Debt restructuring and voting rules
Preliminary and incomplete
Comments welcome [1]

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Abstract

The voting arrangements used by creditors during debt restructuring are prespecified, often by statute. For example, U.S. law stipulates unanimous agreement outside bankruptcy, but allows for a supermajority vote in Chapter 11 bankruptcy. We analyze the effect of voting rules on the welfare of debtors and their creditors in restructuring negotiations. When markets are liquid, the "toughness" engendered by a requirement of unanimous agreement benefits creditors by more than the rise in the probability of disagreement hurts them. Conversely, if markets are illiquid (or creditors non-Bayesian), a unanimity requirement makes successful restructuring almost impossible, hurting creditors and the debtor alike. We apply our results to the choice of securities issued in exchange offers, to U.S. regulations governing debt restructuring, and to the current debate on the desirability of a sovereign debt restructuring mechanism. On a more technical level, our analysis extends the existing strategic voting literature (see especially Feddersen and Pesendorfer 1997) to the case in which the issue being voted over is endogenous to the voting rule used.

1 Introduction

An important literature within financial economics has sought to understand how creditors can protect themselves against a debtor who seeks to renegotiate the amount owed. For example, consider a debtor who owes \$100 and values staying in business at \$100. Suppose moreover that creditors would obtain only \$50 from liquidation. The obvious problem for the creditors is that if the debtor defaults on the \$100 owed, and then suggests restructuring the debt so that only \$50 is owed, it is in the creditors' best interests to accept. The inability of the creditors to credibly commit to punish the debtor in the event of default both directly reduces the amount that can be borrowed, and worsens any moral hazard problems that may exist.

Previous research has highlighted the role played by the size[1] and numerosity[2] of the creditors, along with the structure of the claims held, in strengthening the *ex post* bargaining power of creditors vis-à-vis the debtor. In this paper, we consider a related but largely neglected question: how does the institutional structure of debt renegotiation impact creditors' ability to protect themselves?

In practice, debt renegotiation often takes place in a highly structured way. For example, in U.S. style Chapter 11 proceedings, a debtor proposes a reorganization plan, which is then voted upon. The plan is accepted only if a supermajority of creditors accept. The details of this procedure are fixed by law — and so, in particular, cannot themselves be renegotiated *ex post*.

Debt renegotiation outside bankruptcy is likewise highly structured. In the U.S., the Trust Indenture Act (1939) stipulates that any changes to the principal, interest or maturity of outstanding debt must be unanimously approved by the creditors. (The alternative of exchange offers are afflicted by the well-known hold-up problem. As a consequence, most exchange offers require acceptance by a very large fraction of creditors — often over 95% — turning them into *de facto* votes, with close to unanimity required for acceptance.) In other legal jurisdictions,[3] bond contracts may contain a majority agreement clause, whereby a proposal to change debt terms must be approved only by some (pre-specified) fraction of bondholders.

In all the cases just described, the basic structure of the renegotiation game is the same: the debtor makes a proposal, and the creditors then vote to accept or reject this proposal. If agreement cannot be reached liquidation ensues. It is not at all obvious what agreement rule would be best for creditors, in the sense of maximizing their recovery rate. On the one hand, requiring unanimity among creditors makes agreement harder to obtain. This hurts creditors when liquidation is worse for them than the proposed restructuring. But on the other hand, the fact that agreement is less likely should induce the debtor to propose an offer which delivers more to the creditors.

In this paper, we explicitly model the voting game and establish the net impact of

[1]See Dewatripont and Maskin (1995).

[2]See Bolton and Scharfstein (1990), Berglöf and von Thadden (1994), von Thadden, Berglöf and Roland (2003), and Diamond (2004).

[3]For example, English law and Luxembourg law.

these two countervailing effects. When creditors are Bayesian and financial markets are liquid, creditors recover more under a unanimity agreement rule: the increased probability of failing to reach an agreement is more than compensated for by the improvement in the offer engendered by the tougher bargaining stance. But if either condition is violated — that is, if either creditors are far from rational, or if markets are illiquid — then creditors are better off *ex post* under a majority voting rule. In this second case, agreement is essentially impossible to reach under a unanimous agreement rule.

It is worth stressing from the outset that our focus in this paper is primarily on how agreement rules affect the *ex post* recovery rate of creditors. Of course, *ex post* outcomes have *ex ante* impacts. Here, the improvement in creditors' recovery rate engendered by employing a tougher agreement rule raises the amount that a debtor can borrow *ex ante*. However, it does so at a cost: tougher agreement rules make efficient restructuring harder, and so result in social inefficiency *ex post*. Nevertheless, as we show formally in Section 4, this cost is worth bearing when the debtor is sufficiently credit constrained at the financing stage.[4]

We apply these results to the choice of securities issued in exchange offers, to U.S. restructuring law, and to sovereign debt.

A near-universal feature of exchange offers is that the debtor offers to replace outstanding debt with relatively low risk securities. There is nothing inevitable about this (unless, of course, one believes in market-segmentation). After all, in principle the debtor could offer a package consisting of a large number of out-of-the-money warrants. Our model offers a possible explanation: holding the expected value of an offer constant, dispersed bondholders are more likely to agree to an offer that is of low information-sensitivity.

As already noted, U.S. law embodies two diametrically opposing voting rules: outside bankruptcy, unanimity is required, while inside Chapter 11 bankruptcy a majority rule is in effect. Our results suggest that this combination may be quite effective — provided that filing for bankruptcy is costly to the debtor, creditors are protected outside bankruptcy, but at the same time their is an escape hatch provided for those instances in which unanimity cripples any possibility of agreement.

In recent years a large number of observers have called for the creation of a some form of sovereign debt restructuring mechanism (SDRM). A key component of most proposals is to allow a debtor to put a restructuring agreement to a binding majority vote. Proponents of an SDRM argue that without such a possibility, it is too hard for debtors and creditors to reach agreement. On the other side, critics caution that creditors are already poorly protected in sovereign markets, and that facilitating *ex post* renegotiation will weaken their position still further.[5] Our paper provides a theoretical framework in which to assess these competing effects.

[4]In general, the tension between *ex ante* and *ex post* efficiency is of course well-known — see Starr (1973) for an early analysis, and White (1983) or Adler *et al* (2000) for examples of this insight to the specific context of debt restructuring.

[5]See, e.g., Dooley (2000), or Shleifer (2003).

An overview of the model and its relation to existing literature

Existing models of bargaining in Chapter 11 bankruptcy treat the creditors as a unified actor and hence are not suited for studying the effect of the voting rules on the creditors' welfare (see Baird and Picker 1991, Bebchuk and Chang 1992, Eraslan 2003). One partial exception is Kordana and Posner (1999) who discuss many issues, among which is the choice of voting rule. Haldane *et al* (2003) consider voting in the specific context of sovereign debt restructuring. They restrict themselves to an analysis of majority voting among creditors with private values preferences (see Section 5).

In voting over the debtor's offer, creditors will only vote differently from one and other if they have distinct preferences over the offer versus liquidiation, and/or distinct information about the comparative merits of these two alternatives. Given that the securities offered by the reorganizing debtor will typically be tradeable to at least some extent, the most natural way to introduce heterogeneity among creditors of the same priority level is to take seriously informational differences. A recent political economy literature has analyzed "strategic voting" by differentially informed voters — see in particular Feddersen and Pesendorfer (1996, 1997, 1998).[6] The main findings of this literature, which we review in more detail below, are as follows. First, under any non-unanimity voting rule, information is efficiently aggregated when the number of voters is large. Second, information aggregation fails under the unanimity rule. (As Feddersen and Pesendorfer 1998 observe, these findings suggest that requiring unanimous agreement in jury trials can result in more wrongful convictions.)

In our context, the voting behavior of creditors is only half of the story. Equally important is the debtor's response. The voting papers discussed above all make the assumption that the issue being voted offer is exogenous to the voting rule being used. This is clearly inappropriate in our context; our main interest is precisely in the effect of different voting rules on the debtor's restructuring proposal. Accordingly, we add to the existing voting literature by endogenizing the debtor's offer as a function of the voting rule used.

Our primary finding is that the unanimity rule gives creditors a higher recovery rate. The main intuition is as follows. As noted, information is aggregated efficiently under any non-unanimity voting rule. As such, creditors will accept an offer that gives them only slightly more than the amount they would obtain in liquidation. In contrast, the unanimity rule does not fully aggregate information, and so creditors will sometimes reject offers that give them more than they obtain in liquidation. This forces the debtor to offer a reorganization package that is worth significantly more than the liquidation value of the firm.

To formalize this argument requires two main components. First, we show that although creditors make mistakes under a unanimity rule, as the debtor improves his offer the nature of the mistakes shifts from mistaken rejections of good offers to mistaken acceptances of bad offers. Only if this property is satisfied will a failure of information aggregation actually lead to a better offer. Second, we show that a higher offer actually improves the creditors' recovery rate. Given that the higher offer is a consequence of creditor mistakes, this is by no means obvious; it is quite possible that the main effect of an improved offer is to

[6] See also Austen-Smith and Banks (1996).

cause creditors to accept offers that are worth less than liquidation, thereby lowering their recovery rate.

Within the finance literature, Maug and Yılmaz (2002) is the first published paper to make use of a strategic voting model. They show that information revelation is enhanced if, in Chapter 11, creditors of different seniority levels are grouped into different classes, with a majority acceptance within each class required for reorganization. Likewise, Gilson and Schwartz (2001) apply the results of Feddersen and Pesendorfer (1997) to compare the efficacy of hostile bids and proxy contests as takeover mechanisms. However, in common with the rest of the strategic voting literature, both these papers take the issue being voted over as exogenous. Our paper also has some relation to an older finance literature on takeover offers. In particular, Bagnoli and Lipman (1988), Holmström and Nalebuff (1992) and Gromb (1993) all address how the possibility of being pivotal affects a shareholder's decision to tender.

Paper outline

The paper proceeds as follows. Section 2 outlines the basic model. Section 3 derives the creditors' recovery rate under various voting rules by analyzing the voting behavior of creditors and the debtor's optimal response. Section 4 embeds our analysis of debt restructuring into a simple model of the original financing decision. Section 5 reanalyzes our model under the assumption of illiquid markets for securities in the reorganized firm. Section 6 applies our results to the choice of securities issued in exchange offers, to U.S. regulations governing debt restructuring, and to the current debate on the desirability of a sovereign debt restructuring mechanism. Finally, Section 7 explores a couple of possible extensions to our model.

2 Model

We consider a negotiation between a financially distressed debtor and its n creditors. We will denote a typical creditor by $i \in \{1, \ldots, n\}$. In order to focus on the effect of different voting rules, we abstract from interclass conflicts: each creditor holds an identical claim against the debtor. That is, creditors are owed equal amounts with the same priority.

The reorganization and liquidation values of the firm need not be the same. We assume that a proportion $1-p$ of firms are economically distressed, in the sense that the liquidation value exceeds the reorganization value. The remaining fraction p of firms are not economically distressed. Notationally, we denote the firm's type by $R \in \{H, L\}$, where $R = L$ indicates that the firm is economically distressed and should be liquidated.

In practice, creditors are likely to have different views as to whether the debtor is economically distressed, or is merely financially distressed. Formally, we assume that no creditor can directly observe the firm's type R, but instead each observes only a noisy signal σ_i. Of course, creditors may be able to partially share their assessments of the firm. Bondholder committees constitute one possible forum; to the extent to which debt claims in distressed firms are traded, the market price of debt could be another. However, it seems highly unlikely that creditors are able to share all information. For example, only a small number of bondholders sit on creditor committees. Likewise, given the comparative illiquidity of debt markets, particularly for bonds in distressed companies, the likelihood of prices being fully revealing seems remote. For the remainder of the paper we interpret σ_i as the residual piece of private information of creditor i after all feasible information sharing has occured.

For most of the paper we will assume that the debtor himself does not know whether he is of type H or L. In practice, this will be the case if, for example, there is substantial uncertainty concerning the liquidation value — which the debtor has little incentive to spend resources estimating. On a more technical level, this assumption is useful because it allows us to abstract from signalling issues in the debtor's choice of offer. Nonetheless, it is worth emphasising that the equilibrium we characterize under the assumption that the debtor is uninformed is *also* a pooling equilibrium of the alternate game in which the debtor is fully informed about R. We expand on this observation in Section 7 below.

The relevant uncertainty in our model relates to whether the firm is worth more under reorganization or liquidation. This uncertainty can clearly arise either from uncertainty about the reorganization value, or from uncertainty about the liquidation value, or both. It turns out that these distinctions have no impact on our main results,[7] and so to ease the presentation we assume all the uncertainty relates to the reorganization value, while the liquidation value is fixed. Without loss, we normalize the liquidation value to 1, and assume that the reorganization value of a firm of type H (respectively, L) is simply H (respectively, L). Since only type L firms are economically distressed, $L < 1 < H$.

Throughout, we assume that it is the debtor who proposes the reorganization plan. To start with we consider only proposals that entail replacing the outstanding debt with equity. In this case, the debtor's offer is fully characterized by the proportion of equity in the reorganized firm that he offers to the n creditors. The debtor's offer, which we denote by x, is itself a choice variable; much of our analysis below concerns its determination. Our results would be unchanged if instead the debtor offered, for example, risky debt; the only case which we need to rule out is that the debtor can feasibly offer new *risk-free* bonds. In Section 5 below we return to this issue when we explicitly consider the security design problem of a debtor seeking to restructure his debt.

Following the offer, creditors vote simultaneously on whether to accept or reject the proposal. The final outcome is determined by the voting rule in place. We allow for any voting rule of the type: the offer is accepted if at least a fraction $\alpha \in (0, 1]$ of the n creditors vote in favor of the offer. If the offer is rejected (i.e., if the number of creditors voting for the proposal is less than αn), then we assume for now that the firm is liquidated and the game ends. In Section 6 we extend our analysis to the case in which if the debtor's first offer is rejected he can make a second offer, with the voting rules used for the two offers potentially differing.

Note that when $\alpha = 1/2 + 1/n$ the agreement rule is the simple majority rule, while

[7] A proof of this claim is available from the authors.

when $\alpha = 1$ the agreement rule is the unanimity rule. Throughout, we commonly refer to any non-unanimity rule as a majority rule.

