Equilibrium Analysis, Banking and Financial Instability*

By

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Abstract

This paper first extends the canonical General Equilibrium with Incomplete Markets (GEI) model with money and default to allow for competitive banking and financial instability. Second, it introduces capital requirements for the banking sector to assess the short and medium term macroeconomic consequences of the proposed New Basel Accord. *Monetary Equilibria with Commercial Banks and Default (MECBD)* exist and *financial instability and default emerge as equilibrium phenomena.*
A non-trivial quantity theory of money is derived and the term structure of interest rates incorporates both the 'expectations' and the 'liquidity preference' hypotheses. Thus, monetary, fiscal and regulatory policies necessarily generate real effects. *Non-neutrality* relies upon the real and nominal determinacy of MECBD.
A version of the liquidity trap holds and the Diamond-Dybvig (1983) result is a special case. Finally, because of the presence of capital requirements for banks, a trade off exists between regulatory policy and efficiency.
The model provides a useful analytical device for policy analysis of situations in which *crisis prevention and management* become necessary to reduce the risks and costs of financial instability.

Keywords: Financial instability, competitive banking, capital requirements, Basel accord, regulation, incomplete markets, default, non-neutrality, Gains-to-Trade.

JEL Classification: D52, E4, E5, G1, G2.

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1.Introduction

Recent financial crises in Texas banking during the 1980's, in Scandinavia and the U.K. in the early 90's, in Mexico in 1995, East Asia in 1997, Russia in 1998, the near-collapse of LTCM and Japan has renewed the interest in studying financial instability. The difficulty of analyzing financial instability lies in the fact that most of the crises manifest themselves in a seemingly unique manner and almost always require different policies for their tackling. A potpourri of models, primarily game-theoretic in nature, has been introduced to address financial instability. After the seminal papers of Bryant [7] and Diamond-Dybvig [13], a multitude of papers[1], have attempted to rationalize bank-runs and panics based on some type of co-ordination failure. Most of them depend on asymmetric information and some type of moral hazard friction.

Some additional minimum structural characteristics should be present in any model attempting to capture fundamental aspects of financial instability. First, it should be multiperiod, with aggregate uncertainty and agent heterogeneity. Second, money and liquidity constraints should be explicit, since financial crises evolve from the nominal sector and subsequently spread to the real economy. Third, since the performance of banks is critical for the study of financial instability a banking sector well integrated in the model is indispensable. Finally, the regulatory framework should be clearly defined for policy and sensitivity analysis of various regulatory regimes.

This paper follows a novel approach in modeling financial instability. Almost always a common feature of most crises is increased default and lower profitability in the banking sector. Empirical studies show that the amount of non-performing loans increases precipitously before and during a crisis, and bank profitability falls. A definition of financial instability that depends on increased default by the household and banking sector and reduced bank profitability is suggested. It allows for analysis of financial stability issues as a continuum of possible contingencies whereas standard definitions usually consider only polar situations, which are tantamount to financial crises. Consequently, analyzing financial instability in the continuum implies that crisis prevention/management policies may be readily applied before an actual crisis occurs.

The canonical GEI model with money and default by Dubey and Geanakoplos [16] and Dubey, Geanakoplos and Shubik [17] is extended to incorporate a competitive banking sector and capital requirements. Commercial banks are heterogeneous and maximize their expected profits. They are owned by their shareholders who have bought shares (as in Shubik and Tsomocos [38]). This modeling approach allows for a variety of financial institutions, not just commercial banks. Heterogeneous banks differ among themselves with respect to initial capital endowments, risk preferences (i.e., coefficients of risk aversion) and assessments of future scenarios (i.e., subjective probabilities). The modeling of the banking sector is akin to Tobin [49]: banks borrow from investors/households and from the Central Bank via the interbank credit market[2] and extend credit to them via the consumer credit markets. They also hold a diversified portfolio of securities.

The remaining characteristics of the model are consistent with the standard GEI and its extension to include money and default. The analysis of the proposed New Basel Accord needs the imposition of state dependent capital requirements that may or may not depend on other macroeconomic variables such as output and default. An equity market for ownership shares of commercial banks meets in the first period. The Central Bank interacts with commercial banks via the interbank market in the first period and loan settlement occurs in the second. One intraperiod consumer credit market per state and one interperiod consumer credit market, in which commercial banks extend credit to households, exist. Thus, commercial banks can be viewed as creators of "money" *à la* Tobin [41]. Commodity markets meet in each state and cash-in-advance is needed for all market transactions. Both households and banks are allowed to default on their financial obligations, namely, asset deliveries and loan repayments. They are penalized proportionally to their size of default by subtracting a linear term from their respective objective function.

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[1] For an excellent survey of these models see the textbook of Freixas and Rochet [19], where most of them are presented. Also, see [1] where the recent contributions of Allen and Gale are presented. A by no means exhaustive list of empirical studies includes [31], [34].
[2] The repo and the interbank market are collapsed into one, since my focus is on contagion and financial instability and not on monetary policy.

The closest methodological precursor to this model is the work of Martin Shubik [36], [37], [39] who introduced a Central Bank with exogenously specified stocks of money, and cash-in-advance constraints in a strategic market game.[3] Shubik [37] also emphasized the virtues of explicitly modeling each transaction. As in strategic market games, prices and the rest of the outcomes are formed by the choices of households and banks. However, since I focus on the monetary and liquidity effects and not on the oligopolistic effects, I adopt the continuum formulation. Agents regards prices as fixed as in [15] and [16].

The commercial banking sector of this model follows closely Shubik and Tsomocos [38], who used, however, gold-backed money and modeled a mutual bank with fractional reserves. Grandmont [25], [26], [27], also introduced a banking sector into general equilibrium with overlapping generations and he pointed out the inefficiency of trade with money. The modeling of money and default in an incomplete markets framework follows the models developed by Dubey and Geanakoplos [15], [16] and Dubey, Geanakoplos and Shubik [17]. However, [15] is a one period model with money and default, [16] includes incomplete asset markets and money, and [17] has incomplete asset markets, default and no money. None of the previous papers combines all three ingredients, incorporates a competitive commercial banking sector, and focuses on financial instability. Also, Drèze and Polemarchakis [14] have introduced a monetary sector akin to [16]. Default is modeled as in Shubik [37] and Shubik and Wilson [39], namely by subtracting a linear term from the objective function of the defaulter proportional to the debt outstanding.

In sections 2-4, the model, the budget sets are presented and MECBD is defined. Section 5 derives the quantity theory of money proposition in which both prices and quantities adjust in response to policy changes. This result differs from Lucas [32], [33], because he postulated a "sell-all model" in which every agent sells everything he owns in every period. In this model agents transact only if they wish to do so. In addition, the term structure of interest rates is specified in which the expectations and the liquidity preference hypothesis are accommodated and default influences its shape. Section 6 shows that the linear asset pricing rule fails to hold in equilibrium because of the "liquidity cost" of transactions due to positive interest rates. Positive interest rates induce a "price wedge" between the selling and buying price of an asset equal to $(1 + r)$. Section 7 establishes existence of MECBD provided that the necessary gains-to-trade are present in the initial allocation.

A definition of financial instability is proposed in section 8. A version of the Keynesian liquidity trap holds in which commodity prices stay bounded whereas asset trades tend to infinity whenever monetary policy is loosening. Moreover, this situation corresponds to a financially unstable equilibrium. The seminal Diamond-Dybvig [19] result of bank runs manifests itself in the model under certain assumptions, namely homogeneity of commercial banks. Finally, in sections 9-10 the issues of determinacy, non-neutrality and the relationship among MECBD, GEI and GE are discussed. All the proofs of the theorems, propositions and corollaries are relegated to the appendix.

2. The Model

2.1 The Economy

Consider the canonical general equilibrium with incomplete markets model in which time extends over two time periods. The consolidated government/central bank and the regulator are modeled as "strategic dummies". Households participate in the trade of commodities, assets, consumer loans and shares of commercial banks. Commercial banks lend to the consumer credit markets and admit deposits. Also, they borrow and lend in the interbank credit market. Finally, they invest in the asset market and auction their shares of ownership in the equity market. The consolidated government/central bank operates in the interbank credit market via open market operations[4] (OMOs).[5] The regulator fixes the bankruptcy code for households and commercial banks exogenously and sets the capital-adequacy requirements for the commercial banks.

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[3] The cash-in-advance constraint can be traced at least as far as back as Clower [10], and has been used by Lucas [32], [33].
[4] Alternatively, it sets the interbank interest rate and provides liquidity for interbank reserves, as is usually the current Central Bank practise.
[5] In [23], we allow for fiscal policy, money financed fiscal transfers, taxation and foreign exchange intervention. A more general formulation of the model can be found in [43].

Formally, the notation that will be used henceforth is as follows:
$t \in T = \{0,1\}$ = time periods,
$s \in S = \{1,\ldots,S\}$ = set of states at t=1,
$S^* = \{0\} \cup S$ = set of all states,
$h \in H = \{1,\ldots,H\}$ = set of economic agents (households/investors),
$b \in B = \{1,\ldots,B\}$ = set of commercial banks,
$l \in L = \{1,\ldots,L\}$ = set of commodities,
$R_+^L \times R_+^{SL}$ = commodity space indexed by $\{0,\ldots S\} \times \{1,\ldots L\}$,
$e^h \in R_+^L \times R_+^{SL}$ = endowments of households[6],
$e^b \in R_+^{S*}$ = capital endowments of commercial banks.
$u^h : R_+^L \times R_+^{SL} \rightarrow R$ = utility function of agent $h \in H$,
$\chi_{sl}^h \equiv$ consumption of commodity l in state s by $h \in H$.
The standard assumptions hold:
(A1) $\forall s \in S^*$ and $l \in L$, $\sum_{h \in H} e_{sl}^h > 0$,
(i.e., every commodity is present in the economy.)
(A2) $\forall s \in S^*$ and *h (b)* $\in$ *H (B),* $e_{sl}^h > 0$ ($e_s^b > 0$) for some $l \in L$ ($s \in S^*$),
(i.e., no household (commercial bank) has the null endowment of commodities (capital) in any state of the world.)
(A3) Let A be the maximum amount of any commodity sl that exists and let ***1*** denote the unit vector in R^{SLxL}. Then $\exists\, Q>0 \ni u^h(0,\ldots Q,\ldots 0) > u^h$ for Q in an ordinary component (i.e., strict monotonicity in every component). Also, continuity and concavity are assumed.

The money supply expansion mechanism of the economy highlights the importance of introducing multiple banks with active choice sets. Regulatory intervention in the financial system occurs primarily through the banking industry (e.g., capital-adequacy ratios etc). It is evident that the liquidity of the monetary economy as well as the equilibrium outcomes are affected by the risk profiles of commercial banks. Finally, the heterogeneity of banks is a crucial ingredient of analysing systemic effects of exogenous shocks occurring in a continuum of possible outcomes.

$u^b(\pi_0^b, \pi_1^b, \ldots, \pi_s^b) : R_+^{S*} \rightarrow R$ = utility function of bank b,
π_s^b = monetary holdings of b at $s \in S^*$.
A straightforward assumption is imposed.
(A4) Let A_m be the maximum amount of money present in the economy and let ***1*** denote the unit vector in R^{s^*}. Then $\exists Q > 0 \ni u^b(0\ldots,Q,\ldots 0) > u^b(A_m 1)$
for Q in an arbitrary component (i.e., strict monotonicity in every component). Also continuity and concavity are assumed.

2.2 Government, Central Bank and the Regulator

There is a government sector, which has the capacity to act on the interbank market through, for example, the Central Bank or Federal Reserve. A regulatory agency legislates the bankruptcy code of the economy and fixes the capital adequacy requirements. Both of these institutions' actions are exogenously specified and the consequences of their choices are analysed.

Formally, the vector (M^G, μ^G) gives the government and Central Bank's actions where,
M^G = OMOs on behalf of the government/central bank, and
μ^G = bond sales by the government/Central Bank.

[6] In section 2.5 monetary endowments will be allowed.

Note that the government is not required to spend less than it borrows; the existence of equilibrium is compatible with the Central Bank printing money to finance its expenditures. All the results hold for both cases, (i.e., with or without money financing) except where otherwise stated.[7]

Similarly, the following vector gives the regulator's actions
$(\kappa,\lambda,\omega)\equiv\left((\kappa_s^b)_{b\in B,s\in S};(\lambda_s^h)_{h\in H\cup B,s\in S^*};(\omega_{ij}^b)_{b\in B,i\in S^*,j\in J}\right)$ where,

κ's are the time varying capital-adequacy ratios prevailing in the commercial banking sector, λ's are the bankruptcy penalties imposed upon the parties breaking their contractual obligations and ω's are the time-varying risk weights of bank assets that apply for the calculation of the capital requirements.[8] The analysis of default and bankruptcy is conducted in section 2.7 and of the capital requirements in section 2.8.

2.3 The Time Structure of Markets

At $t=0$, the commodity, asset, equity, credit (long and short) and interbank markets meet. At the end of the first period consumption and settlement (i.e., principal, interest rate and bankruptcy penalties payment) of the one-period loans take place. At $t=1$, the commodity and short-term credit markets meet, and long-term loans and assets are delivered. At the end of the second period consumption and settlement of the interbank, long-term and second period short-term loans defaults take place. Capital requirements need to be met at the end of each period for each state. Figure1 makes the time line of the model explicit.

