Advanced Philosophy of Physics: The Philosophy of Symmetries

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HT25-W1

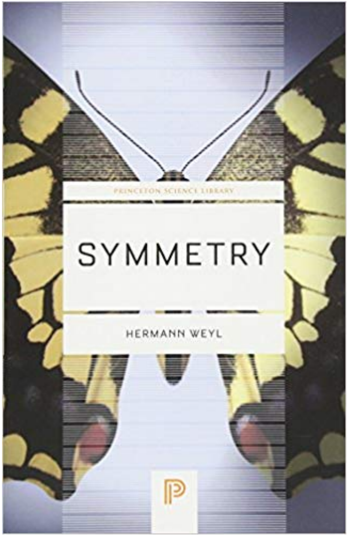
The plan

W1: The philosophy of symmetries

W2: The hole argument

W3: The Aharanov–Bohm effect

W4: The local validity of special relativity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SYMMETRY
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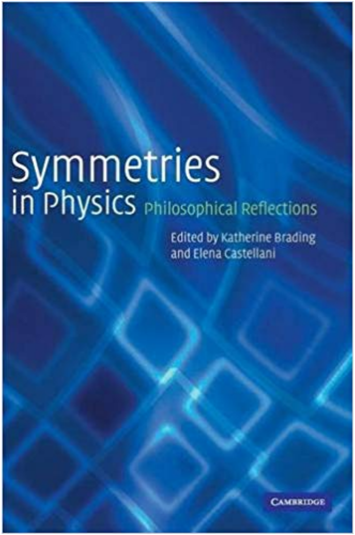
LAWS AND SYMMETRY

CLARENDON PAPERBACKS


Symmetries
in Physics
Philosophical Reflections
Edited by Katherine Brading
and Elena Castellani
CAMBRIDGE

“Symmetries in physics are a guide to reality.” (Dasgupta, 2016)

Today

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Kinematically possible models

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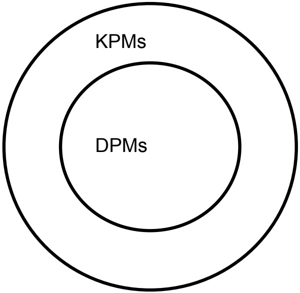
- In Quine's terminology, DPMs pick out a theory's *ideology*.
- A theory's DPMs therefore form a subclass of a theory's KPMs. Sometimes, a theory's DPMs are known as its *solutions*.

Kinematically and dynamically possible models



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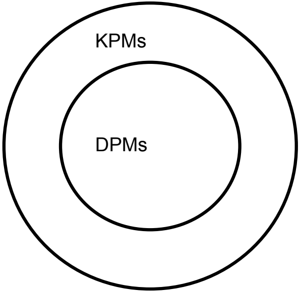

(For more on the kinematics/dynamics distinction, see Curiel (2016) and March (2024).)

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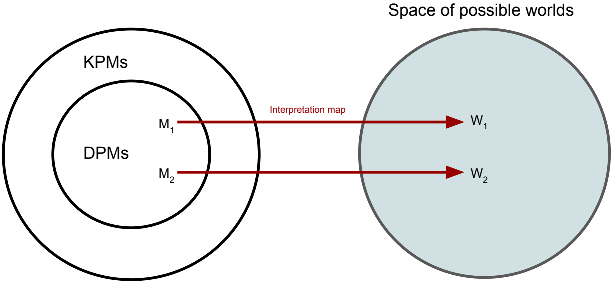

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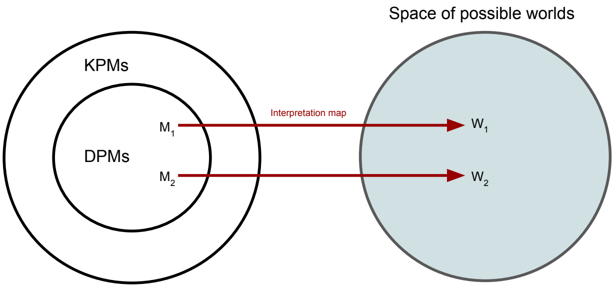

(Note: here, I am eliding certain important points about 'representational contexts', on which see Fletcher (2020) and Pooley & Read (2025).)