For the most part our analysis is conducted under the assumption that all the creditors are of the same size, and so their votes have equal weight in determining the outcome. (See Section 7 for a discussion of how our results might be affected if some creditors were larger than others.) Under our assumptions, each creditor receives a payoff of $\frac{x}{n}E[R|\text{offer accepted}]$ if the debtor's offer of x is accepted, and a payoff of $1/n$ if it is rejected and liquidation occurs. Likewise, the debtor's expected payoff if his offer is accepted and reorganization occurs is $(1-x)E[R|\text{offer } x \text{ is accepted}]$. For simplicity, we assume that the face value of creditor claims exceeds the liquidation value, and so when liquidation occurs the debtor receives nothing.[8]

Formally, we assume that the private signals observed by creditors satsify the following standard assumptions. (a) After conditioning on the true reorganization value, $(\sigma_i)_{i\in 1,\ldots,n}$ are independent and identically distributed random variables. Conditional on the reorganization value R, let $F(\sigma|R)$ and $f(\sigma|R)$ denote the distribution and density functions of σ_i. (b) The density function $f(\sigma|R)$ is a continuous function of σ for both $R \in \{L, H\}$.[9] (c) After either realization of the reorganization value $R \in \{L, H\}$, σ_i has full support over $[\underline{\sigma}, \bar{\sigma}]$. (d) The realizations of σ_i are informative about the true reorganization value, in the sense that the monotone likelihood ratio property (MLRP) holds strictly: $f(\sigma|H)/f(\sigma|L)$ is strictly increasing in $\sigma \in [\underline{\sigma}, \bar{\sigma}]$. (e) Not too much information is revealed by a signal $\sigma \in [\underline{\sigma}, \bar{\sigma}]$: $f(\sigma|H)/f(\sigma|L)$ is bounded away from both 0 and ∞.

3 Voting rules and equilibrium creditor recovery rates

To characterize the outcome of the debt reorganization, we work backwards: we first find the probability that any given offer x is accepted, and then solve for the debtor's preferred offer.

We start by considering the voting behavior of an individual creditor i. Clearly his vote will depend partially on his own assessment σ_i of the firm's type $R \in \{H, L\}$. However, and perhaps less obviously, his vote will also reflect the information of other creditors: when voting, a Bayesian creditor should reason as follows.

> I don't know what signals other creditors have received. However, most of the time my vote doesn't matter. Either αn or more of the other $n-1$ creditors will vote to accept, and the offer will be accepted; or else $\alpha n - 2$ or less vote to accept, and the offer will be rejected. So the only time my vote matters is when I'm pivotal, i.e., when exactly $\alpha n - 1$ of the other $n-1$ creditors vote to accept. So even though I don't know what signals other creditors have received, in deciding how to vote I should think *only* about the states in which $\alpha n - 1$

[8] This assumption has no qualitative impact on our results, and is easily relaxed.

[9] The early papers on strategic voting employ a finite signal space. See Yılmaz (1999) for a prior analysis using continuous signals.

other creditors vote to accept, and try to infer what this implies about their signals.

Essentially, the voting decision mirrors bidding behavior in a common values auction: in deciding how to bid, each bidder must take into account not only his own valuation, but also the information he infers about others' valuations from the fact that he is the winner (which is the only event in which his bid matters). In the formal voting literature on which this paper builds (see especially Feddersen and Pesendorfer 1997), voting behaviour of this type is described as *strategic*; in contrast, voting purely according to one's private information is described as *naive*.[10]

Formally, consider a creditor i who has received a signal σ_i. Let piv denote the event that his vote is pivotal, and $\overline{piv}$ denote the event that his vote is not pivotal. Define $K(\sigma_i) = E\left[\text{payoff}|\overline{piv}\right] \Pr\left(\overline{piv}|\sigma_i\right)$, the expected payoff of creditor i in states in which he is not pivotal. His payoff from voting against the proposal is

$$\frac{1}{n} Pr\left(piv|\sigma_i\right) + K(\sigma_i) \tag{1}$$

while his payoff from voting for the proposal is

$$\frac{1}{n}(xH \Pr\left(R = H, piv|\sigma_i\right) + xL \Pr\left(R = L, piv|\sigma_i\right)) + K(\sigma_i) \tag{2}$$

Thus creditor i votes to accept proposal x if and only if

$$xH \Pr\left(R = H, piv|\sigma_i\right) + xL \Pr\left(R = L, piv|\sigma_i\right) \geq \Pr\left(piv|\sigma_i\right). \tag{3}$$

Just as we described above, condition (3) says that in choosing whether to accept or reject an offer, the creditor focuses on the states in which he is pivotal. Formally, the term $K(\sigma_i)$ cancels from expressions (1) and (2) above.

Voting equilibria

First, observe that each creditor follows a cutoff strategy, in the sense of voting to accept whenever his/her signal exceeds some critical level:

Lemma 1 ***(Cutoff rules)***
In any equilibrium, for each creditor i there must exist a cutoff signal $\sigma_i^*(x, \alpha) \in [\underline{\sigma}, \bar{\sigma}]$ such that creditor i vote to accept proposal x if his signal is more positive than $\sigma_i^*(x, \alpha)$, i.e., $\sigma_i > \sigma_i^*(x, \alpha)$; and votes to reject the proposal if his signal is more negative, i.e., $\sigma_i < \sigma_i^*(x, \alpha)$.

[10]See Maug and Rydqvist (2004) for evidence of strategic voting behavior in the context of shareholder general meetings.

Throughout, we focus on symmetric equilibria in which all creditors follow the same voting strategy. In light of Lemma 1, let $\sigma_n^*(x,\alpha) \in [\underline{\sigma}, \bar{\sigma}]$ denote the common cutoff signal.[11] For clarity of exposition, we will suppress the arguments n, x and α unless needed. Evaluating explicitly, the probability that an agent is pivotal is given by[12]

$$\Pr(piv|R) = \binom{n}{n\alpha - 1}(1 - F(\sigma^*|R))^{n\alpha-1} F(\sigma^*|R)^{n-n\alpha}. \tag{4}$$

Note that since creditors' signal are independent conditional on R,

$$\Pr(R, piv|\sigma_i) = \frac{\Pr(piv|R)\Pr(\sigma_i|R)\Pr(R)}{\Pr(\sigma_i)}. \tag{5}$$

Substituting (4) and (5) into inequality (3), creditor i votes to accept proposal x after observing signal σ_i if and only if

$$\begin{aligned} & (xH-1)\, pf(\sigma_i|H)(1-F(\sigma^*|H))^{n\alpha-1} F(\sigma^*|H)^{n(1-\alpha)} \\ \geq\ & (1-xL)(1-p) f(\sigma_i|L)(1-F(\sigma^*|L))^{n\alpha-1} F(\sigma^*|L)^{n(1-\alpha)}. \end{aligned} \tag{6}$$

If there exists a $\sigma^* \in (\underline{\sigma}, \bar{\sigma})$ such that creditor i is indifferent between accepting and rejecting the offer x exactly when he observes the signal $\sigma_i = \sigma^*$, then the equilibrium can be said to be a *responsive equilibrium*: there is a positive probability that each creditor votes to accept, and a positive probability that each creditor votes to reject. That is, a responsive equilibrium exists whenever the equation

$$\frac{xH-1}{1-xL}\frac{pf(\sigma^*|H)}{(1-p)f(\sigma^*|L)}\frac{1-F(\sigma^*|L)}{1-F(\sigma^*|H)} = \left(\frac{(1-F(\sigma^*|L))^{\alpha}F(\sigma^*|L)^{1-\alpha}}{(1-F(\sigma^*|H))^{\alpha}F(\sigma^*|H)^{1-\alpha}}\right)^n \tag{7}$$

has a solution $\sigma^* \in (\underline{\sigma}, \bar{\sigma})$. Notationally, we represent a responsive equilibrium by its corresponding cutoff value $\sigma^* \in (\underline{\sigma}, \bar{\sigma})$.

We assume throughout that the information content of signals is such that if an individual creditor receives the most negative signal, $\sigma_i = \underline{\sigma}$, and has no other information, then he will decline even the debtor's best possible offer, $x = 1$:

Assumption 1 ***(Reject the best offer given information*** $\sigma_i = \underline{\sigma}$***)***

$$E[R|\underline{\sigma}] = \frac{Hpf(\underline{\sigma}|H) + L(1-p)f(\underline{\sigma}|L)}{pf(\underline{\sigma}|H) + (1-p)f(\underline{\sigma}|L)} < 1.$$

Assumption 1 enables us to establish the existence of responsive equilibria for large enough offers when the number of creditors is sufficiently large.

[11] As we show below, there exists a unique cutoff signal.

[12] Throughout, we ignore the issue of whether or not αn were an integer. This issue could easily be handled formally by replacing αn with $[\alpha n]$ everywhere, where $[\alpha n]$ denotes the smallest integer weakly greater than αn. Since this formality has no impact on our results, we prefer to avoid the extra notation and instead proceed as if αn is an integer.

Lemma 2 ***(Responsive equilibrium existence and uniqueness)***
Fix a voting rule α*, and some* $\varepsilon > 0$*. Then there exists an* N *such that a unique responsive equilibrium* σ^* *exists whenever* $n \geq N$ *and* $x \geq \frac{1}{H} + \varepsilon$*. Furthermore,* σ^* *decreases as* x *increases.*

For any offer such that $xH \leq 1$, voting to accept is clearly a dominated strategy, since liquidation gives at least as much — and maybe more. For such offers, voting to reject with probability 1 is the only equilibrium to survive elimination of dominated strategies.

What about offers such that xH is above 1 but the number of creditors is not large enough for Lemma 2 to hold? In general, *for any offer* x there is an equilibrium in which all creditors vote to reject; and except for the case of unanimity voting, there is also an equilibrium in which all creditors vote to accept. However, when there is no responsive equilibrium, the equilibrium in which all creditors accept cannot be perfect:

Lemma 3 ***(Non-responsive equilibrium)***
Suppose a voting rule $\alpha \geq \frac{1}{2} + \frac{1}{2n}$ *is in effect. Then if no responsive equilibrium exists for an offer* $x \geq \frac{1}{H} + \varepsilon$*, the only symmetric trembling-hand perfect equilibrium is that in which all creditors reject the offer with probability* 1.

In light of Lemmas 2 and 3, we focus on the responsive equilibrium whenever one exists, and otherwise use the rejection equilibrium in which case the cutoff signal is given by $\sigma^* = \bar{\sigma}$. Note that Lemmas 2 and 3 also imply that σ^* is weakly decreasing in x, and so the acceptance probability is weakly increasing in the offer x.

Finally:

Lemma 4 ***(Acceptance is more likely when reorganization value is high)***
For any given offer x*, the acceptance probability is at least weakly higher when the true reorganization value is high than when it is low.*

Majority voting rules

The existing literature on strategic voting has established that for any majority voting rule the private information of voters is effectively aggregated when the number of voters is large. In the current context, when the number of creditors is large then their individual estimates σ_i of the reorganization value jointly reveal the true reorganization value R. Efficient aggregation of information then means that offers such that $xR > 1$ will be accepted almost for sure, while offers such that $xR < 1$ will be rejected almost for sure. Put slightly less formally, asymptotically creditors accept exactly those offers which, given full information, it would be (*ex post*) in their best interests to accept.

A rough intuition for this result is as follows. For concreteness, consider the case of a supermajority rule requiring the consent of two-thirds of the creditors ($\alpha = 2/3$). Clearly there exist values for the cutoff signal σ^* such that when the true reorganization value is L, each creditor votes to accept with a probability a little *less* than 2/3, while when the true liquidation value is H each creditor votes to accept with a probability a little *more* than 2/3. In the former case, an individual creditor is pivotal when an unusually *large* number of creditors receive a high signal; while in the latter case, an individual is pivotal when an unusually *small* number of creditors receive a high signal.

By choosing the cutoff signal σ^* appropriately, it is possible to make the probabilities that an individual creditor is pivotal *arbitrarily close* for the two underlying reorganization values $R = H, L$. This is essentially the nature of the voting equilibrium under a majority rule: if all but one creditors follow the cutoff rule σ^*, the remaining creditor views the probability of high and low reorganization values as roughly equal after conditioning on the event that he turns out to be pivotal. As such, he will then vote to accept when his own signal is high, and to reject when his own signal is low. Information aggregation occurs in this equilibrium since all creditors vote to accept when their private signals are high, and to reject when their private signals are low.

To formally state this result, we need to introduce some additional notation. When the number of creditors is n, let $A_n(x, \alpha)$ denote the event that the offer x is accepted under the agreement rule α.[13] Likewise, let x_n denote the equilibrium offer when there are n creditors.

All our main results in this paper hold whenever the number of creditors, n, is sufficiently large. This raises the following small complication in stating our results: there is no reason to suppose that either the sequence of offers x_n, or the associated acceptance probabilities, has a well-defined limit. Accordingly, we will state almost all of our results in terms of the *limit infimum* ($\liminf$) and *limit supremum* ($\limsup$).[14]

Lemma 5 ***(Acceptance probabilities under majority voting)***
Fix any $\alpha < 1$. Then:

- *(i) If creditors should reject an offer when the true reorganization value is high, then they do indeed reject it in such states: if* $\limsup x_n < 1/H$ *then* $\Pr(A_n | H) \to 0$.
- *(ii) If creditors should accept an offer when the true reorganization value is high, then they do indeed accept it in such states: if* $\liminf x_n > 1/H$ *then* $\Pr(A_n | H) \to 1$.
- *(iii) Creditors reject all offers when the true reorganization value is low (as they should):* $\Pr(A_n | L) \to 0$.

One possibly surprising implication of Lemma 5 that all non-unanimity voting rules have the same asymptotic acceptance probabilities. Thus when the number of creditors

[13] We suppress the arguments x and α whenever possible.

