

# Reproducing Kuang and Nielsen Generalized log normal Chain-Ladder using the apc package

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## 1 Introduction

The purpose of this vignette is to use the `apc` package version 1.3.5 to reproduce some the result in Kuang and Nielsen (2018): *Generalized Log-Normal Chain-Ladder*. This adopts the theory presented in Harnau and Nielsen (2018), from an over-dispersed Poisson model to a log-normal model. There is also a vignette available for that paper. The `apc` package builds on the identification analysis and the forecast theory in Kuang, Nielsen and Nielsen (2008a,b), the development of deviance analysis for general data arrays in Nielsen (2014). The package is discussed in Nielsen (2015).

## 2 Table 1.1: The data

The data set is a reserving triangle from the XL group. It represents US casualty, gross paid and reported loss and allocated loss adjustment expense in 1000 USD.

The data are available in the `apc` package. They can be called with the following command. Note that the output is wide and therefore truncated to fit the page width.

```
> library(apc)
> data <- data.loss.XL()
> data$response[,1:8]
```

	1997	1998	1999	2000	2001	2002	2003	2004
1997	2185	13908	44704	56445	67313	62830	72619	42511
1998	3004	17478	49564	55090	75119	66759	76212	62311
1999	5690	28971	55352	63830	71528	73549	72159	37275
2000	9035	29666	47086	41100	58533	80538	70521	40192
2001	7924	38961	41069	64760	64069	61135	62109	52702
2002	7285	25867	44375	58199	61245	48661	57238	29667
2003	3017	22966	62909	54143	72216	58050	29522	25245
2004	1752	25338	56419	75381	64677	58121	38339	21342
2005	1181	24571	66321	65515	62151	43727	29785	23981
2006	1706	13203	40759	57844	48205	50461	27801	21222
2007	623	14485	27715	52243	60190	45100	31092	22731
2008	338	6254	24473	32314	35698	25849	30407	15335
2009	255	3842	14086	26177	27713	15087	17085	12520
2010	258	7426	22459	28665	32847	28479	24096	NA
2011	1139	10300	19750	32722	41701	29904	NA	NA
2012	381	5671	34139	33735	33191	NA	NA	NA
2013	605	11242	24025	32777	NA	NA	NA	NA
2014	1091	9970	31410	NA	NA	NA	NA	NA
2015	1221	8374	NA	NA	NA	NA	NA	NA
2016	2458	NA	NA	NA	NA	NA	NA	NA

### 3 Table 4.1: Analysis of variance

The deviance table can be reproduced by the following commands. The first call has the APC model as reference. The second call has the AC model as reference. For an overview of the models, see Nielsen (2014). The output is wide, so only selected columns are shown.

```
> apc.fit.table(data, "log.normal.response")[,c(1,2,6,7)]
```

	-2logL	df.residual	F vs.APC	prob(>F)
APC	170.003	153	NaN	NaN
AP	243.531	171	3.564	0.000
AC	179.873	171	0.409	0.984
PC	633.432	171	68.736	0.000
Ad	258.570	189	2.230	0.000
Pd	643.892	189	36.340	0.000
Cd	649.142	189	37.368	0.000
A	357.359	190	5.956	0.000
P	644.176	190	35.412	0.000
C	672.392	190	41.099	0.000
t	664.488	207	27.015	0.000
tA	681.993	208	29.072	0.000
tP	664.746	208	26.560	0.000
tC	686.181	208	29.713	0.000
1	690.399	209	29.830	0.000

```
> apc.fit.table(data, "log.normal.response", "AC")[,c(1,2,6,7)]
```

	-2logL	df.residual	F vs.AC	prob(>F)
AC	179.873	171	NaN	NaN
Ad	258.570	189	4.319	0
Cd	649.142	189	79.257	0
A	357.359	190	11.955	0
C	672.392	190	84.930	0
t	664.488	207	42.993	0
tA	681.993	208	45.869	0
tC	686.181	208	46.886	0
1	690.399	209	46.670	0

Thus, Table 4.1 in the paper is constructed as follows.

```
> table.APC <- apc.fit.table(data, "log.normal.response")
> table.AC <- apc.fit.table(data, "log.normal.response", "AC")
> Table41 <- matrix(NA, nrow=3, ncol=6)
> Table41[1:3,1:2] <- table.APC[c(1,3,5),1:2]
> Table41[2:3,3:4] <- table.APC[c(3,5),6:7]
> Table41[3,5:6] <- table.AC[2,6:7]
```

```
> rownames(Table41) <- c("apc", "ac", "ad")
> colnames(Table41) <- c("-2logL", "df", "F_sup,apc", "p", "F_sup,ac", "p")
> Table41
```