Note that bankruptcy settlements occur in both periods and liquidity injections in the interbank market can be thought of as aggregate Lender of Last Resort support (LOLR) to the market in response to an exogenous adverse shock. An example is Federal Reserve intervention in the aftermath of the 1998 Asian crisis. This type of intervention is tantamount to Emergency Liquidity Assistance (ELA) to the market as a whole.[9]

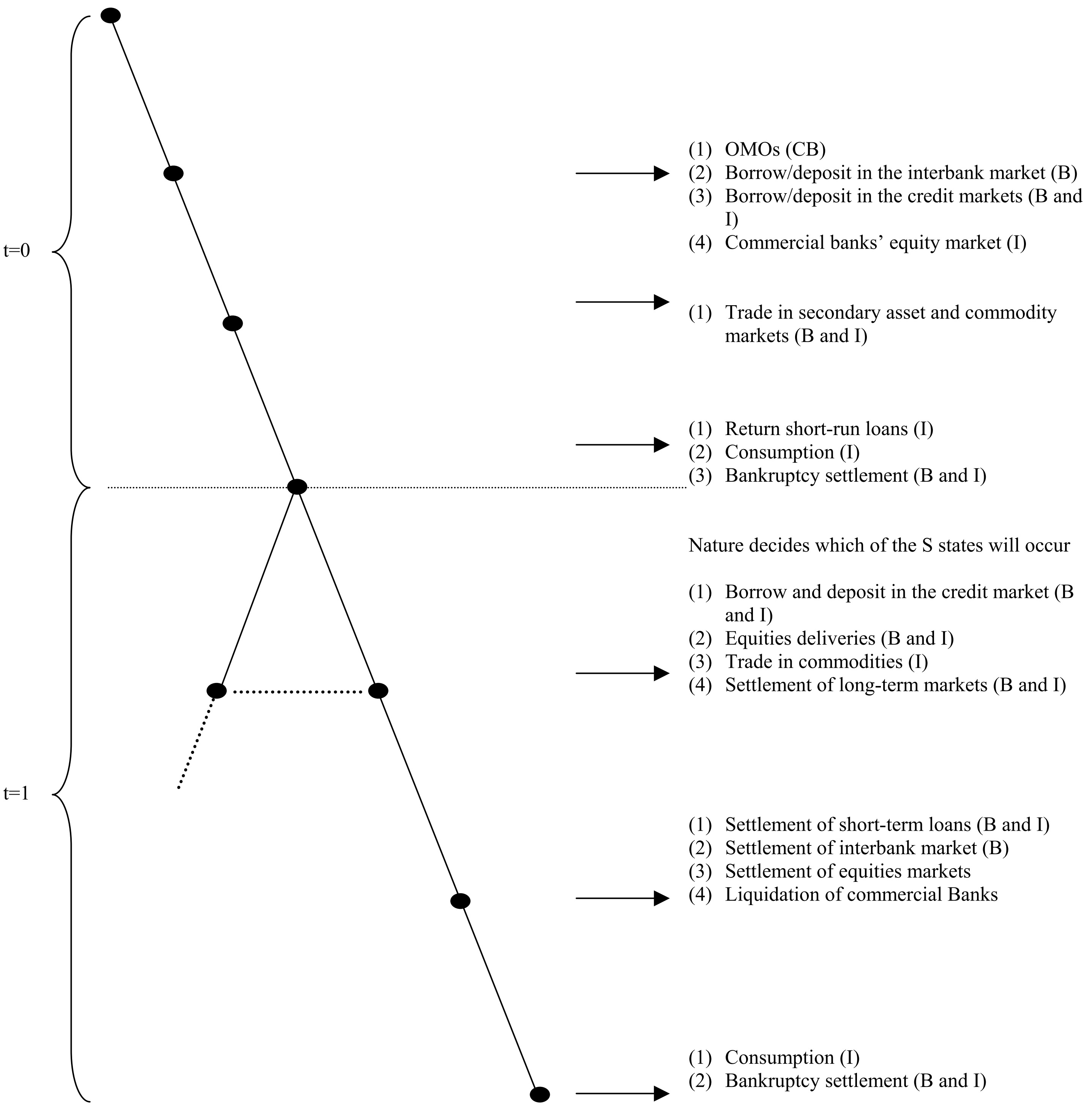
2.4 Asset Markets

The set of assets is $J=\{1,...J\}$. Assets are promises sold by the seller in exchange for a price paid by the buyer today. They are traded at $t=0$ and the contractual obligations are delivered at $t=1$ at a particular state $s\in S$. An asset $j\in J$ is denoted by a vector $A^j\in R_+^{S(L+1)}$ indicating the collection of goods deliverable plus the money at any future state $s\in S$. Therefore, the asset market is summarized by an $((L+1)S)\times J$ matrix $\boldsymbol{A}$. All the deliveries are made in money. When the assets promise commodities the seller of the assets delivers the money equivalent of the value of the agreed commodities at their spot prices at the relevant state. Real assets are necessary for the possibility of the liquidity trap in equilibrium (see section 8.2).
Whenever rank$|J|=$rank$|S|$ the capital markets are said to be complete whereas when rank$|J|<$rank$|S|$ the markets are said to be incomplete. The case when some assets are missing is the most interesting one because in addition to its realism it allows for the study of financial innovation, government intervention and a multitude of other interesting macroeconomic phenomena.

Furthermore,
(A5) $A^j\neq 0,\forall j\in J$.
(i.e., no asset makes zero promises.)
(A6) $A^j\geq 0,\forall j\in J$.
(i.e., asset payoffs are non-negative.)

[7] However, at the end of $t=1$ the government observes its budget constraints. As I discuss in section 9, money non-neutrality depends on the positive interest rate and not exclusively on government financing.
[8] Time varying risk weights are consistent with the proposed new Basel Accord.
[9] In order to minimise moral hazard, central banks do not usually preannounce or set specific criteria for LOLR preferring instead a policy of "constructive ambiguity".

FIGURE 1
TIME STRUCTURE OF THE MODEL



G = Government/Central Bank
CB = Central Bank
B = Commercial Banks
I = Investors

Without loss of generality I present a two-period version of the model. The same sequence of transactions applies for any finite horizon (i.e., $T = (0,1 \ldots T)$).

Finally, agents do not hold positive endowments of assets and thus all asset transactions are short sales. In the spirit of perfect competition asset prices facing individuals are fixed. Let $b_j^h \equiv$ amount of money sent by agent *h* in the market of asset *j*. Also, let $q_j^h \equiv$promises sold of asset *j* by agent $h \in H$. In equilibrium, at positive levels of trade $0 < \theta_j < \propto$,

$$\theta_j = \sum_{h\in H} b_j^h + \sum_{b\in B} b_j^b \Big/ \sum_{h\in H} q_j^h + \sum_{b\in B} q_j^b \;,$$

for $j \in J$, $h \in H$, $b \in B$. All the asset markets meet contemporaneously; hence cash obtained from the sale of asset *j* cannot be used for the purchase of another asset $j' \neq j$. Thus, the volume of trade in the asset market is affected by the overall liquidity of the economy. This way monetary policy interacts with asset markets and influences asset prices.[10]

2.5 Money and Credit Markets

Fiat money is the stipulated means of exchange. All commodities can be traded for money, and (as noted) all assets deliver exclusively in money. Money can be either inside or outside. At the outset NBPS and banks hold net monetary assets -outside money- (i.e., *as if* they hold central bank liabilities). Inside money is credit created by the banking sector through the credit markets in period 0 via monetary policy and is accompanied by debt. Commercial banks receive central bank money as an asset but simultaneously incur a debt liability. In turn, commercial banks lend to the non-bank private sector. This represents an asset of the commercial bank and thus a liability of the non-bank private sector. The net assets of the private sector as a whole remain unchanged. Cash-in-advance is required for any purchase.

Market is regarded as a symmetric exchange between two instruments (for more discussion see Dubey and Geanakoplos [16]). Just as agents cannot "sell" money they do not have in a market, so in the model agents cannot sell commodities they do not have. The only exception is credit markets, where we allow agents to write their own promises (bonds).[11]

Money enters the economy in three ways. First, it may be present in the private endowments of agents and commercial banks. Agent $h \in H$ has an endowment m_s^h of money, for each $s \in S^*$ and commercial banks have initial capital endowment e_s^b, for each $s \in S^*$. Second, when government and/or the Central Bank) purchases commodities and/or bonds with currency, it injects money in the economy. Third, when the government (perhaps through the Central Bank) repays previously issued government bonds, it also injects money in the economy. Money exits the system in two ways: bonds from investors/households and payment on bonds (promises) sold to the government.

Two kinds of bond (equivalently credit or loan) markets can be distinguished. Short-term (intraperiod or overnight) bonds promise 1 unit of fiat money at the end of the same period in which they are taken. Long-term (interperiod) bonds promise 1 unit of money at the beginning of the next period, but after the next short loan begins. Note that one should not expect $r_o = \bar{r}$. Typically, the equilibrium term structure can be quite complex and depends on the multiple factors that influence the demand for money.

Agents are permitted to buy and sell bonds, i.e., to borrow and deposit money in the consumer credit markets. Let $\mu_s^h \left(or \bar{\mu}^h \right)$ be the amount of zero-coupon bonds issued by $h \in H$, or equivalently, the amount of money agent *h* chooses to owe on the short loan (or long loan). Let $d_s^h \left(or \bar{d}^h \right)$ be the amount of money that agent $h \in H$ spends on purchases of short-term (long-term) bonds. If all agents pay exactly what they owe, then we must have that,

[10] Since liquidity affects asset prices, the *asset price inflation* channel of monetary policy is present in the model.
[11] An alternative interpretation of the cash in advance constraint is that financial markets clear immediately whereas commodity markets adjust sluggishly.

$$\left(1+r_s\right)=\sum_{h\in H}\mu_s^h \Big/ \sum_{h\in H} d_s^h + \sum_{b\in B} m_s^b \quad , \quad \forall s \in \{0,1,...,S\}$$

and similarly for the long-term credit market by replacing $(r_s, \mu_s^h, d_s^h, m_s^b)$ with $(\bar{r}, \bar{\mu}^h, \bar{d}^h, \bar{m}^b)$, where, $m_s^b\left(\overline{m}^b\right)$ is the amount of credit that commercial banks extend which is also subject to their capital-adequacy ratio as specified by the regulator.

Bonds, money and assets can be inventoried; they are the only stores of value in our model.

2.6 Default

Let $\Omega=\{N\}\cup\{J\}=\{0^*,\bar{0},0,1,...,S\}\cup\{1,...,J\}$. Ω is the set of all credit markets (including the interbank) and secondary asset markets ($0^* \equiv$ interbank, $\bar{0}\equiv$ long-term credit market, $0,1,\ldots,S \equiv$ short-term credit markets). Let us define $D_{s\omega}^h=(1-v_{s\omega}^h)\mu_s^h$ $\left(D_{s\omega}^b=(1-v_{s\omega}^b)\mu^b\right)$ where $v_{s\omega}^h$ $(v_{s\omega}^b)$ is the rate of repayment by households (banks). $D_{s\omega}^h$ is the nominal value of debt due to default either in the credit markets (analogously in the asset market or deposits and interest rate obligations). In practice, default penalties and the bankruptcy code depend normally on the nominal values of debt and are adjusted at discrete intervals as the general level of prices increases. In the model, nominal values are deflated so as to penalize households and banks on "real" default.[12]

The parameters $\lambda_{s\omega}^h$ $(\lambda_{s\omega}^b)$ to represent the marginal disutility of defaulting for each "real" dollar on assets or loans in state s. Therefore, the payoffs to investors/households and commercial banks will be respectively $\forall$ $s \in S^*$,

$$\Pi_s^h\left(x_s^h,\left(D_{s\omega}^h\right)_{\omega\in\Omega},p_s\right)=u_s^h(x_s)-\frac{\sum_{\omega\in\Omega}\lambda_{s\omega}^h\left[D_{s\omega}^h\right]^+}{p_s g_s}, \text{for } h\in H\cup B.$$

where g_s is the base basket of goods which serves as a price deflator with respect to which the bankruptcy penalty is measured and $[c]^+\equiv\max[0,c]$.

In equilibrium, agents equalize the marginal utility of defaulting with the marginal disutility of the bankruptcy penalty.[13] Thus, expected rates of delivery of interbank, long-term, short-term in $s=0$, short-term loans for all $s\in S$, for assets, and analogously for deposits $R=(\tilde{R}_s,\bar{R}_s,R_s,R_{sj},\tilde{R}_{ds},\bar{R}_{ds},R_{ds})$ $\forall s\in S^*$ and $j\in J$, are equal to actual rates of delivery in equilibrium. This idea is precisely the crucial ingredient of the model. It allows us to establish default as an equilibrium outcome without necessarily destroying the orderly functioning of the financial system.

2.7 Capital Requirements

The regulator sets the banks' minimum capital requirements. Given that the assets of the commercial banks consist of loans (including interest rate payments[14]) and their asset investments, the capital requirement constraint becomes,

[12] Complicated structures of default penalties can be incorporated into the model as long as concavity of the payoff function is maintained. For further details on this topic see [15], [17]. Without loss of generality, a simple specification is used that assumes penalties are adjusted instantaneously following changes in the price level.
[13] It shown in [17] that more lenient default penalties may indeed increase welfare in equilibrium.
[14] Interest rate payments paid on loans are considered assets since they represent investors' obligations payable to the bank. Also, I assume, as it is evident from the budget constraint of the commercial banks (see section 3.2), that they are totally reinvested.

$$k_{bs} \leq \frac{e_s^b + \sum_{h\in H} u_b^h + c_s^b}{w_{ss}^b(\eta,\sigma) R_s m_s^b (1+r_s) + w_{s\bar{0}}^b(\eta,\sigma)\overline{R}_s \overline{m}^b (1+\bar{r}) + \sum_{j\in J} w_{sj}^b(\eta,\sigma) R_{sj}(p_s A_s^j)(\frac{b_j^b}{\theta_j}) + w_{s0^*}^b \widetilde{R}_{ds}(1+\rho)d^b}$$

$\forall s \in S \ , b \in B,$

and

$$k_{b0} \leq \frac{e_0^b + \sum_{h\in H} u_b^h + c_0^b}{w_{00}^b(\eta,\sigma) R_0 m_0^b (1+r_0) + w_{0\bar{0}}^b(\eta,\sigma)\overline{m}^b (1+\bar{r}) + \sum_{j\in J} w_{0j}^b(\eta,\sigma)(p_s A_s^j)(\frac{b_j^b}{\theta_j}) + w_{00^*}^b(\eta,\sigma)(1+\rho)d^b},$$

for $s = 0$,

where, $w_{0s}^b \equiv$ risk weights for short term loans, $w_{s\bar{0}}^b \equiv$ risk weights for long term loans, $w_{s0^*}^b \equiv$ risk weights for interbank loans, w_{sj}, $\forall j \in J \equiv$ risk weights for assets that the regulator establishes for the valuation of the bank's assets, $\eta \equiv$ set of macrovariables, and $\sigma \equiv$ choice variables of households and commercial banks.[15] In addition, $\sum_{h\in H} u_b^h \equiv$ initial equity of commercial banks and $c_s^b \equiv$ capital adjustment. Thus, commercial banks decide on any capital adjustment jointly with the structuring of their portfolio to satisfy their capital requirements constraint. Banks may not necessarily hold the same capital since the precautionary capital over and above the regulatory minimum can vary across banks.[16] Note that credit requirements are calculated with respect to the *realised* asset deliveries in equilibrium and *not the expected* ones. The impact of regulatory policy is similar to the ones of monetary policy since it affects credit extension.