[14] Recall that for any sequence of real numbers z_n, these mathematical objects are defined by

$$\liminf z_n = \lim_{N\to\infty} \inf\{z_n : n \geq N\} \text{ and } \limsup z_n = \lim_{N\to\infty} \sup\{z_n : n \geq N\}.$$

For example, the statement that $\liminf x_n > 1/H$ says that the debtor's offer stays bounded away from $1/H$, in the sense that there exists an $\varepsilon > 0$ such that for all n sufficiently large, $x_n > 1/H + \varepsilon$. This is a stronger statement than simply saying that x_n does not converge to $1/H$.

is large, changing the voting rule from a simple majority to a supermajority rule has no impact on what offers are accepted — and consequently, no impact on the what the debtor proposes either. Numerical simulations suggest that n does not need to be enormously large for this conclusion to hold. For example, Figure 1 displays the results of a typical numerical simulation of the voting subgame for $n = 30$ creditors, and illustrates the conclusions of Lemma 5. The high reorganization value is $H = 3/2$; offers not much above $1/H = 2/3$ are accepted with quite high probability when the true reorganization value is H, and the acceptance probability converges to 1 reasonably quickly as the offer grows larger. On the other hand, under majority voting rules, conditional on the true reorganization value being low, the acceptance probability is always close to 0. Even though the simulation is for only $n = 30$ creditors, the simple majority rule $\alpha = 1/2$ and the supermajority rule $\alpha = 2/3$ already coincide almost completely in terms of their implications for acceptance probabilities.

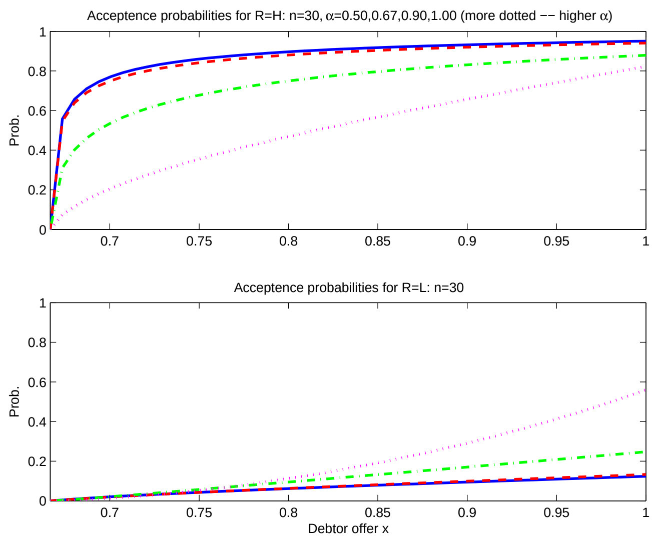

Figure 1: Numerical simulation of the voting subgame for $n = 30$ creditors and reorganization values $L = 1/2$ and $H = 3/2$. The graphs show the acceptance probabilities for different voting rules conditional on true reorganization values H (upper) and L (lower). The solid line represents the voting rule $\alpha = 1/2$, the dashed line represents $\alpha = 2/3$, the dash-dot line represents $\alpha = 0.9$ and the solid line represents $\alpha = 1$ (unanimity).

We now turn to the debtor's optimal chioice of offer. Recall that the liquidation payoff of the debtor is assumed to be zero. Consequently, the debtor's expected payoff if he offers x is given by

$$\Pi_\alpha^D(x) \equiv (1-x)(pH \Pr(A_n(x,\alpha)|H) + (1-p)L \Pr(A_n(x,\alpha)|L))$$

and the debtor chooses the offer x to maximize $\Pi_\alpha^D(x)$. Note that since the equilibrium cutoff signal $\sigma^*(x, \alpha)$ is continuous in x, and the probability of acceptance is continuous in σ^*, this maximization problem has a well-defined solution.

A consequence of Lemma 5 is that the debtor is able to construct a sequence of offers x_n such that these offers converge to $1/H$. Moreover, conditional on the reorganization value being high, these offers are accepted with a probability approaching 1. Since asymptotically no offer is accepted when the true reorganization value is low, this is the best the debtor can do:

Lemma 6 ***(The debtor's offer under a majority voting rule)***
Suppose a majority voting rule $\alpha < 1$ is in effect. Let x_n be a sequence of offers that maximize the debtor's payoff. Then $x_n \to 1/H$. Moreover, the acceptance probability conditional on the true reorganization value being $R = H$ converges to 1.

The unanimity rule

We now turn to an analysis of voting behavior when unanimity is required for agreement. The existing strategic voting literature has established that voting does not aggregate information. As such, the equilibrium acceptance probabilities differ starkly from those arising under *any* majority voting arrangement, even in the limit. Although the limiting acceptance probabilities do not have a convenient analytical form, as they do in Lemma 5, we are nonetheless able to characterize the key qualitative properties. In doing so, we extend the existing voting literature to link the extent to which information aggregation fails with the issue being put to the vote.

As just discussed (Lemma 6), when facing a majority agreement rule, as the number of creditors grows large the debtor is able to have an offer arbitrarily close to $1/H$ accepted with probability arbitrarily close to 1 (conditional on the true reorganization value being H). This is not the case under a requirement of unanimity: if the debtor tries to drive the offer down towards $1/H$ as the number of creditors grows large, that offer will be accepted with a vanishingly small probability.

Does this observation imply that the debtor is doomed to have his offer rejected when he faces a large coalition of creditors bound by a unanimous agreement rule? The answer is no. Provided he makes offers that stay bounded away from $1/H$, those offers will be accepted at least sometimes.

Lemma 7 ***(Acceptance probabilities under unanimity voting)***

(i) *If the debtor's offer converges to $1/H$ the corresponding rejection probabilities converge to 1: if $\limsup x_n \leq 1/H$ then $\Pr(A_n | H) \to 0$.*

(ii) *Creditors accept offers that are bounded away from $1/H$ with strictly positive probability (even asymptotically): if $\liminf x_n > 1/H$ then $\liminf \Pr(A_n | H) > 0$.*

(See Figure 1 for the results of a numerical simulation illustrating Lemma 7. Also, by Lemma 4, part (i) implies that if $\limsup x_n \leq 1/H$ then $\Pr(A_n|\, L) \to 0$. Moreover, clearly if $\liminf x_n > 1/H$ then $\liminf \Pr(A_n) > 0$.)

The difference between Lemma 5 and Lemma 7 is traceable to the fact that information is aggregated poorly under a unanimity voting rule. The reason is that as the number of creditors grows large, for any given cutoff rule σ^* a creditor believes he is vastly more likely to be pivotal when the true reorganization value is H. Consequently, if all but one creditors employs a cutoff rule σ^*, the remaining creditor finds it best to ignore his own signal entirely — reasoning that conditional on actually holding the pivotal vote, the true reorganization value must in fact be H. Given this, the only possibilities for an equilibrium are those in which σ^* converges to a limit of the signal support as the number of creditors grows large. But in these cases, creditors either always vote to accept, or always to reject, independent of their own signal. As a result, information aggregation fails.

Because of the failure of voting to aggregate information, the voting outcome is inefficient *ex post*. In particular, offers such that $xH > 1$ are sometimes rejected even when the true reorganization value is H. The first part of Lemma 7 shows that offers such that xH is close to the liquidation value 1 are almost always mistakenly rejected in this manner. However, the second part of Lemma 7 shows that the incidence of mistakes declines as the debtor improves his offer x.[15]

As a consequence of these observations, the debtor is forced to offer an amount that stays bounded strictly away from $1/H$:

Lemma 8 ***(The debtor's offer under a unanimity rule)***
Suppose a unanimity rule is in effect ($\alpha = 1$). Let x_n be a sequence of offers that maximize the debtor's payoff. Then $\liminf x_n > 1/H$.

Equilibrium recovery rates

The results above characterize creditors' voting responses to a debtor's offer, and the debtor's optimal choice of offer given those voting responses. We turn now to an evaluation of the equilibrium payoffs for the creditors and the debtor.

Given an offer from the debtor of x, let $\Pi^C_\alpha(x)$ denote the creditors' aggregate expected payoff under voting rule α. Decomposing,

$$\Pi^C_\alpha(x) = x\Big(pH \Pr(A_n(x,\alpha)|H) + (1-p)L \Pr(A_n(x,\alpha)|L)\Big) + (1 - \Pr(A_n(x,\alpha))) \quad (8)$$

When creditors employ a majority voting rule, the debtor makes an offer such that xH is just more than the liquidation value 1. With very high probability the creditors vote to accept this offer when the true reorganization value is H, and vote to reject it when the

[15] Although an increase in the offer x generates fewer mistaken rejections when the true reorganization value is high, it also generates more mistaken *acceptances* when the true reorganization value is low. Lemma 10 below gives the net effect.

true reorganization value is L. As such, in either state the creditors are left with a payoff that is close to their liquidation payoff of 1. It immediately follows from Lemma 6 that:

Lemma 9 ***(Creditor recovery under majority voting rule)***
Suppose a majority voting rule $\alpha < 1$ is in effect. Let x_n be a sequence of offers that maximize the debtor's payoff. Then the creditors' payoff satisfies

$$\lim \Pi_{\alpha}^{C}(x_n) = 1$$

In contrast to the situation arising under majority voting, when creditors instead require unanimous agreement to accept the debtor's offer, the debtor responds by offering an amount such that xH stays bounded strictly away from the liquidation value 1 (Lemma 8). At first sight it might seem that this trivially implies that the creditors' recovery is higher under the unanimity requirement. However, matters are not quite that simple. Recall that creditors are receiving an offer of more than $1/H$ precisely because information is aggregated poorly under unanimity voting, and so they sometimes mistakenly reject offers above $1/H$. But by the same token, creditors sometimes wrongly accept the debtor's offer when the reorganization value is L. *A priori* it is by no means clear that the net effect of receiving a higher offer, but mistakenly passing up more liquidation opportunities, is positive.

To deal with this complication, observe first that by part (i) of Lemma 7 an offer of $1/H$ is always rejected by creditors using a unanimity voting rule. Since liquidation always ensues, the creditors expected payoff in this case coincides with the expected liquidation value of 1.

Suppose now that we exogenously increase the offer made to the creditors. How does this affect their payoff? On the one hand, if the creditors accept the offer in exactly the same states an increase in the offer clearly raises welfare. The potential caveat is that the set of states in which the offer is accepted changes. Since acceptance is suboptimal for the creditors when the true reorganization value is L, in principle this change in the acceptance states may lead to a *decrease* in their welfare. However, our next result establishes that the creditors' payoff is increased by an amount *at least* as great as if the acceptance states remained unchanged.

Lemma 10 ***(Effect of raising the offer x)***
Suppose that a unanimity rule is in effect ($\alpha = 1$). Then if an offer x is such that a responsive voting equilibrium exists, the derivative of the creditors' payoff with respect to the offer x is equal to the expected utility of creditors over states in which the offer is accepted.

Lemma 10 is enough to establish that the creditors' payoff is indeed higher under a unanimity rule than under a majority voting rule. If the debtor's offer is exogenously fixed at $1/H$ in the case of unanimity voting, the creditors' welfare asymptotically coincides with their welfare under a majority voting rule and the debtor's optimal offer. In contrast, from Lemma 8 we know the creditors receive an offer above $1/H$ when they use a unanimous agreement rule, and from Lemma 10 we know this increase in the offer does in fact benefit them. Stated formally,

Lemma 11 ***(Creditor recovery under a unanimous rule)***
Suppose a unanimity voting rule is in effect ($\alpha = 1$). Then the creditors' payoff satisfies

$$\liminf \Pi_1^C (x_n) > 1.$$

Lemmas 9 and 11 together give our main result:

Proposition 1 ***(Unanimity better for creditors)***
Consider any majority voting rule $\alpha < 1$. Then whenever the number of creditors is sufficiently large, the creditors' payoff is higher under a requirement of unanimity for acceptance.

Conventional wisdom identifies two opposing effects of adopting a unanimous voting rule. On the one hand, unanimity makes agreement harder to obtain. On the other hand, this "toughness" may be useful in negotiation. Proposition 1 shows that the latter effect dominates: the increase in the debtor's offer relative to that obtained under majority voting more than compensates for the increased probability of mistakenly rejecting the offer.

One way to think about this result is that when creditors vote strategically, the requirement of unanimity is not as inimical to agreement as it might at first seem. Recall that each creditor conditions his or her vote only on the circumstances under which it is actually pivotal. Given a unanimity agreement rule, this means that a creditor considers the impact of voting to accept an offer conditional on all other creditors accepting — in other words, conditional on all other voters viewing the offer as attractive. Such a creditor will vote to accept unless his own signal is very pro-liquidation.

Clearly this result relies heavily on the creditors all placing the same value on securities in the reorganized firm, at least conditional on the true reorganization value. This assumption is appropriate when markets are liquid.[16] In Section 5 below, we revisit the comparison between unanimity and majority rules under the opposite extreme of complete illiquidity.

4 Trading off *ex post* inefficiency and creditor recovery

Proposition 1 tells us that a unanimity rule improves creditor recovery rates. However, it does so at a cost: creditors accept some reorganization offers even when the reorganization value is less than the liquidation value of the firm,[17] and may also reject some offers even

[16] We should emphasize the distinction between the markets for the pre- and post-reorganization securities. Pre-reorganization creditors hold bonds. Institutionally, bond markets are on average relatively illiquid; and markets for distressed debt are likely to be even less liquid than average. With the removal of financial distress, post-reorganization liquidity is likely to improve, especially if the new securities issued are equity claims on the firm.

[17] To see this formally, the equilibrium condition (7) can be rewritten as

$$\frac{xH - 1}{1 - xL} \frac{pf(\sigma^*|H)}{(1-p) f(\sigma^*|L)} \frac{1 - F(\sigma^*|L)}{1 - F(\sigma^*|H)} = \frac{\Pr(A_n|L)}{\Pr(A_n|H)}. \tag{9}$$

For the debtor's optimal offers, the lefthand side is bounded away from 0 — and so the righthand side is also.

when the reorganization value is greater than the liquidation value. In contrast, under majority voting the probability of both types of mistakes approaches 0 as the number of creditors grows large (see Lemma 6).

Clearly, if creditors are able to decide on a voting rule after the debt has been issued, they will opt for a unanimity rule whenever they are sufficiently numerous and sufficiently close to common values. In practice, however, voting rules are likely to be fixed in advance: either by law, or by the terms of the debt contract. Given the cost it imposes in terms of inefficient liquidation *ex post*, is the unanimity rule ever socially efficient? Relatedly, would a prospective borrower ever want to issue debt specifying that unanimous agreement by creditors is required in order to change the terms of the debt agreement?

