Advanced Philosophy of Physics: The Aharanov–Bohm Effect

James Read[1]

[1]Faculty of Philosophy, University of Oxford, UK, OX2 6GG

HT25-W3

The plan

W1: The philosophy of symmetries

W2: The hole argument

W3: The Aharanov–Bohm effect

W4: The local validity of special relativity

Today

Gaugeymmetries

Locality, separability, and determinism

Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

Gauge theories

- Today, we're going to be thinking about some puzzling features of electromagnetism.

Gauge theories

- Today, we're going to be thinking about some puzzling features of electromagnetism.
- These will have to do with the fact that it is a *gauge theory*.

Gauge theories

- Today, we're going to be thinking about some puzzling features of electromagnetism.
- These will have to do with the fact that it is a *gauge theory*.
- But what is a gauge theory? Here the puzzles already begin...

Puzzles over gauge

The word "gauge" is ubiquitous in modern physics. Our best physical theories are described, in various contexts, as "gauge theories." The "gauge argument" allegedly reveals the underlying "logic of nature" (Martin 2002). Our theories regularly exhibit "gauge freedom," "gauge structure," and "gauge dependence." Unfortunately, however, it is far from clear that the term has some univocal meaning across the many contexts in which it appears. It is a bit like "liberal" in American political discourse: it shows up everywhere, and no one knows what it means. (Weatherall 2016, p. 1039)

Three senses of ‘gauge’

There are (at least) three key ways of understanding what it is for a theory to be a ‘gauge theory’—the first is interpretational, and the second and third are mathematical:

Three senses of ‘gauge’

There are (at least) three key ways of understanding what it is for a theory to be a ‘gauge theory’—the first is interpretational, and the second and third are mathematical:

G1: Theories with ‘representational redundancy’, in the sense of their having ‘redundant’, ‘non-physical’ degrees of freedom.

Three senses of ‘gauge’

There are (at least) three key ways of understanding what it is for a theory to be a ‘gauge theory’—the first is interpretational, and the second and third are mathematical:

G1: Theories with ‘representational redundancy’, in the sense of their having ‘redundant’, ‘non-physical’ degrees of freedom.

G2: Theories with symmetries which are *local*, in the sense of being spacetime-dependent.

Three senses of ‘gauge’

There are (at least) three key ways of understanding what it is for a theory to be a ‘gauge theory’—the first is interpretational, and the second and third are mathematical:

G1: Theories with ‘representational redundancy’, in the sense of their having ‘redundant’, ‘non-physical’ degrees of freedom.

G2: Theories with symmetries which are *local*, in the sense of being spacetime-dependent.

G3: Theories with a certain mathematical structure—namely, built using principal fibre bundles.

Three senses of ‘gauge’

There are (at least) three key ways of understanding what it is for a theory to be a ‘gauge theory’—the first is interpretational, and the second and third are mathematical:

- G1: Theories with ‘representational redundancy’, in the sense of their having ‘redundant’, ‘non-physical’ degrees of freedom.
- G2: Theories with symmetries which are *local*, in the sense of being spacetime-dependent.
- G3: Theories with a certain mathematical structure—namely, built using principal fibre bundles.

Electromagnetism can be understood a gauge theory in all of these senses—but actually, the notions can come apart!

Today

Gauge symmetries

Locality, separability, and determinism

Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

Locality, separability, and determinism

The next thing to get clear on are three key properties of physical theories:

1. Locality
2. Separability
3. Determinism

Locality

What does it mean for a theory to be *local*?

Locality

What does it mean for a theory to be *local*?

- The objects of the world can only be affected by objects in the neighbourhood of their location?

Locality

What does it mean for a theory to be *local*?

- The objects of the world can only be affected by objects in the neighbourhood of their location?
- The objects and properties of the world have well defined spatial locations?

Locality

What does it mean for a theory to be *local*?

- The objects of the world can only be affected by objects in the neighbourhood of their location?
- The objects and properties of the world have well defined spatial locations?
- We cannot produce observable effects on objects not in our neighbourhood?

Locality

What does it mean for a theory to be *local*?

- The objects of the world can only be affected by objects in the neighbourhood of their location?
- The objects and properties of the world have well defined spatial locations?
- We cannot produce observable effects on objects not in our neighbourhood?

Is locality the conjunction of these? Or some subset or other?

Action-at-a-distance

- One thing which locality could mean is this: *no action at a distance*.

Action-at-a-distance

- One thing which locality could mean is this: *no action at a distance*.
- If a physical theory allows one system to directly change, influence, alter, or otherwise interact with another system at a remote location, unmediated by some material connection, this is action-at-a-distance.

Two senses of ‘at-a-distance’

Einstein locality: ‘At-a-distance’ means at a spacelike separation between spacetime events. This assumes at minimum special relativity theory.

Two senses of ‘at-a-distance’

Einstein locality: ‘At-a-distance’ means at a spacelike separation between spacetime events. This assumes at minimum special relativity theory.

Bell locality: ‘At-a-distance’ means beyond causal influences recognised by current physical theories.

Two senses of ‘at-a-distance’

Einstein locality: ‘At-a-distance’ means at a spacelike separation between spacetime events. This assumes at minimum special relativity theory.

Bell locality: ‘At-a-distance’ means beyond causal influences recognised by current physical theories.

Einstein locality implies Bell locality but not the other way around. (See Redhead (1987) for more.)

Separability

- Unlike locality (which, roughly, is a thesis about the action that one system can exert upon another distant system), *separability* is a metaphysical thesis.

Separability

- Unlike locality (which, roughly, is a thesis about the action that one system can exert upon another distant system), *separability* is a metaphysical thesis.
- Separability says this: *the properties of a system supervene upon the properties of its subsystems*.

Separability

- Unlike locality (which, roughly, is a thesis about the action that one system can exert upon another distant system), *separability* is a metaphysical thesis.
- Separability says this: *the properties of a system supervene upon the properties of its subsystems*.

For more on separability, see e.g. (Healey & Gomes 2022).

Determinism

- As a first pass, we can say that a physical theory is deterministic iff (a) the laws of that theory, plus (b) a specification of the state of the universe at a particular time, fixes the state of the universe at all other times.

Determinism

- As a first pass, we can say that a physical theory is deterministic iff (a) the laws of that theory, plus (b) a specification of the state of the universe at a particular time, fixes the state of the universe at all other times.
- This is what's sometimes called 'Laplacian determinism'.

Determinism

- As a first pass, we can say that a physical theory is deterministic iff (a) the laws of that theory, plus (b) a specification of the state of the universe at a particular time, fixes the state of the universe at all other times.
- This is what's sometimes called 'Laplacian determinism'.
- There is actually a vast literature on different possible definitions of determinism—Earman (1986) is the *locus classicus*.

