      i. In which case "The concept horse is not a concept" is again false.

ii. Parsons: This is what Frege should have done *if* he insisted on sticking to the definite-article test for completeness.

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Week 4: Frege on sense and reference

1 The motivation for the sense/reference distinction

1. The sense/reference distinction is first mentioned in *Function and Concept*.
   (a) There, Frege had proposed that the following expression is (not only a grammatically correct sentence, but also) true: "(2^2=4)=(2>1)".
      i. Rationale: "2^2=4" denotes the truth-value True. So does "2>1".
   (b) Frege considers the following objection: "(2^2=4)" and "(2>1)" express different thoughts, therefore it is incorrect to say that they are equal.
   (c) His response is that "(2^2=4)" and "(2>1)" do indeed express different thoughts, but that we must separate what a sentence *expresses* from what it *designates/denotes/refers to*.
      i. A sentence has a 'sense' (Sinn) and a 'referent' (Bedeutung). Its sense is the thought it expresses. Its referent is its truth-value.
      ii. It is referents, not senses, that are relevant to the truth of identity claims.
2. In "Sense and reference", essentially the same point is presented as the *puzzle of cognitive value*: if identity is a relation between objects, and it is known to be the relation which every object bears to itself and to no other object, then how can a true identity statement of the form "a=b" (e,g,"Hesperus is Phosphorus") be more informative than the obviously uninformative "a=a" ("Hesperus is Hesperus")?
   (a) In BS, this puzzle had motivated Frege to hold that identity was not a relation between objects, but rather a relation between signs.
   (b) In SR, Frege abondons the BS view, and proposes instead that we must distinguish the 'sense' from the 'referent' of a proper name.
      i. The referent of a name is the name's bearer (e.g. Venus).
      ii. The sense of a name is the names's 'mode of presenting' its referent.
      iii. An identity statement is *true* if the two names have the same *referent*; it is *cognitively non-trivial* if the two names have distinct *senses*. True but cognitively non-trivial identity statements are possible because the sense-reference relation is many–one (i.e. sense determines reference, but not vice versa).

2 Sense, tone/colour/shading and Ideas

1. Three types of 'thing' associated with a linguistic expression:
   (a) The associated "Idea" (Vorstellung) is something in an individual person's mind.
   (b) The "tone" or "colour" or "shade" of an expression is (something like) the constellation of feelings and associations that it elicits in the speaker/hearer.
   (c) The sense of an expression is neither of the above.
2. Distinguishing senses from Ideas
   (a) Ideas are "subjective" in the sense that they are "parts" or "modes" of a person's mind, and thus could not exist if that mind did not exist. In contrast, senses are mind-independent, objectively existing entities.
   (b) It is not possible that two people could have "the same Idea". (Aside: it's not completely clear whether Frege is talking here about *qualitative* identity, or *numerical* identity.) In contrast, it is essential that two people *can* grasp the same sense, otherwise communication would be impossible (it would not be possible that "mankind has a common store of thoughts which can be transmitted from one generation to another" — p.154 in Beaney).)
   (c) The telescope analogy: moon $\sim$ referent, image projected on glass inside telescope $\sim$ sense, retinal image of observer $\sim$ Idea (p. 155).
3. Distinguishing sense from tone/colour/shading
   (a) Tone/colour/shading need not be preserved in translation. In contrast, preservation of sense is the criterion of correctness of translation. E.g.
      i. "Je t'adore" vs "I love you"
      ii. Russian symbolist literature ...

3 The sense/reference distinction and opaque contexts

1. *Principle of substitutivity of coreferentials salva veritate (PSC)*: Let A, B be subsentential expressions, and let $S1, S2$ be sentences. Then if A and B have the same referent as one another, and S2 is obtained by S1 by substituting B for A, then S1 and S2 have the same truth-value as one another.

2. This principle seems to hold in most cases. But in some contexts — so-called “opaque” or “intensional” contexts — it *seems* to fail [on the ‘de dicto’, as opposed to the ‘de re’, reading of the sentences concerned]. For example:

   (a) Belief contexts:

   S1 Lois Lane believes that <u>Superman</u> can fly.

   S2 Lois Lane believes that <u>Clark Kent</u> can fly.

   (b) Indirect speech:

   S1 John said that <u>the guy who scheduled these lectures before midday</u> is an idiot.

   S2 John said that <u>James</u> is an idiot.

   (c) Modal contexts:

   S1 Necessarily, <u>eight</u> is eight.

   S2 Necessarily, <u>the number of planets</u> is eight.

3. *Frege holds that PSC is universally valid.*

4. Frege’s account: in so-called “opaque” contexts, the *reference* of words is their customary sense (i.e. what would be the senses of the names, in an ordinary non-opaque context).

4 Some things to worry about

4.1 Application of the sense/reference distinction to sentences

1. As we’ve seen, Frege insists that the notions of sense and reference apply to *sentences*, as well as to sub-sentential expressions like names and predicates. In particular, he insists that the referent of a sentence is its truth-value: The True or The False.

2. Many people find this counterintuitive. Intuitively, sentences do not “stand for” truth-values in the same way that names “stand for” their bearers.

3. Frege does have an *argument* for applying the notions of sense and reference to sentences. Roughly, his idea is that since we are interested in determining the *referents* of subsentential expressions when and only when we are interested in determining the *truth-value* of the sentence (e.g. when doing science, but not when engaged in literature appreciation), it follows that the truth-value of the sentence is the referent of the sentence.

4. This is probably not a good argument — it has several highly theoretical premises for which Frege supplies no motivation. [Exercise: spell out this argument of Frege’s as precisely as you can, adding any implicit premises that are required to render the argument valid. The relevant extract from ‘On sense and reference’ is pp.156–7 in Beaney.]

5. But a bad argument can have a true conclusion, and it is not clear that the charge of counterintuitiveness is a good objection ...