One instance in which a requirement of unanimous agreement among creditors is likely to be socially valuable is that in which the debtor is credit constrained. Under such a circumstance, unanimity permits a debtor to promise a higher repayment to creditors, thus loosening the credit constraint. Even though the unanimous agreement rule imposes a social cost *ex post*, this cost will be worthwhile provided that relaxing the credit constraint is sufficiently valuable.

To illustrate this point more formally, consider the following simple model of a firm's *ex ante* financing decision. At date 0, a firm (with no resources of its own) seeks to raise funds for a constant returns to scale investment opportunity with maximum size $\bar{I}$. If the firm invests $I \leq \bar{I}$, then with probability $1-Q$ the firm receives ρI at date 1, where $\rho > 1$ is the return. For simplicity, we assume that in this case the continuation value of the firm is simply 0. Moreover, because of the possibility of diversion, the firm can credibly pledge at most K to its investors. It is this limited pledgeability that raises the possibility that the firm is credit-constrained.

With probability Q, the firm has no liquid resources at date 1. It is forced to attempt to restructure is debt. Let μI denote the liquidation value of the firm. If the firm is allowed to continue, it will produce either $H\mu I$ or $L\mu I$ at date 2, with probabilities p and $1-p$ respectively. For simplicity we assume that these date 2 incomes are fully pledgeable, and that there are no further cash flows subsequent to date 2.

Restructuring takes place according to the voting game described in earlier sections. The debtor's and creditors' expected payoffs from restructuring are $\Pi_\alpha^C \mu I$ and $\Pi_\alpha^D \mu I$ respectively.

The firm issues one-period debt at date 0. Let k denote the face value of the debt. Normalizing the market interest rate to 1, the face value must satisfy $(1-Q)\,k + Q\Pi_\alpha^C \mu I = I$, i.e.,

$$k = \frac{1 - Q\Pi_\alpha^C \mu}{1-Q} I.$$

So the firm chooses its project size $I \leq \bar{I}$ to maximize

$$\begin{aligned} V(I) &\equiv (1-Q)\,\rho I - (1-Q)\,k + Q\Pi_\alpha^D \mu I \\ &= (1-Q)\,\rho I - I + Q\left(\Pi_\alpha^C + \Pi_\alpha^D\right)\mu I \end{aligned}$$

subject to the credit constraint

$$I \leq \frac{1-Q}{1-Q\Pi_\alpha^C \mu} K. \tag{10}$$

One immediate implication is:

Proposition 2 ***(Non-binding credit constraint)***
When the credit constraint (10) does not bind, the optimal voting arrangement is one that maximizes the combined ex post *welfare of the creditors and debtor,* $\left(\Pi_\alpha^C + \Pi_\alpha^D\right) \mu I$ *— i.e., a majority rule. A firm is less likely to be credit constrained when K is high, μ is high, and (provided $\Pi_\alpha^C \mu < 1$) when Q is low.*

What happens if the credit constraint (10) is binding? Then moving from a majority voting rule α to a unanimity voting rule increases the creditors' payoff in restructuring from $\Pi_\alpha^C \mu I$ to $\Pi_1^C \mu I$, while reducing the combined *ex post* payoff from $\left(\Pi_\alpha^C + \Pi_\alpha^D\right) \mu I$ to $\left(\Pi_1^C + \Pi_1^D\right) \mu I$. Define

$$\phi = \frac{\left(\Pi_1^C + \Pi_1^D\right) - \left(\Pi_\alpha^C + \Pi_\alpha^D\right)}{\Pi_1^C - \Pi_\alpha^C},$$

i.e., ϕ is the "cost" in social efficiency of increasing the creditors' payoff.

Proposition 3 ***(Ex ante optimality of unanimity rule)***
Suppose that a firm is credit constrained under both majority voting rule α and the unanimity rule. Then the unanimity voting rule maximizes the firm's welfare if and only if

$$\frac{(1-Q)\rho - 1 + Q\left(\Pi_\alpha^C + \Pi_\alpha^D\right)\mu}{1-Q\Pi_\alpha^C \mu} > \phi \tag{11}$$

In particular, the unanimity rule is preferred whenever the investment opportunity is sufficiently productive (ρ large enough).

5 Illiquid markets (or irrational investors)

As noted, we have thus far concerned ourselves with the benchmark case in which the securities issued by the reorganized firm to restructure the original debt are fully liquid. If capital markets are imperfectly liquid, as will often be the case for shares in a distressed company, then creditors holding the same beliefs about the reorganization value may nonetheless value the securities received in reorganization differently. For example, creditors may have different opportunity costs of funds; or they may face different tax rates. If the securities issued by the reorganized firm cannot be easily traded, they will not end up in the hands of those who value them the most. Below, we briefly consider the extreme of completely illiquid markets.[18]

[18] In a companion and more technical paper, we analyze our model for the general case in which markets are neither fully liquid nor fully illiquid.

Our model is easily adapted to handle the case of complete illiquidity: we simply reinterpret each creditor's signal σ_i as his private valuation of the reorganized firm, and assume now that the support $[\underline{\sigma}, \bar{\sigma}]$ from which these valuations are drawn satisfies $\underline{\sigma} < 1 < \bar{\sigma}$. That is, there is a positive probability both that each creditor prefers reorganization to liquidation, and liquidation to reorganization. Under this reinterpretation, MLRP simply says that the private valuation of each individiual creditor is positively correlated with the future cash flows of the firm, which themselves determine the debtor's valuation R.

At a formal level the distinction between fully liquid and fully illiquid markets is exactly analogous to the *common-values/private-values* distinction made in auction theory, and we adopt this terminology for the remainder of the paper. Moreover, private-values preferences can also be viewed as capturing the behavior of non-Bayesian creditors. That is, in our analysis of the voting game in Section 3 above, we have assumed that each creditor is able to correctly infer what signals other creditors must have received if he is pivotal. This is an analogous problem to that faced by a bidder in a common-values auction who wishes to avoid the "winner's curse". At least in some instances it is potentially misleading to assume that creditors possess this degree of rationality.[19] If instead creditors are completely unable infer anything useful from the fact they are pivotal, they will vote only according to their private information. Such behavior is termed *naive* in the voting literature.

Under private values, the analysis is very straightforward. Each creditor votes to accept an offer x if and only if his or her own private valuation of this offer, $x\sigma_i$, exceeds the liquidation value 1. He no longer pays any attention to how other creditors are voting.

Formally, we start by defining $\sigma^{R,\alpha}$ as the signal realization such that when the debtor's type is R there is a probability of α that a creditor receives a signal at least as positive:

$$\Pr\left(\sigma > \sigma^{R,\alpha}|R\right) = 1 - F\left(\sigma^{R,\alpha}|R\right) = \alpha.$$

Consider first an offer x by the debtor satisfying $x\sigma^{R,\alpha} < 1$. The probability that each individual creditor accepts this offer is less than α. By the weak law of large numbers, this implies that the offer x is rejected with a probability that approaches 1 as the number of creditors grows large.

Likewise, an offer x such that $x\sigma^{R,\alpha} > 1$ will be accepted with a probability that approaches 1 as the number of creditors grows large.

Under a unanimity voting rule, $\alpha = 1$ and so $\sigma^{H,\alpha} = \sigma^{L,\alpha} = \underline{\sigma}$. As such, *any* offer x is rejected with probability approaching 1 when the number of creditors is large — independent of whether the reorganization value of L or H. The intuition is straightforward. When there are many creditors, it is almost certain that one of them values reorganization as less desirable than liquidation. As such, agreement under a unanimity rule becomes vanishingly rare.

Lemma 12 ***(Private values creditor recovery under the unanimity rule)***
Suppose a unanimity voting rule is in effect ($\alpha = 1$). Then under any feasible set of offers $x_n \leq 1$, the acceptance probability converges to 0 and the creditor payoff converges to 1.

[19] See Thaler (1991) for evidence of the winner's curse in auctions.

Under a majority voting arrangement, agreement is also harder to obtain when creditors have private values preferences. As we have established (Section 3), when creditors' preferences embody common values, if the true reorganization value is $R = H$ they will accept an offer x such that $xH > 1$ with a probability very to close 1. But if creditors' preferences embody private values, the debtor must make an offer x such that $x\sigma^{H,\alpha} > 1$ to be confident that it will be accepted when the true reorganization value is high. Since we can always choose α such that $\sigma^{H,\alpha} < H$, the offer made in the latter case is higher, at least for *some* majority rule.

Loosely speaking then, private values creditors drive a tougher bargain than common values ones — and are impossible to satisfy if they adopt a unanimous agreement rule, in which case liquidation ensues for sure. Since the creditors are themselves worse off under liquidation, we then obtain the following converse to Proposition 1:

Proposition 4 ***(Unanimity worse for private values creditors)***
Suppose $E[\sigma|H] > 1$. Then when creditors preferences embody private values, there exists a majority voting rule $\alpha < 1$ such that the creditor payoff is higher under that rule than under a unanimity voting rule.

Recall that adopting the unanimity rule has two key effects: unanimity makes agreement harder to obtain but this makes it useful in negotiations by extracting a higher surplus from the debtor. As we have seen, when creditors have common values preferences the latter effect dominates. In contrast, as we move to a situation in which creditors are have private values preferences, we reach a situation in which agreement is indeed extremely difficult to obtain under unanimity: each creditor will vote to accept only if his own valuation of the offer on the table beats the liquidation value. As such, when creditors have private values preferences, a unanimity agreement rule is self-defeatingly tough, and creditors do better by adopting a less stringent rule. To reiterate, this is in sharp contrast to the situation arising when all creditors place the same valuation on a stake in the reorganized firm.

6 Applications

The main finding of our analysis is that when creditor preferences are at the common values benchmark (i.e., full liquidity and rationality) the unanimity rule increases the creditors' payoff in *ex post* restructuring negotiations. In contrast, if creditor preferences are at the private values extreme, then creditors are better off selecting a (suitably chosen) majority voting rule. We have argued that the latter case is likely to arise either when markets are illiquid, or when creditors are non-Bayesian. We turn now to several applications of these results. Unless otherwise stated, for the remainder of the paper we assume that markets are liquid and creditors Bayesian, and so the analysis of Section 3 applies.

Security Design

So far we have restricted attention to restructuring in the form of equity-for-debt swaps. We now relax this assumption and instead allow the debtor to decide what type of securities to issue in response to the voting rule in place.

First, we need to introduce some additional notation. We continue to denote the debtor's offer by x, with the understanding that it now denotes something potentially more complicated than a share of equity in the reorganized firm. Given x, let $V_R(x)$ denote the value of the offer when the true reorganization value is $R \in \{L, H\}$. It is easily checked that all of existing results are unchanged by this generalization. In particular, provided that there is no feasible offer x such that $V_L(x) > 1$, then a debtor facing a non-unanimity voting rule will make an offer such that $E[V_R(x)]$ asymptotically approaches the liquidation value 1; while a debtor facing a unanimity rule will offer more.

An immediate consequence of our analysis is that the debtor's choice of security is asymptotically irrelevant when facing a non-unanimity vote. As such, the rest of the section is concerned with security design under unanimity rule.

Our analysis up to now has not required any assumptions about the volatility of the cash flows, and hence we have worked with the expected firm values. In order to deal with security design issues, we need to make assumptions about the cash flows that generate these expected firm values. We do so by assuming that future cash flows Y are G or B depending on whether the state of the world is "good" or "bad," and that the probability of the good cash flow state when the firm value is $R \in \{L, H\}$ is given by q_R where $q_H > q_L$. Clearly in this highly stylized setup with just two possible cash flow realizations, several real life securities may correspond to same payouts. For example, risky debt with a given face value is equivalent to some combination of safe debt and equity. Given this, we focus on the information sensitivity of the security package offered.

In our framework, a security x is identified by a vector (x_G, x_B) where x_G (x_B) is the fraction of the cash flow paid out to creditors in the good (bad) state of the world. Thus, when the firm value is R, security x yields an expected value of $V_R(x) = q_R G x_G + (1 - q_R) B x_B$ to the creditor, and $R - V_R(x)$ to the debtor.

Given an offer $x = (x_G, x_B)$, the equilibrium condition for the voting subgame can now be written as

$$\frac{(1-p)\Pr(A|L)}{p\Pr(A|H)} = \frac{V_H(x) - 1}{1 - V_L(x)} \frac{f(\sigma^*|H)}{f(\sigma^*|L)} \frac{1 - F(\sigma^*|L)}{1 - F(\sigma^*|H)}.$$

The debtor chooses $x = (x_G, x_B)$ to maximize his payoff

$$E[R - V_R(x)|A]\Pr(A).$$

The derivative of the debtor's payoff with respect to the two components of the offer, x_G and x_B, is then given by

$$-\frac{\partial E[V_R(x)|A]}{\partial x_Y}\Pr(A) + E[R - V_R(x)|A]\frac{\partial \Pr(A)}{\partial x_Y} \text{ for } Y = B, G$$

In words, raising the percentage of cash flows paid out in $Y = B, G$ benefits the debtor by raising the acceptance probability, but of course has the cost of reducing the value of having the offer accepted. The ratio of the marginal benefit to marginal cost of increasing x_Y is

$$\left(\frac{MB}{MC}\right)_{x_Y} \equiv \frac{E\left[R - V_R(x)\,|A\right] \frac{\partial \Pr(A)}{\partial x_Y}}{\frac{\partial E[V_R(x)|A]}{\partial x_Y} \Pr(A)}.$$

Our main result in this subsection is that the debtor benefits relatively more from increasing the share of cash flows paid out in the low state than in the high state. Loosely speaking, by reducing the information sensitivity of the security-package offered to the creditors, the debtor makes the creditors less wary of accepting his offer. Formally,

Proposition 5 ***(Security design)***
Under a unanimity agreement rule ($\alpha = 1$),

$$\left(\frac{MB}{MC}\right)_{x_B} > \left(\frac{MB}{MC}\right)_{x_G}$$

An immediate consequence of Proposition 5 is that the debtor prefers to pay creditors in the form of securities that are as insensitive to information as possible. More specifically, the debtor's highest payoff is obtained by choosing x_B as high as possible. To see this, suppose that $x_B < 1$, and then consider increasing x_B by ϵ while decreasing x_G by $\delta = \epsilon \frac{MC_{x_B}}{MC_{x_G}}$. The resulting change in debtor's payoff is

$$-\delta MB_{x_G} + \delta MC_{x_G} + \epsilon MB_{x_B} - \epsilon MC_{x_B} = \epsilon MB_{x_B}\left(1 - \frac{MC_{x_B}}{MC_{x_G}}\frac{MB_{x_G}}{MB_{x_B}}\right) > 0.$$

The predictions of our model with respect to the security package offered can be summarized as follows. Stated in the simplest possible terms, in an exchange offer the debtor can offer a package of securities which is either *more* or *less* senior to pure equity. Our model predicts that the debtor will always prefer to make the former offer. That is, he will offer to replace outstanding bonds with some mix of new bonds, convertible debt and equity. What he will *not* do is offer the bondholders a package of equity and warrants.

