Advanced Philosophy of Physics: The Local Validity of Special Relativity

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HT25-W4

Today

The equivalence principle

The local validity of SR as a heuristic

Local validity of SR in the completed theory of GR

Interpretative significance of the local validity of SR

Conclusions

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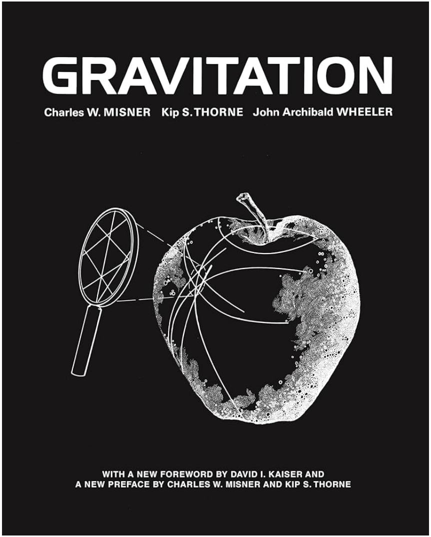
The standard story on the local validity of SR in GR

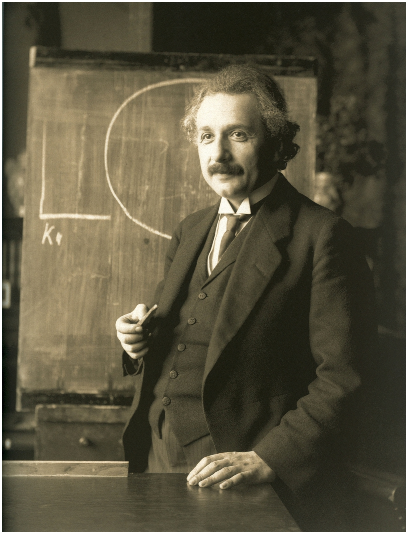
> *General relativistic physics is special relativistic 'for sufficiently small neighbourhoods'.*


GRAVITATION
Charles W. MISNER Kip S. THORNE John Archibald WHEELER
WITH A NEW FOREWORD BY DAVID I. KAISER AND
A NEW PREFACE BY CHARLES W. MISNER AND KIP S. THORNE

Steven Weinberg (1933–2021)

Weinberg

... we formulate the equivalence principle as the statement that at every space-time point in an arbitrary gravitational field it is possible to choose a "locally inertial coordinate system" such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation. There is a little vagueness here about what we mean by "the same form as in unaccelerated Cartesian coordinate systems," so to avoid any possible ambiguity we can specify that by this we mean the form given to the laws of nature by special relativity [...] There is also a question of how small is "sufficiently small." Roughly speaking, we mean that the region must be small enough so that the gravitational field is sensibly constant throughout it, but we cannot be more precise until we learn how to represent the gravitational field mathematically (Weinberg 1972, p. 68)



Albert Einstein (1879–1955)

Einstein

According to the special theory of relativity the coordinates x, y, z, t are directly measurable via clocks at rest with respect to the coordinate system. Thus, the invariant ds, which is defined via the equation $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$, likewise corresponds to a measurement result. The general theory of relativity rests entirely on the premise that each infinitesimal line element of the spacetime manifold physically behaves like the four-dimensional manifold of the special theory of relativity. Thus, there are infinitesimal coordinate systems (inertial systems) with the help of which the ds are to be defined exactly like in the special theory of relativity. The general theory of relativity stands or falls with this interpretation of ds. It depends on the latter just as much as Gauss' infinitesimal geometry of surfaces depends on the premise that an infinitesimal surface element behaves metrically like a flat surface element [...] . (Lehmkuhl 2021, p. 135)

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Today, we'll first look at (1), then look at (2). We won't discuss (3) but I mention it here for completeness/interest.

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- This narrative takes inspiration from...

John Stewart Bell (1928–1990)

9

How to teach special relativity

I have for long thought that if I had the opportunity to teach this subject, I would emphasize the continuity with earlier ideas. Usually it is the discontinuity which is stressed, the radical break with more primitive notions of space and time. Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired[1].