4.2 Unicity of sense

1. Frege talks of 'the' sense of a proper name. But is it obvious that to a given name there corresponds only one sense?
2. In a footnote in 'On sense and reference' (p.153 in Beaney), Frege writes:

   > In the case of an actual proper name such as 'Aristotle' opinions as to the sense may differ. It might, for instance, be taken to be the following: the pupil of Plato and teacher of Alexander the Great. Anybody who does this will attach another sense to the sentence 'Aristotle was born in Stagira' than will a man who takes as the sense of the name: the teacher of Alexander the Great who was born in Stagira. So long as the thing meant remains the same, such variations of sense may be tolerated, although they are to be avoided in the theoretical structure of a demonstrative science and ought not to occur in a perfect language.

3. But recall Frege's insistence on the need for senses to be intersubjectively shared. If (as seems plausible) you and I attach different senses to the name 'Aristotle', (how) do we succeed in communicating, when we take ourselves to be discussing Aristotle?

4.3 Sense without reference/what *is* a sense?

1. Frege says that an expression's sense is its mode of presenting its referent. But he also says that an expression can have a sense but no referent (e.g. 'Odysseus'; 'the celestial body most distant from the Earth'). Is this consistent?
   (a) Response 1: No; the notion of Fregean sense without reference is incoherent.
      i. In that case, Frege is unable to explain how sentences putatively about, e.g., Odysseus/the celestial body most distant from the Earth/the present King of France are even *meaningful*.
   (b) Response 2: Yes; the locution 'mode of presentation of a referent' serves a merely pedagogical purpose, and is not to be taken literally.
      i. In that case, what *is* a sense?
      ii. One answer: the sense of an expression is its 'intension' — a *function from possible worlds to referents*.

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Week 5: Russell's theories of types

A useful overview of this topic: Irving M. Copi, *The theory of logical types*, Routledge and Kegan Paul (1971). See also Russell, *Introduction to Mathematical Philosophy*, esp. chapter 13 (page references below are to the 2008 Spokesman edition.)

1 Preamble: Russell on number

1. Russell, like Frege, is keen to reduce arithmetic to logic.
2. Russell's definition of number is similar to Frege's. The difference is that whereas Frege's theory uses concepts and extensions of concepts, Russell talks explicitly of classes and class-membership:
   (a) The number of a class is the class of all classes that are similar [i.e. equinumerous] to it. (IMP, p.18)
   (b) 0 is the class whose only member is the null-class.
   (c) The successor of the number of terms in the class α is is the number of terms in the class consisting of α together with x, where x is any term not belonging to [α]. (IMP, p.23)
3. Thus Russell's theory, like Frege's, is susceptible in principle to 'Russell's paradox'.
4. Russell's proposal for solving this paradox (and others) is 'type theory', developed initially in Appendix B of his 1903 *Principles of Mathematics*, and further in his 1908 paper 'Mathematical logic as based on the theory of types'.

2 The paradoxes and the Vicious Circle Principle

1. Russell saw 'Russell's paradox' as just one of a family paradoxes with a common cause, calling for a common solution. For example:
   (a) The Liar (Epimenides) paradox: "This statement is false."
      i. Is this statement true, or false?
   (b) Russell's paradox (classes): The class of all classes that are not members of themselves.

    i. Is this class a member of itself?

(c) Russell's paradox (relations): The relation T defined by: T(R,S) iff not R(R,S).

    i. Do we have T(T,T)?

(d) Berry's paradox: The least integer not defineable in fewer than twenty syllables

    i. Is it defineable in fewer than twenty syllables?

(e) Richard's paradox: Let E be the class off all decimals that can be defined in a finite number of words. Then, a finite-length diagonal description can be used to define a decimal not in E.

2. Russell himself thought that these paradoxes all arise for the same reason: *self-reference*. He therefore sought to develop a system obeying "the vicious circle principle", which system would be free from such paradoxes.

    (a) Russell gives several statements of 'the' vicious circle principle:

        i. "No totality can contain members defined in terms of itself" [Russell, MLTT:237].
        ii. "If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total" [Whitehead and Russell, PM].
        iii. "Whatever contains an apparent variable must not be a possible value of that variable." [Russell, MLTT:237]
        iv. "Whatever involves *all* of a collection must not be one of the collection" [Whitehead and Russell, PM].

    (b) These formulations are quite vague. See STT vs. RTT for precisification ...

3 Simple Type Theory (STT) (Russell 1903)

1. Every object, property, relation and function (equivalently: every name, predicate, relation-symbol and function-symbol) has a *type*:

    (a) Individuals/names of individuals (i.e. individual-constants) have type 0
    (b) Properties of individuals/sets of individuals/expressions denoting such entities have type (0)
    (c) Binary relations between individuals/sets of ordered pairs of individuals/expressions denoting such relations have type (0,0)
    (d) Binary relations whose first argument is an individual and whose second argument is an entity of type (0)/expressions denoting such expressions have type (0, (0)) ...

(e) Generally: If entities/expressions A_1, ..., A_n have types t1, ..., tn, and G is an entity/expression capable of taking A_1, ..., A_n as inputs, then G has type $(t_1, ..., t_n)$.

2. STT avoids the 'Russell paradoxes':

   (a) Blocking Russell's paradox for classes: if x has type t, then we are grammatically permitted to write $x \in y$ only if y has type (t). Hence we cannot even *ask whether* any set is or is not a member of itself.

   (b) Blocking Russell's paradox for relations: Every relation is of higher type than its relata. Hence the expression "T(T,T)" cannot be a wff, whatever "T" may be. So, again, STT judges it meaningless to ask whether or not any relation does or does not relate itself to anything.

3. But Ramsey (1926) pointed out that the paradoxes are actually of two quite distinct types: *logical* and *semantic* paradoxes.

   (a) Semantic paradoxes: involve notions of truth, nameability, defineability

      i. Examples: Liar paradox, Berry's paradox, Richard's paradox

   (b) Logical paradoxes: arise even in ("purely logical") systems that do not have expressions for the notions of truth, nameability, defineability

      i. Examples: Russell's paradoxes, Burali-Forti's contradiction

4. Simple Type Theory does *not* block the semantic paradoxes.

4 Simple type theory: objections and replies

1. Simple type theory does not block the semantic paradoxes!

   (a) Reply: This doesn't prevent the theory from providing a foundation for *mathematics* — for that purpose, we can just refrain from talking about semantics at all.