Empirically, this is exactly what we observe (see, e.g., Frank and Torous 1994). Moreover, it is worth stressing that this is *not* a necessary consequence of the seniority of debt claims: in principle it would be quite possible for a debtor to offer to retire outstanding bonds by issuing, for example, a large number of warrants with high strike values. Such a package would be junior to pure equity, and yet could easily be of high enough value to be accepted.[20]

[20] Of course, segmentation of capital markets could also potentially account for why bondholders receive relatively senior securities in debt exchanges.

United States reorganization law

At first sight, U.S. law embodies two diametrically opposing approaches to debt reorganization. On the one hand, the Trust Indenture Act (1939) stipulates that the unanimous agreement of bondholders is required in order to change the terms of a debt agreement.[21] On the other hand, if a firm files for bankruptcy under Chapter 11, debt agreements can be restructured if a supermajority of debtholders agree. Our results help shed some light on these two very different approaches.

We extend our model to allow for the possibility of reorganizing either outside *or* inside Chapter 11 as follows. First, the debtor can choose to make an offer, x^0 say, outside Chapter 11. As discussed, for such an offer to be accepted *all* creditors must agree: that is, the unanimity voting rule is in effect. Our analysis thus far has assumed that if the offer x^0 is rejected, liquidation ensues. However, in practice a debtor who fails to reorganize outside Chapter 11 has the option of trying again *inside* Chapter 11. We denote the debtor's Chapter 11 offer (made only if his first offer is refused) by x^1. In our setting all creditors are the same size and belong to the same class; under Chapter 11 rules, a two-thirds majority is required in order for the debtor's offer to be accepted. If agreement cannot be reached at this stage, the debtor's assets are liquidated.

A large body of empirical research has sought to estimate the costs of Chapter 11. For large firms the direct costs of bankruptcy are estimated to be 3%-5% of the prebankruptcy value of the firm (Altman 1984, Betker 1997, Warner 1977). For smaller firms, the direct costs of bankruptcy are much larger (in the order of 20%, see Lawless, et. al. 1994). In addition there are indirect costs that are hard to estimate precisely but are believed to be large (Altman 1984, Andrade and Kaplan 1998, Cutler and Summers 1988).

Figure 2 summarizes the extended reorganization game when offers inside and outside Chapter 11 are allowed. The parameters γ_{reorg} and γ_{liq} represent the costs of Chapter 11 discussed above: reorganizing under Chapter 11 reduces the debtor's reorganized value by $\gamma_{reorg}R$, while Chapter 11 reduces the liquidation value by γ_{liq}. In our game form, a decision by the debtor to proceed directly to Chapter 11 is equivalent to making an offer x^0 that is so low that it is never accepted. We assume that, even given these costs, reorganization is preferred to liquidation when the true reorganization value is high: $\left(1-\gamma_{reorg}\right)H > 1-\gamma_{liq}$. We also assume that when the true reorganization value is low, then liquidation under Chapter 11 is preferred to reorganization outside Chapter 11: $1-\gamma_L > L$.

What are the creditor and debtor payoffs under this two-stage reorganization procedure? We proceed by backwards induction.

If the debtor's offer x^0 outside Chapter 11 is rejected, he still has an opportunity to make a second offer x^1 under a supermajority voting rule. By Lemma 6, modified to account for the costs of Chapter 11, this offer asymptotically approaches $\left(1-\gamma_{liq}\right)/H\left(1-\gamma_{reorg}\right)$; and is accepted with a probability approaching one if the true reorganization value is

[21]The alternative of exchange offers are afflicted by the well-known hold-up problem. As a consequence, most exchange offers require acceptance by a very large fraction of creditors — often over 95% — turning them in to *de facto* votes, with close to unanimity required for acceptance.

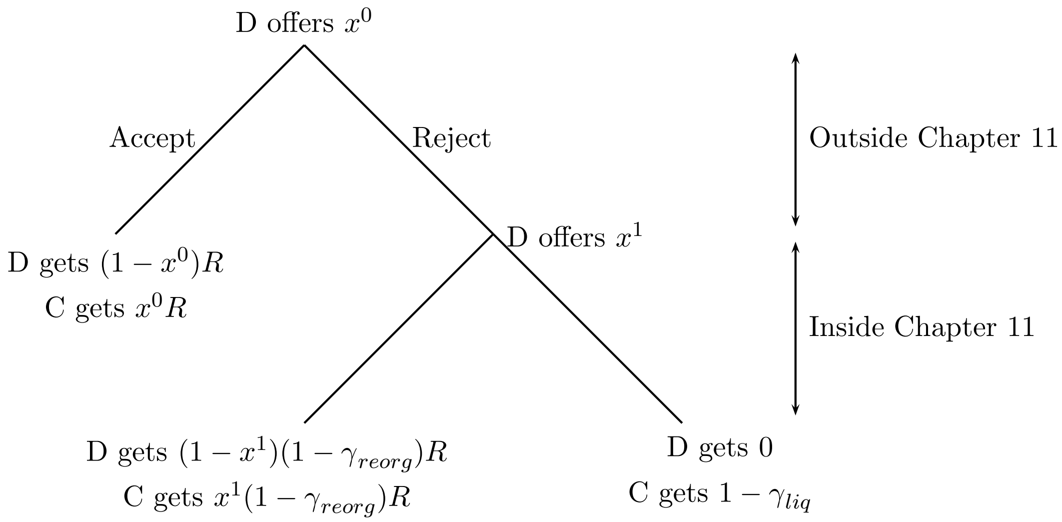

Figure 2: Reorganization outside and inside Chapter 11

H, and with probability zero otherwise.[22] Consequently, a creditor's payoff under the Chapter 11 subgame converges to $1-\gamma_{liq}$; while the debtor's payoff approaches $Z_H \equiv \left(1 - \frac{1-\gamma_{liq}}{(1-\gamma_{reorg})H}\right)(1-\gamma_{reorg})H$ if the true reorganization value is H, and $Z_L \equiv 0$ otherwise.

We turn now to the debtor's offer x^0 outside Chapter 11. It is easily shown[23] that all our previous results extend to the case in which the the debtor receives a payoff in the event of non-agreement of Z_R, where $R = L, H$ is the true reorganization value, provided that $H > 1-\gamma_{liq} + Z_H$ and $1-\gamma_{liq} + Z_L > L$. Both these conditions are satisfied here. It follows that if a majority voting rule is used outside Chapter 11 then the creditors would receive a payoff approaching $1-\gamma_{liq}$ (independent of the true reorganization value); while Proposition 1 implies that a unanimity voting rule outside Chapter 11 gives the creditors an expected payoff that is strictly higher.

This analysis implies that creditors continue to receive some protection from the unanimity requirement even when (costly) Chapter 11 proceedings are available to a debtor. Nonetheless, Chapter 11 does reduce the creditors' payoff: it lowers their outside option to $1-\gamma_{liq}$, and emboldens the debtor to make a lower offer under a unanimity voting rule.[24] (The higher the reorganization cost γ_{reorg}, and the lower the liquidation cost γ_{liq},

[22]Note that since majority voting rules efficiently aggregate information, the outcome of this stage of the reorganization is unaffected by any information the debtor and creditors may be able to infer from the fact that the original offer x^0 was rejected.

[23]The details are available from the authors.

[24]In choosing his offer x^0, the debtor equalizes the marginal cost of making a higher offer (i.e., the reduction in his share of the reorganized firm), with the marginal benefit (i.e., the increased probability that the offer is accepted). Increases in his payoff Z_H reduce the marginal benefit of a higher offer, and so in turn reduce

the smaller is the reduction in the creditors' payoff.[25]) Does Chapter 11 have any offsetting advantages?

Provided that at least some reorganizations are better characterized by the assumption of private values creditors (i.e., illiquid markets after the restructuring and/or non-Bayesian creditors), the answer is yes. From Section 5, we know that a unanimity rule functions very badly in this case: agreement becomes almost impossible, regardless of the true reorganization value. In contrast, an appropriately chosen majority rule can work much better.

Taken together, these results suggest that Chapter 11 may serve as a kind of escape hatch to be used in the case in which creditors' preferences are best described as close to private values — either because of limited liquidity, or because of bounded rationality constraints. At the same time, that fact that Chapter 11 exposes a firm to substantial costs means that at least some of the protection afforded by the Trust Indenture Act's unanimity requirement is preserved. To summarize, our analysis suggests that the combination of unanimity and majority agreement rules outside and inside of Chapter 11 serves to balance the competing aims of (a) protecting creditors by strengthening their bargaining position, and (b) preserving enough flexibility to avoid unnecessary and inefficient liquidations.[26]

Sovereign debt

In recent years, a large number of observers and policymakers have advocated the creation of a Sovereign Debt Restructuring Mechanism (SDRM).[27] A key element of most proposals is that (as in U.S. Chapter 11 reorganizations) a supermajority of creditors would be able to vote to accept new contract terms.[28] At present, the vast majority of sovereign debt issued under U.S. law requires the unanimous consent of bondholders for such a change.[29] In contrast, international debt issued under English law typically *does* allow for a binding supermajority vote to reschedule payments.[30]

Proponents of an SDRM argue that without the possibility of a binding majority vote, it is too hard for debtors and creditors to reach agreement. On the other side, critics caution that creditors are already poorly protected in sovereign markets, and that facilitating *ex post* renegotiation will weaken their position still further.[31] Our paper provides a theoretical framework in which to assess these competing effects, and suggests there is some truth in

his optimal offer.

[25] Note that Z_H is decreasing in γ_{reorg} and increasing in γ_{liq}.

[26] Practitioners have told us that recent Eurobond contracts specify a "two-tier" agreement rule to be used in the event of restructuring under distress. If a first restructuring offer is rejected, the fraction of creditors required to approve a second offer is reduced. Our interpretation of the Trust Indenture Act and Chapter 11 as balancing creditor protection with restructuring efficiency may apply to two-tier restructuring clauses of this kind.

[27] See in particular Krueger (2002).

[28] See, e.g., Eichengreen (2003).

[29] See, e.g., White (2002).

[30] Given that the Trust Indenture Act does not apply to sovereign debt (see, e.g., Bucheit *et al* 2002), the reasons for this sharp contrast across jurisdictions are unclear.

[31] See, e.g., Dooley (2000), or Shleifer (2003).

both positions. The impossibility of holding a majority vote does make agreement harder. But under some circumstances (liquid markets and reasonably rational investors) a majority agreement rule would reduce the creditors' *ex post* recovery. A sovereign borrower who is highly credit constrained and in need of funds would then be hurt by a move to facilitate restructuring (Proposition 3).

As Eichengreen and Mody (2004) observe, the fact that different jurisdictions have adopted different stances on the admissibility of majority agreement rules allows for these issues to be addressed empirically. They find that (I) lower quality borrowers are less likely to issue bonds under English law, and (II) if, nonetheless, low quality borrowers do issue under English law, interest rates are higher — even after controlling for borrower quality. Finding (II) is clearly consistent with our analysis;[32] finding (I) is also, provided that lower quality borrowers are more likely to be credit constrained.[33,34]

Applications to areas other than debt restructuring

Although we have couched our analysis in terms of creditors voting over a restructuring offer made by a debtor, our results have implications for other settings. For example, the case of an employer bargaining with a union is similar in many ways: the employer proposes a wage offer, and union members then vote to accept or reject the offer. Union members potentially differ in their alternative employment opportunities (private values), and in their assessment of future economic conditions (common values). The same tradeoffs between holding a unanimous vote and a majority vote then apply as in the case of debtor-creditor negotiations. To the extent to which union members' preferences are closer to being characterized by private values, our results suggest that they will opt to use a majority vote — exactly as is observed.

[32] Although our analysis is couched in terms of a firm seeking to reorganize, its main results apply to the restructuring of sovereign debt. To see this, consider our extension of the basic model to non-equity securities in Section 7. In the sovereign context, "liquidation" should be understood as the expected recovery rate if the outstanding debt is not restructured. Conversely, if the debt is restructured, bondholders receive new bonds paying $x_G G$ if the country recovers, and with an expected recovery rate of $x_B B$ if the country ends up back in default.

[33] Eichengreen and Mody also find that the very lowest quality borrowers are more likely to issue under U.S. law. Proposition 3 can account for this: when the expected recovery rate should default ensue (μ) is low, borrowers with *higher* default probabilities Q are *less* likely to satisfy condition (11).

[34] Our analysis cannot easily account for Eichengreen and Mody's third main finding: high quality borrowers issuing under English law actually pay *lower* interest rates. However, sovereign bonds issued under English law differ from in ways other than the inclusion of a majority restructuring clause. For example, the ability of an individual creditor to litigate is constrained. Finally, Becker *et al* (2003) also empirically assess the effect of including a majority agreement rule. They find no statistically significant impact.

7 Extensions

An informed debtor

Recall that we have assumed that the debtor is uninformed about the true reorganization value of the firm. What happens if instead the debtor knows the true reorganization value, $R \in \{L, H\}$?

First, observe that since majority voting rules aggregate information, asymptotically whether or not the debtor knows the true reorganization value makes no difference: any offer from a low reorganization value debtor will be rejected, while an offer just above $1/H$ from a high reorganization debtor will be accepted almost for sure. The equilibrium matches that when the debtor is completely uninformed.