Finally, it should be noted that increased default reduces utility but it may also have a counterbalancing effect. If risk weights were countercyclical, higher defaults, as would occur in a recession, would lead banks to *reduce* their risk weights in the expectation that future economic conditions would improve. In turn, if capital constraints were binding, lower risk weights would allow for an increasing credit extension that reduces interest rates and thus facilitates transactions.[17] Thus, utility may very well increase if the latter effect dominates the former. In such a situation capital requirements may be thought of as a built-in-stabiliser of the economy.

2.8 The Commodity Markets

Commodity prices p_{sl} are taken to be fixed by the agents. Let $b_{sl}^h \equiv$ amount of fiat money sent by agent *h* to trade in the market of commodity $sl \in L$ and $q_{sl}^h \equiv$ amount of good *l* offered for sale at state *s* by agent *h*. Also $q_{sl}^h \leq e_{sl}^h$. In equilibrium, at positive levels of trade, $0 < p_{sl} < \infty$,

$$p_{sl} = \sum_{h\in H} b_{sl}^h \Big/ \sum_{h\in H} q_{sl}^h \ .$$

2.9 Commercial Banks

Commercial banks enter the model because of their importance both for the transmission of monetary policy and for financial stability. The risk profiles of commercial banks have an effect on both liquidity and the credit

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[15] The risk weights may or may not be invariant. For example, they may be time varying and depend on other macrovariables or choices of investors and/or banks. Catarineu-Rabell, Jackson and Tsomocos [9] examine the case where they depend on investors' default either procyclically or countercyclically.

[16] Since commercial banks almost always observe their capital requirement constraint, I do not need to establish default penalties for their violation. However, this could have been very easily incorporated into the model. See Goodhart and Tsomocos [24].

[17] See Catarineu-Rabell, Jackson and Tsomocos [9].

expansion of the economy. Consequently, the money supply multiplier of the economy depends on the portfolio of the banking sector. Thus, equilibrium outcomes of the economy and the effectiveness of monetary policy depend on the banking sector.

Let $b \in B = \{1,...,B\}$ be the set of commercial banks. We assume:
(A7) perfectly competitive banking sector (i.e., commercial banks take interbank interest rates and asset prices as fixed).
(A8) perfect financial intermediation (i.e., no market imperfections with respect to information and participation in the capital and credit markets).
An important consequence of (A8) and (A9) is that there is no margin between borrowing and lending rates.

Commercial banks participate in the interbank market by borrowing and lending. They then extend credit to the consumer credit markets and allocate the appropriate capital to satisfy their capital requirement constraint. Moreover, commercial banks diversify their portfolios by also investing in the asset markets. Thus, the model can encompass the interaction between monetary policy and the asset markets. The modeling of the banking behavior is akin to the portfolio balance approach of the banking firm introduced by Tobin [57], [58].

At the beginning of period zero an equity market operates in which banks issue equity to the investors. Shares of ownership of the bank are determined on a prorated manner according to the formula[18],

$$s_b^h = u_b^h \Big/ \sum_{h \in H} u_b^h$$

where, $u_b^h \equiv$ amount of money offered by h for ownership shares of banks $b \in B$.[19]
Finally at $t=1$ the profits of commercial banks are liquidated and distributed back to the individual owners according to their ownership shares. This way I close the model.[20]

An important phenomenon that now appears is that not all the investors will defray their loans and that not all banks might honor their contractual obligations. The different risk-attitudes and the bankruptcy penalties imposed on defaulting banks make this compatible with equilibrium.

A simplified version of the bank's balance sheet is:
(Deposits)+(Equity)≡(Required Reserves)[21] *+ (Loans) + (Interbank Deposits) + (Asset Investments).*

2.10 Interbank Credit Market

The Central Bank conducts its monetary policy through OMO's in the interbank market. Also, interbank lending and borrowing occurs in this market. The existence of this market establishes interbank linkages and in the case of default causes a domino effect. Since perfect financial intermediation is assumed default is prorated among all the lenders. Thus, contagion and interbank linkages are equilibrium phenomena that manifest themselves via the perfect intermediation and do not necessarily require oligopolistic (or monopolistic competition) market structure or other market imperfections.

The interbank interest rate is established in equilibrium at positive levels of trade,

$$(1+\rho) = \sum_{b \in B} \mu^b + \mu^G \Big/ \sum_{b \in B} d^b + M^G$$

where, $\mu^b \equiv$ amount of zero-coupon bonds issued by bank b, or equivalently the amount of money bank b chooses to owe in the interbank credit market, $d^b \equiv$ amount of money that bank b deposits, or equivalently the

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[18] When the model extends over to more than two periods then dilution of the existing ownership structure can be introduced by allowing retrading of the ownership shares of the banks. However, this extension would be more appropriate after I introduce production into the model and analyse the capital structure of banks and firms. See Goodhart and Tsomocos [24].
[19] It would be redundant to allow commercial banks to bid for each other's shares since the ultimate shareholders of the banks are the investors themselves and the horizon is finite.
[20] However, the reader, who is willing to do so, can mentally suppress the banks' equity market without compromising the understanding of the results.
[21] Required reserves need to satisfy the required reserve ratio if the monetary authority has established one. For simplicity, I do not include it in the model.

amount of money that bank b spends on purchases of bonds. Similarly, $\mu^G \equiv$ amount of zero-coupon bonds issued by the central bank and $M^G \equiv$ central bank money supply or equivalently the amount of money the central bank spends on purchases of bonds.[22]

3. The Budget Set

It is assumed that commodities are perishable lasting only one period, and that each market meets once in each period. Aside from putting an upper bound on the velocity of money, the drawback of this simplification is that order in which the markets meet needs to be carefully chosen. The reason of this is to maintain the cash in advance requirement (i.e., to ensure that agents have the money before they spend). Accordingly, at the beginning of each period, intraperiod bank loans are available so that agents can borrow the cash to make purchases. The timing of the interperiod loan does not matter, as long as agents can roll over loans by alternating short and long loans.

3.1 Investors

The macro variables which are determined in equilibrium, and which every agent regards as fixed, are denoted by $\eta = (p, \rho, r_s, \bar{r}, \theta, R)$. The choices of the NBPS $h \in H$, are denoted by $\sigma^h \in \Sigma^h(\eta)$
where,
$\sigma^h = (\chi^h, \bar{\mu}^h, \mu^h, d^h, \bar{d}^h, b^h, q^h, u^h, v^h) \in R_+^{LS^*} \times R_+ \times R_+^{S^*} \times R_+^{S^*} \times R_+ \times R_+^{LS^*+J} \times R_+^{LS^*+J} \times R_+^B \times R_+^{S^*+J+1}$
is the vector of all of investors' decisions. Denote the macro variables which are determined in equilibrium, and which every agent regards as fixed, by $\eta = (p, \rho, r_s, \bar{r}, \theta, R)$. Denote the choices of an investor $h \in H$, $\sigma^h \in \Sigma^h(\eta)$
where,
$\sigma^h = (\chi^h, \bar{\mu}^h, \mu^h, d^h, \bar{d}^h, b^h, q^h, u^h, v^h) \in R_+^{LS^*} \times R_+ \times R_+^{S^*} \times R_+^{S^*} \times R_+ \times R_+^{LS^*+J} \times R_+^{LS^*+J} \times R_+^B \times R_+^{S^*+J+1}$
is the vector of all of his market decisions.
The variables represent the following quantities:
$\chi^h \equiv$ consumption,
$\bar{\mu}^h \equiv$ long-term loan,
$\mu_s^h \equiv$ short-term loans at each $s \in S^*$,
$\bar{d}^h \equiv$ long-term deposits,
$d_s^h \equiv$ short-term deposits,
$b_{sl}^h, (b_j^h) \equiv$ amounts of money offered in the goods (asset) markets,
$q_{sl}^h, (q_j^h) \equiv$ sales of goods and assets,
$u^h \equiv$ bid for ownership shares of the commercial banks, and
$v_{n(j)}^h, (\bar{v}_s^h) \equiv$ percentage deliveries of promised short-term loans, assets and long-term loans.

$B^h(\eta) = \{\sigma^h \in \Sigma^h(\eta) : (1) - (10) \text{ below}\}$ is the budget set, where Δ(i) represents the difference between RHS and LHS of inequality (i).

For $t=0$,

$$d_0^h + \bar{d}^h \le m_0^h \qquad (1^h)$$

(i.e., short-term + long-term deposits ≤ initial private monetary endowment).

[22] Instead of conducting OMOs, the Central Bank could determine the interest rate letting borrowing and lending with commercial banks equilibrate the market. In fact, this is the current practice of implementing monetary policy. The crucial point is that the Central Bank has one degree of freedom and therefore can use only one of the two policy instruments.

$$\sum_{j\in J} b_j^h + \sum_{b\in B} u_b^h + \sum_{l\in L} b_{0l}^h \le \Delta\left(1^h\right) + \frac{\mu_0^h}{\left(1+r_0\right)} + \frac{\overline{\mu}^h}{\left(1+\overline{r}\right)} \qquad (2^h)$$

(i.e., expenditures for commodities, equities and banks' shares of ownership ≤ money at hand + borrowed money in the short and long-term credit markets).

$$q_{0l}^h \le e_{0l}^h \quad , \quad \forall l \in L \qquad (3^h)$$

(i.e., sales of commodities ≤ endowments of commodities).

$$v_0^h \mu_0^h \le \Delta\left(2^h\right) + \sum_{l\in L} p_{0l} q_{0l}^h + R_0 d_0^h \left(1+r_0\right) + \sum_{j\in J} \theta_j q_j^h \qquad (4^h)$$

(i.e., short-term loan repayment ≤ money at hand + receipts from sales of commodities and equities + deposits and interest repayment).

$$x_{0l}^h \le e_{0l}^h - q_{0l}^h + \frac{b_{0l}^h}{p_{0l}} \quad , \quad \forall l \in L \qquad (5^h)$$

(i.e., consumption ≤ initial endowment - sales + purchases).

$\forall s \in S$,

$$d_s^h \le \Delta\left(4^h\right) + m_s^h \qquad (6^h)$$

(i.e., short-term deposits in state s ≤ money at hand + initial private monetary endowment in state s).

$$\sum_{l\in L} b_{sl}^h + \sum_{j\in J} \left(v_{sj}^h p_s A_s^j\right) q_j^h + \overline{v}_s^h \bar{\mu}^h \le \Delta\left(6^h\right) + \frac{\mu_s^h}{\left(1+r_s\right)} + \overline{R_{ds}}\,\overline{d}^h \left(1+\overline{r}\right) \qquad (7^h)$$

(i.e., expenditures for commodities, equities deliveries and long-term loan repayments ≤ money at hand + short-term loan in state s + long-term deposits and interest repayments).

$$q_{sl}^h \le e_{sl}^h \quad , \quad \forall l \in L \qquad (8^h)$$

(i.e., sales of commodities ≤ endowments of commodities).

$$v_s^h \mu_s^h \le \Delta\left(7^h\right) + \sum_{j\in J} \left(R_{sj} p_s A_s^j\right) \left(\frac{b_j^h}{\theta_j}\right) + R_{ds} d_s^h \left(1+r_s\right) + \sum_{b\in B} \left(\frac{u^h}{\sum_{h\in H} u^h}\right) \pi_s^b + \sum_{l\in L} p_{sl} q_{sl}^h \qquad (9^h)$$

(i.e., short-term loan repayment in state s ≤ money at hand + receipts from equities deliveries, sales of commodities, deposits and interest repayment, distribution of commercial banks' profits).

$$x_{sl}^h \le e_{sl}^h - q_{sl}^h + \frac{b_{sl}^h}{p_{sl}} \quad , \quad \forall l \in L \qquad (10^h)$$

(i.e., consumption ≤ initial endowment – sales + purchases).

3.2 Commercial Banks

Denote the choices of a commercial bank $b \in B$, $\sigma^b \in \Sigma^b(\eta)$, where $\sigma^b = \left(\mu^b, d^b, \overline{m}^b, m_s^b, b_j^b, q_j^b, c_s^b, v_s^b, \pi_s^b\right) \in R_+ \times R_+ \times R_+ \times R_+^{S^*} \times R_+^J \times R_+^J \times R_+^{S^*} \times R_+^{S^*+J+2} \times R_+^{S^*}$ is the vector of all its market decisions.

The variables represent the following quantities:

$\mu^b \equiv$ interbank market loans,

$d^b \equiv$ interbank market lending (deposits),

$m_s^b \equiv$ credit extension at the various short-term credit markets,

$m^b \equiv$ credit extension at the long term credit market,

$b_j^b \equiv$ amount of money offered for the purchase of assets,

$q_j^b \equiv$ sales of assets,

$c_s^b \equiv$ capital adjustment,

$v_{s(j)}^b, (\bar{v}_s^b, \bar{\bar{v}}_s^b, \tilde{v}_s^b) \equiv$ percentage deliveries of promised assets, interest rate payments on deposits and interbank loans.

$\pi_s^b \equiv$ final monetary holdings (profits) in every state.

$B^b(\eta) = \{\sigma^b \in \Sigma^b(\eta) : (1)-(7) \text{below}\}$, is the budget set, where Δ(i) represents the difference between RHS and LHS of inequality (i).