2. In STT (and in contrast to, e.g., Frege's system), we have to *assume* an axiom of infinity in order to derive arithmetic (specifically: in order to prove that any two distinct natural numbers have distinct successors)). But the axiom of infinity is not a logical truth. So adopting STT amounts to abandoning logicism.

   (a) Reply 1 (Ramsey): Those parts of arithmetic that depend on the existence of infinitely many numbers are interesting only if there are infinitely many objects; so, it's unobjectionable to assume AI for the purposes of arithmetic even if AI is not a logical truth.

   (b) Reply 2 (Godel, Carnap): We can take the set of all natural numbers as the lowest type.

        i. But this amounts to abandoning logicism.

    (c) Reply 3: *Every* foundation for arithmetic will require an Axiom of Infinity (in some sense), so, it's no objection to STT that STT does too.

3. STT is "self-contradictory", in the sense that: any statement of STT violates the restrictions imposed by STT.

    (a) Reply (Wittgenstein): The Simple Type Theorist doesn't need to *state* the restrictions imposed by STT. She can just *speak* a language that does not violate them.

5 Ramified Type Theory (RTT) (Russell, 1908)

1. RTT is an extension of STT. Hence, RTT blocks the logical paradoxes in exactly the same way as STT. But RTT also imposes additional restrictions that block the semantic paradoxes.

2. The basic idea: Some simple examples

    (a) "Napoleon had all the qualities of a great general."

        i. To formalise this in predicate logic, we could write Pn (Px: x has all the qualities of a great general; n: Napoleon).

        ii. But this predicate P is not a *primitive* predicate: it could be defined as $Px \equiv \underline{\forall\phi}\,(\forall y(Gy \to \phi y) \to \phi x)$ (Gy: y is a great general), where the predicate-variable ϕ ranges over all *primitive* predicates.

            A. Russell will say that P has *order 2*, while primitive predicates have *order 1.*

            B. (If we're using superscripts to indicate types,) we can use subscripts to indicate orders: then our definition of P becomes $P_2x \equiv \forall\phi_1\,(\forall y(G_1y \to \phi_1y) \to \phi_1x)$

    (b) "George is a typical Englishman."

        i. Similarly: we could write Tg (Tx: x is a typical Englishman; g: George).

        ii. A 'paradox': a 'typical Englishman' is presumably (by definition) something like: an Englishman who has all the properties that are possessed by most Englishmen. But there cannot be many Englishmen who have *that* property. Does this mean that a typical Englishman is untypical?

        iii. Ramification to the rescue: a 'typical Englishman' is an Englishman who has all the *primitive*, or *first order*, properties that are possessed by most Englishmen. Typicality is a *second*-order property:

A. T_2x (definition): $\underline{\forall\phi_1}(\phi_1$ is possessed by most Englishmen $\rightarrow \phi_1 x)$

3. Formalisation of the idea: Each predicate has a certain *order*:
   (a) Primitive predicates are order 1.
   (b) Predicates whose definitions involve quantification over order-1 predicates are order 2...
   (c) Predicates whose definitions involve quantification over order-n predicates are order n+1.

4. This hierarchy of orders builds in restrictions that prevent expression of the semantic paradoxes.
   (a) Berry's paradox
      i. To formulate the paradox in the notation of formal logic, we must *define* the predicate 'x is definable [in fewer than twenty syllables]'.
      ii. Intuitively: "x is definable [in a certain way]" means "**there exists** a predicate G such that G defines x [and G has certain further features]".
         A. I.e. we need to **quantify over predicates**.
      iii. So ramification blocks the paradox: 'the least integer not defineable$_n$ in fewer than twenty syllables' is a definition of order $n+1$, *not* a definition of order n. [Exercise (medium-hard): make this rigorous, i.e. introducing definitions and premises as required, show how a contradiction can be formally derived in the absence of ramification, and where ramification blocks that derivation.]
   (b) Richard's Paradox is precisely analogous to Berry's paradox. [Exercise; easy if you've already done Berry's!]
   (c) The Liar paradox: See Copi p.81 (and judge for yourself!).

6 The Axiom of Reducibility

1. As formulated so far, RTT is (it turns out) too weak to recover certain large parts of mathematics.
   (a) Example: It turns out that we cannot prove in RTT that every bounded set of real numbers *of a given order* has a least upper bound *of that same order*.
   (b) Terminology: Theories conforming to the strictures of RTT are said to be *predicative*.

2. Russell's solution: Introduce the *Axiom of Reducibility* (AR):

Axiom of reducibility (AR) : For any predicate, there is an extensionally equivalent first-order predicate.

3. This facilitates the recovery of the desired parts of mathematics, *but* (surprisingly?) does not remove the theory's ability to block the semantic paradoxes. [Exercise: think through why not, in the case of e.g. the Berry paradox.]

7 Criticisms of RTT, and replies

1. RTT, like STT, outlaws completely general statements, and (relatedly) cannot be stated in a way that obeys its own rules.

2. The only motivation for introducing orders is to avoid the semantic paradoxes. But these are no business of logic anyway.

   (a) Possible reply: They *are* the business of logic; logic is concerned with the laws of thought quite generally.

3. The Axiom of Reducibility is not a logical truth. Hence, adopting it amounts to abandoning logicism.

   (a) Russell: Quite so. We must hope that with further work, we can "arrive at a doctrine of classes which does not require such a dubious assumption" (IMP, p.193).

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Week 6: Russell on logical constructions and definite descriptions

1 Logical constructions

Russell, 'Our Knowledge of the External World' (OKEW), esp. chapter 3. (Page references herein are to the Routledge 2009 edition.)

1. Background considerations

   (a) Reason vs experience: arch-rationalism, arch-empiricism and the happy medium

   (b) The bar for justification: setting this too high, too low, and the happy medium

   (c) While very little is actually *certain*, still some beliefs are *more nearly* certain, more directly 'given', than others. These 'data' include beliefs about: particular facts of sense, general truths of logic, some facts of memory, some introspective facts, the existence of other minds, that physical objects continue to exist unperceived, similar facts reported via others' testimony.

   (d) Russell's project: starting from such data, admit such relaxations of standard and such unprovable principles as are *required* to recover most of common sense and science, but no more.