We now turn to the case of a unanimous agreement rule. Note first that there is no non-trivial pure strategy separating equilibrium. Clearly there is no separating equilibrium in which the L-debtor makes an offer that is accepted. But nor can there be an equilibrium in which the L-debtor makes an offer that is rejected, while the H-debtor makes an offer that is accepted with positive probability. So the only possible separating equilibrium is one in which both debtor types make (distinct) offers that are rejected with probability 1.

In contrast, the equilibrium behavior of the game in which the debtor does not know his type forms part of a pooling equilibrium in the game in which the debtor is fully informed. This is easily seen: if creditors interpret any deviation from the pooling offer as being made by a L-debtor, they will reject such an offer. In fact, it is a pooling equilibrium for both debtor types to pool at any offer x. But by construction, pooling at the offer associated with an uninformed debtor is the the equilibrium that maximizes debtor welfare.

There may also exist a semi-separating equilibrium of the following type: the L-debtor offers x^P, while the H-debtor mixes between x^P and $1/H > x^P$. Creditors accept the offer x^P with a probability strictly between 0 and 1, and the offer $1/H$ with probability 1. In such an equilibrium, the conclusion of Proposition 1 continues to obtain: creditors are better off under the unanimity voting rule. This is easily seen. The logic of Proposition 1 directly implies that over the pooling portion of the equilibrium, creditors receive more than the they would under a majority voting rule — where they receive the liquidation value 1. Moreover, in the separating part of equilibrium, creditors must receive at least their liquidation value.

Large creditors

So far we have focused on the benchmark case in which all creditors have the same number of votes. For the case of a unanimity voting rule ($\alpha = 1$), this is without loss: all creditors have the same weight, in the sense that all have veto power. However, for non-unanimity rules the existence of larger creditors receiving greater weight in the voting game would clearly change the equilibrium conditions.

To explore the effects of larger creditors on voting outcomes, we consider the simplest possible such case: there is one large creditor, with $\beta < \alpha$ times as many votes as each of the remaining $(1-\beta)\,n$ creditors. As before, each voter (large or small) bases his voting decision on information he infers from being pivotal. Now, however, the different sized creditors learn different information from being pivotal. In particular, the large creditor is pivotal if between $(\alpha-\beta)\,n$ and $\alpha n-1$ small creditors vote to accept. Conversely, each small creditor is pivotal either if (I) the large creditor votes to accept, as do $\alpha n-\beta n-1$ of the other small creditors, or if (II) the large creditor votes to reject, but $\alpha n-1$ of the other small creditors vote to accept.

Figure 3 displays the equilibrium acceptance probabilities for four different voting arrangements. The solid lines represent the outcomes of a 2/3 majority voting arrangement: the unmarked lines show the benchmark case of 24 equal creditors, while the lines marked with diamonds show the case of 20 equal creditors, and one creditor who has 4 times as many votes.

The dotted lines show the same two scenarios for a unanimity agreement rule: in this case, all creditors have veto power regardless of size, and the only difference between the two scenarios is the total number of creditors (24 and 21 respectively). As is clear from the graphs, the voting equilibria are approximately the same for these two scenarios.[35]

Two conclusions should be tentatively drawn from this numerical exercise. First, even when some creditors have more voting weight than others, it is still the case that, relative to a majority rule, a unanimity rule substantially lowers the probability that a low offer x is accepted (see panel (I)). That is, a unanimity agreement rule still gives creditors a higher payoff *ex post* than would a more permissive arrangement. As such, our main result (Proposition 1) appears robust to heterogeneously-sized creditors.

Second, it is nonetheless the case that moving from a situation with equal-sized creditors to a situation in which one of creditors is more powerful than the others does increase the number of mistaken acceptences and rejections (panels (II) and (III) respectively). In this, the emergence of a larger creditor moves the majority equilibrium outcome in the direction of the unanimity outcome. This is natural: after all, with a large enough creditor, majority rule collapses to unanimity rule.

This effect suggests one possible motivation for the consolidation of debt claims of troubled debtor. At least in our example, by buying the debt claims of other lenders a "vulture" investor can help toughen the combined negotiating stance of the creditor class. We leave a fuller exploration of such a case for future research.

[35] So similar, in fact, that the lines are visually indistinguishable.

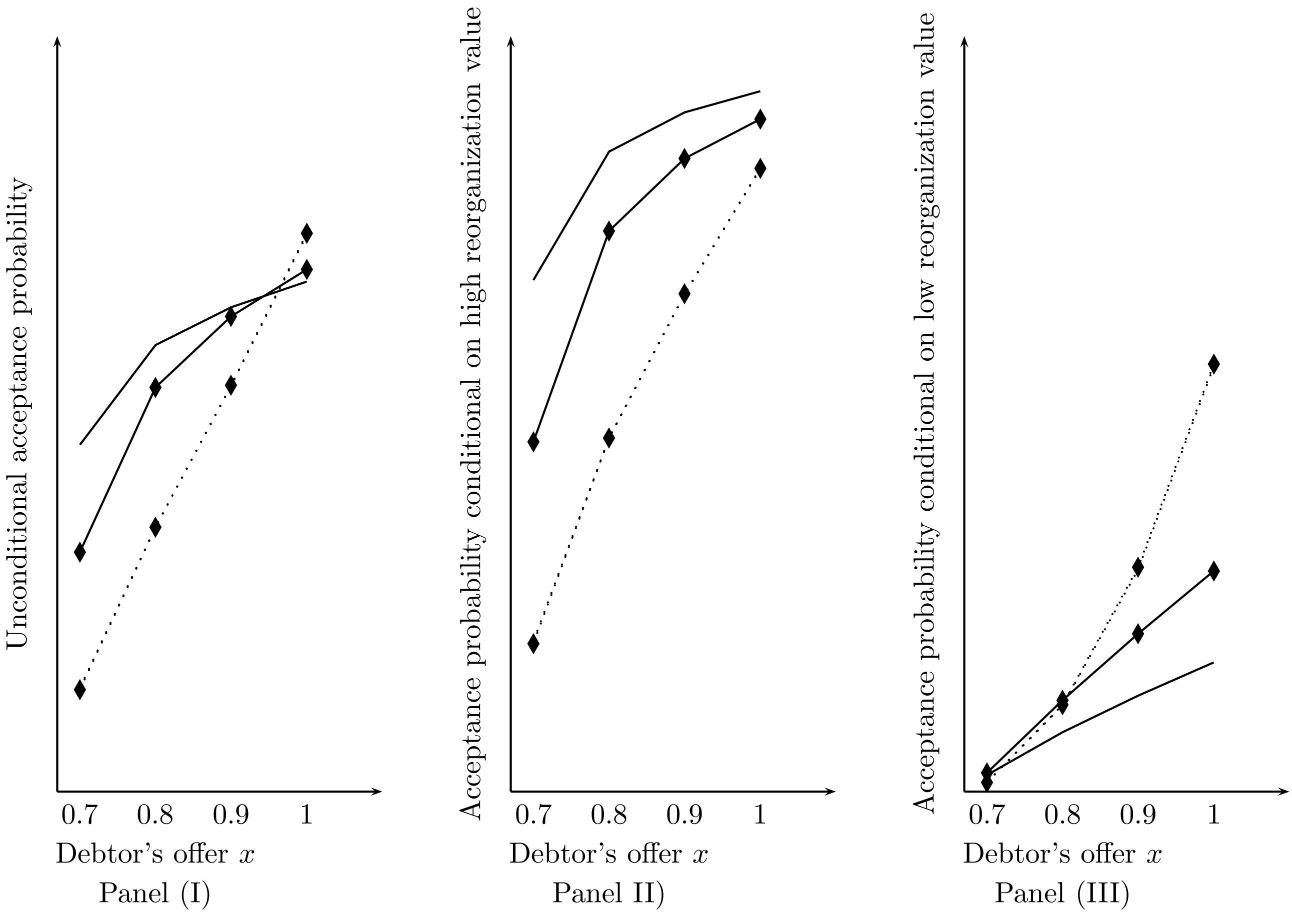

Figure 3: Acceptance probabilities as functions of the debtor's offer x. Solid line is 24 equal sized creditors for $\alpha = 2/3$; dotted line is 20 equal sized creditor and one creditor who is four times as large, for $\alpha = 2/3$; dashed line is 24 equal sized creditors for $\alpha = 1$

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A Appendix

We repeatedly use the following minor observation:

Lemma 13 *$F(\sigma|L)/F(\sigma|H)$ is strictly decreasing in $\sigma \in (\underline{\sigma}, \bar{\sigma})$, and is bounded below by 1. As a consequence, $F(\sigma|H) < F(\sigma|L)$ for $\sigma \in (\underline{\sigma}, \bar{\sigma})$. Finally, $(1 - F(\sigma|L))/(1 - F(\sigma|H))$ is decreasing in σ.*

Proof: Rewriting, we must show that

$$\frac{\int^{\sigma} f(\tilde{\sigma}|H) \frac{f(\tilde{\sigma}|L)}{f(\tilde{\sigma}|H)} d\tilde{\sigma}}{\int^{\sigma} f(\tilde{\sigma}|H) \, d\tilde{\sigma}}$$

is decreasing in σ. Differentiating, we must show

$$f(\sigma|H) \frac{f(\sigma|L)}{f(\sigma|H)} \int^{\sigma} f(\tilde{\sigma}|H) \, d\tilde{\sigma} < f(\sigma|H) \int^{\sigma} f(\tilde{\sigma}|H) \frac{f(\tilde{\sigma}|L)}{f(\tilde{\sigma}|H)} d\tilde{\sigma},$$

which is immediate from MLRP.

The proof that $(1 - F(\sigma|L))/(1 - F(\sigma|H))$ is decreasing is exactly parallel. ∎

In the remainder of the appendix, it will be convenient to define the following functions:

$$G(\sigma) \equiv F(\sigma|L)/F(\sigma|H),$$

$$g(\sigma) \equiv f(\sigma|L)/f(\sigma|H),$$

and

$$J(\sigma) \equiv (1 - F(\sigma|L))/(1 - F(\sigma|H)).$$

Proof of Lemma 1: Note that $\Pr(R, piv|\sigma_i) = \frac{\Pr(R)\Pr(piv|R) f(\sigma_i|R)}{pf(\sigma_i|H) + (1-p) f(\sigma_i|L)}$. Substituting this in equation (3), the claim immediately follows from MLRP. ∎

Proof of Lemma 2: Consider the function defined by

$$Z(\sigma) \equiv (xH - 1) \, pf(\sigma|H) + (xL - 1)(1 - p) f(\sigma|L) \, J(\sigma)^{n\alpha - 1} G(\sigma)^{n(1-\alpha)}$$

A responsive equilibrium exists if and only if $Z(\sigma) = 0$ has a solution in the interval $(\underline{\sigma}, \bar{\sigma})$. Note that as $\sigma^* \to \underline{\sigma}$,

$$Z(\sigma) \to (xH - 1) \, pf(\underline{\sigma}|H) + (xL - 1)(1 - p) f(\underline{\sigma}|L) \, g(\underline{\sigma})^{(1-\alpha)n}.$$

By MLRP, $g(\underline{\sigma}) \geq 1$, and certainly $xL < 1$: so

$$\lim_{\sigma \to \underline{\sigma}} Z(\sigma) \leq (xH - 1) \, pf(\underline{\sigma}|H) + (xL - 1)(1 - p) f(\underline{\sigma}|L),$$

which by Assumption 1 is less than 0. On the other hand, as $\sigma^* \to \bar{\sigma}$,

$$Z(\sigma) \to (xH-1)\,pf(\bar{\sigma}|H) + (xL-1)(1-p)\,f(\bar{\sigma}|L)\,g(\bar{\sigma})^{n\alpha-1}.$$

By assumption, $xH - 1 \geq \varepsilon$. By strict MLRP $g(\bar{\sigma}) < 1$, and so there there exists some N such that $Z(\bar{\sigma}) > 0$ whenever $n \geq N$. Continuity of Z in σ then implies that for all n large enough, there is a value of σ in the range $(\underline{\sigma}, \bar{\sigma})$ such that $Z(\sigma) = 0$.

If a symmetric responsive equilibrium exists, it is the unique such equilibrium: this is immediate from the equilibrium condition (7) and Lemma 13, together with MLRP. By the same token, the associated cutoff value σ^* is a decreasing function of the offer x. ∎

Proof of Lemma 3: Since there is no responsive equilibrium, from the proof of Lemma 2

$$(xH-1)\,pf(\bar{\sigma}|H) + (xL-1)(1-p)\,f(\bar{\sigma}|L)\,g(\bar{\sigma})^{n\alpha-1} < 0 \tag{12}$$

Since $xL < 1$, by MLRP it follows that for all $\sigma_i \in [\underline{\sigma}, \bar{\sigma}]$ and any $m \leq n\alpha - 1$,

$$(xH-1)\,pf(\sigma_i|H) + (xL-1)(1-p)\,f(\sigma_i|L)\,g(\bar{\sigma})^{m} < 0. \tag{13}$$

This last inequality is equivalent to

$$\begin{aligned} &(xH-1)\Pr(H|\text{a signal } \sigma_i \text{ and } m \text{ signals of } \bar{\sigma}) \\ &+ (xL-1)\Pr(L|\text{a signal } \sigma_i \text{ and } m \text{ signals of } \bar{\sigma}) \quad < \quad 0. \end{aligned} \tag{14}$$

In words, inequality (14) says that creditor i, having observed his own signal σ_i, will reject the offer x even if he conditions on the event that $m \leq n\alpha - 1$ other creditors observe the most pro-acceptance signal $\bar{\sigma}$. This has two implications.

First, the equilibrium in which all creditors reject always is a trembling-hand perfect equilibrium: for if all creditors tremble and accept with probability ε independent of their own signal, it remains a best response to reject the offer (this is just inequality (14) with $m = 0$).