	-2logL	df	F_sup,apc	p	F_sup,ac	p
apc	170.003	153	NA	NA	NA	NA
ac	179.873	171	0.409	0.984	NA	NA
ad	258.570	189	2.230	0.000	4.319	0

## 4 Table 4.2: Estimates

The table of estimates can be reproduced by the following commands. As a default the program gives the double differenced time effects. These are referred to as the canonical parameters. Note that in the `apc` package  $\alpha$  is the age or development year  $\beta$  is the period or calendar year,  $\gamma$  is the cohort or policy year

```
> fit <- apc.fit.model(data, "log.normal.response", "AC")
> fit$coefficients.canonical
```

	Estimate	Std. Error	t value	Pr(> t )
level	7.660055032	0.1377951	55.59016605	0.000000e+00
age slope	2.272100342	0.1335080	17.01846386	5.992216e-65
cohort slope	0.288755900	0.1335080	2.16283663	3.055375e-02
DD_age_1999	-1.339569792	0.2328429	-5.75310613	8.761844e-09
DD_age_2000	-0.696924194	0.2386087	-2.92078261	3.491534e-03
DD_age_2001	-0.146719747	0.2453026	-0.59811748	5.497615e-01
DD_age_2002	-0.264930913	0.2527431	-1.04822205	2.945363e-01
DD_age_2003	0.031598844	0.2609768	0.12107911	9.036284e-01
DD_age_2004	-0.283163142	0.2701148	-1.04830652	2.944974e-01
DD_age_2005	0.127081007	0.2803153	0.45335018	6.502966e-01
DD_age_2006	-0.099202405	0.2917899	-0.33997887	7.338724e-01
DD_age_2007	0.210073941	0.3048208	0.68917200	4.907150e-01
DD_age_2008	-0.052407612	0.3197906	-0.16388105	8.698248e-01
DD_age_2009	-0.018395937	0.3372315	-0.05454988	9.564971e-01
DD_age_2010	-0.294857921	0.3579095	-0.82383376	4.100340e-01
DD_age_2011	0.252083441	0.3829750	0.65822430	5.103940e-01
DD_age_2012	0.709064853	0.4142566	1.71165624	8.696004e-02
DD_age_2013	-1.301108834	0.4548907	-2.86026717	4.232842e-03
DD_age_2014	1.011946941	0.5108825	1.98078211	4.761571e-02
DD_age_2015	-0.499717114	0.5959631	-0.83850344	4.017480e-01
DD_age_2016	0.109628839	0.7552897	0.14514807	8.845940e-01
DD_cohort_1999	-0.125331664	0.2328429	-0.53826711	5.903927e-01
DD_cohort_2000	-0.428405722	0.2386087	-1.79543197	7.258490e-02
DD_cohort_2001	0.414810916	0.2453026	1.69101751	9.083346e-02
DD_cohort_2002	-0.524216258	0.2527431	-2.07410692	3.806938e-02

DD_cohort_2003	0.175650935	0.2609768	0.67305179	5.009143e-01
DD_cohort_2004	0.189928763	0.2701148	0.70314081	4.819680e-01
DD_cohort_2005	0.003469177	0.2803153	0.01237598	9.901256e-01
DD_cohort_2006	-0.126934898	0.2917899	-0.43502154	6.635468e-01
DD_cohort_2007	0.110410208	0.3048208	0.36221353	7.171925e-01
DD_cohort_2008	-0.450739627	0.3197906	-1.40948386	1.586921e-01
DD_cohort_2009	0.035029472	0.3372315	0.10387366	9.172696e-01
DD_cohort_2010	0.733084362	0.3579095	2.04823952	4.053654e-02
DD_cohort_2011	0.015034268	0.3829750	0.03925653	9.686859e-01
DD_cohort_2012	-0.579238239	0.4142566	-1.39825961	1.620351e-01
DD_cohort_2013	0.410823816	0.4548907	0.90312651	3.664588e-01
DD_cohort_2014	0.059646180	0.5108825	0.11675127	9.070572e-01
DD_cohort_2015	-0.294450287	0.5959631	-0.49407469	6.212534e-01
DD_cohort_2016	0.965669947	0.7552897	1.27854248	2.010582e-01

The present age-cohort model has no calendar (period) effect, so also the first differences parameters are identified. They are found by an additional identification command. Thus, Table 4.2 in the paper is constructed from the following commands.

```
> apc.identify(fit)$coefficients.dif[,1:2]
```