Foreshadowing some worries about electromagnetism

The local gauge freedom of electromagnetism already invites worries about indeterminism:

Foreshadowing some worries about electromagnetism

The local gauge freedom of electromagnetism already invites worries about indeterminism:

> *Firstly, it seems that the gauge transformations of electrodynamics relate empirically equivalent yet physically distinct states of affairs, implying an underdetermination of empirical facts by the theory's dynamics. Secondly, the fact that gauge symmetries are local means that one can construct analogues of the infamous Hole Argument: transformations that act trivially before some time t, but non-trivially thereafter. This implies a failure of indeterminism.* (Jacobs 2023, p. 34)

Today

Gauge symmetries

Locality, separability, and determinism

Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

Formulations of electromagnetism

In anticipation of discussing the Aharanov–Bohm effect, let's now move on to presenting various different formulations of electromagnetism—to begin with, there will be three which are of interest:

Formulations of electromagnetism

In anticipation of discussing the Aharanov–Bohm effect, let’s now move on to presenting various different formulations of electromagnetism—to begin with, there will be three which are of interest:

1. A-field electromagnetism
2. F-field electromagnetism
3. Holonomy-based electromagnetism

Formulations of electromagnetism

In anticipation of discussing the Aharanov–Bohm effect, let's now move on to presenting various different formulations of electromagnetism—to begin with, there will be three which are of interest:

1. A-field electromagnetism
2. F-field electromagnetism
3. Holonomy-based electromagnetism

I'll go through each of these in turn now. Before I get there, though, a brief general EM recap... (Material which could be found in e.g. (Jackson 1998).)

Maxwell's equations in 3-vector formulation

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

Maxwell's equations in Faraday tensor formulation

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix},$$

$$J^{\mu} = \begin{pmatrix} \rho \\ J^i \end{pmatrix}.$$

Maxwell's equations in Faraday tensor formulation

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix},$$

$$J^\mu = \begin{pmatrix} \rho \\ J^i \end{pmatrix}.$$

Then Maxwell's equations can be written:

$$\eta_{\mu\lambda}\partial^\lambda F^{\mu\nu} = J^\nu,$$

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} =: \partial_{[\mu} F_{\nu\lambda]} = 0.$$

Maxwell's equations in Faraday tensor formulation

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix},$$

$$J^\mu = \begin{pmatrix} \rho \\ J^i \end{pmatrix}.$$

Then Maxwell's equations can be written:

$$\eta_{\mu\lambda}\partial^\lambda F^{\mu\nu} = J^\nu,$$

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} =: \partial_{[\mu} F_{\nu\lambda]} = 0.$$

Or in differential forms:

$$d * F = J,$$

$$dF = 0.$$

Introducing the vector potential

One can define—and, indeed, it turns out to be useful for a lot of physics of EM to do so—a vector field $A^\mu = (\phi, \mathbf{A})$, the 'electromagnetic vector potential', as

$$F^{\mu\nu} =: \partial^\mu A^\nu - \partial^\nu A^\mu.$$

Introducing the vector potential

One can define—and, indeed, it turns out to be useful for a lot of physics of EM to do so—a vector field $A^\mu = (\phi, \mathbf{A})$, the 'electromagnetic vector potential', as

$$F^{\mu\nu} =: \partial^\mu A^\nu - \partial^\nu A^\mu.$$

One can then compute the electric and magnetic fields directly from the vector potential, as

$$\begin{aligned} \mathbf{E} &= -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}, \\ \mathbf{B} &= \nabla \times \mathbf{A}. \end{aligned}$$

Introducing the vector potential

One can define—and, indeed, it turns out to be useful for a lot of physics of EM to do so—a vector field $A^\mu = (\phi, \mathbf{A})$, the 'electromagnetic vector potential', as

$$F^{\mu\nu} =: \partial^\mu A^\nu - \partial^\nu A^\mu.$$

One can then compute the electric and magnetic fields directly from the vector potential, as

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t},$$
$$\mathbf{B} = \nabla \times \mathbf{A}.$$

But there is some representational/gauge redundancy, because if we take

$$A^\mu \to A^\mu + \partial^\mu \Lambda,$$

this will yield the same electric and magnetic fields!

Maxwell's equations in vector potential formulation

In any case, writing Maxwell's equations in terms of the vector potential yields:

$$-\partial^\mu \partial_\mu A^\alpha + \partial^\alpha \partial_\mu A^\mu = J^\alpha.$$

Maxwell's equations in vector potential formulation

In any case, writing Maxwell's equations in terms of the vector potential yields:

$$-\partial^\mu \partial_\mu A^\alpha + \partial^\alpha \partial_\mu A^\mu = J^\alpha.$$

Since in differential forms $F = dA$, it's easy to see that the Maxwell equations in terms of the vector potential should be:

$$\begin{aligned} d * dA &= J, \\ ddA &= 0. \end{aligned}$$

Maxwell's equations in vector potential formulation

In any case, writing Maxwell's equations in terms of the vector potential yields:

$$-\partial^\mu \partial_\mu A^\alpha + \partial^\alpha \partial_\mu A^\mu = J^\alpha.$$

Since in differential forms $F = dA$, it's easy to see that the Maxwell equations in terms of the vector potential should be:

$$\begin{aligned} d * dA &= J, \\ ddA &= 0. \end{aligned}$$

(The latter, recall from Week 1, is a mathematical identity—the twice exterior derivative necessarily vanishes.)

Turning to the formulations

So much for the background. With all of this in hand, I'm now in a position to introduce formally the first two of our three different versions of electromagnetism, which recall again were:

Turning to the formulations

So much for the background. With all of this in hand, I'm now in a position to introduce formally the first two of our three different versions of electromagnetism, which recall again were:

1. A-field electromagnetism
2. F-field electromagnetism
3. Holonomy-based electromagnetism

A-field electromagnetism

Kinematically possible models (KPMs) given by $\langle M, \eta_{ab}, A^a, J^a \rangle$.

A-field electromagnetism

Kinematically possible models (KPMs) given by $\langle M, \eta_{ab}, A^a, J^a \rangle$.

Dynamically possible models (DPMs) given by

$$-\nabla^a \nabla_a A^b + \nabla^b \nabla_a A^a = J^b.$$

A-field electromagnetism

Kinematically possible models (KPMs) given by $\langle M, \eta_{ab}, A^a, J^a \rangle$.

Dynamically possible models (DPMs) given by

$$-\nabla^a \nabla_a A^b + \nabla^b \nabla_a A^a = J^b.$$

This approach is:

- Local
- Separable
- Indeterministic (can generate a version of the hole argument with it; recall the Jacobs quote from before).

F-field electromagnetism

KPMs given by $\langle M, \eta_{ab}, F_{ab}, J^a \rangle$.

F-field electromagnetism

KPMs given by $\langle M, \eta_{ab}, F_{ab}, J^a \rangle$.

DPMs given by

$$\eta_{ab}\nabla^b F^{ac} = J^c,$$
$$\nabla_{[a}F_{bc]} = 0.$$

F-field electromagnetism

KPMs given by $\langle M, \eta_{ab}, F_{ab}, J^a \rangle$.