Second, we claim that the equilibrium in which all creditors accept is not trembling-hand perfect. Recall that a creditor i's vote only matters if $n\alpha - 1$ other creditors vote to accept. This event only arises if $n - n\alpha$ of the $n - 1$ other creditors tremble. By assumption $n - n\alpha \leq n\alpha - 1$. Consequently, inequality (14) implies that *no matter what information* creditor i infers from conditioning on the event that $n - n\alpha$ of the other creditors tremble, it is a best response to reject the offer. As such, the equilibrium in which all creditors accept always cannot be trembling-hand perfect. ∎

Proof of Lemma 4: Immediate from MLRP and the fact that creditors use a cutoff voting rule (Lemma 1). ∎

Proof of Lemma 5:

The proof makes use of the following notation: given a voting rule α and reorganization value $R \in \{L, H\}$, let $\sigma^{R,\alpha}$ be the signal realization such that there is a probability of α that a creditor receives a signal at least as positive:

$$\Pr\left(\sigma > \sigma^{R,\alpha}|R\right) = 1 - F\left(\sigma^{R,\alpha}|R\right) = \alpha.$$

Proof of part (i): If $\limsup x_n < 1/H$ then there exists an $\varepsilon > 0$ such that $x_n \leq 1/H - \varepsilon$ for all n large. In this case, regardless of the true reorganization value creditors prefer liquidation (formally, $x_n H - 1 < 0$ and $x_n L - 1 < 0$) and so there cannot exist a responsive equilibrium. The equilibrium is for all creditors to vote to reject.

Proof of part (ii): We will show that $\liminf x_n > 1/H$ then $\limsup \sigma_n^* < \sigma^{H,\alpha}$. The result then follows immediately.

If $\liminf x_n > 1/H$, there exists ε such that $x_n \geq 1/H + \varepsilon$ for all n large enough. By Lemma 2 a responsive equilibrium exists for all n large. Suppose that contrary to the claimed result $\limsup \sigma_n^* \geq \sigma^{H,\alpha}$. So for any $\delta > 0$, there is a subsequence such that $\sigma_n^* \geq \sigma^{H,\alpha} - \delta$. Next, define

$$\phi \equiv \max_{\sigma \in [\sigma^{H,\alpha}-\delta, \bar{\sigma}]} \frac{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}} = \max_{\sigma \in [\sigma^{H,\alpha}-\delta, \bar{\sigma}]} J(\sigma)^\alpha G(\sigma)^{1-\alpha}$$

Note that the function $(1-q)^\alpha q^{1-\alpha}$ is increasing for $q \in (0, 1-\alpha)$ and decreasing for $q \in (1-\alpha, 1)$. Recall that by definition $F\left(\sigma^{H,\alpha}|H\right) = 1 - \alpha$, and by Lemma 13 $F(\sigma|H) < F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi < 1$ for δ chosen small enough, and so

$$\left(J(\sigma^*)^\alpha G(\sigma^*)^{1-\alpha}\right)^n \leq \phi^n \to 0$$

Since by hypothesis $x_n H - 1$ is bounded away from 0, this contradicts the equilibrium condition (7).

Proof of part (iii): We will show that $\liminf \sigma_n^* > \sigma^{L,\alpha}$. The result then follows immediately.

Suppose to the contrary that $\liminf \sigma_n^* \leq \sigma^{L,\alpha}$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma^{L,\alpha} + \delta$. Consequently we are in a responsive equilibrium for δ chosen small enough.

Next, define

$$\phi \equiv \min_{\sigma \in [\underline{\sigma}, \sigma^{L,\alpha}+\delta]} \frac{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}} = \min_{\sigma \in [\underline{\sigma}, \sigma^{L,\alpha}+\delta]} J(\sigma)^\alpha G(\sigma)^{1-\alpha}$$

Again, note that the function $(1-q)^\alpha q^{1-\alpha}$ is increasing for $q \in (0, 1-\alpha)$ and decreasing for $q \in (1-\alpha, 1)$. Recall that by definition $F\left(\sigma^{L,\alpha}|L\right) = 1 - \alpha$, and by Lemma 13 $F(\sigma|H) < F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi > 1$ for δ chosen small enough, and so

$$\left(J(\sigma^*)^\alpha G(\sigma^*)^{1-\alpha}\right)^n \geq \phi^n \to \infty$$

Since $1 - x_n L$ is certainly bounded away from 0, this contradicts the equilibrium condition (7). ∎

Proof of Lemma 6:
Part 1: $x_n \to 1/H$

We show that $\liminf x_n \geq 1/H \geq \limsup x_n$.

First, we claim that $\liminf x_n \geq 1/H$. If this were not the case, then for some $\varepsilon > 0$ there must exist a subsequence $\tilde{x}_n$ such that $\tilde{x}_n \leq 1/H - \varepsilon$, and thus $\limsup \tilde{x}_n \leq 1/H - \varepsilon$. By Lemma 5, $\Pr(A_n(\tilde{x}_n, \alpha)|H) \to 0$. In contrast, the sequence of offers $\hat{x}_n \equiv 1/H + \kappa$ is accepted with a probability converging to p for any $\kappa \in (0, 1 - 1/H)$. So for any $\kappa < 1 - 1/H$, the debtor's payoff is higher under this alternative offer for all n large enough. But this contradicts the optimality of x_n from the debtor's perspective.

Second, we claim that $\limsup x_n \leq 1/H$. If this were not the case, then for some $\varepsilon > 0$, there must exist a subsequence $\tilde{x}_n$ such that $\tilde{x}_n \geq 1/H + \varepsilon$, and thus $\liminf \tilde{x}_n \geq 1/H + \varepsilon$. By Lemma 5, $\Pr(A_n(\tilde{x}_n, \alpha)|H) \to 1$. However, consider a second sequence of offers $\hat{x}_n \equiv 1/H + \varepsilon/2$. Again by Lemma 5, $\Pr(A_n(\tilde{x}_n, \alpha)|H) \to 1$. The difference in the debtor's payoff under the two sequences of offers is

$$\begin{aligned}
& (1 - \hat{x}_n)(pH\Pr(A_n(\hat{x}_n, \alpha)|H) + (1-p)L\Pr(A_n(\hat{x}_n, \alpha)|L)) \\
& -(1 - \tilde{x}_n)(pH\Pr(A_n(\tilde{x}_n, \alpha)|H) + (1-p)L\Pr(A_n(\tilde{x}_n, \alpha)|L)) \\
= \; & (\tilde{x}_n - \hat{x}_n)(pH\Pr(A_n(\hat{x}_n, \alpha)|H) + (1-p)L\Pr(A_n(\hat{x}_n, \alpha)|L)) \\
& -(1 - \tilde{x}_n)[pH(\Pr(A_n(\tilde{x}_n, \alpha)|H) - \Pr(A_n(\hat{x}_n, \alpha)|H)) \\
& +(1-p)L(\Pr(A_n(\tilde{x}_n, \alpha)|L) - \Pr(A_n(\hat{x}_n, \alpha)|L))]
\end{aligned} \tag{15}$$

From Lemma 5 both $\Pr(A_n(\tilde{x}_n, \alpha)|L)$ and $\Pr(A_n(\hat{x}_n, \alpha)|L)$ converge to 0. Clearly $\tilde{x}_n - \hat{x}_n \geq \varepsilon/2$. Consequently expression (15) is bounded below by $pH\varepsilon/2$ for n large enough. In other words, the offer $\hat{x}_n$ is preferred to the offer $\tilde{x}_n$, contradicting the optimality of $\tilde{x}_n$.
Part 2: $\Pr(A_n(x_n, \alpha)|H) \to 1$

Suppose to the contrary that $\Pr(A_n(x_n, \alpha)|H) \not\to 1$. So there is some $\varepsilon > 0$ and some subsequence $\tilde{x}_n$ for which $\Pr(A_n(\tilde{x}_n, \alpha)|H) \leq 1 - \varepsilon$. Consider the alternative offer sequence $\hat{x}_n \equiv 1/H + \delta$. By Lemma 5, $\Pr(A_n(\hat{x}_n, \alpha)|H) \to 1$, and so the debtor's payoff converges to $(1 - 1/H - \delta)pH$. In contrast, under the subsequence of original offers

$$\limsup\left(\Pi_\alpha^D(x_n)\right) \leq (1-\varepsilon)(1 - 1/H)pH$$

So for δ chosen small enough, the optimality of the original offers is contradicted. ∎

Proof of Lemma 7:

It is fractionally more convenient to prove the parts of the lemma in a different order to that in which they are stated:
Proof of part (ii): By hypothesis, there exists ε such that $x_n \geq 1/H + \varepsilon$ for all n large enough. Then by Lemma 2 it follows that there exists an N such that a responsive equilibrium $\sigma_n^* \geq \underline{\sigma} = \sigma^{H,\alpha}$ exists whenever $n \geq N$. Since $x_n \geq 1/H + \varepsilon$, it follows that

$x_n H - 1 > 0$, and therefore from the equilibrium condition (7), there exists a $\delta > 0$ such that

$$J\left(\sigma_n^*\right)^n \geq \delta \text{ for } n \geq N. \tag{16}$$

Note that (16) implies that

$$\sigma_n^* \to \underline{\sigma} \text{ as } n \to \infty. \tag{17}$$

(It is immediate from Lemma 13 that σ_n^* cannot remain bounded away from both $\underline{\sigma}$ and $\bar{\sigma}$. A straightforward application of l'Hôpital's rule, together with fact that $f\left(\bar{\sigma}|L\right)/f\left(\bar{\sigma}|H\right) < 1$, is then enough to establish (17).)

Given (17), we assume without loss that N was chosen large enough so that if $n \geq N$ then

$$\sigma_n^* \leq \left(\underline{\sigma} + \bar{\sigma}\right)/2. \tag{18}$$

Now suppose to the contrary of our claim that

$$\liminf \Pr\left(A_n|H\right) = 0, \tag{19}$$

which in turn implies that there exists some sequence $m\left(n\right)$ for which $m\left(n\right) \geq N$ and

$$\left(1 - F\left(\sigma_{m(n)}^*|H\right)\right)^{m(n)} \to 0 \text{ as } n \to \infty. \tag{20}$$

Note that $m\left(n\right) \to \infty$: if this were not the case, (20) would imply $\sigma_{m(n)}^* \to \bar{\sigma}$, contradicting (18). As such, to satisfy (16) it must be the case that $\sigma_{m(n)}^* \to \underline{\sigma}$.

Now, the sequence$J\left(\sigma_{m(n)}^*\right)^{m(n)}$ is bounded, and so by Bolzano-Weierstrass has a convergent subsequence. Without loss, assume that the sequence $m\left(n\right)$ was chosen so that it has this property directly. By (16), the limit of $J\left(\sigma_{m(n)}^*\right)^{m(n)}$ is above δ.

By the discrete version of l'Hôpital's rule,

$$\lim \ln\left(1 - F\left(\sigma_{m(n)}^*|H\right)\right)^{m(n)} = \lim \frac{m\left(n+1\right) - m\left(n\right)}{\frac{1}{\ln 1 - F\left(\sigma_{m(n+1)}^*|H\right)} - \frac{1}{\ln 1 - F\left(\sigma_{m(n)}^*|H\right)}}$$

provided the right hands side exists.[36] Expanding, the right hand side is equal to

$$\begin{aligned}
& \lim\left(m\left(n+1\right)-m\left(n\right)\right)\frac{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)} \\
= & \lim\left(m\left(n+1\right)-m\left(n\right)\right)\frac{\ln J\left(\sigma^*_{m(n+1)}\right)\ln J\left(\sigma^*_{m(n)}\right)}{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)} \\
& \times\frac{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)} \\
& \times\frac{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}{\ln J\left(\sigma^*_{m(n+1)}\right)}\frac{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln J\left(\sigma^*_{m(n)}\right)}
\end{aligned}$$

Since $\sigma^*_{m(n)}\to\underline{\sigma}$,[37]

$$\begin{aligned}
\frac{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)} & \to g(\underline{\sigma})-1 \\
\frac{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln J\left(\sigma^*_{m(n)}\right)} & \to (g(\underline{\sigma})-1)^{-1}
\end{aligned}$$

By strict MLRP, $g(\underline{\sigma})>1$. Thus provided

$$\frac{\ln J\left(\sigma^*_{m(n+1)}\right)\ln J\left(\sigma^*_{m(n)}\right)}{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)}$$

[36]The discrete version of l'Hôpital's rule holds provided that $\frac{1}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}\to\infty$, which is satisfied since $\sigma^*_{m(n)}\to\underline{\sigma}$.