If you doubt this, then you might try the experiment of confronting

How to Teach General Relativity

Guy Hetzroni and James Read

Supposing that one is already familiar with special relativistic physics, what constitutes the best route via which to arrive at the architecture of the general theory of relativity? Although the later Einstein would stress the significance of mathematical and theoretical principles in answering this question, in this article we follow the lead of the earlier Einstein (circa 1916) and stress instead how one can go a long way to arriving at the general theory via inductive and empirical principles, without invoking presumptions concerning the geometrical structure of the final theory. We focus on the construction of the kine-

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- A central motivation for Einstein in the development of GR was to extend the relativity principle to frames of reference moving arbitrarily with respect to one another.
- This can be seen in his identification of inertial and gravitational effects in the EEP.

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- Boost to an arbitrary frame of reference.
- Replace the fictitious force terms which appear in said frame of reference with new physical fields. (An 'passive-to-active' transition.)
- Thereby, arrive at the kinematical structure of GR while achieving general covariance.

Warmup case

- The Lagrangian describing the motion of a test particle in an inertial frame of reference in SR is:

$$\mathcal{L} = \eta_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} =: \eta_{\alpha\beta} u^\alpha u^\beta .$$

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- In arbitrary curvilinear coordinates, the same Lagrangian takes the form

$$\mathcal{L} = \omega_{\mu\nu} \frac{d\xi^\mu}{d\lambda} \frac{d\xi^\nu}{d\lambda}, \qquad \omega_{\mu\nu} := \eta_{\alpha\beta} \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{\partial x^\beta}{\partial \xi^\nu} .$$

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- $\omega_{\mu\nu}$ (i) transforms like a metric tensor, (ii) reduces to $\eta_{\mu\nu}$ in a certain special class of frames.
- So now replace this ‘passive’, ‘fictitous force’-like object with a new physical field, $g_{\mu\nu}$.

Passive-to-active bootstrap: warmup

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- The field $g_{\mu\nu}$ has basic mathematical properties necessary to be interpreted as a metric.
- The replacement of $\eta_{\mu\nu}$ with $g_{\mu\nu}$ has achieved three goals:
  1. The theory is now generally covariant.
  2. It now has additional physical content.
  3. The new physical content *explains* the non-invariance of the original theory.

The methodological equivalence principle

Given a non-invariant dynamical law in the sense that its form simplifies maximally in a given preferred class of representations but involves modified/additional expressions in arbitrary representations, construct an invariant law by replacing the modified/additional expressions with new dynamical fields, whose set of possible local values is identical to that of the modified/additional expressions, and which manifest the same representation-to-representation transformation properties. (Hetzroni & Read 2023, pp. 15–16)

The case of field theories

- Consider an arbitrary curvilinear coordinate transformation applied to the Lagrangian of a Klein-Gordon scalar field:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left[\eta^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - m^2\phi^2\right] \\ &= \frac{1}{2}\left[\eta^{\alpha\beta}\left(\frac{\partial\xi^\mu}{\partial x^\alpha}\frac{\partial\phi}{\partial\xi^\mu}\right)\left(\frac{\partial\xi^\nu}{\partial x^\beta}\frac{\partial\phi}{\partial\xi^\nu}\right) - m^2\phi\right] \\ &= \frac{1}{2}\left[\omega^{\mu\nu}\frac{\partial\phi}{\partial\xi^\mu}\frac{\partial\phi}{\partial\xi^\nu} - m^2\phi\right],\end{aligned}$$

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- Applying the Methodological Equivalence Principle by now replacing $\omega^{\mu\nu}$ with $g^{\mu\nu}$, one obtains the Lagrangian

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- Thus, one is once again led to a generally covariant action featuring a new field $g_{\mu\nu}$.

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- Whether or not to adopt a geometrical understanding of the resulting theory is now a matter of various considerations not related to the indispensability of a geometrical perspective for the construction of the theory.
- (This accords with Bell's warnings about "premature philosophising about space and time" in the context of SR.)
- The physical content of general covariance is revealed not as a formal requirement, but rather as a heuristic one, which gains its significance only when applied together with the Methodological Equivalence Principle.