For $t=0$,

$$d^b \le e_0^b \qquad (1^b)$$

(i.e., deposits in the interbank market ≤ initial capital endowment).

$$m_0^b + \bar{m}^b + \sum_{j \in J} b_j^b \le \left(\Delta(1^b) + \frac{\mu^b}{(1+\rho)} + \sum_{h \in H} u_b^h \right) \qquad (2^b)$$

(i.e., credit extension in the short and long-term market + expenditures for equities ≤ money at hand + interbank loan + receipts from ownership shares' sales).

$$(v_0 (1+r_0) \sum_{h \in H} d_0^h) z_0 + c_0^b \le \Delta(2^b) + (R_0 \sum_{h \in H} \mu_0^h) z_0 + \sum_{j \in J} \theta_j q_j^b \qquad (3^b)$$

(i.e., short-term deposits and interest repayment + capital adjustment ≤ money at hand + short-term loans repayment + receipts from equities sales).

$$k_{b0} \le \frac{e_0^b + \sum_{h \in H} u_b^h + c_0^b}{w_{00}^b(\eta,\sigma) R_0 m_0^b (1+r_0) + w_{0\bar{0}}^b(\eta,\sigma) \bar{m}^b (1+\bar{r}) + \sum_{j \in J} w_{0j}^b(\eta,\sigma)(p_s A_s^j)(\frac{b_j^b}{\theta_j}) + w_{00^*}^b(\eta,\sigma)(1+\rho) d^b} \qquad (4^b)$$

(i.e., capital requirements constraint in period 0).

$\forall s \in S$,

$$m_s^b + (\sum_{h \in H} \bar{v}_s^b (1+\bar{r}) \bar{d}^h) \bar{z} \le \Delta(3^b) + (\sum_{h \in H} \bar{R}_s \bar{\mu}^h) \bar{z} + e_s^b = \pi_0^b \qquad (5^b)$$

(i.e., credit extension in the short-term credit market in state s + long-term deposits and interest repayment ≤ money at hand + long-term loan repayment + initial capital endowment in state $s \equiv$ period 0 profits).

$$c_s^b + \sum_{h \in H} v_s^b d_s^h (1+r_s) z_s + (\bar{\bar{v}}_s^b (1+\rho) \sum_{b \in B} d^b) z + \tilde{v}_s^b \mu^b + \sum_{j \in J} v_{sj}^b p_s A_s^j q_j^b \le$$

$$\Delta(5^b) + \sum_{j \in J} (R_j p_s A_s^j) \left(\frac{b_j^b}{\theta_j} \right) + \tilde{R}_{ds} d^b (1+\rho) z + (\sum_{h \in H} R_s \mu_s^h) z_s \qquad (6^b)$$

(i.e., capital adjustment + short-term deposits and interest repayment + interbank loan repayment + expenditures for equities deliveries ≤ money at hand + money received from equities payoffs + interbank deposits and interest repayment + short-term loan repayment).

$$k_{bs} \le \frac{e_s^b + \sum_{h \in H} u_b^h + c_s^b}{w_{ss}^b(\eta,\sigma) R_s m_s^b (1+r_s) + w_{s\bar{0}}^b(\eta,\sigma) \bar{R}_s \bar{m}^b (1+\bar{r}) + \sum_{j \in J} w_{sj}^b(\eta,\sigma) R_{sj} (p_s A_s^j)(\frac{b_j^b}{\theta_j}) + w_{s0^*}^b \tilde{R}_{ds} (1+\rho) d^b} \qquad (7^b)$$

(i.e., capital requirements constraint in state s).

where, $z_s = m_s^b \Big/ (\sum_{h\in H} d^h + \sum_{b\in B} m_s^b), \bar{z} = \overline{m}^b \Big/ (\sum_{h\in H} \bar{d}^h + \sum_{b\in B} \overline{m}^b), z = d^b \Big/ (\sum_{b\in B} d^b + M^G)$, and $\pi_s^b = \Delta(6^b)$.
Note that since the banking sector is perfectly competitive loan repayments are made proportional to the credit issued by each commercial bank relative to the aggregate credit issued by the entire banking sector.

4. MECBD

We say that[23] $\left(n, (\sigma^h)_{h\in H}, (\sigma^b)_{b\in B}\right)$ is a *monetary equilibrium with commercial banks and default* (MECBD) for the economy $E = \left\{(u^h, e^h, m^h)_{h\in H}, (u^b, e^b)_{b\in B}, A, M^G, \mu^G, k, \lambda, \omega\right\}$

iff:

(i) $$p_{sl} = \frac{\left(\sum_{h\in H} b_{sl}^h + m_s^G\right)}{\sum_{h\in H} q_{sl}^h}, \qquad \forall s\in S, l\in L \;;$$

Condition (i) shows that all commodity markets clear (or equivalently that price expectations are rational).

(ii) $$1+\rho = \frac{\left(\sum_{b\in B} \mu^b + \mu^G\right)}{\sum_{b\in B}\left(d^b + M^G\right)} \;;$$

Condition (ii) shows that the interbank credit market clears (or equivalently that interbank interest rate forecasts are rational).

(iii) $$1+r_s = \frac{\sum_{h\in H} \mu_s^h}{\left(\sum_{h\in H} d_s^h + \sum_{b\in B} m_s^b\right)}, \qquad \forall s\in S^* \;;$$

(iiia) $$1+\bar{r} = \frac{\sum_{h\in H} \bar{\mu}^h}{\left(\sum_{h\in H} d^h + \sum_{b\in B} \bar{m}^b\right)} \;;$$

Condition (iii) shows that all short-term (long-term) credit markets clear (or equivalently, that predictions of long-term interest rates are rational).

(iv) $$\theta_j = \frac{\left(\sum_{h\in H} b_j^h + \sum_{b\in B} b_j^b\right)}{\left(\sum_{h\in H} q_j^h + \sum_{b\in B} q_j^b\right)}, \qquad \forall j\in J \;;$$

Condition (iv) shows that asset markets clear (or equivalently, asset price expectations are rational).

(v) $\sum_{h\in H} u_b^h = 1$, ∀ b∈B ;

Condition (v) shows that the equity market for the bank ownership clears (or equivalently bank equity shareholding expectations are rational).

[23] Recall that by assumption $p, \rho, r_s \bar{r}, \theta, R$ are different from 0 and ∞ in each component.

$$\text{(vi)} \quad R_{sj} = \begin{cases} \dfrac{\sum_{h \in H \cup B} \left[v_{sj}^h p_s q_j^h A^j \right]}{\sum_{h \in H \cup B} \left[p_j q_j^h A^j \right]} , & if \quad \sum_{h \in H \cup B} \left[p_j q_j^h A^j \right] > 0 \\ \text{arbitrary} , & if \quad \sum_{h \in H \cup B} \left[p_j q_j^h A^j \right] = 0 \end{cases} ; \forall s \in S, \quad j \in J .$$

Condition(vi) shows that each asset buyer is correct in his expectation about the fraction of assets that will be delivered to him.[24]

$$\text{(vii)-(xii)} \quad R_s(\widetilde{R}_s, \overline{R}_s, R_{ds}, \widetilde{R}_{ds}, \overline{R}_{ds}) = \begin{cases} \dfrac{\sum_{h \in H \cup B} \left[v_s^h \mu_s^h (\mu^h, \overline{\mu}^h, d^h(1+\rho), \overline{d}^h(1+\overline{r}), d_s^h(1+r_s)) \right]}{\sum_{h \in H \cup B} \left[\mu_s^h (\mu^h, \overline{\mu}^h, d^h(1+\rho), \overline{d}^h(1+\overline{r}), d_s^h(1+r_s)) \right]}, \\ \quad if \quad \sum_{h \in H \cup B} \left[\mu_s^h (\mu^h, \overline{\mu}^h, d^h(1+\rho), \overline{d}^h(1+\overline{r}), d_s^h(1+r_s)) \right] > 0 \\ \qquad\qquad\qquad\qquad\qquad\qquad ; \forall s \in S^* \\ \text{arbitrary} , \quad if \quad \sum_{h \in H \cup B} \left[\mu_s^h (\mu^h, \overline{\mu}^h, d^h(1+\rho), \overline{d}^h(1+\overline{r}), d_s^h(1+r_s)) \right] = 0 \end{cases} .$$

Conditions (vii)-(xii) show that the Central Bank and commercial banks are correct in their expectations about the fraction of loans that will be delivered to them. Similarly, investors and commercial banks are correct on their expectations about the fraction of deposits and interest rate payments that will be delivered to them.

$$\text{(a)} \quad \sigma^h \in \underset{\sigma^h \in B^h(\eta)}{Arg \max} \quad \Pi^h\left(x^h\right) ,$$

(xiii) and

$$\text{(b)} \quad \sigma^b \in \underset{\sigma^b \in B^b(\eta)}{Arg \max} \quad \Pi^b\left(\pi^b\right).$$

Condition (xiii) shows that all agents optimise.

In sum, all markets clear and agents optimise given their budget sets. These are the defining properties of a competitive equilibrium.

If a MECBD exists, then default and financial instability are established as equilibrium phenomena that arise in a standard equilibrium framework. Note, also that fiat money has positive value in a finite horizon economy (for an extensive discussion on this see, Dubey and Geanakoplos [15], Grandmont and Younes [27], and Hahn [28] who posed initially this problem). Government and regulator actions are exogenously fixed and are not deduced from optimising behaviour.

[24] Rates of delivery do not depend on anything that agents do themselves. Every agent receives the same rate of delivery. This is in accordance with the spirit of mass anonymous capital and credit markets. Otherwise, I would need to consider complicated strategic consequences, which unnecessarily complicate the model.

Conditions (vi)-(xii) ensure that expected deliveries of assets, loans and deposits are equal to realised deliveries in equilibrium. However, the specification of expectations for inactive markets is arbitrary. Thus, the model does not rule out trivial equilibria in which there is no trade. There are two ways around this case (i.e., exclude trivial equilibria) in order to allow for comparative statics and policy analysis. The first has been introduced by Dubey and Shubik [18] and adds and external agent that always supplies the asset and loan market with a minimal amount ε and never abrogates his contractual obligation. The second has been suggested in Dubey, Geanakoplos and Shubik [17] and offers an equilibrium refinement by forbidding "overly pessimistic" expectations thus guaranteeing full delivery of small promises. I adopt the first way and then let $\varepsilon \to 0$.

Hereafter, I will assume in (vi)-(xii):

Inactive Markets Hypothesis: Whenever credit or asset markets are inactive (i.e., asset supply, credit extension or deposits are 0) the corresponding rates of delivery are set equal to 1.

5. Quantity Theory of Money and the Term Structure of Interest Rates

At each meeting of the market, money is exchanged for another instrument, which can be either commodities or assets or bonds. There are the traditional motives for holding money. Thus, we see that the standard Hicksian IS/LM determinants of money demand, namely interest rates and income, are at work in the model. Nevertheless, it is also easy to see that with our simple specification of the economy, if all the interest rates are positive, then all the money will be spent in each $s \in S$ in the commodity markets. An investor, or a commercial bank, which has cash that he does not wish to spend it will not hold it idle, but he will lend it out to somebody who will spend it. However, this is not the case at $s=0$, since, because of the uncertainty and incomplete markets, investors will spend their cash also in the asset markets and may hold some precautionary reserves.

QUANTITY THEORY OF MONEY PROPOSITION: *In a MECB, if $\rho > 0$ then the aggregate income at $s \in S$, namely the value of all commodities sales is equal to the total credit provided by commercial banks plus asset payoff liquidity. At s=0, aggregate income equals the total central bank money supply plus deposits in the interbank credit market minus expenditures in the asset and equity markets, precautionary reserves and bank profits.*

$$\sum_{h \in H} \sum_{l \in L} p_{sl} q_{sl}^{h} = \sum_{b \in B} m_{s}^{b} + \sum_{h \in H \cup B} \sum_{j \in J} v_{j}^{h} p_{s} q_{j}^{h} A^{j} + \Delta(4^{h}) \quad , \forall s \in S.$$

For s=0,

$$\sum_{h \in H} \sum_{l \in L} p_{0l} q_{0l}^{h} = M^{G} + \sum_{b \in B} d^{b} - \sum_{h \in H \cup B} b_{j}^{h} - \sum_{h \in H} \sum_{b \in B} u_{b}^{h} - \Delta(4^{h}) - \pi_{0}^{b}.$$

It follows from the foregoing that if all interest rates are positive, then in equilibrium the quantity theory of money must hold, with velocity of money fixed at one. At any moment the stock of money will be equal to the value of nominal income. Given the level of the real economic activity, price levels will move in the same direction as the stock of money (as more money chases the same goods). Yet, this is no crude quantity theory in which the demand for money is independent of interest rates; quite the opposite is the case. For example, the "real" velocity of money, that is how many real transactions can be moved by money per unit time, is endogenous.

In equilibrium, the quantity of economic activity, by which I mean the quantity of real goods traded in a period, is endogenous. By contrast, in the model of Lucas and his followers the amount of real economic activity is exogenously specified by the requirement that each agent sells the whole of his endowment in each period. A corollary of the quantity theory of money in our model is that, all others being equal, increases in trading activity in state s, due perhaps to more productivity or lower interest rates, will result in lower state s price levels, given the same money supply in state s. Similarly, the volume of trade in the asset markets affects the prices and has second order effects on the inflation rate.

The model can encompass several monetary theories, if I superimpose structural assumptions and restrictions. However, even at this level of generality, several useful results can be stated.

PROPOSITION 1: At any MECBD, $\bar{r}, r_s, \geq 0 \quad \forall s \in S^*$ and $\bar{r} \geq r_0 \geq 0$.

PROPOSITION 2: *At any MECBD,* $\rho \geq r_s, \bar{r} \quad \forall s \in S^*$.

This proposition holds because the *ex ante* interest rates are considered and they do not incorporate their respective default premia. Thus, borrowing rates have to be higher than lending rates to preclude arbitrage opportunities. It also emphasises the power of the monetary authorities to control the interest rates and influence the term structure of the economy.

TERM STRUCTURE OF INTEREST RATES PROPOSITION: *At any MECBD,* $\forall s \in S$,

$$\left[\sum_{b\in B}\left(m_0^b r_0 + m_s^b r_s + \overline{m}^b \bar{r}\right)\right] = \sum_{h\in H}\left(m_0^h + m_s^h\right)\left(\frac{1}{v_s^h}\right) + \sum_{b\in B}\left(e_0^b + e_s^b\right)\left(\frac{1}{v_s^b}\right)$$

(The analogous equation holds with weak inequality for s=0.)