DPMs given by

$$\eta_{ab} \nabla^b F^{ac} = J^c,$$
$$\nabla_{[a} F_{bc]} = 0.$$

This approach is:

- Non-local (for reasons to do with the AB effect, to be discussed in a minute)
- Separable
- Deterministic

Loop holonomies

To talk about our next approach to electromagnetism—the *holonomy* interpretation—I first need to introduce the notion of a loop holonomy:

Loop holonomies

To talk about our next approach to electromagnetism—the *holonomy* interpretation—I first need to introduce the notion of a loop holonomy:

- For a vector field A_μ, we can defined its *holonomy* as

$$H(l) = \exp\left(-iq\oint_l A_\mu dx^\mu\right).$$

Loop holonomies

To talk about our next approach to electromagnetism—the *holonomy* interpretation—I first need to introduce the notion of a loop holonomy:

- For a vector field A_μ, we can defined its *holonomy* as

$$H(l) = \exp\left(-iq \oint_l A_\mu dx^\mu\right).$$

- In holonomy-based electromagnetism, there is a fundamental non-localised property $H(l)$ associated to every closed curve l in spacetime.

Holonomy-based electromagnetism

KPMs given by $\langle M, H, J^a \rangle$.

Holonomy-based electromagnetism

KPMs given by $\langle M, H, J^a \rangle$.

DPMs given by...

Holonomy-based electromagnetism

KPMs given by $\langle M, H, J^a \rangle$.

DPMs given by... ???

Holonomy-based electromagnetism

KPMs given by $\langle M, H, J^a \rangle$.

DPMs given by... ???

This approach is:

- Local
- Non-separable
- Deterministic

Today

Gauge symmetries

Locality, separability, and determinism

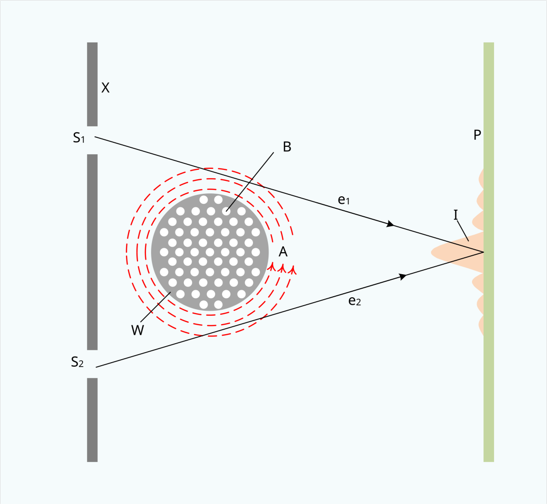
Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

The Aharanov–Bohm effect



The Aharanov–Bohm effect summarised

- The Aharanov–Bohm effect is essentially a modified double slit experiment in which a(n impenetrable) solenoid is placed between the plate and the screen.

The Aharanov–Bohm effect summarised

- The Aharanov–Bohm effect is essentially a modified double slit experiment in which a(n impenetrable) solenoid is placed between the plate and the screen.
- The Aharanov–Bohm effect then refers to the fact that the interference pattern on the screen changes when we let a current run through the solenoid.

The Aharanov–Bohm effect summarised

- The Aharanov–Bohm effect is essentially a modified double slit experiment in which a(n impenetrable) solenoid is placed between the plate and the screen.
- The Aharanov–Bohm effect then refers to the fact that the interference pattern on the screen changes when we let a current run through the solenoid.
- This is the case despite the fact that the electromagnetic field vanishes outside the solenoid!

The Aharanov–Bohm effect summarised

- The Aharanov–Bohm effect is essentially a modified double slit experiment in which a(n impenetrable) solenoid is placed between the plate and the screen.
- The Aharanov–Bohm effect then refers to the fact that the interference pattern on the screen changes when we let a current run through the solenoid.
- This is the case despite the fact that the electromagnetic field vanishes outside the solenoid!
- Hence, we cannot simply understand the effect as a result of the force field acting locally on the matter field.

The Aharanov–Bohm effect summarised

- The Aharanov–Bohm effect is essentially a modified double slit experiment in which a(n impenetrable) solenoid is placed between the plate and the screen.
- The Aharanov–Bohm effect then refers to the fact that the interference pattern on the screen changes when we let a current run through the solenoid.
- This is the case despite the fact that the electromagnetic field vanishes outside the solenoid!
- Hence, we cannot simply understand the effect as a result of the force field acting locally on the matter field.
- This led Aharonov and Bohm (1959) to posit the vector potential A^a—which *doesn't* vanish outside the solenoid—as causally responsible for the effect, despite the fact that it is usually considered to be 'gauge'!

Background on the AB effect

- The time-dependent Schrödinger equation for a single particle of mass m and charge q immersed in an electromagnetic field is

$$i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t} = \left[\frac{1}{2m} \left(-i\hbar \nabla - q\mathbf{A}(\mathbf{x}, t) \right)^2 + q\phi(\mathbf{x}, t) \right] \psi(\mathbf{x}, t).$$

Background on the AB effect

- The time-dependent Schrödinger equation for a single particle of mass m and charge q immersed in an electromagnetic field is

$$i\hbar\frac{\partial\psi\left(\mathbf{x},t\right)}{\partial t}=\left[\frac{1}{2m}\left(-i\hbar\nabla-q\mathbf{A}\left(\mathbf{x},t\right)\right)^{2}+q\phi\left(\mathbf{x},t\right)\right]\psi\left(\mathbf{x},t\right).$$

- This equation is invariant under the gauge transformations:

$$A_\mu=(\phi,\mathbf{A})\rightarrow A_\mu+\frac{1}{q}\partial_\mu\alpha(x),$$
$$\psi\rightarrow e^{iq\alpha(x)}\psi,$$

where $\alpha(x)$ is a function of the spacetime coordinates x.

The question

- For the AB effect, we consider the field

$$\psi(x) = \psi_I(x) + \psi_{II}(x),$$

where ψ_I and ψ_{II} are the components of the field that pass through the left and right slit, respectively.

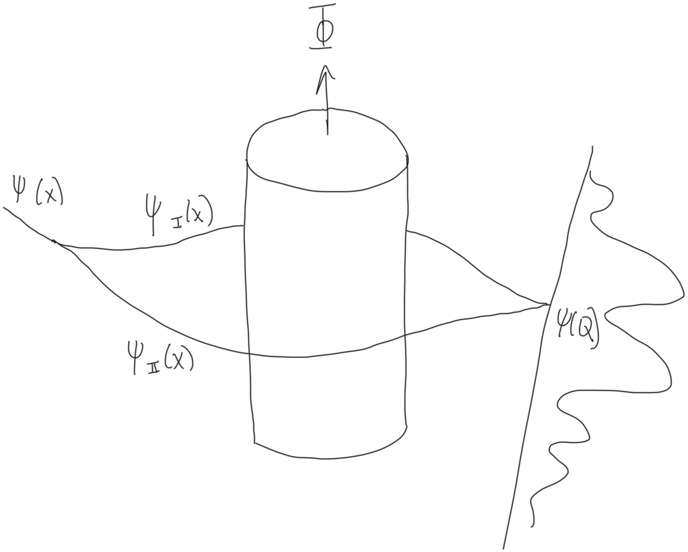
The question

- For the AB effect, we consider the field

$$\psi(x) = \psi_I(x) + \psi_{II}(x),$$

  where ψ_I and ψ_{II} are the components of the field that pass through the left and right slit, respectively.
- Let Q denote an arbitrary point on the screen. What happens to $\psi(Q)$ when we turn on the solenoid?