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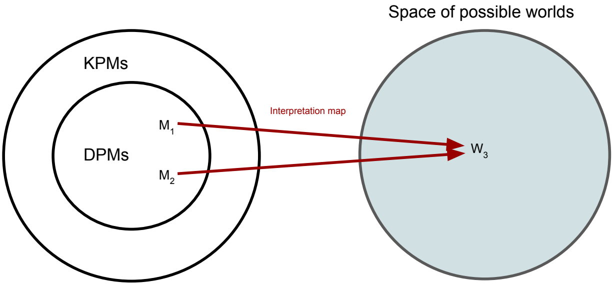

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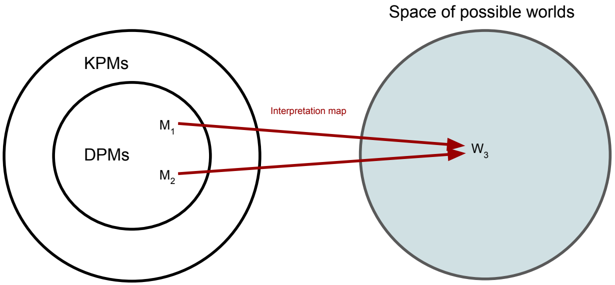

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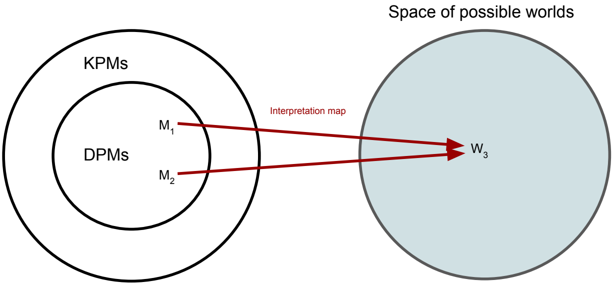

- Question: When is this the case?
- Tentative answer: In the presence of *symmetries*.

Symmetry and interpretation

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1. What *is* a symmetry transformation?
2. Should symmetry-related solutions of a given theory *invariably* be interpreted as representing the same physical state of affairs?
3. *How* do we articulate that putative common ontology of symmetry-related models?

Belot's catastrophic conclusion

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But the combination of D1 and D2 leads to catastrophe!

...So something has gone wrong with the orthodoxy.

Today's project

1. What *is* a symmetry transformation?
2. Should symmetry-related solutions of a given theory *invariably* be interpreted as representing the same physical state of affairs?
3. *How* do we articulate that putative common ontology of symmetry-related models?

We're going to work though the above questions in turn, and try to give more nuanced answers, which (hopefully!) avoid Belot's catastrophe.

Today

Two phases of the interpretative process

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- In the **first phase** of the interpretative process, a model's *empirical* content is fixed: we establish how a world would look, according to that model.
- In the **second phase** of the interpretative process, a model's *physical* content is fixed: we establish the ontology of the world, according to that model.

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Symmetry-to-reality based reasoning

- We've already seen the (widespread) claim that "symmetries are a guide to reality".
- Regardless of how one *defines* symmetries (more on which later), it is the symmetries which relate *empirically equivalent* solutions of a given theory which are involved in symmetry-to-reality based reasoning.
- Any two empirically equivalent solutions should have the structure which varies between them excised—since this structure is *ex hypothesi* undetectable, and should therefore, given Occam's razor, be eliminated. (See Dasgupta (2016).)

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2. Kinematic Leibniz shifts: Boost the entire material content of the universe, such that all bodies now possess an extra velocity component of five metres per second in the North direction.
3. Constant shifts of the gravitational/electrostatic potential.

Today

Theories and interpretation

Further preliminaries

What is a symmetry?

The normative import of symmetries

Articulating common ontology

Conclusions

Three approaches to symmetries

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- **Formal approaches** "define [the notion of symmetry] in purely formal, set-theoretic terms" (Dasgupta, p. 861).
- **Ontic approaches** "define a symmetry of a law to be a function that preserves the law and also preserves ... [salient physical] features F" (Dasgupta, p. 862).
- **Epistemic approaches** define a symmetry such that "given any set of laws, any two situations related by a symmetry of those laws are observationally equivalent" (Dasgupta, p. 866).