The term structure equation is now affected by both liquidity provision by banks and default by households and banks. In the one period case, the interest rate can be specified a priori, independent of the "real" data of the economy. But in a multiperiod setting there are two degrees of freedom since there are *S* equations and *S+2* interest rates to be determined in equilibrium. Therefore, the term structure of interest rates is endogenously determined at *s=0*, and depends on the real data of the economy and is subject to policy interventions. The exception is where all government deficit spending is zero, in which case all interest rates are zero for all states $s \in S^*$.

Finally, since the model has an integral monetary sector, the interest rates determined in equilibrium are in nominal terms. Thus, they depend both on the real interest rates and the prevailing inflation rate. I summarize this intuition in the following proposition.

FISHER EFFECT PROPOSITION: *Suppose that for some* $h \in H$, b_{0l}^h *and* $b_{sl}^h > 0$, *for* $l \in L$ *and* $s \in S$. *Suppose, further that h has some money left over the moment that the long loan comes due at* $s \in S$. *Then, at a MECBD,*

$$(1+\bar{r}) = \left(\left(\frac{\partial u^h(x)}{\partial x_{0l}}\right) \Big/ \left(\frac{\partial u^h(x)}{\partial x_{sl}}\right)\right).(p_{sl}/p_{0l}).$$

Taking the logarithm of both sides and interpreting loosely, this says that the nominal rate of interest is equal to the real rate of interest plus the (expected) rate of inflation.

6. Asset Pricing

Positive interest rates and the liquidity based market transactions introduce a "price wedge" whose size depends on period zero interest rates. The "price wedge" manifests itself both in the commodity and the asset markets. The complication that positive interest rates introduce is the failure of the exact linear pricing rule of assets.

PROPOSITION 3: *Suppose that* $A^j = \lambda_1 A^1 - \lambda_2 A^2$ *and* $\lambda_1, \lambda_2 \geq 0$. *If asset* $j \in J$ *is traded at a MECBD then*

$$\lambda_1\theta_1 - (1 + r_0)\lambda_2\theta_2 \leq \theta_j \leq \lambda_1\theta_1 - (1/1 + r_0)\lambda_2\theta_2 .$$

Note that the linearity principle obtains only if all the private monetary endowments, and initial capital condition of commercial banks are zero. Moreover, the bankruptcy penalties should be such that they preclude bankruptcy altogether. Thus, interest rates will all be zero and linear pricing will obtain.

7. Orderly Function of Markets: Existence

When interest rates are positive (including the interbank interest rate) agents may not be willing to borrow from commercial banks (respectively commercial banks from the Central Bank). Thus, the transaction must be sufficiently profitable to undertake it. The crucial assumption is that there are sufficient gains to trade available to agents, to justify voluntary interest rate payments on grounds of rational behavior. Debreu [12] suggested that an allocation $\chi^h \in R_+^{S*L}$ for each $h \in J \subset H$ is not *δ-Pareto efficient* for $h \in J$, if it is possible to costlessly reallocate the commodity bundle *(1-δ)χ* among those agents and make them all better off than they were at the original allocation. Dubey and Geanakoplos [15] proposed that an allocation permits at least *δ-gains to trade* if starting from that allocation, it is possible to make everybody better off in $J \subset H$ by transferring commodities, even though a fraction δ of the original bundle is lost. Following, Dubey and Geanakoplos [15], [16] who first introduced the gains to trade hypothesis, and Geanakoplos and Tsomocos [23] who used it in a related model, I modify the definition and the hypothesis to the present framework.

Definition:

Let $(\chi^h, \pi^b) \in R_+^{S*\times(L+1)} \quad \forall h \in H, b \in B. \quad \forall \quad \delta > 0$, we will say that

$(\chi^1,..., \chi^h; \pi^1,...\pi^b) \in (R_+^{S*\times L})^H \times R_+^{S*\times B}$ permits at least *δ-gains-to-trade* in state *s* if there exist trades

$\tau^1,..., \tau^H; \tau^1,..., \tau^B \quad in \quad R^{L+1}$ such that

$$\sum_{h\in H} \tau^h + \sum_{b \in B} \tau^B = 0$$

(a) $\chi_s^h + \tau^h \in R_+^L, \quad \forall h \in H$

(b) $\hat{\pi}_s^b + \tau^B \in R, \quad \forall b \in B$

3. (a) $u^h(\overline{x}^h) > u^h(x^h), \quad \forall h \in H$

   (b) $u^b(\overline{\pi}^b) > u^b(\pi^b)$ where,

$$\overline{x}_{tl}^h = \begin{cases} \chi_{tl}^h, & t \in S^* \backslash \{s\} \\ \chi_{tl}^h + \min\{\tau_l^h, \tau_l^h/(1+\delta)\} & for \quad l \in L \quad and \quad t = s \end{cases}$$

$$\overline{\pi}_t^b = \begin{cases} \pi_t^b, t \in S^* \backslash \{s\} \\ \pi_t^b + \min\{\tau^b, \tau^b/(1+\delta\} & for \quad t = s \end{cases}$$

Note that when $\delta > 0, \overline{x}_l^h < \chi_{tl}^h + \tau_l^h, if \quad \tau_l^h > 0 \quad and \quad \overline{x}_{tl}^h = x_{tl}^h + \tau_l^h \quad if \quad \tau_l^h \leq 0$.

Also, $\overline{\pi}_t^b < \pi_t^b + \tau^b, \quad if \quad \tau^b > 0 \quad and \quad \overline{\pi}_t^b + \tau^b, \quad if \quad \tau^b \leq 0$

Formally, the hypothesis I impose on the economy for sufficient gains to trade is:

G to T: *∀s ∈S, the initial endowment* $(e^h, e^b)_{h\in H, b\in B}$ *permits at least* δ_s*-gain to trade in state s, where*

$$\delta_s = \frac{\sum_{h\in H} m_0^h + \sum_{h\in H} m_s^h + \sum_{b\in B} e_0^b + \sum_{b\in B} e_s^b}{M^G}.$$

Condition (G to T) needs to be valid *∀s ∈S* but not for *s=0*. Note that gains from trade should be present in the banking sector even though commercial banks do not consume. But since they are mutually owned and are liquidated at the end of period *1*, rational agents expect these proceeds and incorporate them in the maximisation problem as can be seen from their budget constraint ((9^h) of 3.1). Also (G to T) precludes the case where *L=1, ∀s ∈S**. Moreover, if the initial endowment is not Pareto optimal ∀s∈S, then holding all other government actions fixed as I vary $M^G \to \infty$, (G to T) is automatically satisfied.[25] The following theorem is proved in the Appendix.

THEOREM 1: If in the economy $E = \{(u^h, e^h, m^h)_{h\in H}, (u^b, e^b)_{b\in B}, A, M^G, \mu^G, \kappa, \lambda, \omega\}$,

(1) Gains to Trade and Inactive Markets Hypotheses hold,

(2) $M^G>0$,

(3) $\forall s\in S$, $\sum_{h\in H} m_s^h + \sum_{b\in B} e_s^b > 0$ and

(4) $\lambda>>0$, $\forall h\in H$, $b\in B$

then a MECBD exists.

The proof of this theorem resolves the Hahn problem and also the non-existence example of Hart [29]. Positive interest rates and bankruptcy penalties compactify the choice space by binding the potential transactions in the asset markets. Radner [36], instead, imposed artificial bounds on asset trades to show existence. The proof of the theorem follows the method introduced by Dubey and Geanakoplos [15], [16], who provide an excellent discussion on the issue, also indicates that as the aggregate "outside" money becomes large then the necessary gains to trade for existence becomes arbitrarily large. Therefore as "outside" money grows, prices eventually rise, converging rapidly to infinity. When the ratio of outside to inside money becomes high enough so that equilibrium is unsustainable, hyperinflation occurs and trade collapses.

The main argument of the proof is that the monetary authority backs the fiat money present in the economy and gives it real value. By this is meant that "money is an institutionalised symbol of trust." The government compels the acceptance of money as a final discharge of debt. Investors use this "government-backed" money because the benefit it gives them is greater than the interest rate loss. The interest rate can be thought of as the tax levied by the government for the supply of "trust" and the "system" that it establishes to safeguard trade in assets and commodities. This argument does not rest only on the cash in advance constraint but also on the presence of the government, standing ready to recoup loans, and the potential gains from trade. It is this triplet that guarantees existence and positive value of fiat money in the finite horizon.

8. Financial Instability, Contagion and Systemic Risk

8.1 Financial Instability: Concepts and Definitions

Few attempts have been made to formally define financial stability and characterize it in an analytically meaningful way. Academics and policy-makers have offered various descriptive definitions.[26] A definition of financial instability here is:

Definition: A MECBD is *financially unstable* whenever substantial default of a "number" of households and banks (i.e., a liquidity "crisis"), without necessarily becoming bankrupt, occurs and the aggregate profitability of the banking sector decreases significantly (i.e., a banking "crisis").

Formally, a MECBD $\left(n, (\sigma^h)_{h\in H}, (\sigma^b)_{b\in B}\right)$ is *financially unstable* whenever

$$D_{s\omega}^{h*}, D_{s\omega}^{b*} \geq \overline{D}, \quad \sum_{b\in B} \pi_s^b \leq \overline{\Pi}, \; for \quad |H*| + |B*| \geq \overline{Z}, \text{ and } s \in S^* \; where \quad \overline{Z} \in (0, |H| \cup |B|] \quad and \quad \overline{\Pi}, \overline{D} \in R_{++}.$$

[27]

Thus, financial instability is characterized by both liquidity shortage and banking sector vulnerability. Moreover, it is allowed that the authorities (government and/or the Central Bank) have the liberty to determine the threshold, which is commensurate with the outset of a financially unstable environment. Also note that this definition hinges upon the welfare of the economy and its distributional consequences, when one works with

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[25] Alternatively, if $\sum_{h\in H} m_s^h = 0$ and $\sum_{h\in H} e^b = 0$, $\forall s\in S^*$ then G to T is automatically satisfied. See Shubik and Tsomocos [38], for the argument of this case in a related model.

[26] A survey of these definitions and extensive discussion can be found in Bank for International Settlements [4]. See also Crockett [11] and Mishkin [34] for an 'institutionally' and an 'informationally'based definition respectively.

[27] Π (D) may be exogenously fixed or constructed as a weighted average of expected banking profits (defaults). Alternatively, one can just compare profitability (default) across different equilibria.

explicitly multi-agent models. The standard techniques and the well-known theorems of equilibrium theory can be readily applied. Equilibrium analysis is also amenable to comparative statics, for example, by varying capital requirement rules one can determine the expected default and the welfare effects of a crisis.

A natural question that may be raised is why either one of the conditions is not sufficient to constitute a financially unstable regime. Besides the empirical evidence provided in the introduction, increased default without reduced profitability might be an indicator of increased volatility and risk-taking without necessarily leading to financial instability. On the other hand, lower bank profitability without increased default might be an indicator of a recession in the real economy and not of financial vulnerability. It is the combination of both conditions that destabilizes the financial system and may produce financial crises.
This definition is, I think, flexible enough to encompass most of the recent episodes of financial instability. For example, the Japanese crisis can be thought of as a *bona fide* phenomenon of the Keynesian liquidity trap. The Mexican crisis of the early 1990s is a classic example of liquidity and banking crisis. The late 1990s East Asian crisis was characterized by a banking crisis and economic recession as well as extensive default. Finally, the Russian crisis, the Texas Banking crisis, and the U.S. Stock Market crash of 1987 conformed to the characterization of a financially unstable regime generated by liquidity shortages, extensive default and declines in bank profitability.

8.2 Liquidity Trap

An extreme case of financial instability is the well-known liquidity trap. An economy manifests a liquidity trap whenever financial instability is coupled with monetary policy ineffectiveness. The Keynesian liquidity trap describes a situation in which monetary policy would not affect the nominal variables of the economy because consumers simply hold extra real money balances for speculative purposes. If that interest rates are sufficiently low and investors expect them to go up in the future, then they do not invest into assets like bonds whose value will decrease when interest rates rise. Various authors provide explanations and formalizations of the liquidity trap, e.g. Tobin [40], Grandmont and Laroque [25], and Hool [30] among others, based on non-rational expectations.

Dubey and Geanakoplos [16] provide an alternative explanation based on the incompleteness of asset markets. In the present model the liquidity trap occurs via the same process. However, now commercial banks engage in large asset trades without changing the interest rates of the consumers' credit markets. When the government employs an expansionary monetary policy, commercial banks do not channel the increased liquidity to the consumer credit markets but the asset market, and therefore increased activity is observed in asset transactions. So, commodity prices remain relatively unaffected along with the interest rates of consumer credit markets.

The liquidity trap is observed in the interbank market and not in the rest of capital markets where trades are very large and prices naturally are affected. This is consistent with the intuition that it may be the case that the liquidity trap may occur in an isolated section of the capital markets and then impinges upon the rest of the nominal sector of the economy (e.g., Japan in the late 1990s).[28]

PROPOSITION 4*:* *Suppose that the economy has a riskless asset* A^j_{sm} *=(1,…,1), (i.e., monetary payoffs in every state are equal to one) and* A^j_{sl} *=0,* $\forall s \in S^*$ *and* $l \in L$*. Also consider the case in which the underlying economy has no GEI. Then as* $M^G \to \infty$*, then*

[28] Charles Goodhart made this point to me and it seems that the model accommodates it

$$w^b_{ss}(\eta,\sigma)R_s m^b_s(1+r_s)+w^b_{s\bar{0}}(\eta,\sigma)\overline{R}_s\overline{m}^b(1+\bar{r})+\sum_{j\in J}w^b_{sj}(\eta,\sigma)R_{sj}(p_s A^j_s)(\frac{b^b_j}{\theta_j})+w^b_{s0^*}\widetilde{R}_{ds}(1+\rho)d^b\to\infty,$$

$\forall s\in S^*$,

$$w^b_{00}(\eta,\sigma)R_0 m^b_0(1+r_0)+w^b_{0\bar{0}}(\eta,\sigma)\overline{m}^b(1+\bar{r})+\sum_{j\in J}w^b_{0j}(\eta,\sigma)(p_s A^j_s)(\frac{b^b_j}{\theta_j})+w^b_{00^*}(\eta,\sigma)(1+\rho)d^b\to\infty,$$

for $s=0$,

$M^G/\|p_{ol}\|\to\infty$ *, and* $\left(\sum_{h\in H}q^h_j+\sum_{b\in B}q^b_j\right)\to\infty$.