	Estimate	Std. Error
level	7.660055032	0.1377951
D_age_1998	2.272100342	0.1335080
D_age_1999	0.932530550	0.1362610
D_age_2000	0.235606356	0.1398301
D_age_2001	0.088886609	0.1438733
D_age_2002	-0.176044303	0.1483681
D_age_2003	-0.144445459	0.1533567
D_age_2004	-0.427608601	0.1589136
D_age_2005	-0.300527594	0.1651428
D_age_2006	-0.399729999	0.1721838
D_age_2007	-0.189656058	0.1802245
D_age_2008	-0.242063670	0.1895226
D_age_2009	-0.260459607	0.2004421
D_age_2010	-0.555317528	0.2135164
D_age_2011	-0.303234088	0.2295651
D_age_2012	0.405830766	0.2499291
D_age_2013	-0.895278068	0.2769988
D_age_2014	0.116668873	0.3156054
D_age_2015	-0.383048241	0.3777268
D_age_2016	-0.273419402	0.5083832
D_cohort_1998	0.288755900	0.1335080
D_cohort_1999	0.163424236	0.1362610
D_cohort_2000	-0.264981486	0.1398301
D_cohort_2001	0.149829430	0.1438733

```

D_cohort_2002 -0.374386828  0.1483681
D_cohort_2003 -0.198735893  0.1533567
D_cohort_2004 -0.008807130  0.1589136
D_cohort_2005 -0.005337953  0.1651428
D_cohort_2006 -0.132272851  0.1721838
D_cohort_2007 -0.021862643  0.1802245
D_cohort_2008 -0.472602270  0.1895226
D_cohort_2009 -0.437572798  0.2004421
D_cohort_2010  0.295511564  0.2135164
D_cohort_2011  0.310545832  0.2295651
D_cohort_2012 -0.268692406  0.2499291
D_cohort_2013  0.142131410  0.2769988
D_cohort_2014  0.201777590  0.3156054
D_cohort_2015 -0.092672697  0.3777268
D_cohort_2016  0.872997251  0.5083832

```

```
> fit$s2
```

```
[1] 0.1693316
```

```
> fit$RSS
```

```
[1] 28.9557
```

## 5 Table 4.3: Forecasts

The forecasts are produced as follows. First the log normal forecasts use the fit found above. We want the 99.5% quantile. The reserves are computed by policy year and for the entire lower triangle. Note that the `apc` also allows aggregation by development year and by calendar year as well as single cell forecasts. Aggregation by calendar year would give a cashflow. Further, there are three columns with standard errors: the overall standard error and breakdowns into process error and estimation error.

```

> forecast <- apc.forecast.ac(fit,quantiles=0.995)
> forecast$response.forecast.coh

```

	forecast	se	se.proc	se.est	t-0.995
coh_1998	1871.073	1026.463	707.4405	743.7428	4544.891
coh_1999	5099.330	1874.681	1375.8435	1273.3744	9982.659
coh_2000	7171.317	2123.128	1622.5220	1369.3412	12701.822
coh_2001	11699.350	2984.949	2274.8292	1932.6338	19474.801
coh_2002	13717.388	3345.138	2654.4080	2035.6984	22431.090
coh_2003	14343.522	3188.410	2471.3130	2014.5886	22648.964
coh_2004	18377.001	3834.057	2910.9751	2495.2390	28364.281
coh_2005	25488.052	5241.618	3976.5389	3414.9225	39141.867

```

coh_2006 30524.942 6213.652 4662.3320 4107.5694 46710.794
coh_2007 40078.245 8115.990 5976.5789 5490.8835 61219.471
coh_2008 32680.319 6603.511 4727.4210 4610.6241 49881.712
coh_2009 28509.077 5895.265 4143.1332 4193.8760 43865.568
coh_2010 51760.526 11013.030 7540.3989 8026.7807 80448.208
coh_2011 98747.731 22063.641 14798.3216 16365.0210 156220.991
coh_2012 100330.677 23254.845 14704.7084 18015.5316 160906.889
coh_2013 149813.314 36629.836 21310.2885 29792.8931 245229.846
coh_2014 221549.649 58610.037 29815.3239 50459.7158 374222.093
coh_2015 229480.904 69931.745 29102.9866 63588.2473 411645.102
coh_2016 575343.178 235016.967 70362.1087 224236.8135 1187535.497

```

```
> forecast$response.forecast.all
```

```

forecast      se se.proc  se.est t-0.995
all 1656586 267445.9 88190.59 252487.1 2353252

```

We compare with the standard chain ladder using the over-dispersed Poisson distribution forecasts of Harnau and Nielsen (2018). Here, the standard error has a third component, tau.est.

```

> CL.fit <- apc.fit.model(data,"od.poisson.response","AC")
> CL.forecast <- apc.forecast.ac(CL.fit,quantiles=0.995)
> CL.forecast$response.forecast.coh

