Advanced Philosophy of Physics: The Hole Argument

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HT25-W2

The plan

W1: The philosophy of symmetries
W2: The hole argument
W3: The Aharanov–Bohm effect
W4: The local validity of special relativity

Is general relativity radically indeterministic?

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Should we, therefore, be relationalists about general relativity? (Whatever that means.)

Today

Background on spacetime theories

The hole argument and responses

Shift arguments in Newtonian mechanics

Mathematical representation

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Coordinate and abstract indices

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2. g_{ab} refers directly to a geometrical object itself—the $a, b, \dots$ are just there to remind us how many components this object *would* have, were we to write it in a coordinate basis. Here, $a, b, \dots$ are *abstract indices*.

Spacetime theories

- The *kinematically possible models* (KPMs) of a given theory are picked out by tuples $\langle M, \Phi_1, \ldots, \Phi_n \rangle$, with
  1. a manifold, M;
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  1. a manifold, M;
  2. fields on M, $\Phi_1, \ldots, \Phi_n$.
- The *dynamically possible models* (DPMs) of a given theory are those KPMs in which the Φ_i satisfy certain dynamical equations.

Dynamical versus fixed fields

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  - E.g. g_{ab} in GR.
- *Fixed fields* are fixed identically in all KPMs, and do not have their own associated dynamical equations.
  - E.g. η_{ab} in SR.

Special relativity

- KPMs of SR are picked out by triples $\langle M, \eta_{ab}, \Phi \rangle$, where M is the manifold, η_{ab} is a fixed Minkowski metric field on M, and Φ is a placeholder for matter fields.

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- DPMs of SR are picked out by dynamical equations which manifest the same symmetries as η_{ab} (i.e., which are Poncaré invariant).

Example: Maxwell theory

- KPMs picked out by $\langle M, \eta_{ab}, F_{ab}, J^a \rangle$.
- DPMs picked out by

$$
\begin{aligned}
\nabla_a F^{ab} &= J^b \\
\nabla_{[a} F_{bc]} &= 0.
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General relativity

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- DPMs of GR are picked out by the Einstein equation,

$$G_{ab} = 8\pi T_{ab},$$

plus dynamical equations for the matter fields Φ.

Example: Einstein-Maxwell theory

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- DPMs picked out by

$$\begin{aligned} G_{ab} &= 8\pi T_{ab} \\ \nabla_a F^{ab} &= J^b \\ \nabla_{[a} F_{bc]} &= 0. \end{aligned}$$

General covariance

Definition

(General covariance) *A formulation of a theory is generally covariant iff the equations expressing its laws are written in a form that holds with respect to all members of a set of coordinate systems that are related by smooth but otherwise arbitrary transformations.* (Pooley 2017, p. 115)

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- Thought: GR is privileged in virtue of its general covariance.
- Kretchmann objection: *Any* theory admits of a generally covariant formulation.

Diffeomorphism invariance

Definition

(Diffeomorphism invariance) *A theory $\mathcal{T}$ is diffeomorphism invariant iff, if $\langle M, F_1, \ldots, F_n, D_1, \ldots, D_m \rangle$ is a DPM of $\mathcal{T}$ (where the F_i are fixed fields and the D_i are dynamical fields), then so is $\langle M, F_1, \ldots, F_n, d_* D_1, \ldots, d_* D_m \rangle$, for all $d \in Diff(M)$.* (Pooley 2017, p. 117)

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- SR theories such as Maxwell theory are not diffeomorphism invariant.
- GR theories such as Einstein-Maxwell theory *are* diffeomorphism invariant.

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1. *Passively*: The diffeomorphism just alters the coordinate system used to describe the physical system under consideration—it leaves the physical system itself unchanged.
2. *Actively*: The diffeomorphism alters the physical system under consideration, while leaving the coordinate system unchanged.

A warning...

I defy anyone to avoid getting confused by active versus passive transformations. (Graeme Segal, 2006)

Today

Diffeomorphism invariance and interpretation

- By the diffeomorphism invariance of GR, if $\mathcal{M}_1 = \langle M, g_{ab}, \Phi \rangle$ is a solution, then so too is $\mathcal{M}_2 = \langle M, d_* g_{ab}, d_* \Phi \rangle$, for arbitrary diffeomorphism d.