Deriving dynamics

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- So far, we've used the MEP as a heuristic to obtain the *kinematics* of general relativity. Obtaining the dynamics (i.e., the EFEs) will require further reasoning.
- There are many ways in which one might select a suitable dynamics, and indeed such dynamical choices might be guided and constrained via other reasoning.

Examples of derivations of the dynamics

- One example of such a path leading to the EFEs is based on 'Lovelock's theorem', which states that from a local action which contains at most second derivatives of $g_{\mu\nu}$, the only possible Euler-Lagrange equations are the EFEs.

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- Alternatively, one could take an effective field theory approach, considering all possible dynamical couplings in a Lagrangian describing local fields, and then identifying those terms relevant at a certain energy scale.
  - Explicitly, one writes down an action of the form

$$S = \int d^4x \sqrt{g} \left(\frac{1}{16\pi G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + \mathcal{L}_{\text{matter}} \right),$$

    before arguing that higher-order terms (i.e., those with coefficients $c_1, c_2, \ldots$) are irrelevant at low energies. In this way, one can pick out the EFEs as the first-order result in an infinite energy expansion.

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The story so far

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- Whether and how one is to make sense of the local validity of SR in the completed theory of GR is a different story—one to which we'll now turn.

A recent debate

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- Linnemann, Read & Teh (2024), defend the standard story from a ‘scale-relative’ perspective.
- On the other hand, Fletcher & Weatherall (2023a) find claims about the local validity of SR in GR to be problematic.

LRT's ‘scale-relative’ outlook

LRT claim that a ‘scale-relative’ outlook explicates the standard story satisfactorily.

Orthonormal lab frame

- Relative to a base point x', define an orthonormal 'lab' frame $e_I{}^\mu$ such that

$$g_{\mu\nu} e_I{}^\mu e_J{}^\nu (x') = \eta_{IJ}(x'),$$

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- At this stage, just think of $e_I{}^{\mu}$ as a change-of-basis matrix—the point is, one can always find coordinates at any point $p \in M$ in which the metric $g_{\mu\nu}$ takes its simple, 'special relativistic', diagonal form.

Orthonormal lab frame ($'$-convention)

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Synge's world function

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- Denote their unique connecting geodesic by $z^\mu(\lambda)$ with λ the affine parameter ranging from λ_0 to λ_1 such that $z(\lambda_0) = x'$ and $z(\lambda_1) = x$.
- The world function relative to the base point x' and its neighbouring point x is defined as

$$\sigma(x, x') = \frac{1}{2}(\lambda_1 - \lambda_0) \int_{\lambda_0}^{\lambda_1} g_{\mu\nu}(z(\lambda)) t^\mu t^\nu d\lambda,$$

with $t^\mu := \frac{dz^\mu}{d\lambda}$ tangent to the geodesic.

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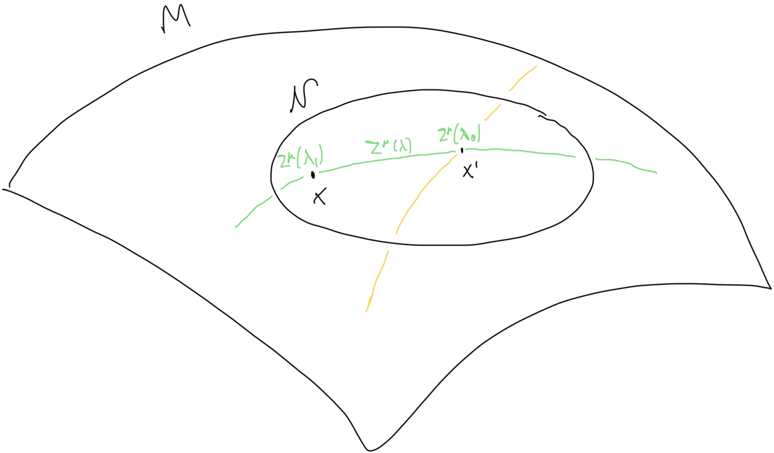

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- The world function $\sigma(x, x')$ is tantamount to a Lagrangian. In combination with Hamilton's principle, it encodes the **geodesic equations**.
- The derivative of the world function at base point, x', seen as a function of x, i.e., $\sigma^{a'}_{x'}(x) := \nabla^{a'}\sigma(x, x')$ encodes **geodesic deviation** structure, in the sense that it is associated to Jacobi vector fields.