The Aharanov–Bohm effect



Mathematical warmup

- Consider a region of space R which is simply connected (no ‘holes’), so that all closed curves in R can be continuously deformed to a point.

Mathematical warmup

- Consider a region of space R which is simply connected (no ‘holes’), so that all closed curves in R can be continuously deformed to a point.
- Suppose the magnetic field strength vanishes everywhere in R:

$$\mathbf{B} = \nabla \times \mathbf{A} = 0.$$

Mathematical warmup

- Consider a region of space R which is simply connected (no ‘holes’), so that all closed curves in R can be continuously deformed to a point.
- Suppose the magnetic field strength vanishes everywhere in R:
$$\mathbf{B} = \nabla \times \mathbf{A} = 0.$$
- Recall that the following is a mathematical identity for any scalar f:
$$\nabla \times \nabla f = 0.$$

Mathematical warmup

- Consider a region of space R which is simply connected (no 'holes'), so that all closed curves in R can be continuously deformed to a point.
- Suppose the magnetic field strength vanishes everywhere in R:
$$\mathbf{B} = \nabla \times \mathbf{A} = 0.$$
- Recall that the following is a mathematical identity for any scalar f:
$$\nabla \times \nabla f = 0.$$
- So when $\mathbf{B} = 0$, we can write $\mathbf{A} = \nabla f$.

Mathematical warmup

- Consider a region of space R which is simply connected (no 'holes'), so that all closed curves in R can be continuously deformed to a point.
- Suppose the magnetic field strength vanishes everywhere in R:
$$\mathbf{B} = \nabla \times \mathbf{A} = 0.$$
- Recall that the following is a mathematical identity for any scalar f:
$$\nabla \times \nabla f = 0.$$
- So when $\mathbf{B} = 0$, we can write $\mathbf{A} = \nabla f$.
- But given the gauge freedom $\mathbf{A} \to \mathbf{A} + \nabla\kappa$, it is clear that f is defined only up to a constant additive factor.

Mathematical warmup

- Consider a region of space R which is simply connected (no 'holes'), so that all closed curves in R can be continuously deformed to a point.
- Suppose the magnetic field strength vanishes everywhere in R:

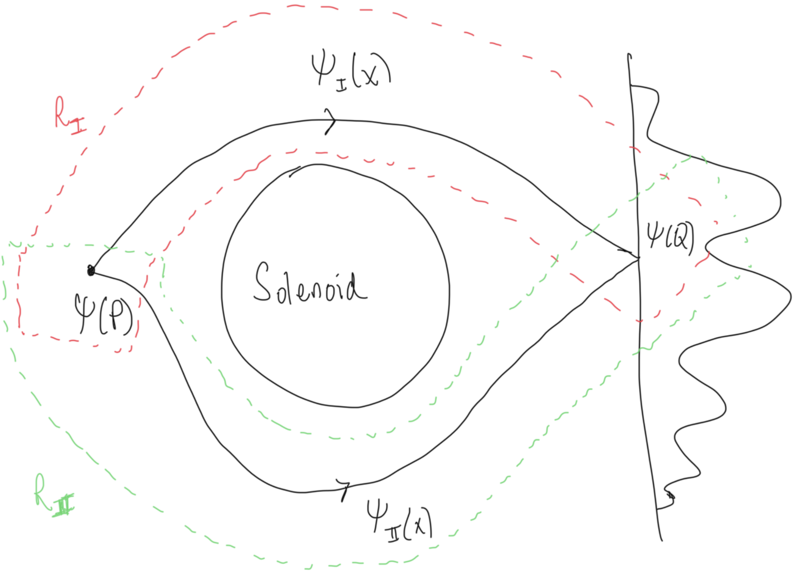
$$\mathbf{B} = \nabla \times \mathbf{A} = 0.$$

- Recall that the following is a mathematical identity for any scalar f:

$$\nabla \times \nabla f = 0.$$

- So when $\mathbf{B} = 0$, we can write $\mathbf{A} = \nabla f$.
- But given the gauge freedom $\mathbf{A} \to \mathbf{A} + \nabla\kappa$, it is clear that f is defined only up to a constant additive factor.
- Given this, for any given origin point $P \in R$, we can always set f such that $f(P) = 0$; let us do so.

The Aharanov–Bohm effect



Mathematical warmup continued

- Consider now the line integral $\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}$, defined in relation to some curve joining P and Q in region R.

Mathematical warmup continued

- Consider now the line integral $\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}$, defined in relation to some curve joining P and Q in region R.
- Note that

$$\mathbf{A} \cdot \mathrm{d}\mathbf{x} = \nabla f \cdot \mathrm{d}\mathbf{x} = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z = \mathrm{d}f.$$

Mathematical warmup continued

- Consider now the line integral $\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}$, defined in relation to some curve joining P and Q in region R.
- Note that

$$\mathbf{A} \cdot \mathrm{d}\mathbf{x} = \nabla f \cdot \mathrm{d}\mathbf{x} = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z = \mathrm{d}f.$$

- So,

$$\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x} = \int_P^Q \mathrm{d}f = f(Q) - f(P) = f(Q).$$

Mathematical warmup continued

- Consider now the line integral $\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}$, defined in relation to some curve joining P and Q in region R.
- Note that

$$\mathbf{A} \cdot \mathrm{d}\mathbf{x} = \nabla f \cdot \mathrm{d}\mathbf{x} = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z = \mathrm{d}f.$$

- So,

$$\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x} = \int_P^Q \mathrm{d}f = f(Q) - f(P) = f(Q).$$

- Thus, the line integral is path-independent, and for fixed P can be taken to be a function $f(Q)$.

Mathematical warmup continued

- Consider now the line integral $\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}$, defined in relation to some curve joining P and Q in region R.
- Note that

$$\mathbf{A} \cdot \mathrm{d}\mathbf{x} = \nabla f \cdot \mathrm{d}\mathbf{x} = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + \frac{\partial f}{\partial z}\mathrm{d}z = \mathrm{d}f.$$

- So,

$$\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x} = \int_P^Q \mathrm{d}f = f(Q) - f(P) = f(Q).$$

- Thus, the line integral is path-independent, and for fixed P can be taken to be a function $f(Q)$.
- Under gauge transformations, then, the wavefunction transforms as

$$\begin{aligned}\psi(Q) \to \psi'(Q) &= \exp\left(iqf(Q)/\hbar\right)\psi \\ &= \exp\left(\frac{iq}{\hbar}\int_P^Q \mathbf{A} \cdot \mathrm{d}\mathbf{x}\right)\psi(Q).\end{aligned}$$

The AB effect proper

- Suppose first that there is zero current in the solenoid. We can write the value of the wavefunction of each electron at point Q on the screen as

$$\psi^0(Q) = \psi_I^0(Q) + \psi_{II}^0(Q).$$

The AB effect proper

- Suppose first that there is zero current in the solenoid. We can write the value of the wavefunction of each electron at point Q on the screen as

$$\psi^0(Q) = \psi_I^0(Q) + \psi_{II}^0(Q).$$

- Now turn on the current.

