Using Options on Greeks as Liquidity Protection

David Bakstein* Sam Howison†

Mathematical Finance Group, University of Oxford,
Mathematical Institute, 24–29 St. Giles', Oxford OX1 3LB, UK.

Abstract

In this paper we suggest and describe the valuation of derivative contracts related to their own Greeks and those of other options and how they can be used to manage the risk of a portfolio of derivatives. We further describe certain types of these options, namely those related to the Delta and Gamma, which can be regarded as a form of insurance against liquidity holes and transaction costs for the writer of the contract representing the underlying.

Keywords: Option pricing, Greeks, Liquidity options, Liquidity, Transaction costs

1 Introduction

In the financial literature and practice the sensitivity of portfolios of derivatives with respect to parameters like the underlying spot prices or their volatilities are referred to as the Greeks. Mathematically the latter are calculated as various partial derivatives of the portfolio value with respect to the parameters. Important ones are the Delta and the Gamma, the first and second derivative with respect to the underlying spot price, respectively. Another important one is the Vega, or the sensitivity with respect to changes in the volatility of the underlying. One of the main tasks of

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†contact: howison@maths.ox.ac.uk, tel +44(0)1865 270500, fax +44(0)1865 270515

traders of structured derivatives is to manage the risk of their book. This is done by closely observing their portfolio Greeks and keeping them in acceptable ranges. If the underlying is a traded asset, e.g. a stock, it will usually be used as one of the main hedging instruments. Then, under standard Black-Scholes assumptions, the Delta represents the fraction of underlying notional that the trader has to hold against the portfolio to make it instantaneously hedged against changes in spot. The Gamma is then closely related to the amount of the underlying that has to be added to or removed from the current Delta in response to the change. Similarly, as an additional way of hedging Greeks the Gamma and the Vega can further be balanced by adding suitable options to the portfolio as it is linear and the Greeks thus additive.

In reality, however, contrary to the Black-Scholes assumptions, (re-)hedging will necessarily take place at discrete-time intervals and, if the market for the underlying is illiquid, it may be expensive because of transaction costs. Furthermore, there may be 'liquidity holes' during which it is impossible to trade at all or only with vastly increased bid-ask spreads; and yet further, the process of hedging may aggravate this lack of liquidity.[1] These considerations suggest that there is a market need for some form of 'liquidity contract' and indeed this proposal was recently made by Scholes [99]. Such contracts would offer opportunities to market participants with different levels of access to the market. Those facing high levels of transaction costs, or who are concerned about lack of liquidity, could pay a small liquidity premium to other participants with lower costs or access to liquidity elsewhere in their book. A very simple example of such a contract would be an 'American (or Bermudan) forward'[2] - a forward contract that is exercisable by its holder at any time (or exercisable according to a set schedule).

In a Black-Scholes world it is easy to show that, depending on circumstances, this contract is optimally exercised either as early as possible (a dividend capture strategy) or as late as possible (Mitton [02]). In some cases its value is the same as a European forward, but in others the possibility of early exercise adds value. In the real world, however, issues such as transaction costs and, especially, liquidity should add a small premium to the Black-Scholes value. This premium would be set by the market, and different participants would find different aspects (long, short, holder, writer) of the contract attractive depending on their own model (however imprecise) of the costs

[1] One particular such incident occurred on the London Stock Exchange with its blue-chip FTSE 100 index on Friday, 20 September 2002, which was a so-called 'triple-witching day', when options on both single equity and the index, as well as the index future expire and their settlement prices are determined by the average price within a 20 minute interval. On this particular day the swings in the index amounted to almost 8%, with some components swinging almost 100% within seconds.

[2] In the market jargon an early exercise feature is also called cancellable or callable.

and risks associated with illiquidity.

A rather more complex (and common) situation arises when a trader has a position which, for certain future scenarios, may have a large Gamma, with attendant liquidity and/or transaction costs risks as the Delta is adjusted. For example, a European Call has large Gamma if the asset is close to the strike at expiry. A liquidity derivative based on the Gamma of such an option would allow this liquidity risk to be transferred to a counterparty in exchange for a premium, as mentioned above. The counterparty in this case would probably have greater access to the market and/or internal synergies (with other positions elsewhere) not available to the original trader. Another way of saying this is that liquidity will obviously be cheaper for, say, a large investment bank than for one of their corporate clients, and thus Pareto efficient, i.e. non-zero sum, contracts may exist.