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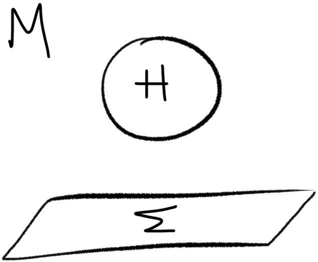
- Natural interpretation (under an active interpretation diffeomorphisms): If $\mathcal{M}_1$ represents a physically possible situation, then so too does $\mathcal{M}_2$.
- **Import of the hole argument:** This seems to render GR radically indeterministic.

Hole diffeomorphisms



- Let $d = \text{Id}$ for all of M outside a given region $H \subset M$ (the 'hole'), but $d \neq \text{Id}$ inside H.
- Suppose M admits a foliation by global spacelike hypersurfaces (relative to the structure of $\mathcal{M}_1$).
- Suppose H is to the future of Σ.
- $\mathcal{M}_1$ and $\mathcal{M}_2$ are identical up to Σ but diverge thereafter.

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- Suppose that W_1 and W_2 are identical up to some time $t := t_\Sigma$ but differ after t.
- A complete specification of a possible world up to t does not distinguish between W_1 and W_2.
- Hence the history of a world up to t, together with $\mathcal{T}$, can fail to fix the future of that world.
- Thus, in light of its diffeomorphism invariance, GR appears to be radically indeterministic.

What's wrong with indeterminism?

Our argument does not stem from a conviction that determinism is or ought to be true [...] Rather our point is this. If a metaphysics which forces all our theories to be deterministic, is unacceptable, then equally a metaphysics which automatically decides in favour of indeterminism, is unacceptable. Determinism may fail, but if it fails, it should fail for a reason of physics, not because of commitment to substantival properties which can be eradicated without affecting the empirical consequences of the theory. (Earman and Norton 1987, p. 524)

Substantivalism and relationalism

SUB: A complete catalogue of the fundamental objects in the universe lists, in addition to the elementary constituents of material entities, the basic parts of spacetime.

REL: Claims apparently about spacetime itself are ultimately to be understood as claims about material entities and the possible patterns of spatiotemporal relations they can instantiate.

Three claims regarding the hole argument

HAE: $\mathcal{M}_1$ and $\mathcal{M}_2$ (can be taken to) represent distinct physically possible worlds.

LZE: $\mathcal{M}_1$ and $\mathcal{M}_2$ (must be taken to) represent the same possible world.

ONE: If $\mathcal{M}_1$ is taken to represent a possible world, then $\mathcal{M}_2$ does not represent a possible world.

Earman and Norton's argument

1. SUB $\rightarrow$ HAE. ("The acid test.")
2. HAE $\rightarrow$ Indeterminism.
3. $\neg$Indeterminism.
4. $\therefore \neg$SUB.

Responses to the argument

- Reject (1): I.e., reject the acid test.
- Reject (2): I.e., question whether HAE leads to indeterminism.
- Reject (3): I.e., accept that GR is indeterministic.
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- That is, unlike SUB, REL commit us only to LZE, not to HAE.

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- A proposal to fill this lacuna was made in (Earman 1989), where appeal was made to 'Einstein algebras'.
- The jury is out on whether a formulation of general relativistic models using such algebras can provide a metaphysically perspicuous characterisation of general relativistic worlds without M: Dasgupta (2011), for example, argues that it cannot.

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- But what of Earman and Norton's claim that determinism should fail "for reasons of physics"?
- If another option were available which would allow us to avoid this indeterminism, that would certainly seem to be preferable.

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- As we will see, this is Maudlin's strategy.

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- ...but is this view plausible?

Earman on metric essentialism

> *[M]etric essentialism must resort to unnatural contortions to explain the most striking feature of Einstein's GTR: the dynamic character of the space-time metric. The most straightforward way to say what this feature means is to assert, for example, that if some extra mass were brought close to some point, then the curvature at that very point would be different. But the metrical essentialist views such assertions as literally self-contradictory.* (Earman 1989, p. 201)

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But is this just a restatement of the position as an objection?

Norton's challenge

Norton's challenge (1989) to the metric essentialist is to identify *which* of a class of isomorphic models can represent a possibility—why *this particular model*, and how can we distinguish this model from an 'imposter'?