```

```

forecast      se se.proc  se.est tau.est t-0.995
coh_1998 1367.774 2472.419 1719.626 1776.238 26.88900 7808.143
coh_1999 4475.781 4120.885 3110.722 2701.363 87.98918 15210.216
coh_2000 6924.767 4745.192 3869.273 2743.546 136.13369 19285.449
coh_2001 10975.055 5928.969 4871.134 3373.155 215.75814 26419.342
coh_2002 14940.740 6519.633 5683.460 3180.821 293.71938 31923.638
coh_2003 18337.446 7135.410 6296.456 3337.478 360.49508 36924.373
coh_2004 24486.906 8225.669 7276.017 3806.503 481.38705 45913.833
coh_2005 31875.928 9355.281 8301.531 4267.690 626.64752 56245.364
coh_2006 35566.882 9836.955 8768.992 4402.451 699.20783 61191.025
coh_2007 48594.889 11673.490 10249.957 5504.188 955.32486 79002.994
coh_2008 42027.151 10902.427 9532.169 5226.666 826.20999 70426.726
coh_2009 37113.693 10490.973 8957.645 5411.911 729.61652 64441.479
coh_2010 66977.208 14927.051 12033.453 8733.793 1316.70209 105860.468
coh_2011 102982.095 20300.137 14921.333 13614.356 2024.52064 155861.631
coh_2012 136646.512 26549.216 17188.027 20055.329 2686.32798 205804.182
coh_2013 164317.838 35454.109 18848.167 29854.740 3230.31740 256671.737
coh_2014 218873.833 55148.569 21753.227 50494.034 4302.83140 362529.547
coh_2015 166119.642 82217.051 18951.224 79936.409 3265.73900 380285.655
coh_2016 337001.247 325178.113 26992.463 323988.148 6625.09322 1184053.039

```

```
> CL.forecast$response.forecast.all
```



```

forecast      se  se.proc  se.est  tau.est t-0.995
all 1469605 350536.3 56367.29 344766.2 28890.91 2382712

```

We also look at the bootstrap forecast. This uses the bootstrap command in the `ChainLadder` package. That packages takes a cumulative triangle as input, which we can form in the `apc` package using the command `triangle.cumulative`. This command operates both on an `apc.data.list` and on a matrix. The output is a matrix.

```
> m.cum <- triangle.cumulative(data)
```

We then call the bootstrap command in the `ChainLadder` package. In the paper we have  $10^5$  bootstrap repetitions. In the following code this is reduced to  $10^3$  repetitions.

```

> library(ChainLadder)
> BS <- BootChainLadder(m.cum, R = 10^3, process.distr=c("od.pois"))
> summary(BS)

```

\$ByOrigin

	Latest	Mean Ultimate	Mean IBNR	SD IBNR	IBNR 75%	IBNR 95%
1997	508553	508553.0	0.000	0.000	0.00	0.00
1998	538683	540121.8	1438.783	2636.212	2149.00	6954.15
1999	573322	577821.2	4499.237	4331.812	6568.50	12438.30
2000	485497	492469.3	6972.303	4933.756	9678.75	16299.60
2001	535844	546570.7	10726.683	6013.204	14126.25	22134.65
2002	413177	427970.2	14793.178	7016.100	18423.50	27971.80
2003	387065	405293.2	18228.156	7410.975	22558.75	31915.50
2004	388149	412508.4	24359.429	8333.566	29232.25	40036.85
2005	363024	394422.7	31398.670	9633.731	37442.00	48418.30
2006	295505	330689.6	35184.649	9985.929	41278.75	52904.30
2007	292145	340571.6	48426.591	11985.280	55552.75	69159.35
2008	186365	228295.2	41930.167	11013.365	48587.00	60974.35
2009	116765	153545.4	36780.413	10757.306	43414.00	55961.45
2010	144230	210176.8	65946.845	15388.979	74407.00	93213.60
2011	135516	238164.8	102648.841	20292.174	115158.75	137151.50
2012	107117	242083.4	134966.378	27929.806	152693.50	183418.65
2013	68649	232112.5	163463.543	33533.336	184214.75	221314.30
2014	42471	261520.1	219049.116	57387.838	250797.50	324329.65
2015	9595	176376.5	166781.464	82263.397	215798.50	318111.30
2016	2458	338915.4	336457.379	352532.830	524820.75	1001131.10

\$Totals

	Totals
Latest:	5594130.0
Mean Ultimate:	7058181.8
Mean IBNR:	1464051.8
SD IBNR:	369276.7
Total IBNR 75%:	1673747.0
Total IBNR 95%:	2124707.5