2. A hypothetical construction for our consideration: The "system of perspectives"

   (a) Heuristically: There is one *perspective* corresponding to each triple <point of space, instant of time, spatial direction> - roughly, the view that *would be* seen by a person located at that point of space, at that instant of time, looking in that direction.

   (b) We are initially given *only* this collection of perspectives.

      i. Each individual perspective contains a "private space", consisting of the spatial relations between the things in that perspective.

   (c) But certain relations of *similarity* obtain between pairs of perspectives. On the basis of these, we can construct another space — 'perspective space':

      i. If we treat the similarity relations obtaining between perspectives as spatial distance relations (where very similar perspectives are regarded as being a very short distance apart), we will thereby construct a three-dimensional space, each point of which is itself an infinite collection of perspectives (*heuristically*: all the perspectives that are views from a given point of ordinary three-dimensional space).

   ii. This constructed three-dimensional perspective space is to be *identified with* the ordinary space of common sense and of physics.

(d) Similarly, we can construct ordinary objects ("things"):

   i. Russell seems to take it for granted that a given perspective can be carved up into "sensible objects" (aka "aspects" of ordinary objects e.g. tables, pot plants) in a natural way.

   ii. These sensible objects also bear similarity relations to one another.

   iii. A class of sensible objects linked by relations of similarity is *identified* with an ordinary object ("thing"). Russell's example: a penny.

      A. We have an infinite collection of perspectives, some containing a circular sensible object [a penny viewed from above or below], containing a rectangular sensible object [a penny viewed from the side], and many containing an elliptical sensible object [a penny viewed from any other angle].

      B. We notice that these form a collection linked by a continuous series of similarity relations.

      C. As a result, we collect all the sensible objects in question into one class, and call the resulting 'logical fiction' an object.

(e) We want to talk of e.g. tables being located at particular points in space. But this requires correlating the private space of a single perspective with perspective space. How is that to be done? The penny again:

   i. The perspectives containing circular penny-aspects lie along a line, arranged from smallest to largest.

   ii. Similarly for the perspectives containing rectangular penny-aspects.

   iii. "These two lines will meet in a certain place in perspective space, i.e. in a certain perspective, which may be *defined as* 'the place (in perspective space) where the penny is."' (OKEW, p.73; emphasis added)

(f) We are now in a position to make sense of the majority, if not all, of our common-sense talk of ordinary objects:

   i. "The chair is 3 metres from the table."

   ii. "One's private world is inside one's head."

   iii. "The aspect which a given thing presents at a given place is affected by the intervening medium."

iv. Define "here" as the place in perspective space occupied by our [the speaker's] private world. Then we also can make sense of talk of things being near to or far from here.

(g) Russell's reflections on the status of this construction

i. The construction is *consistent with* our "hard data".

ii. It is not *logically required by* (i.e. strictly inferable from) our hard data.

iii. But good epistemology recommends believing that the world really is like this: "When some set of supposed entities [e.g. everyday physical objects] has neat logical properties, it turns out, in a great many instances, that the supposed entities can be replaced by purely logical structures composed of entities which have not such neat properties... This is an economy... The principle may be stated in the form: 'Whenever possible, substitute constructions out of known entities for inferences to unknown entities'." (Russell, *The Philosophy of Logical Atomism*; p.160 in the 1985 Pears edition.)

2 Russell on Descriptions

Russell, *On Denoting*, 1905; see also his *Introduction to Mathematical Philosophy* (1919), ch. 19.

2.1 Historical background to Russell's 1905 paper "On denoting"

1. Russell's paper "On denoting" is written in opposition to Russell's own earlier views concerning "denoting phrases" (as expressed in his 1903 book *Principles of Mathematics (PoM).*)

2. "Denoting phrases" are phrases in which only words for *concepts* appear, but that nevertheless somehow succeed in picking out *objects*: e.g. "all men", "a man", "the man who bought the first ticket".

   (a) Something needs to be said about

      i. the details of how they do that;

      ii. *which* object each such phrase denotes.

2.2 Grammatical form vs. Logical form

1. The innovation of "On denoting" centres on the distinction between *grammatical form* and *logical form.*

   (a) The *grammatical form* of a sentence is the structure that builds it up in accordance with the rules for grammatical well-formedness.

(b) The *logical form* of a sentence is the structure that exhibits its inferential relationships to other sentences.

(c) The distinction between grammatical and logical form is required to explain why e.g. the following argument is not valid:

   i. This dog is yours.

   ii. This dog is a father.

   iii. Therefore, this dog is your father.

2. Russell's point is that the grammatical and logical forms of sentences containing "denoting phrases" are distinct.

   (a) A *propositional function* is a function whose values are propositions [NB not truth-values!].

   (b) The logical form of "All men are mad" is "The propositional function 'if x is a man then x is mad' is always true."

   (c) The logical form of "Some man is mad" is "The propositional function 'if x is a man then x is mad' is sometimes true."

2.3 The 1905 theory of "the": Russell's theory of definite descriptions

1. The theory: The logical form of "The F is G" is "Exactly one object is F, and that object is G".

   (a) Example: The logical form of "The King of France is bald" is "Exactly one object is a King of France, and that object is bald."

2. This theory permits a straightforward explanation of what is going on in several otherwise puzzling cases:

   (a) Empty descriptions: "The King of France is bald" is *meaningful*, even though there is no King of France. (No need for Meinongian 'non-subsistent entities' or Fregean 'senses'.)

   (b) Cognitively nontrivial identity statements: "The morning star is the evening star" means "one and only one object is a morning star, and that object is an evening star, and no other object is an evening star". (No need for Fregean senses.)

   (c) Apparent failures of the principle of substitutivity of coreferentials salva veritate:

      i. How can e.g. "George IV believes that Scott is Scott" and "George IV believes that Scott is the author of Waverley" differ in truth-value?

      ii. Russell's answer: this is not a case of substituting coreferentials, since "the author of Waverley" is an incomplete symbol (does not have a referent on its own). (Again, no need for Fregean senses.)