Overall, options on the Greeks of portfolios may embed a combination of static and dynamic hedging strategies in a single contract and thus represent a form of 'pre-packaged' liquidity, as mentioned by Merton [00]. As outlined above, the writer of a liquidity derivative will first exploit all the netting-off and static hedging possibilities elswhere in their portfolio, and then price and hedge the residual risk with an appropriate liquidity and/or transaction cost model (see for example Leland [85] and Hoggard, Whalley & Wilmott [94], Rutkowski [98] for the former, and Jarrow [94], Frey [96], Schönbucher & Wilmott [00], Bakstein & Howison [02], etc. for the latter). The Leland model is one such, and since for single-signed Gamma it reduces to the Black-Scholes model with an adjusted vol, we begin in section 2 with the simple case of liquidity derivatives in a Black-Scholes world. We recognise the limitations of this model but the alternatives are too varied and model-dependent to be addressed in detail here. In section 3 we will address the American early-exercise version of these contracts. Finally section 4 concludes the paper.

2 Valuation of Options related to Greeks

In a Black-Scholes world of perfect liquidity the options do not complete the market, and are thus redundant. But the same could be said about contracts like volatility swaps, although they are traded. A justification for this is that Black-Scholes assumptions of for example constant volatility or zero transaction costs do not hold in reality and the contract as such, even when it can be replicated by portfolios of simple options, offers an efficient 'pre-packaged' way of isolating a particular risk factor. Notwithstanding this, we still commence by analysing options on Greeks in a Black-Scholes world, as it provides us with a number of closed-form solutions wherein, when introducing some particular market models of finite liquidity, only the volatility parameter needs to be modified.

For simplicity we assume that the underlying asset, with spot price S, is traded and thus can be employed to hedge a portfolio of derivatives with value $v(S,t)$ written on it, which we refer to as the underlying derivative or option. We assume that S follows standard geometric Brownian motion

$$dS = \mu S dt + \sigma S dW, \tag{1}$$

with drift μ, volatility σ and a Wiener process dW. If the underlying derivative has a sufficiently simple structure, then it satisfies the standard Black & Scholes [73] partial differential equation (PDE)

$$\mathcal{L}_{S,t} v = \frac{\partial v}{\partial t} + rS\frac{\partial v}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 v}{\partial S^2} - rv = 0, \tag{2}$$

along with the payoff $v(S, T_v)$; here r is the risk-free rate. A second option $V(\Gamma_v, t)$ on, say, the Gamma $\Gamma_v(S,t) = \partial^2 v/\partial S^2$ of the portfolio v then also satisfies

$$\mathcal{L}_{S,t} V = 0$$

with a terminal value $V(\Gamma_v, T_V)$ to be specified. For the contract to have a sensible interpretation, the respective expiries must satisfy $T_v \geq T_V$; the contract could also have an American early-exercise clause as will be discussed in section 3. Since V satisfies the Black-Scholes PDE, it can, in general, be calculated through the Feynman-Kač formula

$$V(S,t) = e^{-r(T_V - t)} E[V(\Gamma_v, T_V)], \tag{3}$$

where the expectation $E[\cdot]$ is taken with respect to the risk-neutral probability, i.e. with the moments of S derived from (1) with $\mu = r$. This pricing framework could be extended to a vast number of options on different Greeks of a potentially unlimited number of underlying portfolios. If the formulae for the Greeks exist in a closed form, then the option values are usually tractable as well. Otherwise standard numerical techniques like finite differences or Monte-Carlo methods can be applied in a straightforward way. We now illustrate these general remarks with some specific contracts.

2.1 Options on Gamma

The most immediate possibility for an option on a Greek is one on the Gamma, although many others are possible. Under Black-Scholes assumptions of perfect liquidity and continuous-time trading a hedger holds $S\Delta_v$ per unit notional to replicate an option position v. The total cost (or revenue) of the amount of stock that needs to be added (or removed) in time dt to rebalance the Gamma of the position is

$$Sd\Delta_v = S(\Gamma_v dS + O(dt)), \tag{4}$$

where $\Gamma_v = \partial^2 v/\partial S^2$ and dS is proportional to S. In order to obtain a 'Dollar'-value for the Gamma contract, i.e. one that can be compared with the option value itself, we multiply $S\Gamma_v$ by S again and consider if a contract V with $S^2\Gamma_v$ as its underlying 'asset', which we will term 'cash Gamma'. If, for example, a Call with payoff

$$V(\Gamma_v, T_V) = \max(S^2\Gamma_v - K_V, 0),$$

were available, the writer or holder of v could employ V to rehedge their positions in the event that the asset price path crosses the 'high-Gamma' region. Explicit solutions for this contract exist for most options that have closed-form formulas for their Gamma terms. For instance the Gamma of a vanilla Call with strike K_v expiring at T_v is given by

$$\Gamma_v(S,t) = \frac{1}{S\sigma\sqrt{2\pi(T_v - t)}} e^{-\frac{1}{2}(d_1(S,t))^2}.$$