The information from the above methods are now combined into Table 4.3.

```

> Table_4_3 <- matrix(NA,nrow=20,ncol=9)
> rownames(Table_4_3) <- c(as.character(2:20),"total")
> col.3 <- c("Res","se/Res","99.5%/Res")
> colnames(Table_4_3) <- c(col.3,col.3,col.3)
> # Table 4.3, part I, log normal Chain Ladder
> forecast.coh <- forecast$response.forecast.coh
> forecast.all <- forecast$response.forecast.all
> Table_4_3[1:19,1] <- forecast.coh[,1]
> Table_4_3[1:19,2:3] <- forecast.coh[,c(2,5)]/forecast.coh[,1]
> Table_4_3[20,1] <- forecast.all[,1]
> Table_4_3[20,2:3] <- forecast.all[,c(2,5)]/forecast.all[,1]
> # Table 4.3, part II, standard Chain Ladder
> CL.forecast.coh <- CL.forecast$response.forecast.coh
> CL.forecast.all <- CL.forecast$response.forecast.all
> Table_4_3[1:19,4] <- CL.forecast.coh[,1]
> Table_4_3[1:19,5:6] <- CL.forecast.coh[,c(2,6)]/CL.forecast.coh[,1]
> Table_4_3[20,4] <- CL.forecast.all[,1]
> Table_4_3[20,5:6] <- CL.forecast.all[,c(2,6)]/CL.forecast.all[,1]
> # Table 4.3, part III, bootstrap
> sum.bs.B <- summary(BS)$ByOrigin
> sum.bs.T <- summary(BS)$Totals
> qua.bs.B <- quantile(BS, c(0.995))$ByOrigin
> qua.bs.T <- quantile(BS, c(0.995))$Totals
> Table_4_3[1:19,7] <- sum.bs.B[2:20,3]
> Table_4_3[1:19,8] <- sum.bs.B[2:20,4]/sum.bs.B[2:20,3]
> Table_4_3[1:19,9] <- qua.bs.B[2:20,1]/sum.bs.B[2:20,3]
> Table_4_3[20,7] <- sum.bs.T[3,]
> Table_4_3[20,8] <- sum.bs.T[4,]/sum.bs.T[3,]
> Table_4_3[20,9] <- qua.bs.T[1,1]/sum.bs.T[3,]
> Table_4_3

```

	Res	se/Res	99.5%/Res	Res	se/Res	99.5%/Res
2	1871.073	0.5485955	2.429029	1367.774	1.8076232	5.708652
3	5099.330	0.3676327	1.957641	4475.781	0.9207076	3.398338
4	7171.317	0.2960583	1.771198	6924.767	0.6852493	2.784996
5	11699.350	0.2551380	1.664605	10975.055	0.5402223	2.407217
6	13717.388	0.2438612	1.635230	14940.740	0.4363661	2.136684
7	14343.522	0.2222892	1.579038	18337.446	0.3891169	2.013605
8	18377.001	0.2086334	1.543466	24486.906	0.3359211	1.875036
9	25488.052	0.2056500	1.535695	31875.928	0.2934905	1.764509
10	30524.942	0.2035598	1.530250	35566.882	0.2765763	1.720449
11	40078.245	0.2025036	1.527499	48594.889	0.2402205	1.625747
12	32680.319	0.2020639	1.526353	42027.151	0.2594139	1.675744
13	28509.077	0.2067855	1.538653	37113.693	0.2826712	1.736326

14	51760.526	0.2127689	1.554239	66977.208	0.2228676	1.580545
15	98747.731	0.2234344	1.582021	102982.095	0.1971230	1.513483
16	100330.677	0.2317820	1.603766	136646.512	0.1942912	1.506106
17	149813.314	0.2445032	1.636903	164317.838	0.2157654	1.562044
18	221549.649	0.2645458	1.689112	218873.833	0.2519651	1.656340
19	229480.904	0.3047388	1.793810	166119.642	0.4949267	2.289228
20	575343.178	0.4084814	2.064047	337001.247	0.9649166	3.513497
total	1656585.594	0.1614440	1.420543	1469605.388	0.2385241	1.621328
	Res	se/Res	99.5%/Res			
2	1438.783	1.8322515	9.197290			
3	4499.237	0.9627881	4.795710			
4	6972.303	0.7076221	3.572634			
5	10726.683	0.5605837	2.896122			
6	14793.178	0.4742795	2.642276			
7	18228.156	0.4065675	2.365781			
8	24359.429	0.3421084	2.180079			
9	31398.670	0.3068197	1.898241			
10	35184.649	0.2838149	1.842862			
11	48426.591	0.2474938	1.800757			
12	41930.167	0.2626597	1.852801			
13	36780.413	0.2924738	1.903523			
14	65946.845	0.2333543	1.798227			
15	102648.841	0.1976854	1.579510			
16	134966.378	0.2069390	1.641663			
17	163463.543	0.2051426	1.621643			
18	219049.116	0.2619862	1.875492			
19	166781.464	0.4932406	2.498628			
20	336457.379	1.0477786	4.779893			
total	1464051.825	0.2522293	1.865851			