Dasgupta on formal definitions of symmetries

The central problem raised by Dasgupta against formal approaches to symmetries is as follows. Any such definition

> *... must imply that given any set of laws, any two systems related by a symmetry of those laws will be observationally equivalent. And it is (to put it mildly) extremely hard to see how any purely formal definition could have this consequence.* (Dasgupta, p. 861)

Responses

1. Why assume that all symmetry-related models must be empirically equivalent? In other words: why assume that all symmetries must be involved in symmetry-to-reality based reasoning?

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2. Who said that establishing the observational equivalence of models was easy?

Ontic definitions

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- Examples include dynamical symmetries, Lagrangian symmetries, etc.

Dasgupta on ontic definitions of symmetries

Dasgupta's central problem with ontic definitions is what he dubs the 'problem of inferential circularity':

> *The objection is that they get the order of justification backwards: we often use premises about symmetries in order to work out which physical features fix the data, so we cannot at the same time define symmetries to be those operations that preserve features that fix the data.* (Dasgupta 2016, p. 865)

Responses

1. Not all ontic definitions need be involved in symmetry-to-reality based reasoning (only the ones which preserve the observational data).
2. We don't fix *all* the physical quantities when we offer an ontic definition—just some. So it's not clear that the problem of inferential circularity is damning.

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(For more on these responses, see Read & Møller-Nielsen (2020b).)

Dasgupta on epistemic definitions of symmetries

In light of the (apparent) problems for formal and ontic definitions, Dasgupta proposes that we should embrace an *epistemic* approach to symmetries: symmetries *just are* transformations between empirically equivalent solutions.

Responses

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A normative question

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Should symmetry-related solutions be interpreted *ab initio* as being physically equivalent? (Møller-Nielsen 2017)

Of course, this isn't to say that there is no value to reformulating a theory's formalism in such a way that the surplus structure is made manifest, so that we can move to a formalism in which it is expunged entirely. Such a presentation lets us see what it is we are committed to by our (qualified) realism about the theory; if we want to know the answers to specific questions about the nature of a theory's ontology and ideology, then this is invaluable. ... But if we lack the mathematical tools to do so, then I maintain that there is nothing wrong with recognising that one's realism will only extend to structures that are invariant under the symmetries—whatever those may turn out to be. (Dewar 2015, pp. 326-7)

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I argue, contrary to current orthodoxy, that the variance of a quantity under a theory's symmetries is not a sufficient basis for interpreting that theory as being uncommitted to the reality of that quantity. Rather, I argue, the variance of a quantity under symmetries only ever serves as a motivation to refrain from any commitment to the quantity in question. (Møller-Nielsen 2017, p. 1253)

Interpretation and motivation

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- On the **interpretational approach** (Dewar), one may regard symmetry-related models *ab initio* as being physically equivalent.
- On the **motivational approach** (Møller-Nielsen), symmetry-related models may *only* be regarded as being physically equivalent once one has to hand a perspicuous explication of their common ontology.

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Møller-Nielsen: We can only declare these shifted models to represent the same physical state of affairs once we've done the metaphysical hard graft of figuring out what that physical state of affairs *actually is*.

Interpretation/motivation and shifts

> *... [W]e can draw the conclusion of the inference [viz., that shifted solutions should be regarded as physically equivalent] only when we have the alternative theory in hand and have shown that all else is equal. This explains why it was rational for Newton to believe in absolute velocity even though he knew that it was variant ... and undetectable. The reason this was rational for him was that he had no good alternative theory to hand. He had good reason (his bucket argument) to think that relationalism was not empirically adequate. And relationalism was the only alternative view he knew of (he was not aware of Galilean space-time structures in which there is a well-defined feature of absolute acceleration ... but no absolute velocity). So for Newton, all else was not equal and he was rational to believe in absolute velocity.* (Dasgupta 2016, p. 854)

Two challenges for the interpretational approach

1. How are we to identify the common structure associated with symmetry-related models—and have we any reason to think that such structure is always there to be found?
2. Even supposing that such structure can be found, does it invariably admit of a coherent physical interpretation?