Then, again employing (3), after some cumbersome arithmetic, the solution emerges as

$$V(S,t) = S^2\Gamma_v(S, T_V)N(d) - K_V e^{-r(T_V - t)}N(d_V), \tag{5}$$

where $d_1(S,t)$ and d_V are as above and

$$d = \frac{\ln(K_v/S^*)(T_V - t) + \ln(S/S^*)(T_v - T_V)}{\sigma\sqrt{(T_v - T_V)(T_V - t)(T_v - t)}},$$

with S^* such that $(S^*)^2\Gamma_v(S^*, T_V) - K_V = 0$. It can be observed that if $T_V = T_v$ the contract has zero value, unless $S = K_v$, when the contract has infinite value, as the Gamma of a Call option is a Dirac-delta function at expiry. This problem can be avoided easily by choosing $T_V < T_v$. (Note also that although $S^2\Gamma_v$ is a solution of the Black-Scholes equation, it does not have a natural financial interpretation, the least artificial probably being in terms of the strike sensitivity of the corresponding asset-or-nothing option or an Arrow-Debreu price.)

Let us now consider how such a contract might be used. Suppose that at any point in time there were a number of liquidly traded contracts v, for example index options, of different expiries T_v and strikes K_v. Each of those could have a number of contracts on its $S^2\Gamma_v$ with different expiries T_V and strikes K_V. Then a trader who is exposed to a particular v, depending on her individual needs and usage of models, could buy or sell an amount c in the Gamma contracts per unit underlying option to manage the risk of her original position. As an example of an application of this contract

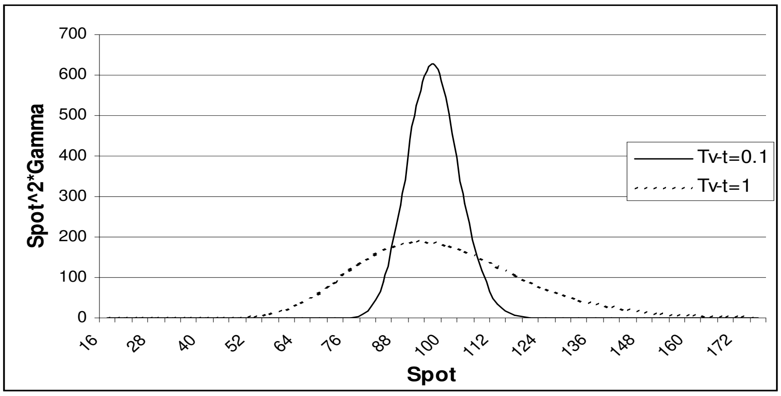

Figure 1: Shape of $S^2\Gamma$ of underlying Call with $r = 0.05$, $\sigma = 0.2$, $K_v = 100$.

we modify the Black-Scholes assumptions using the Leland model. As is well known transaction costs are an increasing function of the Gamma. Therefore, by employing cash Gamma options the writer of v can cap her exposure to the liquidity of the market and the associated transaction costs when rehedging. For this purpose the hedger has to choose both an appropriate strike K_V and amount of options c.

Firstly, to find a suitable strike level K_V, the hedger could, as a point of reference, observe the maximum value that $S^2\Gamma_v$ may take at T_V. Figure 1 shows the generic shape of $S^2\Gamma_v(S,t)$ for an underlying Call option. Its maximum occurs at

$$S_c = K_v e^{\left(\frac{1}{2}\sigma^2 - r\right)(T_v - T_V)},$$

which after substitution results in the maximum of

$$S_c^2\Gamma_v(S_c, T_V) = \frac{K_v e^{-r(T_v - T_V)}}{\sigma\sqrt{2\pi(T_v - T_V)}}.$$

So the hedger could choose her K_V as a fraction of this maximum attainable value.