Riemann normal coordinates

- Relative to the fixed base point x' and the tetrad $e_I{}^{a'}$, the world function can be used to assign coordinates to a neighbouring point x of form

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- The Riemann normal coordinates are a down-projection of the (coordinate-independent) bitensorial object $\sigma^{a'}(x, x')$.
- Thus, these charts bear a direct relation to the coordinate-independent geometric structure.

Riemann normal coordinates: expansion of the metric

Riemann normal coordinates provide an asymptotic expansion of the metric:

$$g_{IJ} = \eta_{IJ} - \delta^2 \frac{1}{3} R_{ILJK} \hat{x}^L \hat{x}^K + O(x^3)$$

where $R_{ILJK} := R_{a'b'c'd'} e_I{}^{a'} e_L{}^{b'} e_J{}^{c'} e_K{}^{d'}$ and $\delta := \frac{l_{\text{probing}}}{l_{\text{curvature}}}$, i.e. the ratio of characteristic lengths for probing and curvature.

World-function-based expansion of the metric

Asymptotic expansion in terms of the derivative of the world function:

$$g_{ac} = \eta_{ac} - \delta^2 \frac{1}{3} R_{a'b'c'd'} \sigma^{b'} \sigma^{d'} \sigma^{a'}{}_{a} \sigma^{c'}{}_{c} + O((\nabla\sigma)^3).$$

In particular, δ is independent of the specific tetradic down-projection and thus intrinsic to the geometric structure of the general relativistic model under consideration.

An EFT-inspired/scale-relative perspective

- The expansion around Minkowski is natural from a 'scale-relative' point of view.
  - Thanks to the world function, we can think of the expansion purely geometrically.
  - This provides a clear sense—LRT claim!—in which GR is locally SR.

An EFT-like/scale-relative aspect is already familiar!

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Approximate Killing vectors in CST:

- Notion of approximate Killing and approximate Rindler observer key to thermodynamic-'derivation' of GR (Jacobson 1995)
- Control through δ allows for seeing that only GR (but not higher-order corrections) has a meaningful thermodynamic reinterpretation (Jacobson 2012).

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Geometrical optics-limit in CST:

- The high-frequency limit for EM waves in CST involves the idealisation of letting δ go to zero (MTW 1973, §22.5).

Questioning the standard story

So much for the ‘standard story’ and an articulation of it via a ‘scale-relative’ perspective. Let’s turn now to scepticism about it from Fletcher & Weatherall (2023a).

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- From (2), they show that it follows that ultimately, to first order, every metric is approximately like every other.
- Together, these results seem to temper straightforward claims about the local validity of SR in GR.

Fletcher & Weatherall on local flatness

Theorem 1 *(Local Flatness). Given any spacetime* (M, g_{ab})*, any embedded curve* $\gamma : I \to M$ *therein, and any point* $p \in \gamma[I]$*, there exists, on some neighbourhood* O *containing* p*, a flat metric* $\bar{g}_{ab}$*, such that on* $\gamma[I] \cap O$*, (a)* $g_{ab} = \bar{g}_{ab}$*, and (b)* $\nabla = \bar{\nabla}$*, where* ∇ *and* $\bar{\nabla}$ *are the Levi-Civita derivative operators associated with* g_{ab} *and* $\bar{g}_{ab}$*, respectively.*