Moreover, there exist

$\overline{D}$ *and* $\overline{\Pi}$ *such that* $D^{h^*}_{s\omega}, D^{b^*}_{s\omega}>\overline{D}>0$ *for some* $b^*\in B$, $h^*\in H$, *and* $\sum_{b\in B}\pi^b_s\le\overline{\Pi}$.

Note that regulatory policy may "break" this liquidity trap by imposing harsh capital requirements (or very high w_j's) and so, blocking excessive trading activity in the asset markets. Thus, extra liquidity will be available for credit extension and further gains to trade will be materialized by the households due to lower interest rates.

8.3 Bank Runs

Perhaps, one of the most noteworthy features of this definition is that it encompasses a wide spectrum of financially unstable regimes and treats the original result by Diamond and Dybvig as a special case. The situation that corresponds to default of the entire banking sector is tantamount to the Diamond-Dybvig inefficient outcome.

PROPOSITION 5: *Suppose that at a MECBD* λ*'s* $<<\infty$, $M^G<<\infty$ *and* $\kappa\le 1$. *Further, suppose* $q^b_j=0,\forall b\in B$. *Also, there exists* $s\in S^*$ *such that* $\sum_{h\in H}\mu^h_s>\sum_{h\in H}d^h_s$. *Then there exists* $b\in B$ *such that* $D^b_{s\omega}>0$ *in* s.

COROLLARY 1 (Diamond-Dybvig): *If together with the assumptions of the previous proposition we have* $u^b=u$, $\lambda^b_s=\lambda$ *and* $\sum_{h\in H}\mu^h_s>\sum_{h\in H}d^h_s,\forall s\in S^*$ *and* $b\in B$, *then* $D^b_{s\omega}>0,\ \forall\ b\in B$ *and* $s\in S^*$.

This corollary that captures the essence of the logic in the Diamond and Dybvig result highlights the special nature of their model and the importance of bank homogeneity. Moreover, all commercial banks are identical and default is pooled. Thus, all depositors rationally expect that the commercial banking sector will uniformly default on its obligations in all future states.

COROLLARY 2: *Let the same assumptions of Corollary 1 hold and k=1. Then there exists* $\overline{\lambda^b}$ *such that* $D^b_{s\omega}=0,\ \forall\ b\in B,\ s\in S^*$. *In addition, there exists* $\overline{\lambda}^h$ *such that* $D^b_{s\omega}=0\ \forall\ h\in H$.

Put differently, Corollary 2 underlines the importance of capital requirements for financial stability. Therefore, whenever credit is fully collateralized, the regulator guarantees future financial stability. But, there is a trade off between financial stability and efficiency since stricter capital requirements generate higher interest rates and thus reduce efficient trade and limit banks' risk taking behavior. Thus, λ's, κ's and monetary policy should be studied contemporaneously when optimal regulatory policy is designed.

9. Non-Neutrality of Monetary and Regulatory Policy

The economic reason that determinacy, both real and nominal, obtains is that positive liquid wealth (i.e., outside money in the economy) anchors interest rates by the term structure of interest rates proposition and thus uniquely (locally) determines the rest of the variables in the model (i.e., prices, consumption, default). Indeed, theorem 2 is shown in Tsomocos [42]. I show in Tsomocos [42] that MECBD are typically finite. Therefore, prices, interest rates and consumption are almost always determinate with respect to the data of the economy. This is in contrast with the real indeterminacy theorem of Geanakoplos and Mas-Collel [22] and Balasko and Cass [3] but consistent with the determinacy theorem of Dubey and Geanakoplos [15].

THEOREM 2: *Let* $\sum_{h\in H} m_s^h + \sum_{b\in B} e^b > 0$ *or* $\lambda^b < \overline{\lambda}^b$ *and* $M^G > 0$. *Then for "generic" economies E the set of MECBD is finite.*

This theorem allows us to determine the impact of monetary and regulatory changes in the economy. The "no-money illusion" property easily follows:

PROPOSITION 6 (No Money illusion): *A proportional increase of all*

$(m_s^h)_{h\in H, s\in S*}, (e_s^b)_{b\in B, s\in S^*}$ *and* (M^G, μ^G) *whereas the regulators' choices of* κ *stay fixed while* λ*'s scaled down proportionally, does not affect the real variables of MECBD.*

Monetary policy usually changes the ratio of bank money to private endowments and initial endowments of commercial banks. I interpret monetary policy as a change in M^G or μ^G (i.e., open market operations). The following proposition demonstrates the non-neutrality of monetary and regulatory policy in a case that can be analysed via the first order condition of equilibrium.[29]

I define a MECBD to be *indecomposable* if for any $s \in S^*$ and any partition of goods into disjoint sets L_1 and L_2 there is some agent $h \in H$ who transacts in at least one commodity from each set in state $s \in S^*$.

PROPOSITION 7: *Suppose that all* u^h, u^b *are differentiable and* $m_s^h, e_s^b > 0$ *or* $\lambda^b < \overline{\lambda}^b$ *and* $\lambda^h < \overline{\lambda}^h$ *for all* $h \in H$, $b \in B$ *and* $s \in S^*$. *Suppose at an indecomposable MECBD at every* $s \in S^*$ *all* $h \in H$ *consume positive amounts of all goods* $l \in L$ *and that some* $h \in H$ *carries over money from period 0 to 1. Then, any change by the government or the regulator (except the one described by Proposition 6) results into a different MECBD in which for some* $h \in H$ *consumption is different.*

Since increases in the stock of bank money, roughly speaking, (i.e., expansionary monetary policy) ultimately move the economy closer to a competitive equilibrium, and hence closer to Pareto efficiency (not necessarily monotonically), one question is why does not the government drastically increase this expansion? It may be because of political, budgetary or creating public expectations for continued expansionary policy. Finally, I remark that the effect of market prices, lower interest rates and subsequent increased trading activity is tantamount to Keynesian monetary policy whereas fiscal policy will raise prices and possibly inflation and increase interest rates (see Tsomocos [42]). This in turn reduces trading activity (crowding out) which is tantamount to Keynesian fiscal policy.

10. MECBD vs. GEI and GE

Recall, $\left(p, \pi\left(\chi^h, \varphi^h\right)_{h\in H}\right)$ is a GEI for the underlying economy E=$\left(\left(u^h, e^h\right)_{h\in H}, A\right)$ iff:

[29] The general argument of monetary and regulatory policy non-neutrality is presented in Tsomocos [42] where it is shown that typically regulatory and monetary changes have non-neutral effects.

(xi) $\sum_{h \in H} \chi^h = \sum_{h \in H} e^h$

$\sum_{h \in H} \varphi^h = 0$

$(\chi^h, \varphi^h) \in B^h(p, \pi) = \left\{ (\chi, \varphi) \in R^{S^* \times L} : p(\chi_0 - e_0^h) + \pi\varphi \leq 0, \ \ and \ \ \forall s \in S \quad p_s(\chi_s - e_s^h) \leq \sum_{j \in J} \sum_{l \in L} p_{sl} A_{sl}^j \varphi_j \right\}$

$(\chi, \varphi) \in B^h(p, \pi) \Rightarrow u^h(\chi) \leq u^h(\chi^h)$

PROPOSITION 8: *Suppose that* $\sum_{h \in H} m_s^h + \sum_{b \in B} e^b = 0, \ \forall s \in S^*, \ \ \kappa = 0, \ \ and \ \ \lambda = +\infty$

Moreover, there exists an asset $A^j = (1,...,1)$. *Then MECBD and GEI coincide.*

COROLLARY 1: *If the assumptions of Proposition 8 hold but there does not exist a risk less asset and the long-term credit market is closed, then MECBD and GEI coincide with respect to prices, net asset trades and final consumption.*

Finally, so long as the quantity of outside money is positive (i.e., $\sum_{h \in H} m_s^h + \sum_{b \in B} e^b > 0$) the GEI is obtained as a limiting case of MECBD model without long-term loans. The next corollary formalises this intuition.

COROLLARY 2: *Suppose that the assumptions of Proposition 8 hold except that* $\sum_{h \in H} m_s^h + \sum_{b \in B} e^b > 0$ *and that all assets are numeraire*

(i.e., $\exists \ \bar{l} \in L \ni \left(A_{s\bar{l}}^j\right)_{s \in S} \neq 0, \forall j \in J \ \ , \ \ (A_{sl}^j)_{s \in S} = 0, \ \ \forall \hat{l} \in L \setminus \{\bar{l}\}$*) and that* A^j*'s are linearly independent. Also, let the long-term credit market be closed. Then as* $M^G \to \infty$, *the MECB attains the GEI prices, net asset trades and final consumption in the limit.*[30] *Finally, if I impose the restriction that span [A]=S (i.e., complete markets) the Arrow theorem [2] holds.*

10.1 The optimum quantity of money

A well-known argument in monetary economics is the optimum quantity of money proposition introduced by Friedman [25]. He argues that efficiency in a monetary economy can be established only if nominal interest rates are equal to zero. In other words, assuming positive real interest rates, optimality is achieved when the growth rate of money supply equals the deflation rate of the economy necessary for zero nominal interest rates. Friedman's argument implies an optimal monetary policy such that inflation should equal the pure rate of time preference of the consumers.[31] My model implies that the Friedman rule holds only if markets are complete, default is precluded and outside money is zero. Alternatively, if outside money is positive, in view of Corollary 2, efficiency is established in the limit as $M^G \to \infty$. When default occurs in equilibrium then interest rates rise and the corresponding default premium is positive and thus there exists a deadweight loss in the economy. Naturally market incompleteness typically generates inefficient equilibria.

11. References

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[30] The reason that I require linear independence in numeraire asset matrix is to ensure that net asset trades remain bounded and that the Hart counterexample does not occur. For more see, Geanakoplos and Polemarchakis [21]

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Appendix

Proof of Quantity Theory of Money Proposition:
If $\rho>0$ then banks that hold money either they could have deposited it in the interbank market, extend it in the credit market or alternatively invest it in the asset market. Otherwise, they could have reduced their borrowing by ε, increase π_s^b and then increase their payoff by $\nabla\Pi_s^b\left(\pi_s^b\right)\cdot\varepsilon>0$.[32] Similarly, households if they are borrowers will spend all of their cash; or else they should not have borrowed since $\sum_{h\in H} u_b^h = 1, \forall b \in B$. Finally, after adjusting for asset default the proposition obtains.
In $s=0$, all unused cash will be preserved and spent in the next period. □

Proof of Proposition 1:
The fact that r_s, $\bar{r} \geq 0$ is proved in step *2* of the Theorem l's proof. If $\bar{r} < r_0$, then any h∈H would improve by borrowing ε more on long-term loan than the short-term and thus $(\sum_{b\in B} d_s^b + \sum_{b\in B} m_s^b) > (\sum_{h\in H\setminus\{h\}} \mu_s^h - \varepsilon) \big/ (1+r_0)$. So the short-term credit market would not clear, a contradiction.
This transaction is feasible since money is perfectly durable.□

Proof of Proposition 2:
If $\rho < \bar{r}$ or $\rho < r_s$ for some $s \in S$ then let any $b \in B$ borrow Δ more on the interbank market and extend Δ more on the corresponding credit market. Then, *b* will realise $(\bar{r}-\rho)\,\Delta$ $((r_s-\rho)\,\Delta)$ more profits and improve Π_s^b, a contradiction. □

Proof of Term Structure of Interest Rates Proposition:
By theorem *1*, money has a positive value, i.e. *p*'s and *r*'s stay bounded. Thus, in section 3.1, $\Delta(9^h) = 0,\ \forall h \in H$. Otherwise *h* could have borrowed $\Delta/(1+r_s) > 0$ more and spend it on commodities and improve his utility by $\nabla\Pi_s^h(x_s^h)\cdot\left(\frac{\Delta}{(1+r_s)p_{sl}}\right)>0$, for some $l \in L$ and $s \in S$. Finally, *h* would have used his left over cash $\Delta = \Delta(9^h)$, w.l.o.g. , to defray his loans. Similarly, no *h* returns more than what he owes. Thus all money in the economy is returned to the commercial banks, after adjusting for bankruptcy and the equality follows.□

Proof of Fisher Effect Proposition:
It follows immediately from the optimization conditions.□

Proof of Proposition 3:
Suppose $\theta_j < \lambda_1\theta_1 - (1+r_0)\,\lambda_2\theta_2$. Then, let a seller of asset *j* reduce his sale by ε and borrow $\varepsilon\lambda_2$ more. He can use the money obtained from the loan to buy $\varepsilon\lambda_2$ more of asset *2* and sell $\varepsilon\lambda_1$ more of asset *1*.Then *h* has to deliver $\varepsilon(\lambda_1 A^1 - \lambda_2 A^2)$ less but he also receives εA^j less. So, his net future deliveries remain unaffected.

[32] I will be using the following notation hereafter: $\nabla f_{x_i} = \frac{\partial f(x_1,\ldots,x_n)}{\partial x_i}$.