Secondly, to determine the amount c that she needs to buy, we try to capture the magnitude of the frictions in the market which this contract is intended to address. For this we rewrite the strategy (4) for discrete time intervals:

$$S\delta\tilde{\Delta}_v = S(\tilde{\Gamma}_v \delta S + O(\delta t)), \tag{6}$$

where $\tilde{\Delta}_v$ and $\tilde{\Gamma}_v$ are the discrete time modifications of Δ_v and Γ_v, which are model-dependent (and chosen by the individual trader). Since we assume that transaction costs are a fixed percentage k of

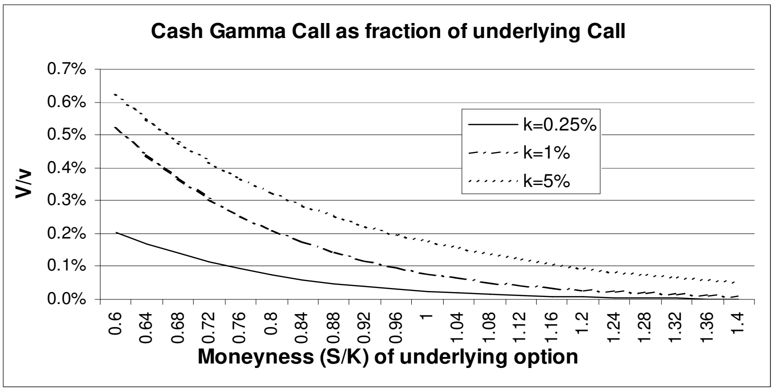

Figure 2: Value of a cash Gamma Call relative to its underlying Call with $r = 0.05$, $\sigma = 0.2$, $K_v = 100$, $K_v = 100$, $T_v = 1$, $\delta t = 1/365$, $T_V = T_v - \delta t$.

the total notional traded over δt their expected value to leading order is

$$kSE[|\delta\tilde{\Delta}_v|] = kSE[|\tilde{\Gamma}_v\delta S|]. \tag{7}$$

Now, Leland [85] shows that for positions with positive Gamma (e.g. long Call or Put) we then obtain

$$kSE[|\tilde{\Gamma}_v\delta S|] = kS\tilde{\Gamma}E[|\delta S|] = k\sigma\sqrt{\frac{2\delta t}{\pi}}S^2\tilde{\Gamma}_v, \tag{8}$$

and the modified Delta and Gamma are given by replacing their Black-Scholes volatility with

$$\tilde{\sigma} = \sigma\sqrt{1 \mp \frac{k}{\sigma}\sqrt{\frac{2}{\pi\delta t}}}. \tag{9}$$

Hence for the purpose of protecting against transaction costs a reasonable strategy would be to buy $c = k\sigma\sqrt{2\delta t/\pi}$ contracts V with strike K_V. Figure 2 shows the value of such an insurance relative to the underlying position for representative parameter values.

Although the values shown are small, it would be necessary to hold several such contracts to obtain adequate coverage over the period leading to expiry. As an alternative these contracts could indeed be bundled together in an OTC transaction as a series of strips or a swing-style option, where the holder can, say, choose m out of n given expiry dates. An additional enhancement would include making the contract American, which is discussed in a subsequent section.

2.2 Greek caps and barriers

A further possibility to deal with the liquidity risk of a position due to hedging would be for the buyer and writer to agree on limits on the theoretical Gamma of a contract, in the form of Gamma caps or barriers. Depending on the triggering conditions and effects, this may make the contract cheaper or transactionally more efficient. For example it could be agreed that the option should settle early at its theoretical Black-Scholes value once its Gamma reaches a certain specified level Γ_0. This would, in effect, be a 'capped Gamma' contract. Now in a Black-Scholes world of no transaction costs and perfect liquidity, the value of this contract would be exactly the Black-Scholes value. In practice, however, this contract removes the uncertainty for both the buyer and the seller as to whether the contract will be in or out of the money as expiry approaches, and may thus save on large transaction costs and slippage effects due to hedging activities.

As a valuation example, if we resort to the Leland model, then the cost of the discrete hedging strategy, hence of the option, will correspond to modifying the Black-Scholes volatility as in (9) when valuing the option and hence the hedger, by using a modified volatility, will have a nontrivial problem to solve. Indeed, if we still refer to the theoretical Black-Scholes Gamma as the barrier, then the contract becomes cheaper, because it now represents an indirect cap on the Gamma, rather than a contract whose own Gamma is capped. Again assuming that rehedging takes place daily, table 1 gives the Black-Scholes value, the Leland transaction cost premium and the reduced premium with the barrier contract clause for a European Call. It becomes apparent that the cap is relatively cheap. Further modifications to optimise the contract according to the counterparties' requirements may include e.g. making the barrier level a function of expiry etc.