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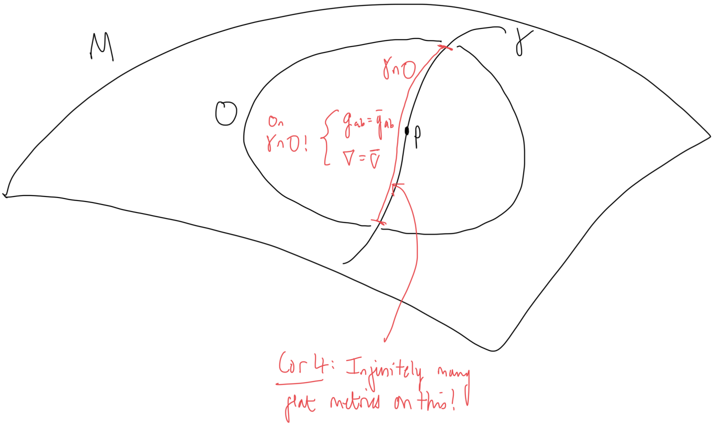
So, this tells us that locally any (possibly curved) metric agrees (non-canonically—see below) with some flat metric.

Fletcher & Weatherall on local flatness



Non-uniqueness of local flatness

Corollary 4. *In general, for any sufficiently small neighborhood of any point on the image of an embedded curve in a spacetime, there are (infinitely) many flat metrics with the properties described in theorem 1.*

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> ***Corollary 4.*** *In general, for any sufficiently small neighborhood of any point on the image of an embedded curve in a spacetime, there are (infinitely) many flat metrics with the properties described in theorem 1.*

From this, F&W conclude:

> *It follows that while every spacetime is locally approximately flat, none is canonically so, as there are many flat metrics locally approximating any given metric at any given point. This failure of uniqueness is obscured, in our view, by approaches that focus on particular construction procedures, or on the existence of certain coordinates, because not all such flat metrics (or normal coordinates) arise from a single construction procedure.*

F&W's scheme for approximate local flatness

Choose any smooth Riemannian metric h_{ab} on U. Physically, any such metric can be determined by a smooth, orthonormal frame field $\{\overset{i}{u}{}^a\}_{i\in\{0,\ldots 3\}}$ on U, as $\sum_{i=0}^{3} \overset{i}{u}_a \overset{i}{u}_b$ is a smooth Riemannian metric. [...] We may then define, relative to h^{ab}, a norm on covariant tensors $f_{a_1\cdots a_n}$ at a point by: [footnote suppressed]

$$|f|_h = \left| h^{a_1 b_1} \cdots h^{a_n b_n} f_{a_1 \cdots a_n} f_{b_1 \cdots b_n} \right|^{1/2}$$

[...] Using this family of norms, we can define a family of distance functions on tensors as:

$$d_U(f, f'; h, k) = \max_{j\in\{0,\ldots,k\}} \sup_U \left| (\nabla)^j (f - f') \right|_h,$$

where $(\nabla)^j$ abbreviates "acts with j derivatives," ∇ is the Levi-Civita derivative operator determined by h_{ab}, and $(f - f')$ abbreviates $f_{a_1\cdots a_n} - f'_{a_1\cdots a_n}$. What this distance function does is return the greatest distance, relative to h_{ab}, between f and f' or any of their first k derivatives, ranging over all points in U. [...]

F&W's scheme for approximate local flatness

The case of greatest interest here will be when we use distance functions defined in this way to measure distances between different Lorentzian metrics on U. Indeed, let g_{ab} and $\bar{g}_{ab}$, and O be as in the statement of Theorem 1. Then it immediately follows from the smoothness of g_{ab} and $\bar{g}_{ab}$ that for any h_{ab} on O and any $\epsilon > 0$, there exists a neighbourhood $U \subseteq O$ such that $d_U(f, f'; h, k) < \epsilon$. Thus we see that not only do the two metrics coincide at p, but they also approximate one another, to first order, arbitrarily well in sufficiently small neighborhoods of p.

Fletcher (2020) on approximate isometry

a local diffeomorphism $\psi : U \to V$ (of course with $U, V \subseteq M$) is an (h, ϵ)-spacetime symmetry to order k on U when $d_U(g, \psi^*(g); h, k) < \epsilon$. (Note that when ψ is a member of a one-parameter family of local diffeomorphisms generated by a local Killing vector field κ, this is equivalent to the condition that $\sup_U \left|\mathcal{L}_\kappa \nabla^{(j)} g\right|_h \leq \epsilon$.)