More nuanced positions in the debate

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Luc (2023) identifies three salient questions in the interpretationalism/motivationalism debate:

1. What should our *initial* reaction towards symmetry-related models of T be—should we regard them as physically equivalent or as physically inequivalent?
2. Should we look for a perspicuous account of the ontology shared by symmetry-related models of T?
3. How should we update our interpretation of symmetry-related models of T depending on the outcomes of the research mentioned in question (2)?

More fine-grained options in the debate

A. Motivationalism
B. Interpretationalism without motivation
C. Steadfast interpretationalism with motivation
D. Concessive interpretationalism with motivation
E. Graded interpretationalism with motivation

A: Motivationalism

Motivationalism gives us determinate answers to each of the above three questions:

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1. We should initially interpret symmetry-related models of T as physically inequivalent.
2. Yes.
3. If we find such an account, we should change our initial interpretation and begin to regard symmetry-related models of T as physically equivalent; if we do not find such an account, we should retain our initial interpretation.

B: Interpretationalism without motivation

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Interpretationalism without motivation gives us determinate answers to each of the above three questions:

1. We should initially interpret symmetry-related models of T as physically equivalent.
2. There is no need to do this.
3. Irrespective of whether we find such an account or not, we should retain our initial interpretation.

C: Steadfast interpretationalism with motivation

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Steadfast interpretationalism with motivation gives us determinate answers to each of the above three questions:

1. We should initially interpret symmetry-related models of T as physically equivalent.
2. Yes.
3. Irrespective of whether we find such an account or not, we should retain our initial interpretation.

D: Concessive interpretationalism with motivation

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Concessive interpretationalism with motivation gives us determinate answers to each of the above three questions:

1. We should initially interpret symmetry-related models of T as physically equivalent.
2. Yes.
3. If we find such an account, we should retain our initial interpretation, whereas if despite lots of effort we do not succeed in finding it, we should change our interpretation and begin to regard symmetry-related models of T as physically inequivalent.

E: Graded interpretationalism with motivation

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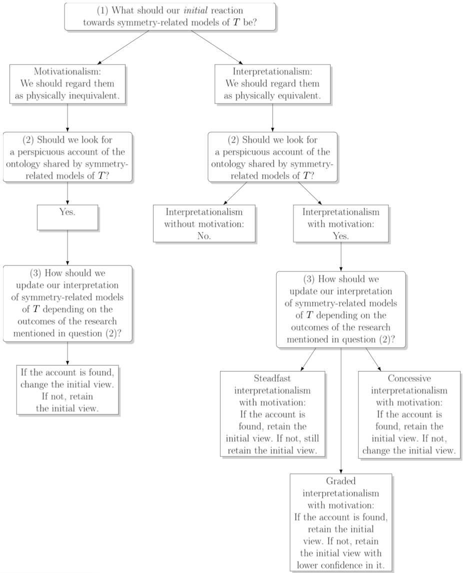
Graded interpretationalism with motivation gives us determinate answers to each of the above three questions:

1. We should initially interpret symmetry-related models of T as physically equivalent.
2. Yes.
3. If we find such an account, we should retain our initial interpretation, whereas if despite lots of effort we do not succeed in finding it, we should still retain our initial interpretation as more plausible than its opposite, but we should significantly decrease our confidence about this interpretation.

A helpful chart



Rapprochement in the debate?

> *Therefore, concessive interpretationalism with motivation is as close to motivationalism as interpretationalism can be. It differs from motivationalism only in what it claims to be the most reasonable prima facie attitude towards models of T that we have recently discovered to be symmetry-related.* (Luc 2023, p. 20)

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I, for one, would be happy to sign up to this—but you can make up your own minds!

Cases where motivation is lacking

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- For example, Belot (2018) considers models of general relativity which differ by boundary diffeomorphisms: these are symmetry-related, but if we regard them as being physically equivalent then we lose the ability to define conserved quantities.
- (For more, see Luc (2022) and Read (2023, ch. 2).)

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An interpretative question

How are we to articulate the common ontology of symmetry-related solutions?

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How are we to articulate the common ontology of symmetry-related solutions?

