Advanced Philosophy of Physics: General Covariance and Background Independence

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MT20-W3

The plan

- W1: The philosophy of symmetries
- W2: The hole argument
- W3: General covariance and background independence
- W4: The dynamical approach to spacetime

What's special about general relativity?

Today

Preliminaries

General covariance

Diffeomorphism invariance

Absolute objects

Variational principles

Belot's approach

Fixed fields

Absolute fields

Conclusions

Today

Models

- The *kinematically possible models* (KPMs) of a given theory are picked out by tuples $\langle M, \Phi_1, \ldots, \Phi_n \rangle$, with
    1. a manifold, M;
    2. fields on M, $\Phi_1, \ldots, \Phi_n$.

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  1. a manifold, M;
  2. fields on M, $\Phi_1, \ldots, \Phi_n$.
- The *dynamically possible models* (DPMs) of a given theory are those KPMs in which the Φ_i satisfy certain dynamical equations.

Dynamical versus fixed fields

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  - E.g. g_{ab} in GR.
- *Fixed fields* are fixed identically in all KPMs, and do not have their own associated dynamical equations.
  - E.g. η_{ab} in SR.

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- DPMs picked out by $\eta_{ab} \nabla^a \nabla^b \varphi = 0$.
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SR2:

- KPMs: $\langle M, g_{ab}, \varphi \rangle$.
- DPMs picked out by $g_{ab}\nabla^a\nabla^b\varphi = 0$ *and* $R^a{}_{bcd} = 0$.
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- DPMs picked out by $g_{ab} \nabla^a \nabla^b \varphi = 0$ *and* $G_{ab} = 8\pi T_{ab}$.

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Definition

(General covariance) *A formulation of a theory is generally covariant iff the equations expressing its laws are written in a form that holds with respect to all members of a set of coordinate systems that are related by smooth but otherwise arbitrary transformations.*

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- Einstein would then convince himself, via his 'point coincidence argument', that the hole argument was not problematic; he thus returned to general covariance and finalised general relativity.
- The hole argument was, more or less, lost to history until it was revived in the latter half of the 20th Century by philosophers of physics.

The Kretschmann objection

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- In 1917, Kretschmann argued to Einstein that *any* theory can be written in a generally covariant form.

By means of a purely mathematical reformulation of the equations representing the theory, and with, at most, mathematical complications connected with that reformulation, any physical theory can be brought into agreement with any, arbitrary relativity postulate, even the most general one, and this without modifying any of its content that can be tested by observation. (Kretschmann 1917, trans. Norton 1993, p. 818)

Modern endorsements

The Kretschmann point has become orthodoxy. For example, in 1983, Friedman wrote:

> *... the principle of general covariance has no physical content whatever: it specifies no particular physical theory; rather it merely expresses our commitment to a certain style of formulating physical theories.* (Friedman 1983, p. 55)

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There are two different ways in which the Kretschmann point can made, which I'll now go over.

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2. Transform those laws into an arbitrary coordinate system.
3. The resulting equations might end up being more complex, but by construction they hold in all coordinate systems!
4. So this is the generally covariant version of the theory under consideration.

Example: Klein-Gordon equation

First, we'll write the Klein-Gordon equation (a wave equation) in index notation. (This is just a presentational reformulation.)

$$-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0$$

$$\Rightarrow \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \phi = 0$$

$$\Rightarrow \eta_{\mu\nu}\partial^\mu\partial^\nu\phi = 0.$$

Example: Klein-Gordon equation

Then, we'll transform the Klein-Gordon equation to an arbitrary coordinate system:

$$\begin{aligned}
\eta_{\mu\nu}\partial^{\mu}\partial^{\nu}\varphi &= 0\\
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\longrightarrow \eta_{\mu\nu}\frac{\partial x_{\mu'}}{\partial x_{\mu}}\frac{\partial}{\partial x_{\mu'}}\left(\frac{\partial x_{\nu'}}{\partial x_{\nu}}\frac{\partial}{\partial x_{\nu'}}\varphi\right) &= 0\\
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\end{aligned}
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This is a generally covariant version of our theory! Note the extra term in the non-inertial frame (cf. fictitious forces).

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Examples

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- We saw a coordinate independent presentation of Newtonian mechanics last week:
  - KPMs: $\langle M, t_{ab}, h^{ab}, \nabla, \rho, \varphi \rangle$.
  - DPMs: $h^{ab}\nabla_a \nabla_b \varphi = 4\pi\rho$.