Upshots from the approximation scheme

Theorem 5. *Given any spacetime* (M, g_{ab})*, embedded curve* $\gamma : I \to M$*, point* $p \in \gamma[I]$*, compact neighbourhood* U *of* p*, Riemannian metric* h_{ab} *on* U*, real* $\epsilon > 0$*, spacetime* (M', g'_{ab})*, and point* $p' \in M'$*, there exist neighbourhoods* $O \ni p$ *and* $O' \ni p'$*, an embedded curve* $\gamma' : I' \to M'$ *with* $p' \in \gamma'[I']$*, and an* $(h, 1, \epsilon)$*-isometry* $\chi : O' \to O$ *between* (O, g_{ab}) *and* (O', g'_{ab}) *satisfying* $\chi \circ \gamma' = \gamma$ *on* I' *and* $\chi^*(g_{ab}) = g'_{ab}$ *on* $\gamma'[I']$*.*

Upshots from the approximation scheme

If we call any spacetime fulfilling the role of Minkowski spacetime [...] a universal locally approximating spacetime, then Theorem 5 shows that every spacetime is a universal locally approximating spacetime. For example, one could equally well take (anti-)de Sitter spacetime or Schwarzschild spacetime to play this role. So, it may be misleading to assert that "free-falling observers see no effect of gravity in their immediate vicinity" (Poisson 2004, 11); one might just as well say "free-falling observers see the local effects of a large cosmological constant" or "free-falling observers see the local effects of being inside a rotating black hole." (FW 2023a, p. 14)

Immediate worry: physical significance of h_{ab} and ϵ

Such a distance function has, I think, severe problems of physical interpretation, which can in large part be traced to the fact that the positive-definite metrics themselves used to fix the distance function have no physical significance (Geroch 1967, 1971; Curiel 2015). Unless the authors explain why we should take these constructions as illuminating or capturing something of physical or conceptual importance, it seems that they are doing only *geometry here, not foundational work. [...] I want to see an argument that explains and justifies the physical significance of these objects, in a way relevant to the foundational aims of the paper: what does such an h_{ab} have to do with* spacetime *structure and its conceptual and physical interpretation?* (Erik Curiel, personal communication)

What to do?

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- ...so how to reconcile FW's results with our defence of the 'standard story' seen previously?

How to proceed?

The following passage from Fletcher & Weatherall (2023a, pp. 14–5) is important:

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> *All this said, the fact that other spacetimes are universal locally approximating does not imply that Minkowski spacetime is not—and so one might ask whether there are other reasons to think that Minkowski spacetime has a distinguished role to play (beyond its pragmatic advantages already noted). One possible answer would return to an issue we raised previously, [...] in some discussions of local (approximate) flatness, authors present particular constructions of normal coordinates, or flat approximating metrics, motivated by physical considerations. [... T]his sort of argument purchases a special status for Minkowski spacetime at the cost of assuming a special status for a particular coordinate construction procedure.*

A possible line of response

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- But surely there is still something privileged (i.e., more physically interesting) about the LRT construction:
  - the 'base point' is associated with the tangent space Minkowski metric, and
  - the expansion is straightforwardly adapted (via the world function) to the intrinsic geometry of the problem.

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  - the expansion is straightforwardly adapted (via the world function) to the intrinsic geometry of the problem.
- Approximation doesn't always track what's of physical interest...
- In some sense, FW push all of the decisions about what expansions people choose to use into pragmatics—but doesn't this in some sense leave interesting questions unanswered?
  - E.g., Why do physicists tend to think in terms of the LRT construction rather than some other?

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- However, it's not itself sufficient to overcome to FW worries.
- FW are likely correct to call for some criteria for when material bodies behave 'as if' they're special relativistic—this is likely where most of the action is (this, indeed, is the topic of their (2023b)).