## 6 Table 4.4: Forecasts

In this table the analysis is redone when dropping the last calendar (period) year and when dropping the two last calendar years. `apc` can extract subsets of the data with the following commands.

```
> data.1 <- apc.data.list.subset(data,0,0,0,1,0,0)
```

```
WARNING apc.data.list.subset: cuts in arguments are:
```

```
[1] 0 0 0 1 0 0
```

```
have been modified to:
```

```
[1] 0 1 0 1 0 1
```

```
WARNING apc.data.list.subset: coordinates changed to "AC" & data.format changed to "t"
```

```
> data.2 <- apc.data.list.subset(data,0,0,0,2,0,0)
```

WARNING apc.data.list.subset: cuts in arguments are:

```
[1] 0 0 0 2 0 0
```

have been modified to:

```
[1] 0 2 0 2 0 2
```

WARNING apc.data.list.subset: coordinates changed to "AC" & data.format changed to "t

Table 4.4 is built up from 9 subpanels, but the last row with the bootstrap panels. We start by defining the 6 top panels, labelled a,b,...,f.

```
> # Define panels
> Table_4_4a <- matrix(NA,nrow=6,ncol=3)
> Table_4_4a[1:5,1] <- 16:20
> colnames(Table_4_4a) <- c("i","se/Res","99.5%/Res")
> Table_4_4b <- Table_4_4c <- Table_4_4d <- Table_4_4a
> Table_4_4b[1:5,1] <- 15:19
> Table_4_4c[1:5,1] <- 14:18
> Table_4_4e <- Table_4_4b
> Table_4_4f <- Table_4_4c
```

We then fill in the forecast information similar to Table 4.3.

```
> # Panel a. log normal Chain Ladder, no cut
> for.coh <- forecast$response.forecast.coh
> for.all <- forecast$response.forecast.all
> Table_4_4a[1:5,2:3] <- for.coh[15:19,c(2,5)]/for.coh[15:19,1]
> Table_4_4a[ 6,2:3] <- for.all[ ,c(2,5)]/for.all[ ,1]
> # Panel b. log normal Chain Ladder, cut 1 calendar year
> fit.1 <- apc.fit.model(data.1,"log.normal.response","AC")
> forecast.1 <- apc.forecast.ac(fit.1,quantiles=c(0.995))
> for.1.coh <- forecast.1$response.forecast.coh
> for.1.all <- forecast.1$response.forecast.all
> Table_4_4b[1:5,2:3] <- for.1.coh[14:18,c(2,5)]/for.1.coh[14:18,1]
> Table_4_4b[ 6,2:3] <- for.1.all[ ,c(2,5)]/for.1.all[ ,1]
> # Panel c. log normal Chain Ladder, cut 2 calendar years
> fit.2 <- apc.fit.model(data.2,"log.normal.response","AC")
> forecast.2 <- apc.forecast.ac(fit.2,quantiles=c(0.995))
> for.2.coh <- forecast.2$response.forecast.coh
> for.2.all <- forecast.2$response.forecast.all
> Table_4_4c[1:5,2:3] <- for.2.coh[13:17,c(2,5)]/for.2.coh[13:17,1]
> Table_4_4c[ 6,2:3] <- for.2.all[ ,c(2,5)]/for.2.all[ ,1]
> # Panel d. Standard Chain Ladder, no cut
> CL.for.coh <- CL.forecast$response.forecast.coh
> CL.for.all <- CL.forecast$response.forecast.all
> Table_4_4d[1:5,2:3] <- CL.for.coh[15:19,c(2,6)]/CL.for.coh[15:19,1]
> Table_4_4d[ 6,2:3] <- CL.for.all[ ,c(2,6)]/CL.for.all[ ,1]
> # Panel e. Standard Chain Ladder, cut 1 calendar year
```

```

> CL.fit.1      <- apc.fit.model(data.1,"od.poisson.response","AC")
> CL.forecast.1 <- apc.forecast.ac(CL.fit.1,quantiles=c(0.995))
> CL.for.coh <- CL.forecast.1$response.forecast.coh
> CL.for.all <- CL.forecast.1$response.forecast.all
> Table_4_4e[1:5,2:3] <- CL.for.coh[14:18,c(2,6)]/CL.for.coh[14:18,1]
> Table_4_4e[ 6,2:3] <- CL.for.all[      ,c(2,6)]/CL.for.all[      ,1]
> # Panel f. Standard Chain Ladder, cut 2 calendar years
> CL.fit.2      <- apc.fit.model(data.2,"od.poisson.response","AC")
> CL.forecast.2 <- apc.forecast.ac(CL.fit.2,quantiles=c(0.995))
> CL.for.coh <- CL.forecast.2$response.forecast.coh
> CL.for.all <- CL.forecast.2$response.forecast.all
> Table_4_4f[1:5,2:3] <- CL.for.coh[13:17,c(2,6)]/CL.for.coh[13:17,1]
> Table_4_4f[ 6,2:3] <- CL.for.all[      ,c(2,6)]/CL.for.all[      ,1]
> # Combine table
> rbind(cbind(Table_4_4a,Table_4_4b,Table_4_4c),
+       cbind(Table_4_4d,Table_4_4e,Table_4_4f))