Pooley's response

Pooley responds to Norton on behalf of the metric essentialist as follows:

> *Abstracting from the pragmatics of representation, all isomorphic models are equally suited to represent the same spacetime. But, in practical situations, some model or other will be singled out, normally quite arbitrarily, to represent a physical possibility. The advocate of [Maudlin's position] claims only that, relative to such a choice of one model, the others must be viewed either as representing impossible worlds (per the haecceitist essentialist) or as representing nothing at all (per the anti-haecceitist).* (Pooley 2002, p. 101)

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For more recent work on metric essentialism, see (Teitel 2019).

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Maudlin on the acid test

If one really believes in event locations, believes that there is a deep ontological fact about at which space-time point a particular event occurred, then one ought to be able to discuss the possibility of that event ... occurring somewhere else. (Maudlin 1989, p. 315)

Haecceitism reprise

Haecceitism is the view that possible worlds can differ solely over which objects instantiate which properties. Two possible worlds that contain exactly the same individuals and the same patterns of property instantiation, but that have different individuals instantiating certain properties, differ purely haecceitistically.

Sophisticated substantivalism

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- *Sophisticated* or *anti-haecceitist* substantivalists deny the primitive identity of spacetime points across possibilities.
- This allows them to remain substantivalists, but to embrace LZE and reject HAE.
- In this way, they can argue that substantivalism does not lead to indeterminism.

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Mathematical representation

Newtonian gravitation theory

- KPMs $\langle M, t_a, h^{ab}, \nabla_a, \sigma^a, \varphi, \rho \rangle$, with

$$\begin{aligned} \nabla_a h^{bc} &= 0, \\ \nabla_a t_b &= 0, \\ h^{ab} t_b &= 0, \\ \sigma^a t_a &= 1. \end{aligned}$$

- DPMs picked out by

$$\begin{aligned} h^{ab} \nabla_a \nabla_b \varphi &= 4\pi\rho, \\ R^a{}_{bcd} &= 0. \end{aligned}$$

Static shifts

- If $\mathcal{M} = \langle M, t_a, h^{ab}, \nabla_a, \sigma^a, \varphi, \rho \rangle$ is a DPM of NGT, then so too is $\mathcal{M}_{\text{stat}} = \langle M, t_a, h^{ab}, \nabla_a, \sigma^a, d_*\varphi, d_*\rho \rangle$, where d is a diffeomorphism which implements a time-independent translation of the material content of the entire universe.

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- $\mathcal{M}$ and $\mathcal{M}_{\text{stat}}$ are said to be related by a *static shift*.
- Following Leibniz, we would ideally want to interpret solutions of NGT related by a static shift as representing the same physical state of affairs.

Sophisticated substantivalism, reprise

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- Statically-shifted solutions of NGT again differ merely haecceitistically.
- So the sophisticated substantivalist strategy is once again available in order to argue that such solutions of NGT do not represent distinct possibilities.

Maudlin's epistemological argument

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> *If Clarke is right, the material universe* could have been *located elsewhere in absolute space—that is, located some other place than it is, keeping all the relative positions the same. But we do not need to make any observation to know that this did not actually happen: by hypothesis, the other placement of matter is counterfactual.* (Maudlin 2012, p. 46)

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- Maudlin's point is that static shifts do not create an *epistemological* problem, for (he claims) we *know* we are in *this* world, rather than in a statically-shifted world.

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- Question: Could Maudlin deploy the same strategy to argue that the hole argument is unproblematic?

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- This would be the exactly analogue of the static shift in NGT. It would lead to a *prima facie* problem of *underdetermination*, to which Maudlin could issue his epistemological argument in response.
- The diffeomorphism invariance of GR also allows us to generate two models of the theory which differ only *locally*, to the future of Σ. This is *not* possible in NGT.

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- In GR, we *could* act with a diffeomorphism d to effect a global, non-trivial, time-independent transformation relating $\mathcal{M} = \langle M, g_{ab}, \Phi \rangle$ to $\mathcal{M}' = \langle M, d_* g_{ab}, d_* \Phi \rangle$.
- This would be the exactly analogue of the static shift in NGT. It would lead to a *prima facie* problem of *underdetermination*, to which Maudlin could issue his epistemological argument in response.
- The diffeomorphism invariance of GR also allows us to generate two models of the theory which differ only *locally*, to the future of Σ. This is *not* possible in NGT.
- This generates a problem of *indeterminism*, to which Maudlin's epistemological response is not appropriate.

Indeterminism and underdetermination

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- This generates a problem of *indeterminism*, to which Maudlin's epistemological response is not appropriate.
- As a result (in my reconstruction), Maudlin adopts a different tactic here: metric essentialism.