The AB effect proper

- Suppose first that there is zero current in the solenoid. We can write the value of the wavefunction of each electron at point Q on the screen as

$$\psi^0(Q) = \psi_I^0(Q) + \psi_{II}^0(Q).$$

- Now turn on the current.
- Consider the simply connected region R_I, containing point P and Q and slit I.

The AB effect proper

- Suppose first that there is zero current in the solenoid. We can write the value of the wavefunction of each electron at point Q on the screen as

$$\psi^0(Q) = \psi^0_I(Q) + \psi^0_{II}(Q).$$

- Now turn on the current.
- Consider the simply connected region R_I, containing point P and Q and slit I.
- Since $\mathbf{B} = \nabla \times \mathbf{A} = 0$ everywhere in this region, we can write:

$$\psi_I(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot d\mathbf{x}_I\right)\psi^0_I(Q).$$

The AB effect proper

- Suppose first that there is zero current in the solenoid. We can write the value of the wavefunction of each electron at point Q on the screen as

$$\psi^0(Q) = \psi^0_I(Q) + \psi^0_{II}(Q).$$

- Now turn on the current.
- Consider the simply connected region R_I, containing point P and Q and slit I.
- Since $\mathbf{B} = \nabla \times \mathbf{A} = 0$ everywhere in this region, we can write:

$$\psi_I(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I\right)\psi^0_I(Q).$$

- We have an analogous equation for region II, so overall we have

$$\psi(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I\right)\psi^0_I(Q) + \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_{II}\right)\psi^0_{II}(Q).$$

The AB effect proper

- It is crucial to recognise that this is *not* a mere gauge transformation!

The AB effect proper

- It is crucial to recognise that this is *not* a mere gauge transformation!
- We can rewrite the previous expression as

$$\psi(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I\right)\left[\psi_I^0(Q) + \exp\left\{\left(\frac{ie}{\hbar}\right)\left(-\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_{II}\right)\right\}\psi_{II}^0(Q)\right].$$

The AB effect proper

- It is crucial to recognise that this is *not* a mere gauge transformation!
- We can rewrite the previous expression as

$$\psi(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I\right)\left[\psi_I^0(Q) + \exp\left\{\left(\frac{ie}{\hbar}\right)\left(-\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_{II}\right)\right\}\psi_{II}^0(Q)\right].$$

- Now,

$$\begin{aligned} -\int_P^Q \mathbf{A}\cdot d\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot d\mathbf{x}_{II} &= \int_Q^P \mathbf{A}\cdot d\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot d\mathbf{x}_{II} \\ &= \oint \mathbf{A}\cdot d\mathbf{x}. \end{aligned}$$

The AB effect proper

- It is crucial to recognise that this is *not* a mere gauge transformation!
- We can rewrite the previous expression as

$$\psi(Q) = \exp\left(\frac{ie}{\hbar}\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I\right)\left[\psi_I^0(Q) + \exp\left\{\left(\frac{ie}{\hbar}\right)\left(-\int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_{II}\right)\right\}\psi_{II}^0(Q)\right].$$

- Now,

$$\begin{aligned} -\int_P^Q \mathbf{A}\cdot d\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot d\mathbf{x}_{II} &= \int_Q^P \mathbf{A}\cdot d\mathbf{x}_I + \int_P^Q \mathbf{A}\cdot d\mathbf{x}_{II} \\ &= \oint \mathbf{A}\cdot d\mathbf{x}. \end{aligned}$$

- This loop integral involves both slits I and II, and can be computed via Stokes' theorem and shown to be equal to the gauge-independent magnetic flux Φ:

$$\oint \mathbf{A}\cdot \mathrm{d}\mathbf{x} = \iint (\nabla\times\mathbf{A})\cdot \mathbf{n}\mathrm{d}S = \iint \mathbf{B}\cdot\mathbf{n}\mathrm{d}S = \Phi.$$

The AB effect: arriving at the result

- Putting this all together, we have:

$$\begin{aligned}\psi(Q) &= e^{i\alpha}\left[\psi_I^0(Q) + \exp\left(\frac{ie}{\hbar}\oint \mathbf{A}\cdot \mathrm{d}\mathbf{x}\right)\psi_{II}^0(Q)\right] \\ &= e^{i\alpha}\left[\psi_I^0(Q) + \exp\left(\frac{ie\Phi}{\hbar}\right)\psi_{II}^0(Q)\right],\end{aligned}$$

where $\alpha := \int_P^Q \mathbf{A}\cdot \mathrm{d}\mathbf{x}_I$.

The AB effect: arriving at the result

- Putting this all together, we have:

$$\begin{aligned}\psi(Q) &= e^{i\alpha}\left[\psi_I^0(Q) + \exp\left(\frac{ie}{\hbar}\oint \mathbf{A}\cdot d\mathbf{x}\right)\psi_{II}^0(Q)\right] \\ &= e^{i\alpha}\left[\psi_I^0(Q) + \exp\left(\frac{ie\Phi}{\hbar}\right)\psi_{II}^0(Q)\right],\end{aligned}$$

  where $\alpha := \int_P^Q \mathbf{A}\cdot d\mathbf{x}_I$.
- The total phase factor $e^{i\alpha}$ of course has no effect on the probability density $|\psi(Q)|^2$, but because of the *relative* phase factor $\exp(ie\Phi/\hbar)$, the interference term contributing to that of $|\psi(Q)|^2$ is not equal to that of $\left|\psi^0(Q)\right|^2$.

The AB effect: arriving at the result

- Specifically, writing the wavefunctions in the polar form $\psi_I^0 = R_I^0 \exp(iS_I/\hbar)$, etc., we have at the point Q

  $$|\psi|^2 = (R_I^0)^2 + (R_{II}^0)^2 + 2R_I^0 R_{II}^0 \cos(\theta + \delta),$$

  where $\theta := (S_{II} - S_I)/\hbar$ and $\delta := e\Phi/\hbar$.

The AB effect: arriving at the result

- Specifically, writing the wavefunctions in the polar form $\psi_I^0 = R_I^0 \exp(iS_I/\hbar)$, etc., we have at the point Q

$$|\psi|^2 = (R_I^0)^2 + (R_{II}^0)^2 + 2R_I^0 R_{II}^0 \cos(\theta + \delta),$$

where $\theta := (S_{II} - S_I)/\hbar$ and $\delta := e\Phi/\hbar$.

- On the other hand,

$$|\psi^0|^2 = (R_I^0)^2 + (R_{II}^0)^2 + 2R_I^0 R_{II}^0 \cos(\theta).$$

The AB effect: arriving at the result

- Specifically, writing the wavefunctions in the polar form $\psi_I^0 = R_I^0 \exp\left(iS_I/\hbar\right)$, etc., we have at the point Q

$$|\psi|^2 = (R_I^0)^2 + (R_{II}^0)^2 + 2R_I^0 R_{II}^0 \cos\left(\theta + \delta\right),$$

  where $\theta := \left(S_{II} - S_I\right)/\hbar$ and $\delta := e\Phi/\hbar$.

- On the other hand,

$$|\psi^0|^2 = (R_I^0)^2 + (R_{II}^0)^2 + 2R_I^0 R_{II}^0 \cos(\theta).$$

- The effect of turning on the magnetic flux is not to shift the diffraction envelope, but the position of the interference fringes within it.