3. Epistemological motivations for Russell's theory of descriptions
    (a) How can I know anything about – how can I even succeed in thinking about – a person or object with which I am not *acquainted*?
    (b) TBC!

2.4 Russell's arguments against rival theories

1. Meinong's theory
    (a) According to Russell, Meinong interprets any grammatically correct denoting phrase as standing for an object (including such "empty" denoting phrases as "the present King of France").
    (b) Thus Meinong avoids the problem of empty descriptions, since, according to his theory, *there aren't any.*
    (c) Meinong thus has available to him a very straightforward semantics for discourse involving "empty" descriptions: such descriptions can straightforwardly *refer to objects*, in the same way that ordinary names are generally thought to.
2. Russell against Meinong
    (a) Preliminary: according to Russell, Meinong has it that the objects referred to by "empty" descriptions may not "subsist", but they "are objects". Russell seems to read the latter as implying that these objects "exist".
    (b) Russell's objection: Meinongian objects "are apt to infringe the law of contradiction":
        i. The existent present King of France exists, and also does not exist.
        ii. The round square is round, and also not round.
3. Frege's theory
    (a) Sentences involving empty descriptions have a sense, but no reference.
4. Russell against Frege
    (a) This means that one of the fundamental principles of logic, the Law of Excluded Middle, fails on Frege's theory. This is to be avoided if possible.

CL 117 Frege, Russell, Wittgenstein
Dr Hilary Greaves
Michaelmas Term 2010

Week 7: Russell on knowledge by acquaintance/knowledge by description

Russell, Knowledge by Acquaintance and Knowledge by Description, *Proceedings of the Aristotelian Society New Series* v.XI 1910-11: pp108-128. Reprinted in *Mysticism and logic* [but with different page numbering].

1 Acquaintance – a first pass

1. Acquaintance is a *binary relation* between subject and object: "Subject S is acquainted with object O."
2. On *which* objects a given subject is acquainted with:
   (a) "I say that I am acquainted with an object when I have a direct cognitive relation to that object, i.e. when I am directly aware of the object itself." (Russell, KA&KD, p.108)
   (b) Intuitive *intial* suggestion: we are acquainted with people we have met and ordinary objects that we have come into 'direct contact' with (e.g. the Principal of my College, this table).
3. The alternative: Knowledge of objects 'by description'
   (a) "Say that we have *merely* descriptive knowledge of the so-and-so when, although we know that the so-and-so exists, and although we may possibly be acquainted with the object which is, in fact, the so-and-so, yet we do not know any proposition "A is the so-and-so," where A is something with which we are acquainted." (Russell, KA&KD, p.113) E.g.:
   (b) I know that the student who scores the highest marks in PPE Finals will get a first, but I don't know which student this is. So I have knowledge merely by description of the student who will score the highest marks in PPE Finals.
   (c) I know *that the Prime Minister studied PPE at Oxford*, but I am (probably) not acquainted with David Cameron. In that case, even though I also know *that David Cameron is the Prime Minister*, I have knowledge merely by description of the Prime Minister.
4. Some background: Russellian propositions

(a) According to Russell, propositions are complex (non-linguistic) entities built from objects, properties, relations etc, just as sentences are complex (linguistic) entities built from names, predicates, relation-expressions etc.

2 The Principle of Acquaintance

1. The *Principle of Acquaintance*: "Every proposition which we can understand must be composed wholly of constituents with which we are acquainted." (KA&KD, p.117)

   (a) This principle states the semantic [*not* epistemological!] role of acquaintance.

2. Russell's motivation for this principle is that "This is merely to say that we cannot make a judgment or a supposition without knowing what it is that we are making a judgment or a supposition about... It seems to me that the truth of this principle is evident as soon as the principle is understood."

   (a) (*Is* it?)

3. This entails that ordinary sentences don't wear their propositional structures on their sleeves: they must generally be riddled with abbreviations. This in turn leads to a project of *analysis*: making the structure of the proposition explicit.

4. Example: "The Queen visited Cheltenham Races last year."

   (a) Intuitively, I can understand [the proposition expressed by] this sentence.

   (b) However, I am not *acquainted with* the Queen (I haven't met her), or the Cheltenham Races (I haven't been to them). Thus, by the Principle of Acquaintance, neither the Queen nor the Races can be a constituent of the proposition that is expressed when I utter that sentence.

   (c) However, various expressions in the above sentence can be understood as abbreviations for definite descriptions made up of universals that I am acquainted with:

      i. "The Queen" may be a shorthand for "the present queen of England"; I am (perhaps?) acquainted with queenhood, presentness, and England...

      ii. "Cheltenham" may be shorthand for "the largish town on the A40 between Oxford and Gloucester"; I am (perhaps?) acquainted with size, townhood, the A40, Oxford and Gloucester...

iii. "Races" may be short for "event at which horses race and people bet money"; I am acquainted with horsehood, racehood, personhood, betting and money...

3 Acquaintance with universals

1. It is *relatively* straightforward to say what acquaintance with an *object* might consist in: e.g. having met that object.
2. What is acquaintance with universals supposed to consist in?
   (a) Russell doesn't tell us.
   (b) Suggested answer: Acquaintance with a universal consists in acquaintance with some object that instantiates that universal. (?)

4 A stricter account of acquaintance

1. A residual worry with the intuitive notion of 'acquaintance' that we have worked with so far: even the notion of "having met" is too vague to carry the weight that anything like a Russellian doctrine of acquaintance seems to require.
   (a) Have I "met" people if I have talked to them only over Skype? If I've received phone calls from? Letters? If I've seen them on TV/read their articles in the paper? If I've been affected by a storm that (as chance would have it) was caused by them wiggling their little finger?
2. When Russell moves on from introductory exposition and states his official doctrine of acquaintance, he insists that we are not in fact acquainted with ordinary objects, or other people at all. Rather, we are acquainted with
   (a) Our own sense-data, and parts thereof;
   (b) Ourselves (perhaps);
   (c) Universals (i.e. properties and relations).
3. Russell's own motivation for this insistence is apparently (??) that, since acquaintance is supposed to be a relation between the subject and the object, acquaintance with O seems to preclude any possibility that the O does not exist
   (a) If so, it is not a good argument! I cannot be *causally affected by/10 metres from* some object unless that object exists either, but it does not follow that I cannot be causally affected by/10 metres from anything of whose existence I am not certain.)
4. A better motivation might be that the resulting theory avoids the unprincipled line-drawing that we complained of above.