3 American options on Greeks

If options on the Gamma of another portfolio of options are indeed regarded as a liquidity protection, then the holder would certainly prefer to be able to choose the moment of exercise. We mentioned the swing-style contract where exercise may be chosen at discrete points in time; an extension is a Bermudan contract where there exist time windows of exercise; finally in the limit an American contract grants the possibility of exercise at any time. Under the Black-Scholes assumptions, there is a simple test to examine whether early exercise is never optimal (as for a vanilla Call with no dividends) or may be optimal (as for a vanilla Put). Writing the the payoff of an option V as $P(S,t)$, k=0.25%

moneyness	Gamma barrier 0.2	0.15	0.1
0.8	0.15%	0.34%	0.66%
0.84	0.12%	0.27%	0.54%
0.88	0.09%	0.22%	0.43%
0.92	0.07%	0.17%	0.34%
0.96	0.06%	0.14%	0.27%
1	0.05%	0.11%	0.21%
1.04	0.04%	0.08%	0.16%
1.08	0.03%	0.06%	0.12%
1.12	0.02%	0.05%	0.10%
1.16	0.02%	0.04%	0.07%
1.2	0.01%	0.03%	0.05%

k=1%

moneyness	Gamma barrier 0.2	0.15	0.1
0.8	0.35%	0.80%	1.61%
0.84	0.29%	0.67%	1.34%
0.88	0.24%	0.55%	1.10%
0.92	0.19%	0.45%	0.91%
0.96	0.16%	0.37%	0.75%
1	0.13%	0.30%	0.61%
1.04	0.11%	0.25%	0.50%
1.08	0.09%	0.20%	0.40%
1.12	0.07%	0.16%	0.33%
1.16	0.06%	0.13%	0.26%
1.2	0.05%	0.11%	0.21%

k=5%

moneyness	Gamma barrier 0.2	0.15	0.1
0.8	0.48%	1.15%	2.43%
0.84	0.43%	1.01%	2.15%
0.88	0.38%	0.89%	1.90%
0.92	0.33%	0.79%	1.68%
0.96	0.29%	0.70%	1.48%
1	0.26%	0.62%	1.31%
1.04	0.23%	0.55%	1.16%
1.08	0.20%	0.48%	1.03%
1.12	0.18%	0.43%	0.91%
1.16	0.16%	0.38%	0.81%
1.2	0.14%	0.34%	0.71%

Table 1: Relative Gamma cap discount of a European Call under Leland transaction cost with $r = 0.05$, $\sigma = 0.2$, $K_v = 100$, $T_v = 1$, $\delta t = 1/365$.

which can be time-dependent, the linear complementarity formulation, see e.g. Wilmott et al. [93], is given by

$$\mathcal{L}_{S,t}V \leq 0 \qquad \text{and} \qquad V \geq P(S,t), \tag{10}$$

with

$$\mathcal{L}_{S,t}V(V-P) = 0 \qquad \text{and} \qquad V(S,T) = P(S,T).$$

Now, because in the early exercise region $\mathcal{R}$ we necessarily have $V(\mathcal{R}) = P(\mathcal{R})$, if $\mathcal{L}_{S,t}V(\mathcal{R}) = \mathcal{L}_{S,t}P(\mathcal{R}) > 0$, then this represents a contradiction to (10). Therefore in this case early exercise can never be optimal, as for instance for a Call option with payoff $P(S,T) = \max(S-K,0)$ and

$$\mathcal{L}_{S,t}\max(S-K,0) = \frac{1}{2}\sigma^2K^2\delta(S-K) + rK\mathcal{H}(S-K) \geq 0,$$

where δ and $\mathcal{H}$ represent the Dirac-Delta and Heaviside functions, respectively. Conversely, if $V(\mathcal{A}) < P(\mathcal{A})$ for some region $\mathcal{A}$, then $\mathcal{A} \subseteq \mathcal{R}$, i.e. it would certainly be optimal to exercise within this region and probably even within a larger one. As an example, for a Put option with $P(S,T) = \max(K-S,0)$ we have that

$$V(0,t) = Ke^{-r(T-t)} < P(0,T) = K.$$

For options on Greeks we can check for non-optimality of early exercise (the first possibility above), by deriving the PDEs for the respective Greeks. For Γ_v this is done by taking twice the partial derivative with respect to S of (2), resulting in

$$\frac{\partial\Gamma_v}{\partial t} + (r+2\sigma^2)S\frac{\partial\Gamma_v}{\partial S} + \frac{1}{2}\sigma^2S^2\frac{\partial^2\Gamma_v}{\partial S^2} + (r+\sigma^2)\Gamma_v = 0.$$