The puzzle

- The puzzling fact is that Φ depends only on F_{ab}—which vanishes outside the solenoid!

The puzzle

- The puzzling fact is that Φ depends only on F_{ab}—which vanishes outside the solenoid!
- *The phase shift of the matter field seems to depend causally on a field with which it cannot interact locally!*

The puzzle

- The puzzling fact is that Φ depends only on F_{ab}—which vanishes outside the solenoid!
- *The phase shift of the matter field seems to depend causally on a field with which it cannot interact locally!*
- ...hence the move to reify A^a—to think, in the words of Maudlin (1998), that there is 'one true gauge'.

The puzzle

- The puzzling fact is that Φ depends only on F_{ab}—which vanishes outside the solenoid!
- *The phase shift of the matter field seems to depend causally on a field with which it cannot interact locally!*
- ...hence the move to reify A^a—to think, in the words of Maudlin (1998), that there is 'one true gauge'.
- But, of course, reifying A^a invites underdetermination and indeterminism! (Maudlin 1998)

The puzzle

- The puzzling fact is that Φ depends only on F_{ab}—which vanishes outside the solenoid!
- *The phase shift of the matter field seems to depend causally on a field with which it cannot interact locally!*
- ...hence the move to reify A^a—to think, in the words of Maudlin (1998), that there is 'one true gauge'.
- But, of course, reifying A^a invites underdetermination and indeterminism! (Maudlin 1998)
- Is there another way out?

Comparison with the hole argument

- There are clear affinities here with the hole argument.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.
    - Hole diffeomorphic models are isomorphic—they differ merely haecceitistically.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.
  - Hole diffeomorphic models are isomorphic—they differ merely haecceitistically.
  - So (mainstream claim) anti-haecceitism suffices to overcome the indeterminism.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.
  - Hole diffeomorphic models are isomorphic—they differ merely haecceitistically.
  - So (mainstream claim) anti-haecceitism suffices to overcome the indeterminism.
- In the AB effect, the claim is: if you reify A^a, then indeterminism follows, given its local gauge symmetry.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.
  - Hole diffeomorphic models are isomorphic—they differ merely haecceitistically.
  - So (mainstream claim) anti-haecceitism suffices to overcome the indeterminism.
- In the AB effect, the claim is: if you reify A^a, then indeterminism follows, given its local gauge symmetry.
  - Gauge-related models of electromagnetism are *not* isomorphic.

Comparison with the hole argument

- There are clear affinities here with the hole argument.
- In the hole argument, the claim is: if you reify M, indeterminism follows.
  - Hole diffeomorphic models are isomorphic—they differ merely haecceitistically.
  - So (mainstream claim) anti-haecceitism suffices to overcome the indeterminism.
- In the AB effect, the claim is: if you reify A^a, then indeterminism follows, given its local gauge symmetry.
  - Gauge-related models of electromagnetism are *not* isomorphic.
  - So some more involved interpretational strategy is going to be needed to (a) overcome the indeterminism, while (b) giving a local narrative about the AB effect.

Today

Gauge symmetries

Locality, separability, and determinism

Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

Verdicts on the effect

What I'm going to do now is think through what the three versions of electromagnetism introduced before have to say about the AB effect.

Verdicts on the effect

What I'm going to do now is think through what the three versions of electromagnetism introduced before have to say about the AB effect.

I'll also be explicit about the shortcomings of each of these approaches.

Verdicts on the effect

What I'm going to do now is think through what the three versions of electromagnetism introduced before have to say about the AB effect.

I'll also be explicit about the shortcomings of each of these approaches.

After doing this, I'll present the 'field monism' approach of Wallace (2014).

A-field electromagnetism

- We've already seen the problems for A-field electromagnetism in the context of the AB effect.

A-field electromagnetism

- We've already seen the problems for A-field electromagnetism in the context of the AB effect.
- It offers an ontology which is local (because A^a doesn't vanish even outside of the solenoid) and separable, but which is radically indeterministic.

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.
- However, this approach still offers an ontology which is separable and deterministic.

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.
- However, this approach still offers an ontology which is separable and deterministic.
- Stepping back, there are some physics reasons to be concerned about F-field electromagnetism:

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.
- However, this approach still offers an ontology which is separable and deterministic.
- Stepping back, there are some physics reasons to be concerned about F-field electromagnetism:
  - $dF = 0$ is not a mathematical identity! (Jacobs 2023)

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.
- However, this approach still offers an ontology which is separable and deterministic.
- Stepping back, there are some physics reasons to be concerned about F-field electromagnetism:
  - $dF = 0$ is not a mathematical identity! (Jacobs 2023)
  - Difficult to couple subsystems without A^a (Rovelli 2014).

F-field electromagnetism

- F-field electromagnetism explain the AB effect in a non-local way: the interference pattern is affected by the F_{ab}, even though this vanishes outside of the solenoid!
- This has struck many as rebarbative.
- However, this approach still offers an ontology which is separable and deterministic.
- Stepping back, there are some physics reasons to be concerned about F-field electromagnetism:
  - $dF = 0$ is not a mathematical identity! (Jacobs 2023)
  - Difficult to couple subsystems without A^a (Rovelli 2014).
  - Problems with quantisation etc.

Holonomy-based electromagnetism

Holonomy-based electromagnetism gives the following account of the AB effect:

Holonomy-based electromagnetism

Holonomy-based electromagnetism gives the following account of the AB effect:

> *On this picture, there is a fundamental non-localised property associated to every closed curve in spacetime. Holonomies are not composed of the field-values at each spacetime point, but attach to curves as a whole. When a matter field interacts with these holonomies, it does so 'at once' around a loop. Since holonomies overlap with the matter field, interactions are local.* (Jacobs 2023, p. 36)

Holonomy-based electromagnetism

Holonomy-based electromagnetism gives the following account of the AB effect:

> *On this picture, there is a fundamental non-localised property associated to every closed curve in spacetime. Holonomies are not composed of the field-values at each spacetime point, but attach to curves as a whole. When a matter field interacts with these holonomies, it does so 'at once' around a loop. Since holonomies overlap with the matter field, interactions are local.* (Jacobs 2023, p. 36)

This offers a local ontology which avoids indeterminism (since it is gauge-invariant, 'reduced' version of electromagnetism).

Holonomy-based electromagnetism

Holonomy-based electromagnetism gives the following account of the AB effect:

> *On this picture, there is a fundamental non-localised property associated to every closed curve in spacetime. Holonomies are not composed of the field-values at each spacetime point, but attach to curves as a whole. When a matter field interacts with these holonomies, it does so 'at once' around a loop. Since holonomies overlap with the matter field, interactions are local.* (Jacobs 2023, p. 36)

This offers a local ontology which avoids indeterminism (since it is gauge-invariant, 'reduced' version of electromagnetism).

However, the holonomy approach has problems:

Holonomy-based electromagnetism

Holonomy-based electromagnetism gives the following account of the AB effect:

> *On this picture, there is a fundamental non-localised property associated to every closed curve in spacetime. Holonomies are not composed of the field-values at each spacetime point, but attach to curves as a whole. When a matter field interacts with these holonomies, it does so 'at once' around a loop. Since holonomies overlap with the matter field, interactions are local.* (Jacobs 2023, p. 36)

This offers a local ontology which avoids indeterminism (since it is gauge-invariant, 'reduced' version of electromagnetism).