[37]This follows since

$$\begin{aligned}
\ln\left(1-F\left(\sigma|R\right)\right) &= -F\left(\sigma|R\right)+o\left(F\left(\sigma|R\right)^2\right) \\
F\left(\sigma|R\right) &= \left(\sigma-\underline{\sigma}\right)f\left(\underline{\sigma}|R\right)+o\left(\left(\sigma-\underline{\sigma}\right)^2\right)
\end{aligned}$$

and so

$$\ln\left(1-F\left(\sigma|R\right)\right)=-\left(\sigma-\underline{\sigma}\right)f\left(\underline{\sigma}|R\right)+o\left(\left(\sigma-\underline{\sigma}\right)^2\right).$$

exists and is finite, the right hand side limit exists and is finite. But again by the discrete version of l'Hôpital's rule,

$$\lim \ln J\left(\sigma^*_{m(n)}\right)^{m(n)} = \lim\left(m\left(n+1\right)-m\left(n\right)\right)\frac{\ln J\left(\sigma^*_{m(n+1)}\right)\ln J\left(\sigma^*_{m(n)}\right)}{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)}$$

which is finite by assumption. Thus $\lim \ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)^{m(n)}$ exists and is finite, contradicting (19) and thus completing the proof. ∎

Proof of part (i): Since $\liminf \Pr\left(A_n|\, H\right) \geq 0$, it suffices to show that $\limsup \Pr\left(A_n|\, H\right) \leq 0$. Suppose to the contrary that $\limsup \Pr\left(A_n|\, H\right) > 0$. So there exists N such that $\inf_{n\geq N}\Pr\left(A_n\right) > 0$. By hypothesis, we can construct a sequence $m\left(n\right)$ for which $m\left(n\right)\geq N$, with $m\left(n\right)\to\infty$ and

$$\inf_n x_{m(n)} - 1/H \leq 0.$$

By Bolzano-Weierstrass, $x_{m(n)} - 1/H$ must have a convergent subsequence. Likewise, $\Pr\left(A_{m(n)}|x_{m(n)}\right)$ must have a convergent subsequence. Thus without loss, assume that $m\left(n\right)$ is chosen so that

$$\begin{aligned}\lim x_{m(n)} - 1/H &\leq 0 &(21)\\ \lim \Pr\left(A_{m(n)}|x_{m(n)}\right) &> 0 &(22)\end{aligned}$$

Similar to in the proof of part (ii), from (22), $\sigma^*_{m(n)} \to \underline{\sigma}$ since $m\left(n\right)\to\infty$. Given this, equation (21) and the equilibrium condition (7) together imply $J\left(\sigma^*_{m(n)}\right)^{m(n)} \to 0$ as $n\to\infty$. By the discrete version of l'Hôpital's rule,

$$\lim \ln J\left(\sigma^*_{m(n)}\right) = \lim \frac{m\left(n+1\right)-m\left(n\right)}{\frac{1}{\ln J\left(\sigma^*_{m(n+1)}\right)}-\frac{1}{\ln J\left(\sigma^*_{m(n)}\right)}}$$

provided the right hand side exists. Expanding in the same manner as in the proof of part (ii), the right hand side is equal to

$$\begin{aligned}&\lim\left(m\left(n+1\right)-m\left(n\right)\right)\\ &\times\frac{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}\\ &\times\frac{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)-\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}{\ln J\left(\sigma^*_{m(n)}\right)-\ln J\left(\sigma^*_{m(n+1)}\right)}\\ &\times\frac{\ln J\left(\sigma^*_{m(n+1)}\right)}{\ln\left(1-F\left(\sigma^*_{m(n+1)}|H\right)\right)}\frac{\ln J\left(\sigma^*_{m(n)}\right)}{\ln\left(1-F\left(\sigma^*_{m(n)}|H\right)\right)}\end{aligned}$$

Again, as in the proof of part (ii), this limit exists and is finite provided

$$\lim \left(1 - F\left(\sigma^*_{m(n)}|H\right)\right)^{m(n)} > 0,$$

which is the case by (22). But this contradicts that $J\left(\sigma^*_{m(n)}\right)^{m(n)} \to 0$ as $n \to \infty$, completing the proof. ∎

Proof of Lemma 8: Suppose otherwise, i.e., $\liminf x_n - 1/H \leq 0$. By supposition, there exists a subsequence $m(n)$ such that $\lim x_{m(n)} - 1/H \leq 0$. By part (i) of Lemma 7, $\limsup \Pr(A_{m(n)}) = 0$. In contrast, for any constant offer $\xi > 1/H$ part (ii) of Lemma 7 implies that the acceptance probability stays bounded away from 0, and so the debtor's payoff does also. But then the offers $x_{m(n)}$ would be suboptimal for all n large enough. ∎

Proof of Lemma 10: The derivative of the creditor's payoff with respect to x is

$$E\left[R|A_n\right]\Pr\left(A_n\right) + p\left(xH-1\right)\frac{\partial}{\partial x}\Pr\left(A_n|H\right) + \left(1-p\right)\left(xL-1\right)\frac{\partial}{\partial x}\Pr\left(A_n|L\right).$$

Observe that

$$\frac{\partial}{\partial x}\Pr\left(A_n|R\right) = \frac{\partial}{\partial x}\left(1 - F\left(\sigma^*|R\right)\right)^n = -n\frac{\partial \sigma^*}{\partial x} f\left(\sigma^*|R\right)\left(1 - F\left(\sigma^*|R\right)\right)^{n-1},$$

and so

$$\frac{\frac{\partial}{\partial x}\Pr\left(A_n|L\right)}{\frac{\partial}{\partial x}\Pr\left(A_n|H\right)} = \frac{f\left(\sigma^*|L\right)\left(1 - F\left(\sigma^*|L\right)\right)^{n-1}}{f\left(\sigma^*|H\right)\left(1 - F\left(\sigma^*|H\right)\right)^{n-1}}.$$

From the equilibrium condition (7), the derivative of the creditor's payoff with respect to x then reduces to simply $E\left[R|A_n\right]\Pr\left(A_n\right)$. ∎

Proof of Lemma 11: From Lemma 8 and part (ii) of Lemma 7, we know that the debtor's optimal offers and associated acceptance probabilities satisfy

$$\begin{aligned} \liminf x_n &> 1/H \\ \liminf \Pr\left(A_n\left(x_n, \alpha\right)\right) &> 0. \end{aligned}$$

Define a new set of offers by $\hat{x}_n = \frac{1}{2}\left(x_n + 1/H\right)$. Thus $\liminf \hat{x}_n - 1/H > 0$ and so by Lemma 7, it follows that $\liminf \Pr\left(A_n\left(\hat{x}_n, \alpha\right)\right) > 0$. Since the acceptance probabilities are positive, we are in a responsive voting equilibrium, and Lemma 10 implies that there exists an $\varepsilon > 0$ such that for all n large enough,

$$\frac{\partial}{\partial x}\Pi_1^C\left(\hat{x}_n\right) \geq \varepsilon.$$

It follows that

$$\begin{aligned}\Pi_1^C(x_n) &\geq \Pi_1^C(1/H) + (x_n - \hat{x}_n)\Pr(A_n(\hat{x}_n,\alpha)) \\ &= \Pi_1^C(1/H) + \frac{\varepsilon}{2}(x_n - 1/H). \end{aligned} \tag{23}$$

By part (i) of Lemma 7, the offer $1/H$ is rejected with probability approaching 1 as the number of creditors grows large. So

$$\lim \Pi_1^C(1/H) = 1.$$

So from expression (23),

$$\liminf \Pi_1^C(x_n) > 1,$$

completing the proof. ∎

Proof of Proposition 3: Since the firm is credit constrained, project size I is determined by the credit constraint (10) at equality. The derivative of the firm's investment with respect to a change in the creditors' payoff Π_α^C is

$$\frac{\partial I}{\partial \Pi_\alpha^C} = \frac{\partial}{\partial \Pi_\alpha^C}\frac{1-Q}{1-Q\Pi_\alpha^C\mu}K = \frac{1-Q}{(1-Q\Pi_\alpha^C\mu)^2}KQ\mu = \frac{Q\mu I}{1-Q\Pi_\alpha^C\mu}$$

First, consider a small increase ε in the creditors' reorganization payoff Π_α^C, accompanied by a reduction of $(\phi+1)\varepsilon$ in the firm's payoff Π_α^D. The effect on the firm's payoff is

$$\begin{aligned} &\varepsilon\left(\left((1-Q)\rho - 1 + Q\left(\Pi_\alpha^C + \Pi_\alpha^D\right)\mu\right)\frac{\partial I}{\partial \Pi_\alpha^C} - \phi Q\mu I\right) \\ =\ &\varepsilon Q\mu I\left(\left((1-Q)\rho - 1 + Q\left(\Pi_\alpha^C + \Pi_\alpha^D\right)\mu\right)\frac{1}{1-Q\Pi_\alpha^C\mu} - \phi\right) \end{aligned} \tag{24}$$

Second, we claim that if (24) is positive for some Π_α^C and Π_α^D, the same is true for $\Pi_\alpha^C + \varepsilon$ and $\Pi_\alpha^D - (\phi+1)\varepsilon$ for any $\varepsilon > 0$. To see this, observe that a further increase of ε in Π_α^C, accompanied by a decrease of $(\phi+1)\varepsilon$ in Π_α^D, increases the term in parentheses by

$$\begin{aligned} &\varepsilon\left(-\frac{\phi Q\mu}{1-Q\Pi_\alpha^C\mu} + \frac{Q\mu}{(1-Q\Pi_\alpha^C\mu)^2}\left((1-Q)\rho - 1 + Q\left(\Pi_\alpha^C + \Pi_\alpha^D\right)\mu\right)\right) \\ \geq\ &\varepsilon\left(-\frac{\phi Q\mu}{1-Q\Pi_\alpha^C\mu} + \frac{Q\mu}{(1-Q\Pi_\alpha^C\mu)^2}\phi\left(1-Q\Pi_\alpha^C\mu\right)\right) \\ =\ &0. \end{aligned}$$

Third, an exactly parallel argument establishes that if (24) is negative for some Π_α^C and Π_α^D, the same is true for $\Pi_\alpha^C + \varepsilon$ and $\Pi_\alpha^D - (\phi+1)\varepsilon$ for any $\varepsilon > 0$. ∎

Proof of Lemma 12: See main text immediately prior to statement of result. ∎

Proof of Proposition 4: Given strict MLRP, $\sigma^{H,\alpha} > \sigma^{L,\alpha}$ for any voting rule $\alpha \in (0,1)$. To see this, suppose to the contrary that $\sigma^{H,\alpha} \leq \sigma^{L,\alpha}$. Then $1-\alpha = F\left(\sigma^{H,\alpha}|H\right) \leq F\left(\sigma^{L,\alpha}|H\right) < F\left(\sigma^{L,\alpha}|L\right) = 1-\alpha$ which is not possible. (The second inequality here is a consequence of $\alpha \in (0,1)$ and strict MLRP — see Lemma 13 in the appendix.)

Moreover, $\sigma^{R,\alpha}$ continuous and decreasing in α, with $\sigma^{R,\alpha=0} = \bar{\sigma} > 1 > \underline{\sigma} = \sigma^{R,\alpha=1}$. Consequently is it always possible to a voting rule $\hat{\alpha}$ such that $E[\sigma|H] > \sigma^{H,\hat{\alpha}} > 1 > \sigma^{L,\hat{\alpha}}$.

By the remarks in the main text preceding the statement of Proposition **4**, under the voting rule $\hat{\alpha}$ the debtor's offer must converge to $1/\sigma^{H,\hat{\alpha}}$ as the number of creditors grows large. To see this, just observe that any offer strictly above $1/\sigma^{H,\hat{\alpha}}$ is accepted with probability converging to p, while any offer strictly below this is rejected with probability converging to 1. Since $\sigma^{L,\hat{\alpha}} < 1$, the debtor cannot make a feasible offer that raises the limit acceptance probability above p. Parallel arguments to those made in the proof of Lemma 6 then imply that the debtor's best offer converges to $1/\sigma^{H,\hat{\alpha}}$; and moreover, that the acceptance probability conditional on the true reorganization value being H (respectively, L) converges to 1 (respectively, 0).

It then follows that the creditors' payoff under the voting rule $\hat{\alpha}$ converges to

$$p\frac{1}{\sigma^{H,\hat{\alpha}}}E[\sigma|H] + (1-p)\,.$$

On the other hand, under a unanimity rule the creditors' payoff converges to the liquidation value, 1, since asymptotically agreement becomes impossible. Since by construction $E[\sigma|H] > \sigma^{H,\hat{\alpha}}$, the result follows. ∎

Proof of Proposition 5: Start by noting that the offer x *only* affects the acceptance probability through the quantity

$$\theta(x) \equiv \frac{V_H(x)-1}{1-V_L(x)},$$

which is the ratio of the gain from accepting the offer when the reorganization value is H to the loss to accepting the offer when the reorganization value is L. Consequently

$$\left(\frac{MB}{MC}\right)_{x_Y} = \frac{E\left[R - V_R\left(x\right)|A\right]\frac{\partial\theta}{\partial x_Y}\frac{\partial \Pr(A)}{\partial\theta}}{\frac{\partial E[V_R(x)|A]}{\partial x_Y}\Pr\left(A\right)}$$

and so

$$\frac{\left(\frac{MB}{MC}\right)_{x_G}}{\left(\frac{MB}{MC}\right)_{x_B}} = \frac{\frac{\partial\theta}{\partial x_G}}{\frac{\partial\theta}{\partial x_B}} \Bigg/ \frac{\frac{\partial E[V_R(x)|A]}{\partial x_G}}{\frac{\partial E[V_R(x)|A]}{\partial x_B}}.$$

It is straightforward to show

$$\frac{\frac{\partial\theta}{\partial x_G}}{\frac{\partial\theta}{\partial x_B}} = \frac{G}{B}\frac{q_H + q_L\theta(x)}{(1-q_H)+(1-q_L)\theta(x)}.$$

and

$$\frac{\frac{\partial E[V_R(x)|A]}{\partial x_B}}{\frac{\partial E[V_R(x)|A]}{\partial x_G}} = \frac{G}{B}\frac{q_H \Pr(H|A) + q_L \Pr(L|A)}{(1-q_H)\Pr(H|A) + (1-q_L)\Pr(L|A)} = \frac{G}{B}\frac{q_H + q_L z(x)}{(1-q_H) + (1-q_L)z(x)}.$$

where

$$z(x) \equiv \frac{\Pr(L|A)}{\Pr(H|A)} = \frac{(1-p)\Pr(A|L)}{p\Pr(A|H)}.$$

Since the function $\frac{q_H+q_L y}{(1-q_H)+(1-q_L)y}$ is decreasing in y, to complete the proof it suffices to show that $\theta(x) > z(x)$.

For this, observe that when $\alpha = 1$, from the equilibrium condition for the voting subgame

$$z(x) = \theta(x)\frac{f(\sigma^*|H)}{f(\sigma^*|L)}\frac{1-F(\sigma^*|L)}{1-F(\sigma^*|H)}$$

and thus, it suffices to show that

$$\frac{f(\sigma^*|H)}{f(\sigma^*|L)}\frac{1-F(\sigma^*|L)}{1-F(\sigma^*|H)} < 1.$$

Note that

$$\frac{1-F(\sigma^*|L)}{1-F(\sigma^*|H)} = \frac{\int_{\sigma^*}^{\bar{\sigma}} f(\sigma|L)d\sigma}{\int_{\sigma^*}^{\bar{\sigma}} f(\sigma|H)d\sigma} = \frac{\int_{\sigma^*}^{\bar{\sigma}} f(\sigma|L)d\sigma}{\int_{\sigma^*}^{\bar{\sigma}} \frac{f(\sigma|H)}{f(\sigma|L)} f(\sigma|L)d\sigma} \leq \frac{f(\sigma^*|L)}{f(\sigma^*|H)}$$

where the last inequality follows from MLRP. ∎