As an example we consider a cash Gamma Put. In this case the option satisfies

$$\mathcal{L}_{S,t}v \leq 0 \qquad \text{and} \qquad v \geq \max(K_V - S^2\Gamma_v, 0),$$

but upon early exercise we get

$$\begin{aligned}
& \mathcal{L}_{S,t}\max(K_V - S^2\Gamma_v, 0) \\
= \quad & -S^2\left(\frac{\partial^2\mathcal{L}_{S,t}v}{\partial S^2}\right) - rK_V\mathcal{H}\left(K_V - S^2\Gamma_v\right) + \frac{1}{2}\sigma^2S^2\delta\left(K_V - S^2\Gamma_v\right)\left(-2S\Gamma_v - S^2\frac{\partial\Gamma_v}{\partial S}\right)^2 \\
= \quad & -rK_V\mathcal{H}\left(K_V - S^2\Gamma_v\right) + \frac{1}{2}\sigma^2S^2\delta\left(K_V - S^2\Gamma_v\right)\left(-2S\Gamma_v - S^2\frac{\partial\Gamma_v}{\partial S}\right)^2.
\end{aligned} \tag{11}$$

Here (11) can be negative, hence for this contract it may be optimal to exercise prior to expiry and this is indeed the case. Figure 3 shows the relative early exercise premium, calculated numerically through finite difference methods.

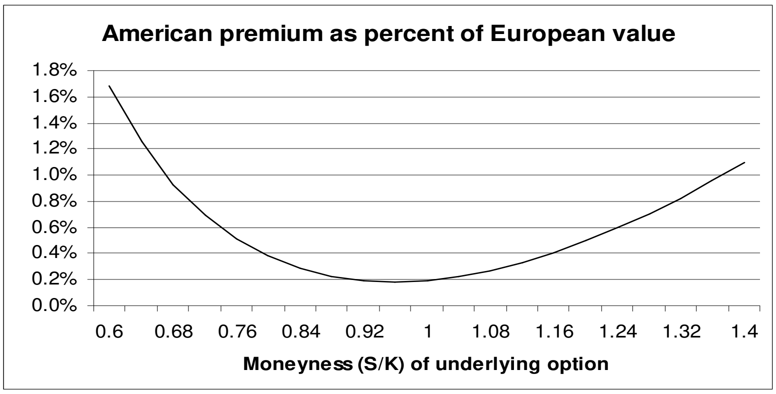

Figure 3: Early exercise premium of American cash Gamma Put on underlying Call as fraction of European cash Gamma Put with $r = 0.05$, $\sigma = 0.2$, $K_V = 50$, $K_v = 100$, $T_v = 1$, $\delta t = 1/365$, $T_V = T_v - \delta t$.

4 Conclusion

We have discussed options on the Greeks of derivative portfolios and presented simple applications to the dynamic hedging of the Gamma of vanilla options under both the Black-Scholes assumptions of continuous-time trading, no transaction costs and perfect liquidity as well as under the Leland [85] transaction cost model. But given the vast number of portfolios, Greeks and models that may describe the risk characteristics of a particular portfolio, the applications, designs of particular contracts and approaches to their valuation seem unlimited. For example one other possibility of hedging these contracts that we did not discuss is the use of static hedges in liquidly traded options, with the aim of replicating the payoff of the option as closely as possible. In this case a local volatility grid for the hedging instruments would be required, but should be straightforward. In fact, combination trades in simple options are very common on derivatives exchanges and their volatilities are usually lower than the sum of their parts, so that it may indeed represent a suitable alternative hedging strategy of these contracts.

However, one apparent problem that would arise with all options on Greeks is to find a consensus measure of unobservable variables and functions thereof. The solution to this would necessary involve proxies, as for instance known closed form formulae for the Greeks, implied and/or realised volatilities observed in the market at particular times etc. Obviously all of these would need to be agreed by the counterparties a priori and eventually standardised. But with industry having already standardised contracts such as volatility swaps and the fact that in reality there does not even exist something as simple as 'one' spot price, this problem could be overcome and is more legal than mathematical.

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