5. Notice that combining the Principle of Acquaintance with this stricter doctrine of which objects we are acquainted with changes the project of analysis: the propositions we are able to express now cannot contain any particulars except (ourselves and) *our own sense-data*.
    (a) Thus Russell's theory of how ordinary objects, people etc might be defined in terms of sense-data (cf. 'Logical constructions', from last week) has a crucial role to play.

5 The description theory of names

1. Since we are not acquainted with other people, cities, ordinary physical objects, etc, it follows (by the Principle of Acquaintance) that our apparent names for such objects cannot really be names, in the sense of expressions whose semantic contribution to the proposition expressed is literally the object in question – we would not be able to understand any proposition of which such objects were really constituents.
2. It follows *(inter alia)* that we are unable even to express many of the propositions that we would intuitively like to assert: "[W]hen we say anything about Bismarck, we should like, if we could, to make the judgment which Bismarck alone can make, namely, the judgment of which he himself is a constituent. In this we are necessarily defeated..." (KA&KD, p. 116)
    (a) Bismarck himself can *express* the proposition of which he is a constituent.
    (b) The rest of us can only *describe* that proposition.
3. Then what *is* going on when other people use the apparent-name 'Bismarck'? Answering this question leads Russell to the *description theory of names*: the theory that ordinary proper names – like "Bismarck", in the mouth of anyone but Bismarck – are abbreviations for definite descriptions.
    (a) All I know about Bismarck is that he was a 19^{th} century statesman, and that his name was "Bismarck". So when I use the name "Bismarck", this name is an abbreviation for the definite description "the C19 statesman named 'Bismarck"'.
    (b) Suppose you are generally a better historian than I am, but you can't remember when Bismarck lived. Then, in your mouth, "Bismarck" may be an abbreviation for e.g. "The Prussian prime minister who oversaw the unification of Germany and was named 'Bismarck"'.
4. Russell on logically proper names
    (a) "Logically proper names": names that are genuinely names, i.e. that really contribute the name-bearer itself to the proposition expressed.

(b) "There are only two words which are strictly proper names of particulars, namely, "I" and "this"." (KA&KD, p.121)

(c) It's not clear that Russell really wants to be quite as austere as this (cf. p. 109).

6 Knowledge of things vs knowledge of truths

1. Russell says that the considerations he is discussing belong to "epistemology". But so far it has been semantics rather than epistemology – we have exclusively been discussing the question of which propositions we can *understand.* Genuinely epistemological considerations have entered only in the discussion of which objects we should take ourselves to be acquainted with.

2. Russell himself (in KA&KD) does not say anything about what his distinction between knowledge by acquaintance and knowledge by description has to do with propositional knowledge, i.e. knowledge-that rather than knowledge-of.

   (a) In particular, he does not explain *how* we are supposed to attain knowledge description of any object (recall that this requires that we know *that* one and only one object possesses a given property).

7 Objections

1. Objection 1: Given his insistence that we are acquainted with relatively few particulars (only sense-data etc), Russell cannot account for *de re* propositional-attitude attributions in which the object that someone is said to have a de re attitude towards is another person or an ordinary physical object.

   (a) We ordinarily distinguish between a *de dicto* and a *de re* reading of such propositional-attitude ascriptions as "George IV wished to know whether the author of Waverley was Scott."

      i. *De dicto* reading: George IV wished to know whether or not the following is true: Scott wrote Waverley and no-one else wrote Waverley.

      ii. *De re* reading: George IV wished to know, of man who is in fact the author of Waverley, whether *he* was Scott.

   (b) On the *de re* reading of the attitude attribution, what George IV is said to have wished to know is a *singular proposition* about Scott, i.e. a proposition of which Scott himself is a component.

   (c) But (recall) according to Russell, such singular propositions cannot be expressed (except possibly by the person the proposition contains), and so presumably can't be wondered about either.

2. Possible responses to Objection 1

    (a) Response 1: Deny that there are any such singular propositions.
        i. This may not be plausible (there certainly *seem* to be! - ?).
        ii. It is clear that Russell himself did not intend this:
            A. "When we say 'George IV wisked to know whether Scott was the author of Waverley', we ... [*may*] mean ... 'George IV wished to know, concerning the man who in fact wrote Waverley, whether he was Scott'. This would be true, for example, if George IV had seen Scott at a distance, and has asked 'Is that Scott'?" (Russell, *On denoting*, Mind, New Series, Vol. 14, No. 56, (Oct., 1905); p.489.)
    (b) Response 2: Retain the framework of acquaintance-theory, but replace Russell's sense-data account with a more liberal account of which particulars we are acquainted with.
        i. E.g. a causal theory of acquaintance/reference?

3. Objection 2: The description theory of names is false (Kripke)

    (a) Modal objection to the description theory: In modal contexts, names do not behave like any definite description. Contrast e.g.
        i. "It might not have been the case that Milo is the cheekiest kitten in Old Marston."
        ii. "It might not have been the case that the cheekiest kitten in Old Marston is the cheekiest kitten in Old Marston."
    (b) Epistemological objection to the description theory: Sentences of the form "[Name] is [definite description]" – e.g. "Milo is the cheekiest kitten in Old Marston" – are not (except in special cases, e.g. "3 is the result of adding 1 and 2") knowable *a priori*. But if the name were merely an abbreviation for the definite description, they would be *a priori*.
    (c) In some cases, there doesn't seem to be any description available as a plausible candidate for abbreviation.
        i. Kripke's example: most people couldn't give you any uniquely identifying description for Feynman, but they can still competently use the name "Feynman".
        ii. This motivates something more like a *causal theory of names*.

4. Objection 3: The project of analysis-in-terms-of-sense-data that is required by Russell's theory is hopeless, to the theory that requires such analyses must be false.

    (a) TBC ...