Enter Wallace (2017)

Einstein Studies
13

Dennis Lehmkuhl
Gregor Schiemann
Erhard Scholz
Editors

Towards a Theory of Spacetime Theories

Birkhäuser

Scale-relativity and Wallace

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Here’s Wallace:

> *So for any given level of tolerable deviation from the Minkowski metric, we can find a tube around the black hole such that the spacetime outside that tube approximates the Minkowski metric to that level.* (Wallace 2017, p. 263)

Then the rough idea is that, if we ‘zoom out’ far enough, that system will be approximately Minkowskian.

LRT on the physical meaning of spacetime points

This seems to chime well with LRT's 'scale-relative' perspective and their understanding of the physical meaning of spacetime points:

> *At this stage, as we are apparently dealing with a lab frame defined at a 'point', it is worth contemplating what is meant by a 'point' in GR to begin with. Notably, a manifold point $p \in M$ does not represent an extensionless event—rather, it can be understood as a mathematical notion that we bring to bear upon some particular modelling context. The context that will concern us in this article is that in which the characteristic length scale of some object (a black hole, a lump of matter, and so on) is sufficiently small relative to some other relevant background length scale that the object is well modelled by a (mathematical) point. Thus understood, the point is rather analogous to point particles.* (Linnemann *et al.* 2024, p. 5)

Back to Wallace

We can coherently talk about isolated systems in general relativity because, as a matter of dynamics, there exist a large number of solutions to the equations—including ones which represent stars, planets, black holes, etc., as well as interacting sets of these—where the curvature and matter are concentrated in some finite region and far outside that region the spacetime is approximately empty and flat. This allows us to paste such solutions together, to form regions of spacetime consisting of a number of isolated subsystems embedded in approximately flat spacetime. Because of the Poincaré symmetry of flat spacetime, we can perform a Poincaré transformation on one of the subsystems without violating the boundary conditions between subsystems; hence, the relativity principle applies for collections of such subsystems ...

Back to Wallace

> *... In turn, regions of effectively flat spacetime can always be found in a given spacetime, provided we are prepared to make those regions sufficiently small. If "sufficiently small" is nonetheless large compared to the effective size of the subsystems we are interested in, then (a) we can apply the above argument for the relativity principle to isolated systems in a curved spacetime; (b) we can embed such systems in any such effectively flat region without affecting their internal dynamics, since their Minkowski boundary conditions are compatible with any region flat on sufficiently large lengthscales.* (Wallace 2017, p. 265)

Stepping back, reflecting

- This seems to imply that it is properties of the solutions used to model the systems under considerations—e.g., the asymptotic flatness of solutions describing black holes—which allows one to say that those systems are aptly described by SR as one ‘zooms out’.

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- As such, Wallace perhaps gives us the resources to weave all of this together?

Today

The equivalence principle

The local validity of SR as a heuristic

Local validity of SR in the completed theory of GR

Interpretative significance of the local validity of SR

Conclusions

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Here, for example, is a well-known quote from Brown:

> *It is because of minimal coupling and local Lorentz covariance that rods and clocks, built out of the matter fields which display that symmetry, behave as if they were reading aspects of the metric field and in so doing confer on this field a geometric meaning. That light rays trace out null geodesics of the field is again a consequence of the strong equivalence principle, which asserts that locally Maxwell's equations of electrodynamics are valid.* (Brown 2005, p. 176)

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- If the claim is that the SEP affords g_{ab} a “geometric meaning” in GR... well, we saw previously in our discussion of heuristics that we didn’t *need* to give this field a geometric interpretation.
- If the claim (which one can find in later works such as Read *et al.* (2018)) is that g_{ab} acquires its ‘chronogeometric significance’ via the SEP, then on reflection this also seems questionable, on various grounds...

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3. One might in general, with FW, find SEP to be intolerably unclear—cf. Fletcher (2020) and Weatherall (2020).
4. There are other ways in which g_{ab} can come to be ‘read off’ by material bodies—e.g., the EPS (1972) construction, on which see e.g. Adlam *et al.* (2025).

Tentative conclusion on interpretative significance

Perhaps SEP is not required for the physical interpretation/chronogeometric significance of the metric field g_{ab} in GR after all, *pace* Brown (2005).

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- The necessity of SEP for the interpretation of GR seems questionable.

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