However, the holonomy approach has problems:

1. No dynamics.
2. Non-separability.
3. Cosmic coincidences.

Problem 1: no dynamics

- On the holonomy interpretation, dynamics are still expressed in terms of A^a, not in terms of H.

Problem 1: no dynamics

- On the holonomy interpretation, dynamics are still expressed in terms of A^a, not in terms of H.
- Nobody has yet succeeded in an *intrinsic* formulation of e.g. the Lagrangian directly in terms of only holonomies.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.
  - But now consider the union $X \cup Y$. This region does enclose the solenoid, so it has a non-zero holonomy value.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.
  - But now consider the union $X \cup Y$. This region does enclose the solenoid, so it has a non-zero holonomy value.
  - Therefore, the intrinsic facts about X and Y fail to determine the intrinsic facts about $X \cup Y$: separability fails.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.
  - But now consider the union $X \cup Y$. This region does enclose the solenoid, so it has a non-zero holonomy value.
  - Therefore, the intrinsic facts about X and Y fail to determine the intrinsic facts about $X \cup Y$: separability fails.
- But is this a problem, given that e.g. many approaches to quantum mechanics are non-separable?

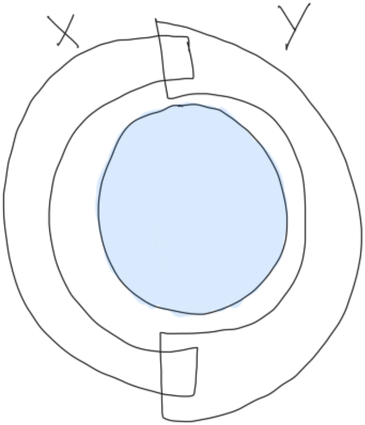
Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.
  - But now consider the union $X \cup Y$. This region does enclose the solenoid, so it has a non-zero holonomy value.
  - Therefore, the intrinsic facts about X and Y fail to determine the intrinsic facts about $X \cup Y$: separability fails.
- But is this a problem, given that e.g. many approaches to quantum mechanics are non-separable?
  - Maudlin (1998) argues that the entanglement here is of a different nature.

Problem 2: non-separability

- The holonomy interpretation is non-separable: the intrinsic facts about a region X and about another region Y don't determine all intrinsic facts about the joint region $X \cup Y$.
- We can easily see that separability fails when we consider two partially overlapping regions X and Y close to the solenoid in the Aharonov-Bohm effect.
  - Since X does not enclose the solenoid, the flux through its surface is zero, likewise for Y.
  - But now consider the union $X \cup Y$. This region does enclose the solenoid, so it has a non-zero holonomy value.
  - Therefore, the intrinsic facts about X and Y fail to determine the intrinsic facts about $X \cup Y$: separability fails.
- But is this a problem, given that e.g. many approaches to quantum mechanics are non-separable?
  - Maudlin (1998) argues that the entanglement here is of a different nature.
  - Dougherty (2017) argues that an version of the holonomy interpretation is separable.

Problem 2: non-separability



Problem 3: cosmic coincidences

- Holonomies in the holonomy interpretation must satisfy

$$H(l_1 \circ l_2) = H(l_1)\, H(l_2)\,, \qquad (*)$$

where $l_1 \circ l_2$ denotes the concatenation of two loops, i.e., the result of first going around l_1 and then going around l_2.

Problem 3: cosmic coincidences

- Holonomies in the holonomy interpretation must satisfy

$$H(l_1 \circ l_2) = H(l_1)\,H(l_2)\,, \qquad (*)$$

  where $l_1 \circ l_2$ denotes the concatenation of two loops, i.e., the result of first going around l_1 and then going around l_2.
- Arntzenius (2012, p. 195): "a fairly obvious explanation of why [this] hold[s] is that the map H is, roughly speaking, the integration of a connection around a loop".

Problem 3: cosmic coincidences

- Holonomies in the holonomy interpretation must satisfy

$$H(l_1 \circ l_2) = H(l_1)\, H(l_2)\,, \qquad (*)$$

  where $l_1 \circ l_2$ denotes the concatenation of two loops, i.e., the result of first going around l_1 and then going around l_2.
- Arntzenius (2012, p. 195): “a fairly obvious explanation of why [this] hold[s] is that the map H is, roughly speaking, the integration of a connection around a loop”.
- But of course, one can’t say this if one takes holonomies to be the primitives in one’s theory.

Problem 3: cosmic coincidences

- As an illustration of Arntzenius' point: note that if we define holonomies in terms of loop integrals of vector potentials as before, then:

$$
\begin{aligned}
H(l_1)H(l_2) &= \exp\left(-iq\oint_{l_1} A_\mu dx^\mu\right)\cdot\exp\left(-iq\oint_{l_2} A_\mu dx^\mu\right) \\
&= \exp\left(-iq\oint_{l_1\circ l_2} A_\mu dx^\mu\right) \\
&= H(l_1\circ l_2).
\end{aligned}
$$

Problem 3: cosmic coincidences

- As an illustration of Arntzenius' point: note that if we define holonomies in terms of loop integrals of vector potentials as before, then:

$$
\begin{aligned}
H(l_1)H(l_2) &= \exp\left(-iq \oint_{l_1} A_\mu dx^\mu\right) \cdot \exp\left(-iq \oint_{l_2} A_\mu dx^\mu\right) \\
&= \exp\left(-iq \oint_{l_1 \circ l_2} A_\mu dx^\mu\right) \\
&= H(l_1 \circ l_2).
\end{aligned}
$$

- Hence we can explain $(*)$ if we posit the existence of a local four-potential, but it is a 'cosmic coincidence' on the holonomy interpretation.

Summarising so far...

Summarising so far, we have the following table of verdicts for our three different approaches to electromagnetism:

	A-field	F-field	Holonomy
Local	Y	N	Y
Separable	Y	Y	N
Deterministic	N	Y	Y

Summarising so far...

Summarising so far, we have the following table of verdicts for our three different approaches to electromagnetism:

	A-field	F-field	Holonomy
Local	Y	N	Y
Separable	Y	Y	N
Deterministic	N	Y	Y

(This, of course, sets aside the various intrinsic challenges for each of the approaches.)

Summarising so far...

Summarising so far, we have the following table of verdicts for our three different approaches to electromagnetism:

	A-field	F-field	Holonomy
Local	Y	N	Y
Separable	Y	Y	N
Deterministic	N	Y	Y

(This, of course, sets aside the various intrinsic challenges for each of the approaches.)

Next, I want to turn to turn to a different approach: Wallace's 'field monism'.

Introducing field monism

- ‘Field monism’ isn’t an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).

Introducing field monism

- ‘Field monism’ isn’t an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).
- On this view, “the electromagnetic and scalar fields cannot be thought of as separate entities”, but jointly “[represent] aspects of a single entity” (Wallace 2014, p. 15).