(Note that this is distinct from our previous, normative question.)

Dewar on this issue

> *It is often claimed that the symmetries of a theory reveal "surplus structure": structure which, in some sense, the theory could do without. For example, the boost symmetry of Newtonian mechanics indicates the superfluousness of absolute velocities; the gauge symmetry of electromagnetism reveals the superfluousness of absolute potentials; and so on and so forth. Moreover, it is widely held that if this is the case, then some modification of one's theory is appropriate, so as to make explicit what structure is not surplus (e.g. the replacement of Newtonian by Galilean spacetime, in response to the boost symmetry of Newtonian mechanics). [...] I compare and contrast two ways of making such a modification. The first is to replace the theory by (what I shall call) a reduced theory: a theory that deals only in quantities which are invariant under the relevant symmetry. The second is to replace the theory by (what I shall call) a sophisticated theory: a theory in which models related by a symmetry are isomorphic.* (Dewar 2019, pp. 485-6)

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Reduction and sophistication

- **Reduction**: “the idea is that we (i) identify some collection of invariants of the original theory; (ii) specify a theory in terms of those invariants; and (iii) show that the new theory captures all the symmetry-invariant content of the old theory.” (Dewar 2019, pp. 492-3)

Reduction and sophistication

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- **Sophistication**: “the idea is that we need not insist on finding a theory whose models are invariant under the application of the symmetry transformation [...] the proposal is that we instead look for a theory such that [symmetry-related models of the original theory] *M* and *N* give rise to distinct but isomorphic models.” (Dewar 2019, p. 498)

A threefold distinction, summarised

- **Reduction**: In the presence of empirically equivalent symmetry-related models of some theory T, construct some new theory T' such that those symmetry-related models of T all map to some unique model of T'.
- **Internal sophistication**: In the presence of empirically equivalent symmetry-related models of some theory T, 'forget' structure while retaining the same number of models, in order to construct some new theory T' such that the symmetries of T under consideration (which needn't act as isomorphisms) act as isomorphisms of T'.
- **External sophistication**: In the presence of empirically equivalent symmetry-related models of some theory T, treat those models of T 'as if' they are isomorphic.

Two orthogonal debates

1. Interpretation + reduction (Caulton 2015)
2. Interpretation + sophistication (Dewar 2019)
3. Motivation + reduction (Dasgupta 2016)
4. Motivation + sophistication (Jacobs 2021)
5. Some more complicated cocktail (Møller-Nielsen 2017)

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- Consider vector potential-based, source-free electromagnetism—KPMs are given by $\langle M, \eta_{ab}, A^a \rangle$; DPMs are given by the dynamical equations $d * dA = 0$ and $ddA = 0$ (the latter of which is in fact a mathematical identity by the Poincaré lemma).

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- This theory has a *gauge symmetry*, $A \mapsto A + d\Lambda$; these symmetries do not relate isomorphic models.

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- This is indeed a reduced theory, since gauge fields of A-EM map to unique models of F-EM.

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- Gauge symmetries now take $\langle M, \eta_{ab}, P, \omega \rangle$ to $\langle M, \eta_{ab}, P, d_* \omega \rangle$—so act as isomorphisms!

Internal sophistication and the need for anti-haecceitism/anti-quidditism

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- The situation when we move to fibre bundle EM is similar—but now the diffeomorphism d is on the value space, not spacetime!
- To deny that points in value space have primitive identities is 'anti-quidditism'; embracing this is again sufficient to assert that these symmetry-related models represent the same possible world.

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(To be clear: I like internal sophistication but not external sophistication!)

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There’s very likely still more to be worked out here...

Today

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- ...you're welcome to adopt a different package!

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2. **Interpretation/motivation:** (J.R. & T.M-N., 2020a)
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3. **Reduction/sophistication:** (N.M. & J.R., 2020)
   - When symmetry-related solutions are isomorphic, we can appeal to anti-haecceitism/anti-quidditism to explicate the common ontology of those solutions.
   - When symmetry-related solutions are not isomorphic, we must appeal to reduction or internal sophistication to explicate the common ontology of those solutions.
   - As a means of explicating the ontology of symmetry-related solutions, external sophistication is unperspicuous.

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