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Week 8: Russell's logical atomism

Russell, *The philosophy of logical atomism* (PLA), reprinted in 'Russell's logical atomism', David Pears (ed.), Fontana/Collins (1972). See also the Stanford Encyclopaedia article "Russell's logical atomism".

1 Monism and the doctrine of internal relations

1. Logical atomism is in significant part a reaction against *monism,* and the associated *doctrine of internal relations.*
2. The Doctrine of Internal Relations (DIR):
   (a) Definition
      i. Say that a relation is *internal* if it supervenes on the *nature* (i.e. on the intrinsic properties) of the relata taken one by one; otherwise, say that the relation in question is *external.*
      ii. 'Intrinsic property': ϕ is an *intrinsic* property of an object C iff, necessarily, any *duplicate* of C would possess ϕ.
   (b) *Prima facie* examples
      i. Some reasonably plausible examples of internal relations:
         A. x is warmer than y
         B. x is larger than y
         C. x is the same colour as y
      ii. Some reasonably plausible examples of external relations
         A. x is married to y
         B. x is 5m from y
   (c) The Doctrine of Internal Relations asserts that, apparent counterexamples notwithstanding, *there are no external relations.*
3. DIR leads to monism:
   (a) If there are no external relations, the apparent counterexamples must be explained away somehow.
   (b) The now-natural move is to postulate either
      i. that the intrinsic nature of each and every object is vastly more complex than one might at first assume, so as to already include all its relations to all other objects in the universe; or
      ii. *monism*: the view that there is only one object – the universe.

A. If A and B cannot stand in an external relation of being the same colour, perhaps there is a complex object that is composed of A and B and that has a monadic property corresponding to having-parts-of-the-same-colour...

B. Taking this line of thought to its logical conclusion leads to monism.

4. Russell found that he needed external relations in order to provide a foundation for arithmetic.

   (a) It is difficult to explain away apparent external relations in terms of properties-possessed-by-complexes if the original relation is asymmetric: the complex composed of A and B is the same as the complex composed of B and A...

   (b) Asymmetric relations (e.g. the successor relation) are of paramount importance in arithmetic.

5. The path to logical atomism

   (a) Russell thinks that the Doctrine of Internal Relations is founded on confusions about logic: in particular, on the mistaken idea that all propositions are fundamentally of subject-predicate form.

   (b) But in that case, much of traditional metaphysics is based on the same mistake.

   (c) Logical atomism is Russell's project of rethinking metaphysics on the basis of a correct account of relations.

6. Aside: the relationship to acquaintance

   (a) The pseudo-monists hold that we do not truly know an object unless we know all of its intrinsic nature, and hence (given their account of intrinsic natures) unless we know the complete description of the universe.

   (b) This (obviously!) makes knowledge of any object extremely hard to attain. (Scepticism?)

   (c) Russell's notion of acquaintance provides an apparently far less demanding account of knowing an object.

      i. "There is... a logical theory which is quite opposed to [Russell's] view, a logical theory according to which, if you really understood any one thing, you would understand everything. I think that rests upon a certain confusion of ideas. When you have acquaintance with a particular, you understand that particular itself quite fully, independently of the fact that there are a great many propositions about it that you do not know." (PLA, p.59)

2 The metaphysics of logical atomism

1. Propositions and facts

   (a) Russell's earlier views (in *Principles of Mathematics* (1903))

      i. Propositions are structured entities consisting of objects, properties and relations.

      ii. For any (e.g.) binary relation R and objects a and b, there is a proposition Rab composed of these three entities: the proposition that a bears relation R to b.

         A. E.g. "Jenny is taller than Margaret" – this expresses a proposition composed of the objects Jenny and Margaret and the binary relation taller-than.

   (b) Objection to the *Principles of Mathematics* view

      i. By the time of PLA, Russell objects to his earlier theory, on the grounds that it cannot distinguish between true and false propositions.

         A. If *there is* a complex Rba as well as a concept Rab, how can one account for the fact that one could have e.g. "Jenny is taller than Margaret" true and "Margaret is taller than Jenny" false?

         B. Russell could not simply say that there is no complex Rba in such cases, as then he would not be able to account for the *meaningfulness* of false sentences.

   (c) Russell's later view:

      i. In PLA, Russellian *facts* seem to be entities much like the earlier Russellian propositions.

      ii. Russell now talks of *propositions* as though these are items of *language*. (It is not clear whether this is to be taken seriously.)

2. The logical atomist's hierarchy of propositions

   (a) Russell envisages the world as being built up from elementary objects and their properties and relations, in much the way that sentences of a formal (predicate-logic) language are built up in introductory logic classes.

   (b) Atomic propositions: the simple case

      i. Atomic propositions correspond to the logician's atomic sentences: they are composed of an n-ary relation-expression (n$\geq$1) and n object-names.

      ii. True atomic propositions are made true by atomic facts.

         A. E.g. If "Jenny is tall" were a true fundamental proposition, it would be made true by the atomic fact whose constituents are Jenny and tallness.

(c) Negative propositions and negative facts

   i. There are several possible ways of dealing with false propositions and propositions that are true but 'negative'.
   ii. False (but 'positive') propositions: Is 'Jenny is short' made false by
      A. a fact [Jenny, not-short]?
      B. the *absence* of any fact [Jenny, short]?
      C. a *negative fact* [Jenny, short]$_-$?
   iii. Similarly: If Tom is not stupid, is 'Tom is not stupid' made true by
      A. a fact [Tom, not-stupid]?
      B. the absence of any fact [Tom, stupid]?
      C. a negative fact [Tom, stupid]$_-$?
   iv. In PLA, Russell answers these questions by the negative-fact theory (apparently because he does not want to postulate negative properties, and because the absence-of-fact theory has not occurred to him (?)). But he does acknowledge some metaphysical squeamishness concerning negative facts, and (much) later he changes his mind.

(d) Molecular propositions

   i. Molecular propositions correspond to the sentences that the logician builds from atomic (and general) sentences and sentence-connectives.
   ii. Examples:
      A. "Milo is cheeky and Tang likes milk."
      B. "If it is eighth week, then the pain will end soon."
   iii. Russell does not believe in molecular facts. A molecular proposition is made true or false by *atomic* facts (in a way determined by the usual truth-table rules).