The upshot

It's not at all obvious that general covariance affords an adequate means of distinguishing general relativity from other spacetime theories.

Today

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Definition

(Diffeomorphism invariance) *A theory $\mathcal{T}$ is diffeomorphism invariant iff, if $\langle M, F_1, \ldots, F_n, D_1, \ldots, D_m \rangle$ is a DPM of $\mathcal{T}$ (where the F_i are fixed fields and the D_i are dynamical fields), then so is $\langle M, F_1, \ldots, F_n, d^* D_1, \ldots, d^* D_m \rangle$, for all $d \in Diff(M)$.*

Assessing diffeomorphism invariance

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- General relativistic theories such as **GR1** are diffeomorphism invariant.
- **SR1** fails to be diffeomorphism invariant, but **SR2** *is* diffeomorphism invariant.
- Therefore, diffeomorphism invariance does not serve to distinguish special from general relativistic theories.

Background independence

Thought: If what distinguishes general relativistic theories isn’t their general covariance, and it isn’t their diffeomorphism invariance, it must be some other property. We’ll label this *background independence*, as a placeholder.

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But what *is* background independence?

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Absolute objects

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Definition

(Absolute object) *A geometrical object the same (up to isomorphism) in all DPMs of a theory.*

Definition

(Background independence, absolute objects) *A theory is background independent iff it has no absolute objects in its formulation.*

Problems for the absolute objects proposal

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There are three kinds of problem case for the absolute object programme:

1. Cases in which structure that intuitively should count as background is not classified as absolute.
2. Cases in which structure that intuitively should not count as background is classified as absolute.
3. The observation that general relativity itself seems to have absolute objects.

Type I problems: Torretti constant curvature

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Type I problems: Torretti constant curvature

- Torretti considers a modified Newtonian theory, in which each model's spatial metric has constant curvature, but different models have different values of that curvature.
- Because the spatial metric in every model has constant curvature, "such a theory has something rather like an absolute object in it" (Pitts 2006, p. 363).
- Nevertheless, the failure of the metrics to be locally diffeomorphically equivalent for distinct curvature values entails that the metric tensor does not satisfy Anderson's definition of an absolute object.

Responses to type I problems

- Pitts (2006, pp. 16-17) argues that Torretti's example does not succeed, for if one decomposes the spatial metric into a conformal spatial metric density and a scalar density, then the former *is* an absolute object.

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- Pitts (2006, pp. 16-17) argues that Torretti's example does not succeed, for if one decomposes the spatial metric into a conformal spatial metric density and a scalar density, then the former *is* an absolute object.
- More generally, Torretti's intuition about background structure would seem to over-generate, for almost all theories have some models in which a given piece of structure does not vary with respect to some given (possibly temporal) parameter. (Consider, e.g., a $3 + 1$ decomposition of general relativity with constant curvature of the induced metric on the spatial slices.)

Type II problems: Jones-Geroch dust

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- Consider a general relativistic theory in which the metric field is coupled to matter characterised only by a 4-velocity field U^a and a mass density.
- Then, as Friedman states, “since any two timelike, nowhere-vanishing vector fields defined on a relativistic space-time are *d*-equivalent, it follows that any such vector field counts as an absolute object ... and this is surely counter-intuitive” (Friedman 1983, p. 59).

Type III problems: GR's absolute object

In GR, we can always decompose the metric as

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\sqrt{-g}^{2/n},$$

where $\hat{g}_{\mu\nu}$ is a conformal metric density, and $\sqrt{-g}$ is the square root of the (negative of the) determinant of the metric.

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> *GR has an absolute object! This absolute object [$\sqrt{-g}$] is a scalar density of nonzero weight, because every neighbourhood in every model spacetime admits coordinates (at least locally) in which the component of the scalar density has a value of -1.* (Pitts 2006, p. 366)

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In any neighbourhood of the manifold in any model of GR, one can find a coordinate system such that the object $\sqrt{-g}$ takes the value -1. Thus, this is a geometrical object which is the same (up to isomorphism) in all DPMs of the theory.

Today

Variational principles

Pooley (2017) proposes a different definition of background independence, in terms of variational principles:

Definition

(Background independence, variational) *A theory is background independent iff its solution space is determined by a generally-covariant action, (i) all of whose dependent variables are subject to Hamilton's principle, and (ii) all of whose dependent variables represent physical fields.*

Assessing the variational principle definition

- General relativity satisfies this definition: the field equations can be derived from the *Einstein-Hilbert action*:

$$S_{\text{EH}} = \frac{1}{16\pi} \int_M d^4x \sqrt{-g} R$$

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- **SR1** violates the definition: η_{ab} is not subject to Hamilton's principle.
- **SR2**...?