However, since $\varepsilon\,\lambda_1\theta_1$ - $\varepsilon\,(1+r_0)\,\lambda_2\theta_2>\varepsilon\theta_j>0$, he can pay back his loan and use his remaining savings to pay back an extra loan that he can use to increase his consumption, a contradiction with optimization. For the second part of the inequality, suppose $\theta_j>\lambda_1\theta_1 - (1/(1+r_0))\,\lambda_2\theta_2$ and apply the reverse argument.□

Proof of Theorem 1:

Let $M^*\equiv M^G+\sum_{h\in H}\sum_{s\in S^*} m_s^h+\sum_{b\in B}\sum_{s\in S^*} e_s^b$ be the total quantity of money ever appearing in the economy. For $h\in H$, $b\in B$ and $\varepsilon>0$ let

$$\sum\nolimits_\varepsilon^h=\{\left(x^h,\bar{\mu}^h,\mu^h,d^h,\bar{d}^h,b^h,q^h,u^h,v^h\right)\in\Re_+^{LS^*}\times\Re_+\times\Re_+^{S^*}\times\Re_+^{S^*}\times\Re_+\times\Re_+^{LS^*+J}\times\Re_+^{LS^*+J}\times\Re_+^{B}\times\Re_+^{S^*+J+1}:$$

$$0\le x^h\le 2A1,\varepsilon m_s^h\le\mu_s^h\le\frac{1}{\varepsilon},\varepsilon m_0^h\le\bar{\mu}_s^h\le\frac{1}{\varepsilon},0\le d_s^h\le M^*,0\le\bar{d}^h\le M^*,$$

$$\varepsilon m_s^h\le b_{sl}^h(b_j^h)\le\frac{1}{\varepsilon},\varepsilon\ \le q_{sl}^h(q_j^h)\le\frac{1}{\varepsilon},\varepsilon m_0^h\le u^h\le\frac{1}{\varepsilon},\varepsilon\le v^h\le 1\}$$

and

$$\sum\nolimits_\varepsilon^b=\{\left(\mu^b,d^b,\bar{m}^b,m_s^b,b_j^b,q_j^b,c_s^b,v_s^b,\pi_s^b\right)\in\Re_+\times\Re_+\times\Re_+\times\Re_+^{S^*}\times\Re_+^{J}\times\Re_+^{J}\times\Re_+^{s^*+J+2}\times\Re_+^{s^*}:$$

$$\varepsilon e_0^b\le\mu^b\le\frac{1}{\varepsilon},0\le d^b\le M^*,\varepsilon e_0^b\le\bar{m}^b\le\frac{1}{\varepsilon},\varepsilon e_s^b\le m_s^b\le\frac{1}{\varepsilon},\varepsilon e_0^b\le b_j^b\le\frac{1}{\varepsilon},\varepsilon\le q_j^b\le\frac{1}{\varepsilon},$$

$$\varepsilon\le c_s^b\le M^*,\varepsilon\le v_s^b\le 1,\varepsilon e_s^b\le\pi_s^b\le M^*\}$$

which are both compact and convex.

Let the typical element of $\sum_\varepsilon^h(\sum_\varepsilon^b)$ be σ^h (σ^b) $\in\sum_\varepsilon^h(\sum_\varepsilon^b)$. Define $B_\varepsilon^h(\eta)=B^h(\eta)\cap\sum_\varepsilon^h$ and $B_\varepsilon^b(\eta)=B^b(\eta)\cap\sum_\varepsilon^h$. Also let $\sigma=(\sigma^1,\ldots,\sigma^H,\ldots,\sigma^{B+H})\in\Sigma_\varepsilon=X_{h\in H\cup B}\sum_\varepsilon^h$. Define the map $\Psi_\varepsilon:\Sigma_\varepsilon\to N$, where

$N=\{\eta=(p,\rho,r_s,\bar{r},\theta,R)\in(\Re_{++}^{LS^*}\times\Re_{++}\times\Re_{++}^{s^*}\times\Re_{++}\times\Re_{++}^{J}\times\Re_+^{S^*+1+J}\}$

and Ψ_ε is defined by equation (i)-(ix). In addition define (η,σ) to be an *ε-MECBD* if $\eta=\Psi_\varepsilon(\sigma)$ and (x), (i.e. (a) $\sigma^h\in\operatorname*{Arg\,max}_{\sigma^h\in B_\varepsilon^h(\eta)}\Pi^h(x^h(\sigma^h))$ and (b) $\sigma^b\in\operatorname*{Arg\,max}_{\sigma^b\in B_\varepsilon^b(\eta)}\Pi^b(x^b(\sigma^b)))$. Note also that all elements of $\Psi_\varepsilon(\sigma)=\eta$ are continuous functions of σ, since in each market some agents are bidding (offering) strictly positive amounts and repayments are bounded away from *0* by the Inactive Market Hypothesis.

Furthermore, define

$G:N\rightrightarrows\underset{h\in H\cup B}{X}\sum_\varepsilon^h=\Sigma_\varepsilon$, where

$G^h=\sigma^h\in\operatorname*{Arg\,max}_{\sigma^h\in B_\varepsilon^h(\eta)}\Pi^h(x^h(\sigma^h))$ and $G=\underset{h\in H}{X}G^h$.

Finally, let $F=G\circ\Psi:\Sigma_\varepsilon\rightrightarrows\Sigma_\varepsilon$. G is convex-valued since $\sigma\to u^h(x^h(\sigma^h))$ is concave. Recall, $\sigma^h\to x^h(\sigma^h)$ is linear, and that $B_\varepsilon^h(\eta)$ is convex. Since Ψ is a function, $F=G\circ\Psi$ is also convex valued. Moreover, if ε is sufficiently small, G is non-empty, since $m_s^h,e_s^b>0\ \ \forall h\in H,b\in B$. When $\varepsilon>0$, p_{sl}, θ, r_s, r, $R>0$, and since e^h, $e^b\neq 0$, $B_\varepsilon^h(\eta)$ for $h\in H\cup B$ is a continuous correspondence. Hence, by the Maximum Theorem, G is compact-valued and upper semicontinuous, and therefore so is F. Note that since we have restricted the domain of Ψ to Σ_ε and since for each good and money, some $h\in H\cup B$ has a strictly positive endowment, the restriction Ψ to strictly positive prices, and interest rates strictly greater than -1 is legitimate. The same applies for R's since an external agent always guarantees a minimum repayment $\varepsilon>0$ by the Inactive Markets Hypothesis. Finally, observe that the total amount of money is bounded above. Commodity prices, $p_{sl}\le(M^*+\varepsilon/L{\cdot}S^*/)/\varepsilon\le 2M^*/\varepsilon\ \ \forall sl$. Thus, $p_{sl}A_j$ is bounded above and so the external agent never delivers more $\frac{|L\cdot S^*|\varepsilon^2 2M^*}{2M^*\cdot\varepsilon}\le\varepsilon$ units of money. Thus the total amount of money is $M^*+(/L{\cdot}S^*/+/J/)\varepsilon\le 2M^*$.

<u>Step 1:</u> An *ε-MECBD* exists for any sufficiently small $\varepsilon>0$.

Proof: The map *F* satisfies all the conditions of the Kakutani fixed point theorem, and therefore admits a fixed point $F(\sigma) \ni \sigma$ which satisfies *(i)-(x)* for an *ε-MECBD*.
For every small $\varepsilon>0$, let $(\eta(\varepsilon), \sigma_\varepsilon)$ denote the corresponding *ε-MECBD*.
Step 2: At any *ε-MECBD*, $r_s(\varepsilon)$, $\bar{r}(\varepsilon)$, $p(\varepsilon) \geq 0 \quad \forall s \in S^*$.
Proof: By Propositions *1* and *2*.
Step 3: At any *ε-MECBD* $\exists I, Z<\infty \ni r_s(\varepsilon)$, $\bar{r}(\varepsilon)$, $\rho(\varepsilon)<I$ and $\bar{\mu}^h(\varepsilon), \mu_s^h(\varepsilon), \mu^b(\varepsilon) \leq Z$, $\forall s \in S^*$, $h \in H$, $b \in B$.
Proof: Suppose that $r_s(\varepsilon) \to \infty$. Then $\exists h \in H$ such that $\mu_s^h(\varepsilon) \to \infty$ and consequently
$D_{sN(s)}^h = (\mu_s^h - M^*) \to \infty$. Then, since $\lambda >> 0$, $\frac{\nabla \pi_{sl}^h}{p_{sl}(\varepsilon)} << \lambda\ D_{sN(s)}^h$. Thus, *h* could have been better off by reducing $\mu_s^h(\varepsilon)$ by Δ, a contradiction. Similarly, for $\bar{r}(\varepsilon)$, $\rho(\varepsilon)$, $\mu_s^h(\varepsilon)$, $\mu^b(\varepsilon)$.
Step 4: For any *ε-MECBD*, $\exists c>0 \ni p_{sl}(\varepsilon) > c$, $\forall s \in S^*$, $l \in L$.
Proof: Suppose that $p_{sl}(\varepsilon) \to 0$ for some $s \in S^*$, $l \in L$. Then choose $h \in H$. He could have borrowed Δ more to buy $\Delta / p_{sl}(\varepsilon) \to \infty$. His net gain in utility would be $\left(\frac{\nabla \pi_{sl}^h}{p_{sl}} - \lambda_{sl}^h r_s\right)\Delta > 0$ since $\lambda_{sl}^h r_s < \infty$ and by (A3), $\pi^h(0, \ldots, Q, \ldots, 0) > u^h(A1)$ with *Q* in the *sl* th place. Thus, $p_{sl} \geq \nabla \Pi_{sl}^h / \lambda_{sl}^h r_s > c > 0$.
Step 5: For any *ε-MECBD* $\exists \Gamma \ni q_j^h(\varepsilon) < \Gamma$, $\forall j \in J$, $h \in H \cup B$.
Proof: Suppose that for some $j \in J$, $q_j^h(\varepsilon) \to \infty$ and $A_{sl}^j > 0$, for some $l \in L$. Then *h* can deliver at most $q_j^h A_{sl}^j \leq \max\limits_{\substack{1 \leq l \leq L \\ s \in S^*}} \sum\limits_{h \in H} e_{sl}^h = \bar{e}$, and therefore his disutility from default would be $(\lambda_j^h p_{sl} q_j^h A_{sl}^j - M^*)/p_s g_s < u(A1)$. Otherwise, $\sigma^h(\varepsilon)$ are not optimal.
Similarly, suppose that $A_{s,m}^j > 0$. Again *h* can deliver at most $q_j^h A_{S,m} \leq \bar{e}$ and then his disutility from default would be $(q_j^h A_{S,m})/p_s g_s \leq u(A1)$. Otherwise, he would not have optimized.
Step 6: For all $h \in H \cup B$, $d_s^h, \bar{d}^h, b_{sl}^h, b_j^h, u^h \leq 2M^*, \bar{m}^b, m_s^b, c_s^b \leq 2M^*$ and $/R/ < \varepsilon$.
Proof: All variables are constrained by the total amount of money present in the economy and *R* by assumption.
Step 7: For all $h \in H \cup B$, $\sigma_\varepsilon^h = \arg\max\limits_{\sigma_\varepsilon^h \in B^h(\eta(\varepsilon))} \pi^h(x^h(\sigma^h))$, for sufficiently small $\varepsilon>0$.
Proof: From steps *2-6* and the budget constraints (3^h), (8^h) of *3.1* and (1^h) of *3.2,* the ε-constraint is not binding thus concavity of payoffs guarantees the optimality of $\sigma^h(\varepsilon)$.
MECBD (η, σ) will be constructed by taking the limit of *ε-MECBD* $(\eta(\varepsilon), \sigma(\varepsilon))$, as $\varepsilon \to 0$. This is achieved by taking limits of sequences of ε and subsequences of subsequences.
Step 8: If for some $\bar{s}\bar{l}$, $p_{\bar{s}\bar{l}}(\varepsilon) \to \infty$, then $p_{sl}(\varepsilon) \to \infty$ $\forall s \in S$, $l \in L$. Also, if either $\theta_j(\varepsilon) \to \infty$, or $p_{0l}(\varepsilon) \to \infty$ then $p_{sl} \to \infty$, $\forall l \in L$, $s \in S^*$.
Proof: Some *h* owns $e_{\bar{s}\bar{l}}^h > 0$. If $p_{sl}(\varepsilon)$ stays bounded on some subsequence, then by borrowing very large $\bar{\mu}^h$ or μ_0^h if $s=0$, *h* can use it to buy *Q* units of *sl*. Then since $\bar{r}(\varepsilon)$, $r_s(\varepsilon)<I$, *h* can sell Δ of $\bar{s}\bar{l}$ acquire $\Delta\, p_{\bar{s}\bar{l}}(\varepsilon) \to \infty$ to defray his loan and improve his utility, a contradiction.
If $\theta_j(\varepsilon) \to \infty$ for some $j \in J$ and $p_{sl}(\varepsilon) < \infty$, let h borrow $\Delta\theta_j(\varepsilon)/(1+r_0(\varepsilon))$ and buy $\Delta q_j(\varepsilon)/(1+r_0(\varepsilon))\, p_{sl}(\varepsilon)$ of *sl* and improve his utility. If $p_{0\bar{l}}(\varepsilon) \to \infty$, as previously argued then $p_{ol}(\varepsilon) \to \infty$, $\forall l \in L$. Then, by selling Δ of $o\bar{l}$ *h* can acquire $\Delta\, p_{0l}(\varepsilon) \to \infty$. If any of $p_{sl}(\varepsilon) \nrightarrow \infty$, $\forall s \in S$ then by inventorying money he can improve upon his utility.
Step 9: $\exists K>0 \ni p_{sl}(\varepsilon)/p_{sk}(\varepsilon) < k$, $\forall l, k \in L$, $s \in S$.
Proof: Suppose the opposite. Then take *h* with $e_{sl}^h > 0$. Let him reduce Δ his sales of *sl* and lose $\Delta(\Pi^h(A1) - \Pi^h(0))$ at most. Then he could buy more *sk* buy borrowing $\Delta p_{sl}(\varepsilon)/(1+r_s(\varepsilon))$ and sell Δ of *sl*. His net gain in utility would be

$$\Delta(\varepsilon)\left\{\frac{p_{sl}(\varepsilon)}{(1+r_s(\varepsilon)\ p_{sk}(\varepsilon)}\left(\nabla\Pi^h_{sk}(x^h)\right)-(\Pi(A1)-\Pi(0)\right\}>0$$

since $p_{sl}(\varepsilon)/p_{sk}(\varepsilon)\to\infty$ and by step *3*, $r_s(\varepsilon) < I$.

<u>Step 10</u>: $\exists\ K'>0 \ni p_{ol}(\varepsilon)/p_{sl}(\varepsilon) < K',\ \forall s\in S^*,\ l\in L$.

Proof: If $s=0$ then step *9* obtains. Otherwise, set Δ (4^h) of *3.1* equal to $\Delta p_{sl}(\varepsilon)/1+r_s(\varepsilon)$.

<u>Step 11:</u> $\theta_j(\varepsilon)\Big/\sum_{l\in L} p_{ol}(\varepsilon)$ □∞, $\forall j\in J$.