Assessing the epistemological argument

- Dasgupta (2015) objects to Maudlin's epistemological argument: my being able to identify indexically my spacetime location does not (Dasgupta claims) eliminate there being a fact about *which* spacetime point at which I located, of which I am ignorant.

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- It's not obvious that Dasgutpa's responses to Maudlin succeed here: see (Perry 2017) and (Cheng & Read 2022) for further discussion.

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- Saunders points out that one can use the invariance of Newtonian physics under arbitrary non-rotating accelerative transformations in order to generate a Newtonian version of the hole argument: let the acceleration associated with this transformation be trivial up to Σ, and non-trivial thereafter!

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    - There are facts of which agents 'embedded' in these worlds are ignorant (e.g. their absolute acceleration profiles), so indexical identification is not possible.
- So neither of Maudlin's preferred arguments seem to work here.

Today

Background on spacetime theories

The hole argument and responses

Shift arguments in Newtonian mechanics

Mathematical representation

Weatherall on the hole argument

The basic thesis of the present article is that Einstein and the generations of physicists and mathematicians after him were right to reject the hole argument. It is based on a misleading use of the mathematical formalism of general relativity. If one is attentive to mathematical practice, I will argue, the hole argument is blocked. (Weatherall 2018, p. 330)

Weatherall's argument

1. Models $\mathcal{M}_1$ and $\mathcal{M}_2$ of GR related by a hole diffeomorphism are isomorphic, and the map which witnesses this isometry is called $\tilde{\psi}$.

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3. Given that we use this standard of identity in our interpretation of these models, the hole argument is blocked: we cannot articulate a difference between the worlds represented by these models.
4. The only way in which we *can* articulate a difference between the worlds represented by these models is to use a different standard of identity, the identity map 1_M, which 'cuts finer' than isomorphism.

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3. Given that we use this standard of identity in our interpretation of these models, the hole argument is blocked: we cannot articulate a difference between the worlds represented by these models.
4. The only way in which we *can* articulate a difference between the worlds represented by these models is to use a different standard of identity, the identity map 1_M, which 'cuts finer' than isomorphism.
5. But to use this map (as is done in traditional discussions of the hole argument) is to not understand the models as Lorentzian manifolds, and so is illegitimate.

Insofar as one wants to claim that these Lorentzian manifolds are physically equivalent, or agree on all observable/physical structure, one has to use $\tilde{\psi}$ *to establish a standard of comparison between points. And relative to this standard, the two Lorentzian manifolds agree on the metric at every point—there is no ambiguity, and no indeterminism. (This is just what it means to say that they are isometric.) Meanwhile, insofar as one wants to claim that these Lorentzian manifolds assign different values of the metric to each point, one must use a different standard of comparison. And relative to this standard—that given by* 1_M*—the two Lorentzian manifolds are not equivalent. One way or the other, the hole argument seems to be blocked.* (Weatherall, pp. 338-9)

Weatherall's dichotomy: second horn

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- But this ignores the indeterminism problem, which still arises even if 1_M is used as the standard of comparison and the epistemological argument is deployed!
- Let us, then, turn to the first horn of Weatherall's dichotomy.

Weatherall's dichotomy: first horn

- Weatherall is correct that one cannot articulate a difference between the worlds represented by $\mathcal{M}_1$ and $\mathcal{M}_2$—and so that GR does not *generate* a metaphysical problem—if isomorphism is used as the standard of identity.

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- Weatherall is correct that one cannot articulate a difference between the worlds represented by $\mathcal{M}_1$ and $\mathcal{M}_2$—and so that GR does not *generate* a metaphysical problem—if isomorphism is used as the standard of identity.
- But is it reasonable to insist on this *ab initio*?
- Arguably, no: rather, *per* the motivationalist, one should only regard isomorphism as being the appropriate standard of comparison of models once one has procured an understanding of the models of GR—e.g., sophisticated substantivalism—which underpins this verdict.

For more discussion of Weatherall's response to the hole argument, see (Fletcher 2020) and (Pooley & Read 2025).

Conclusions

In this lecture, we've:

1. Introduced the hole argument in GR.
2. Discussed a range of classic responses to the hole argument.
3. Considered the possibility of Newtonian versions of the hole argument.
4. Discussed some aspects of Weatherall's mathematical/formalist response to the hole argument.

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