Variational principles

Rosen action for **SR2**:

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- All dependent variables subject to Hamilton's principle.
- But **SR2** still not background independent, if Θ^{abcd} regarded as 'unphysical'.

Variational principles

Further worries:

Variational principles

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- Parameterised versions of special relativity, written in terms of *clock fields* X^{a}. ($\eta_{\mu\nu} := \eta_{\mathsf{ab}}\partial_\mu X^{\mathsf{a}}\partial_\nu X^{\mathsf{b}}$.)

Variational principles

Further worries:

- Parameterised versions of special relativity, written in terms of *clock fields* X^{a}. ($\eta_{\mu\nu} := \eta_{\mathsf{ab}}\partial_\mu X^{\mathsf{a}}\partial_\nu X^{\mathsf{b}}$.)
- This definition assumes that every background independent theory admits of a Lagrangian formulation.

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Belot's account

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- *Geometrical degrees of freedom* are degrees of freedom needed to parameterise variation of O^G across DPMs.
- *Physical degrees of freedom* are degrees of freedom needed to parameterise variation of $\langle O^G, \ldots, O_{n-1} \rangle$ across DPMs (modulo gauge equivalence).

Belot's account

Definition

(Full background dependence, Belot) *A field theory is fully background dependent if it has no geometrical degrees of freedom: every solution is assigned the same spacetime geometry as every other solution.*

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(Full background independence, Belot) *A field theory is fully background independent if all of its physical degrees of freedom correspond to geometrical degrees of freedom.*

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- In **SR1** and **SR2**, the metric field is fixed (up to isomorphism) in all DPMs.
- So there are no geometrical degrees of freedom.
- So these theories are background dependent, for Belot.

Belot's account

- *Prima facie*, general relativity is background independent: each configuration of the matter fields looks to correspond to a unique configuration of the metric field ('geometry'), via the Einstein equations,

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- But: If $\langle M, g_{ab}, F_{ab} \rangle$ is a model of Einstein-Maxwell theory, then so is $\langle M, g_{ab}, {}^*F_{ab} \rangle$. Different matter, same geometry.

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Proposed fix: *Each* geometrical field must differ between *each* non-gauge-equivalent model.

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Fixed fields

Definition

(Background independence, fixed fields) *A theory is background independent iff it has no formulation which features fixed fields.*

Fixed fields

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(Equivalent theory formulations) *Two theories, $\mathcal{T}_1$ and $\mathcal{T}_2$, are equivalent formulations of the same theory when:*

1. *KPMs of $\mathcal{T}_1$ and $\mathcal{T}_2$ involve the same types of geometric object.*
2. *DPMs of $\mathcal{T}_1$ and $\mathcal{T}_2$ are isomorphic, up to classes of diffeomorphism-related models.*

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2. *DPMs of $\mathcal{T}_1$ and $\mathcal{T}_2$ are isomorphic, up to classes of diffeomorphism-related models.*

(One could, alternatively, appeal to one of the other notions of theoretical equivalence presented in (Weatherall 2019ab).)

Fixed fields

- Doesn't this render **SR2** background independent?
- Not if we say this is another formulation of **SR1**:

Definition

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Potential problem: how do we know that the theory in question has no formulation featuring fixed fields?

Today

Absolute fields

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Definition

(Background independence, absolute fields) *A theory is background independent iff it has no absolute fields.*

Assessing absolute fields

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Assessing absolute fields

- The idea would be that e.g. $\sqrt{-g}$ in GR doesn't qualify as an absolute field, because it's not directly written in the KPMs of the theory.
- On the other hand, η_{ab} in **SR1**, and g_{ab} in **SR2**, do qualify as absolute fields.
- Worry: Doesn't this make background independence too dependent upon (syntactic?) choices about the objects which constitute the KPMs of any given theory?

Today

Summary

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- Since its inception, physicists and philosophers have debated what's special about general relativity.
- This began as a debate regarding 'substantive general covariance', and has evolved into a debate regarding 'background independence'.
- There are many interesting and illuminating ways of defining background independence—although arguably none without its problems.

Questions

1. Which of these proposals is the most promising?
2. Should we accept a plurality of definitions?
3. At what point does the task of trying to identify 'what's special about GR' become a futile one?

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