```

	i	se/Res	99.5%/Res	i	se/Res	99.5%/Res	i	se/Res	99.5%/Res
[1,]	16	0.2317820	1.603766	15	0.2330485	1.607871	14	0.2325165	1.607441
[2,]	17	0.2445032	1.636903	16	0.2454914	1.640326	15	0.2453367	1.640933
[3,]	18	0.2645458	1.689112	17	0.2653584	1.692146	16	0.2653280	1.693160
[4,]	19	0.3047388	1.793810	18	0.3054005	1.796590	17	0.3054789	1.798052
[5,]	20	0.4084814	2.064047	19	0.4090368	2.066909	18	0.4091658	2.068930
[6,]	NA	0.1614440	1.420543	NA	0.1252687	1.326744	NA	0.1181010	1.308534
[7,]	16	0.1942912	1.506106	15	0.2028927	1.529214	14	0.2237359	1.584502
[8,]	17	0.2157654	1.562044	16	0.2158180	1.562928	15	0.2387938	1.623840
[9,]	18	0.2519651	1.656340	17	0.2831099	1.738448	16	0.2766660	1.722780
[10,]	19	0.4949267	2.289228	18	0.4784675	2.248008	17	0.4775386	2.247552
[11,]	20	0.9649166	3.513497	19	1.4000506	4.651814	18	1.5254827	4.985267
[12,]	NA	0.2385241	1.621328	NA	0.2070006	1.539929	NA	0.2020588	1.527871

## 7 Table 4.5: Bartlett tests

Here we have a variety of specification tests. They are not coded in `apc` as yet, so they require a bit of work. We start with Bartlett's test and define a function that can take 2 or 3 subsets.

```

> Bartlett <- function(data,model.family,model.design,s.1,s.2,s.3=NULL)
+ # data is an apc.data.list
+ # s are subset indices, that is sets of six numbers
+ { fit.model <- function(data,model.family,model.design,s)
+   { data.s <- apc.data.list.subset(data,s[1],s[2],s[3],s[4],s[5],s[6],
+     suppress.warning=TRUE)
+     fit <- apc.fit.model(data.s,model.family,model.design)
+     dev <- fit$deviance

```

```

+     if(model.family=="log.normal.response")
+         dev <- fit$RSS
+     return(list(dev=dev, df=fit$df.residual))
+ }
+ fit <- fit.model(data,model.family,model.design,s.1)
+ dev <- fit$dev
+ df <- fit$df
+ fit <- fit.model(data,model.family,model.design,s.2)
+ dev <- c(dev,fit$dev)
+ df <- c(df,fit$df )
+ m <- 2
+ if(!is.null(s.3))
+ {   fit <- fit.model(data,model.family,model.design,s.3)
+     dev <- c(dev,fit$dev)
+     df <- c(df,fit$df )
+     m <- 3
+ }
+ dev.<- sum(dev)
+ df.<- sum(df)
+ LR <- df.*log(dev./df.)-sum(df*log(dev/df))
+ C <- 1+(1/3/(m-1))*(sum(1/df)-1/df.)
+ t <- LR/C
+ p <- pchisq(LR/C,m-1,0,FALSE)
+ return(list(t=t,p=p))
+ }

```

The next function is for testing the mean of subsets using F tests when the variance is common.

```

> Ftest <- function(data,model.family,model.design,s.1,s.2,s.3=NULL)
+ # data is an apc.data.list
+ # s are subset indices, that is sets of six numbers
+ {   append <- function(data,model.design,s,v=NULL,d=NULL)
+     {   data.s <- apc.data.list.subset(data,s[1],s[2],s[3],s[4],s[5],s[6],
+         suppress.warning=TRUE)
+         index <- apc.get.index(data.s)
+         v1 <- index$response[index$index.data]
+         d1 <- apc.get.design(index,model.design)$design
+         if(is.null(v)) v <- v1
+         else v <- c(v,v1)
+         if(is.null(d)) d <- d1
+         else
+         {   d0 <- matrix(0,nrow(d),ncol(d1))
+             d10 <- matrix(0,nrow(d1),ncol(d))
+             d <- rbind(cbind(d,d0),cbind(d10,d1))
+         }
+     }
+ }