Proof: Suppose the contrary. Let *h* sell $\dfrac{\Delta}{(1+\bar r(\varepsilon))}$ of *j* and borrow $\dfrac{\Delta\cdot\theta_j(\varepsilon)}{(1+\bar r(\varepsilon))}$ more.

Let him consume $\dfrac{\Delta\cdot\theta_j}{(1+\bar r(\varepsilon))p_{ol}(\varepsilon)}$ more ol for some $l\in L$ in $s=0$.

Then *h* can use the proceeds of the asset sale to defray the loan. His net gain of this action will be

$$\Delta\left(\frac{\theta_j}{(1+\bar r(\varepsilon)p_{ol}}-\frac{A^j_{sl}}{(1+\bar r(\varepsilon)}\right)>0$$

since $\dfrac{\theta_j(\varepsilon)}{p_{01}(\varepsilon)}\to\infty$.

<u>Step 12</u>: If $\theta_j(\varepsilon)\Big/\sum_{l\in L} p_{0l}(\varepsilon)\to 0$ then $A^j_{sl}=0$, $\forall l\in L$ and $\sum_{l\in L} p_{01}(\varepsilon)\to\infty$, whenever $A^j_{sm}>0$, $\forall s\in S$.

Proof: Suppose $A^j_{sl}>0$ for some $s\in S,\ l\in L$.

Choose $h\in H$ with $e^h_{01}>0$ for some $l\in L$. Let h sell $\dfrac{\Delta}{(1+\bar r(\varepsilon))}$ more of *ol* and increase his loan by $\left(\dfrac{\Delta}{(1+\bar r(\varepsilon))}\right)p_{0l}$. Then he could purchase $\dfrac{\Delta\cdot p_{01}}{(1+\bar r(\varepsilon))(1+r_0(\varepsilon))\theta_j(\varepsilon)}$ of *j*. Then, by borrowing in *s* and defraying his loan by asset deliveries he can improve his payoff, a contradiction. The same argument applies if $A^j_{sm}>0$ and $\sum_{l\in L} p_{01}(\varepsilon)\not\to\infty$.

<u>Step 13:</u> There exists $K\ni p_{sl}(\varepsilon)<K\ \forall s\in S^*,\ l\in L$.

Proof: Suppose the contrary and w.l.o.g. suppose that $p_{\bar s\bar l}\to\infty$ for some $\bar s\in S^*, \bar l\in L$.

Since $p_{sl}(\varepsilon)=\dfrac{\sum_{h\in H} b^h_{sl}(\varepsilon)}{\sum_{h\in H} q^h_{sl}(\varepsilon)}\le\dfrac{M^*}{\sum_{h\in H} q^h_{sl}(\varepsilon)}$, it must necessarily be $q^h_{sl}\underset{\varepsilon\to 0}{\to}0$ for all $s\in S^*,\ l\in L$ by step *8*. At any *ε-MECBD,* $\bar r(\varepsilon), r_s(\varepsilon), \rho(\varepsilon)<\delta_s$, by step 3. Hence, at any *ε-MECBD*, there are less than δ_s-gains to trade. By continuity, there are less than δ_s- gains to trade at $(e^h)_{h\in H}$.. However, G-to-T hypothesis guarantees that there are more than δ_s-gains to trade $\forall s\in S$, a contradiction.

<u>Step 14:</u> $\eta=\lim_{\varepsilon\to 0}\eta(\varepsilon)$ and $\lim_{\varepsilon\to 0}(\eta(\varepsilon),\sigma(\varepsilon))=(\eta,\sigma)$.

Proof: From the previous steps, $\eta(\varepsilon)$ is bounded in all components. The same applies for $\sigma(\varepsilon)$. Thus, a convergent subsequence can be selected that obtains (η, σ) in the limit. By continuity of $\Pi^h(\sigma^h)$ and $\Pi^b(\sigma^b)$, (η, σ) is a *MECBD*, and the artificial upper and lower bounds on choices are irrelevant since they are not binding and payoff functions are concave in actions.□

Proof of Proposition 4:

Let $M^G\to\infty$ and consider bounded asset trades. Then by choosing subsequences and further subsequences select a subsequence along which all relative σ's and η's converge. By Proposition *8* and its Corollary 2, the limit of the last subsequence coincides with a GEI, a contradiction. Thus, $(\sum_{h\in H} q_j^h + \sum_{b\in B} q_j^b) \to \infty$. Thus, by feasibility, $M^G / \|\theta_j\| \to \infty$ and

$$\left[w_s(\eta,\sigma)R_s m_s^b(1+r_s) + w(\eta,\sigma)\overline{R}\ \overline{m}^b(1+\overline{r}) + \sum_{j\in J} w_j(\eta,\sigma)R_{sj}(p_{sl}A^j)(b_j^b/\theta_j) \right] \to \infty,\ \forall s \in S^*,\ b \in B.$$

Finally, by relative boundedness of η's (Theorem 1, step *14*), $M^G / \|p_{0l}\| \to \infty$.

Also, $\exists\ \overline{Z}\ \ni \nabla\Pi_s^{b^*}(\pi_s^b) > \lambda_s^{b^*}$ for some $b^* \in B$, by Shubik and Tsomocos [38]. Similarly, for some $h^* \in H$. Interiority of the maximum on π_s^b and x_s^h, $\forall s \in S^*,\ b \in B,\ h \in H$ guarantees bounded aggregate profits and consumptions.□

Proof of Proposition 5:
From step 4 of Theorem 1, $p_{sl} > c,\ \forall s \in S^*,\ l \in L$. Let

$$\overline{Q} = 1 + \max\left\{ \left\{ \frac{\nabla\Pi_s^b(\pi_s^b)}{\nabla\Pi_{s'}^b(\pi_{s'}^b)} : s \in S^*, b \in B, \pi_s^b \in \Diamond \right\}, \left\{ \frac{\nabla\Pi_s^h(x_s^h)}{\nabla\Pi_{s'}^h(x_{s'}^h)} : s \in S^*, h \in H, x_s^h < A \right\} \right\},$$

where $\Diamond = \left\{ \pi_s^b \in \Re_+^{s+1} : \pi_s^b \le 1 + \max_{0\le s\le S} \sum_{b\in B} e_s^b + \max_{0\le s\le S} \sum_{h\in H} m_s^h \right\}$.

If some h goes bankrupt then he can reduce his bid on a commodity l by ε and use this amount to defray his loan. His gain in utility will be $\varepsilon\lambda_{sl}^h$ whereas his loss will be at most $\frac{\varepsilon\nabla\Pi_l^h(x_l^h)}{p_l} \le \frac{\varepsilon\nabla\Pi_l^h(x_l^h)\overline{Q}}{c}$

Thus, if $\lambda^* = \frac{\overline{Q}}{c} \max \left\{ \nabla\Pi_l^h(x_l^h, h \in H, l \in L, x_l^h \in \Diamond \right\}$ then as long as $\lambda > \lambda^*$ no h will go bankrupt.

Conversely, if $\sum_{h\in H} \mu_s^h > \sum_{h\in H} d_s^h$ *and* $\lambda << \lambda^*$, by continuity $\exists\ \underline{\lambda} \ni D_s^h > 0$ for some h.

Now since, $q_j^b = 0,\ \forall j \in J,\ b \in B$ *for* $\underline{\lambda}\ \exists\ b \in B \ni D_{sw}^b > 0$.□

Proof of Corollary 1 to Proposition 5:
Since $u^b = u$ *and* $\lambda_s^b = \lambda,\ \forall b \in B$, *let* $U^B = |B|u^b = |B|u$. Then, $D_{sw}^b > 0$ implies, $D_{sw}^B > 0$, for some $s \in S$.
Also, because loans exceed deposits $\forall s \in S^*$, $D_{sw}^B > 0$, $\forall s \in S^*$.□

Proof of Corollary 2 to Proposition 5:
Take $\overline{\lambda} = \overline{\lambda}^b = \overline{\lambda}^h > \lambda^*$ of the proof of proposition *5* and the result follows immediately.□

Proof of Theorem 2:
See Tsomocos[59]. □

Proof of Proposition 6:
From the definition of a *MECBD*, if we double w.l.o.g. all nominal variables while we half λ's then by doubling all prices we maintain the same consumptions, with double profits for commercial banks.□

Proof of Proposition 7:
Under the maintained hypotheses, let at the original *MECBD* agent h buy x_{sl}^h and sell $x_{sl'}^h$. From Theorem *1*, step *4*, $p_{sl} > c,\ \forall s \in S^*,\ l \in L$. Let $\forall h \in H$, $J^h = \left\{ sl = s \in S^* \text{and } l \in L : b_{sl}^h > 0 \right\}$ and
$L^h = \left\{ sl = s \in S^* \text{and } l \in L : q_{sl}^h > 0 \right\}$. Since $p_{sl} > 0,\ \forall s \in S^*,\ l \in L$ and strategy sets are bounded below by ε, $\exists\ h,\ h',\ \forall sl \ni J^h \cap L^{h'} \ne \varnothing$ (or equivalently $J^{h'} \cap L^h \ne \varnothing$). Otherwise, all traders would be buying or selling the

same commodities and then markets would be lopsided. So, $\exists h, h'$ involved in reverse transactions when one considers trade in all commodities. Else, either one trader would violate his budget constraint or be left with unused cash.

Then from his optimization condition, for any $s \in S^*$, and w.l.o.g. assume no bankruptcy,

$$\frac{\nabla\Pi_s^h(x_{sl}^h)}{p_{sl}} = \frac{\nabla\Pi_s^h(x_{sl'}^h)}{p_{sl'}}(1+r_s).$$

If *LHS* > *RHS* then *h* should have borrowed $\varepsilon{\cdot}p_{sl}$ more on the s^{th} credit market, bought ε units of *sl,* sold $(\varepsilon{\cdot}p_{sl}/p_{sl'})\ (1+r_s)$ *of* sl' to defray the loan and be better off. Alternatively, if *LHS* < *RHS*, then *h* should have spent $\varepsilon{\cdot}p_{sl}$ less on good *sl*, deposited the money in the intertemporal credit market (or borrowed $\varepsilon{\cdot}p_{sl}$ less, instead), sold $(\varepsilon{\cdot}p_{sl}/p_{sl'})\ (1+\bar{r})$ less of sl' and ended up better off. Note that this last option was feasible, since by hypothesis the agent carries over cash from *0* to *1* (i.e. $\Delta\ (4^h)$ of $3.1 > 0$).

After the change in monetary policy or the regulator's choices by the term structure of interest rates proposition one interest rate must change. Suppose that r_s increases yet all *h* do not change their consumptions. By indecomposability, since every *h* is buying, and nothing can be bought unless it is sold, some *h* is selling as well as buying. Thus, for any pair *sl* and sl' that are bought and sold, respectively, by the same *h*, $p_{sl}/p_{sl'}$ should fall. But then *sl* must have a seller, who buys another good *sn.* So, p_{sn}/p_{sl} must also fall. Continuing in this fashion, a commodity *sa* will be reached eventually that has already been mentioned, and then $(p_{sa}/p_{sb})\ (p_{sb}/p_{sc})\ldots(p_{sz}/p_{sa}) = 1$ should be falling, a contradiction.

Suppose, instead that $\bar{r}$ increases. If $h \in H$ has borrowed on the intertemporal credit market, then $\forall\, 0l$ and *sk*, $s \in S$ that he buys we must have,

$$\frac{\nabla\Pi_s^h(x_{0l}^h)}{p_{0l}} = \frac{\nabla\Pi_s^h(x_{sk}^h)}{p_{sk}}(1+\bar{r}).$$

Hence, p_{0l}/p_{sk} must fall if *h* maintains his consumption. From indecomposability and the previous argument, if all consumptions and *r*'s stay the same, then all relative prices at period *0* and $s \in S$ must stay the same. Thus, $\forall\, 0l$ and *sk*, $s \in S$, p_{0l}/p_{sk} must fall. But for $h' \in H$ who carries money from *0* to *1*, let *on* and *sj* be commodities he buys. If h' does not alter his consumption, and if r_s stays fixed, then p_{on}/p_{sj} must stay fixed, a contradiction.

Remark: If, under the maintained hypothesis, the MECBD involves bankruptcy then the previous arguments are reinforced because the bankruptcy penalties, if λ's>>0, affects Π_s^h's. □

Proof of Proposition 8:

From proposition *3* and Term Structure of Interest Rates Proposition, $r_s = 0,\ \forall s \in S^*$ and $\bar{r} = 0$. Then, from the definition of *MECBD* and *GEI* the Proposition follows immediately.□

Proof of Corollary 1 to Proposition 8:

Again since all *r*'s are equal to zero and *no* $h \in H$ can carry forward money via assets, the Proposition follows immediately from the definition of *MECBD* and *GEI*.□

Proof of Corollary 2 to Proposition 8:

Asset trades stay bounded. From step 4 of Theorem 1, $p_{s\bar{l}} > c$. Also, since there does not exist any bankruptcy by hypothesis, a buyer of asset *j* would be able to consume $A_{sl}^j / (1+r_s)$ units more of *l.* Since by the Term Structure of Interest Rates proposition, $r_s \to 0$ and the numeraire asset payoffs are linearly independent by assumption, if for some h, $q_j^h \to \infty$ or $b_j^h \to \infty$ as $M^G \to \infty$, then $\exists\ \hat{s} \in S \ni$

$$\left| \sum_{j\in J}\left(\frac{b_j^h}{\theta_j} A_{s\bar{l}}^j\right) \Big/ (1+r_s) - \sum_{j\in J}\left(q_j^h A_{s\bar{l}}^j\right) \right| \to \infty.$$

Thus, *h*, given bounded relative prices, either he could buy the whole economy or bankrupt, a contradiction.

Since, by Theorem *1* all the net trades of the agents, $(p_{0l}/\|p_{0l}\|)$, $(p_{sl}/\|p_{sl}\|)$ converge along convergent subsequences $\forall s \in S^*$ and by the Term Structure of Interest Rates all r's $\to 0$, these limiting trades would constitute a *GEI*. Finally, note that the (G to T) hypothesis is automatically satisfied as long as $M^G \to \infty$. The Arrow theorem of complete markets obtains whenever A is of full column rank and *GEI* coincides with *GE*.□