(e) Quantified propositions

   i. Quantified propositions correspond to the sentences that the logician builds from atomic (and molecular) sentences using the quantifiers $(\forall, \exists)$.
   ii. Account 1: General propositions are made true by atomic facts (according to the now-usual semantics for predicate logic).
   iii. Account 2 (Russell's account in PLA):
      A. In addition to atomic facts, there are general facts. "When you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are."

B. So there is e.g. *the fact that all men are mortal*, as well as the fact that Socrates is a man and the fact that Socrates is mortal and the fact that Plato is a man and the fact that Plato is mortal and...

C. General propositions are made true by general facts.

iv. Objection: A list does not fail to be complete just because it has not been stated to be complete.

(f) An open problem: Propositional attitude reports

i. Propositional attitude reports, such as "James believes that Kath is ill" and "Tara hopes that Dennis will go away" present a tricky case for the logical atomist.

ii. Attempt 1

A. A natural analysis would be: such reports assert the holding of a binary relation between the subject doing the believing/hoping/commanding/asking/wishing/etc and the proposition believed/hoped/etc.

B. Russell objects to that analysis because it requires the reality of propositions, and he does not think that propositions are real.

> . . . [I]t does not seem to me plausible to say that in addition to facts there are also these curious shadowy things going about such as 'that today is Wednesday' when in fact it is Tuesday. . . . It is more than one can manage to believe, and I do think no person with a vivid sense of reality can imagine it. (PLA, p.79)

C. Clearly belief/hope/etc cannot be relations between a subject and a *fact*, because one can have false beliefs/dashed hopes/etc.

iii. Attempt 2: Behaviourism

A. Behaviourists seek to *define* belief in terms of behaviour. "Suppose, e.g., that you are said to believe that there is a train at 10.25. This means... that you start for the station at a certain time. When you reach the station you see it is 10.24 and you run. That behaviour *constitutes* your belief that there is a train at that time." (PLA, pp.76-7; emphasis added)

B. Russell "[does] not [himself] feel that that view of things is tenable" (PLA, p.77), but doesn't give his reasons here. (His "On the nature of acquaintance" argues against "neutral monism", which he says is closely related.)

iv. Attempt 3: Russell's (earlier) "multiple relation theory of judgment"

A. If I say "I believe that this lecture is fascinating", I assert the holding of a three-place belief relation between me, this lecture and the property of being fascinating.
B. If I say "Mike believes that Frege is smarter than Russell", I assert the holding of a four-place belief relation between Mike, Frege, Russell and the binary relation smarter-than.
C. We will similarly need five-, six- etc. place belief relations.
D. In PLA, Russell expresses (but does not explain – p.83) pessimism about the prospects for this (i.e. his own earlier) theory, arising from its need to "put the subordinate verb on a level with its terms as an object term in the belief." (?)

3 The methodology of logical atomism: the project of analysis

1. Logical atomism recommends clarifying a chosen domain of discourse via a two-stage process:

   (a) "Analytic" phase: Identify the fundamental concepts and principles for the domain of beliefs in question.

   (b) "Synthetic" phase: reconstruct the original body of beliefs from the fundamental stuff of the first phase, using appropriate definitions and deduction.

2. Examples of the application of this procedure:

   (a) Peano's axiomatisation of arithmetic;

   (b) Frege's and Russell's reduction of arithmetic;

   (c) Russell's attempt to analyse physics in terms of sense-data.

3. This process takes us from ordinary natural language towards the logically perfect language that would be best suited for science:

   "In a logically perfect language, there will be one word and no more for every simple object, and everything that is not simple will be expressed by a combination of words, by a combination derived, of course, from the words for the simple things that enter in, one word for each component. A language of that sort will be completely analytic, and will show at a glance the logical structure of the facts asserted or denied. The language which is set forth in *Principia Mathematica* is intended to be a language of that sort. [A]ctual languages are not logically perfect in this sense, and they cannot possibly be, if they are to serve the purposes of daily life." (PLA, pp.52-3)

4 Objections to logical atomism

1. Objection: It follows from logical atomism (combined with the claim that the atomic constituents are sense-data and properties thereof) that communication between distinct people is impossible.
   (a) Russell does not think this follows: "When one person uses a word, he does not mean the same thing by it that another person means by it. I have often heard it said that that is a misfortune. That is a mistake. It would be absolutely fatal if people meant the same things by their words. It would make all intercourse impossible..." (PLA, p.50)
   (b) This may be acceptable, but it raises a need for a non-trivial theory of communication: if communication does not proceed via the speaker expressing a proposition that the listener understands, how *does* it proceed? (Russell doesn't answer this question.)
2. Objection: The analyses recommended by the logical atomist do not in fact exist.
   (a) Urmson's example: What is the final analysis of "England declared war in 1939"?
   (b) Urmson objects that any such analysis would have to be "indefinitely long".
      i. This could mean *of an unknown and very large length*, or *infinitely long.*
      ii. But the former would constitute no objection to logical atomism, and there is no evidence or argument for the latter.
3. The "paradox of analysis"
   (a) If an analysis consists in providing (more structurally perspicuous) sentences that *mean the same thing* as the sentences they are supposed to be analyses of, how can we explain the fact that correct analyses are not immediately evident to every competent speaker of the language?
   (b) If that is *not* what correct analysis consists in, what *is* its status?

5 The influence of logical atomism

1. Logical atomism was one of the key ideas behind the hugely important logical positivist movement of the 1920s and 1930s: the positivists took *the* task of philosophy to be the provision of logical analyses of the language of science.

2. The idea that a fundamental understanding of the world (or of part of the world, or of some domain of discourse) consists in the identification of the fundamental objects, properties and relations still dominates metaphysics today.

3. "The linguistic turn": Much of analytic philosophy still subscribes to the idea that we approach an understanding of the structure of reality (insofar as that is possible at all) via analysis of the structure of language.

4. Much of analytic philosophy still subscribes to a methodology of *conceptual analysis* that is not unlike the logical atomists' notion of analysis. (Cf. 21^{st}-century analyses of knowledge, perception, mental state attributions, moral discourse...)