```

```

+       return(list(v=v,d=d))
+   }
+   a <- append(data,model.design,s.1)
+   v <- a$v; d <- a$d
+   a <- append(data,model.design,s.2,v,d)
+   v <- a$v; d <- a$d
+   if(!is.null(s.3))
+   {   a <- append(data,model.design,s.3,v,d)
+       v <- a$v; d <- a$d   }
+   fit.R <- apc.fit.model(data,model.family,model.design)
+   if(model.family=="log.normal.response")
+   {   fit.R$deviance <- fit.R$RSS
+       fit.U <- glm.fit(d,log(v),family=gaussian(link = "identity"))
+   }
+   if(model.family=="od.poisson.response")
+   {   fit.U <- glm.fit(d,v,family=quasipoisson(link = "log"))
+   }
+   dev.R <- fit.R$deviance
+   dev.U <- fit.U$deviance
+   df.R <- fit.R$df.residual
+   df.U <- fit.U$df.residual
+   F <- (dev.R-dev.U)/(df.R-df.U)/(dev.U/df.U)
+   p <- pf(F,df.R-df.U,df.U,lower.tail=FALSE)
+ #   #   reproducing typo
+ #   F <- (dev.R/df.R)/(dev.U/df.U)
+ #   p <- pf(F,df.R,df.U,lower.tail=FALSE)
+   return(list(F=F,p=p))
+ }

```

We now define Table 4.5 and input the results from Bartlett's test.

```

> dim.names <- list(c("a","b","c"),c("LR/C","p","F","p","LR/C","p","F","p"))
> Table_4_5 <- matrix(NA,nrow=3,ncol=8,dimnames=dim.names)
> s1 <- c(0,0,0,0,0,14)
> s2 <- c(0,0,0,0,6,0)
> Bt <- Bartlett(data,"log.normal.response","AC",s1,s2)
> Ft <- Ftest(data,"log.normal.response","AC",s1,s2)
> Table_4_5[1,1:4] <- c(Bt$t,Bt$p,Ft$F,Ft$p)
> Bt <- Bartlett(data,"od.poisson.response","AC",s1,s2)
> Ft <- Ftest(data,"od.poisson.response","AC",s1,s2)
> Table_4_5[1,5:8] <- c(Bt$t,Bt$p,Ft$F,Ft$p)
> s1 <- c(0,0,0,10,0,0)
> s2 <- c(0,0,10,0,0,10)
> s3 <- c(0,0,0,0,10,0)
> Bt <- Bartlett(data,"log.normal.response","AC",s1,s2,s3)
> Ft <- Ftest(data,"log.normal.response","AC",s1,s2,s3)
> Table_4_5[2,1:4] <- c(Bt$t,Bt$p,Ft$F,Ft$p)

```

```

> Bt <- Bartlett(data,"od.poisson.response","AC",s1,s2,s3)
> Ft <- Ftest(data,"od.poisson.response","AC",s1,s2,s3)
> Table_4_5[2,5:8] <- c(Bt$t,Bt$p,Ft$F,Ft$p)
> s1 <- c(0,0,0,6,0,0)
> s2 <- c(0,0,14,0,0,0)
> Bt <- Bartlett(data,"log.normal.response","AC",s1,s2)
> Ft <- Ftest(data,"log.normal.response","AC",s1,s2)
> Table_4_5[3,1:4] <- c(Bt$t,Bt$p,Ft$F,Ft$p)
> Bt <- Bartlett(data,"od.poisson.response","AC",s1,s2)
> Ft <- Ftest(data,"od.poisson.response","AC",s1,s2)
> Table_4_5[3,5:8] <- c(Bt$t,Bt$p,Ft$F,Ft$p)
> Table_4_5

```

	LR/C	p	F	p	LR/C	p	F
a	6.287150	0.01216164	5.504489	3.481042e-08	11.67530	0.0006333518	6.627022
b	4.703779	0.09518913	4.484162	1.685108e-09	11.63477	0.0029753818	6.033364
c	1.116055	0.29076951	3.080728	7.938771e-06	15.07004	0.0001035946	2.504775
		p					
a		5.142369e-10					
b		2.717596e-13					
c		2.616378e-04					

## References

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