Introducing field monism

- 'Field monism' isn't an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).
- On this view, "the electromagnetic and scalar fields cannot be thought of as separate entities", but jointly "[represent] aspects of a single entity" (Wallace 2014, p. 15).
- Instead of a complex matter field ψ, Wallace's fundamental fields are the real scalar field $\rho = |\psi|$ and the covariant derivative $D_\mu\psi = \partial_\mu\theta - A_\mu$, where θ is the phase of ψ.

Introducing field monism

- ‘Field monism’ isn’t an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).
- On this view, “the electromagnetic and scalar fields cannot be thought of as separate entities”, but jointly “[represent] aspects of a single entity” (Wallace 2014, p. 15).
- Instead of a complex matter field ψ, Wallace’s fundamental fields are the real scalar field $\rho = |\psi|$ and the covariant derivative $D_\mu\psi = \partial_\mu\theta - A_\mu$, where θ is the phase of ψ.
- Since ρ and $D_\mu\theta$ are both gauge-invariant quantities defined in terms of the old theory’s variant quantities, field monism is another example of reduction.

Introducing field monism

- 'Field monism' isn't an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).
- On this view, "the electromagnetic and scalar fields cannot be thought of as separate entities", but jointly "[represent] aspects of a single entity" (Wallace 2014, p. 15).
- Instead of a complex matter field ψ, Wallace's fundamental fields are the real scalar field $\rho = |\psi|$ and the covariant derivative $D_\mu\psi = \partial_\mu\theta - A_\mu$, where θ is the phase of ψ.
- Since ρ and $D_\mu\theta$ are both gauge-invariant quantities defined in terms of the old theory's variant quantities, field monism is another example of reduction.
- The joint distribution of these fields uniquely corresponds to equivalence classes of gauge-related models of electromagnetism.

Introducing field monism

- ‘Field monism’ isn’t an interpretation of *pure* EM, but an interpretation of EM *coupled to the wavefunction* ψ (which of course is the case under consideration in the AB effect anyway).
- On this view, “the electromagnetic and scalar fields cannot be thought of as separate entities”, but jointly “[represent] aspects of a single entity” (Wallace 2014, p. 15).
- Instead of a complex matter field ψ, Wallace’s fundamental fields are the real scalar field $\rho = |\psi|$ and the covariant derivative $D_\mu\psi = \partial_\mu\theta - A_\mu$, where θ is the phase of ψ.
- Since ρ and $D_\mu\theta$ are both gauge-invariant quantities defined in terms of the old theory’s variant quantities, field monism is another example of reduction.
- The joint distribution of these fields uniquely corresponds to equivalence classes of gauge-related models of electromagnetism.
- Wallace (2014) argues that field monism is local, separable, and deterministic!

First worry: lack of generality

But the main issue with field monism is that it does not easily extend to more complex gauge theories. Wallace [...] admits that this is a problem, writing that "in general, I know of no comparably simple set of local gauge-invariant quantities in the non-Abelian case that can serve as a gauge-invariant representation". This suggests that it is no more than a fortunate accident that we can represent the simple $U(1)$ *gauge theory Wallace considers in terms of a unique set of gauge-invariant local quantities.* (Jacobs 2023, p. 37)

Second worry: lack of unique representation

Jacobs (2023, p. 37) also worries that even more complex *Abelian* theories needn't have a unique gauge-invariant representation:

Second worry: lack of unique representation

Jacobs (2023, p. 37) also worries that even more complex *Abelian* theories needn't have a unique gauge-invariant representation:

- Consider a pair of complex-valued fields ψ_1 and ψ_2 with different charges, both of which are coupled to A^a.

Second worry: lack of unique representation

Jacobs (2023, p. 37) also worries that even more complex *Abelian* theories needn't have a unique gauge-invariant representation:

- Consider a pair of complex-valued fields ψ_1 and ψ_2 with different charges, both of which are coupled to A^a.
- Here, there are multiple possible gauge-invariant 'field monist' ontologies, depending upon how we combine the relevant fields.

Jacobs on the second worry

> *Therefore, on one way of understanding Wallace's observation with respect to the unitary gauge, it implies a form of theoretical underdetermination: the choice between these two ontologies is arbitrary. This is hardly better than the underdetermination of theory by the empirical data implied by the existence of gauge symmetries. Therefore, in these more complex scenarios Wallace's account for finding a gauge-invariant representation is inadequate.* (Jacobs 2023, p. 37)

Today

Gauge symmetries

Locality, separability, and determinism

Formulations of electromagnetism

The Aharanov–Bohm effect

Verdicts on the effect

Wrapping up

Summary so far

- In light of the AB effect, none of the original three approaches to EM which we considered (A-field, F-field, holonomies) had all of the features which one would ideally desire of a physical theory (i.e., locality, separability, and determinism).

Summary so far

- In light of the AB effect, none of the original three approaches to EM which we considered (A-field, F-field, holonomies) had all of the features which one would ideally desire of a physical theory (i.e., locality, separability, and determinism).
- Moreover, these approaches has their own intrinsic problems (e.g., lack of dynamics for the holonomy approach).

Summary so far

- In light of the AB effect, none of the original three approaches to EM which we considered (A-field, F-field, holonomies) had all of the features which one would ideally desire of a physical theory (i.e., locality, separability, and determinism).
- Moreover, these approaches has their own intrinsic problems (e.g., lack of dynamics for the holonomy approach).
- Wallace's field monism seems to meet all three desiderata, but it only works in some very specific contexts.

Summary so far

- In light of the AB effect, none of the original three approaches to EM which we considered (A-field, F-field, holonomies) had all of the features which one would ideally desire of a physical theory (i.e., locality, separability, and determinism).
- Moreover, these approaches has their own intrinsic problems (e.g., lack of dynamics for the holonomy approach).
- Wallace's field monism seems to meet all three desiderata, but it only works in some very specific contexts.
- Are we at an an impasse, then?

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.
  - In realistic, de-idealised scenarios, will F_{ab} *really* vanish outside of the solenoid?

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.
  - In realistic, de-idealised scenarios, will F_{ab} *really* vanish outside of the solenoid?
  - So, is F-field electromagnetism really non-local?

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.
  - In realistic, de-idealised scenarios, will F_{ab} *really* vanish outside of the solenoid?
  - So, is F-field electromagnetism really non-local?
  - So can't we just hang onto this more 'standard' understanding of EM?

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.
  - In realistic, de-idealised scenarios, will F_{ab} *really* vanish outside of the solenoid?
  - So, is F-field electromagnetism really non-local?
  - So can't we just hang onto this more 'standard' understanding of EM?
- Non-quantum AB effects? Non-Abelian AB effects? With trivial topologies? (Gomes 2025)

Looking forward

- Jacobs (2023) makes a compelling case that another version of electromagnetism—written using fibre bundles—is the best way of interpreting EM.
  - It is local, deterministic, and separable.
  - It's a 'sophisticated' theory (in the sense of Week 1)—so a datum in favour of the merits of sophisticating symmetries (in the sense of Dewar (2019)).
- Others, e.g. Krátký (2024), have questioned the idealisations involved in the setup of the AB effect.
  - In realistic, de-idealised scenarios, will F_{ab} *really* vanish outside of the solenoid?
  - So, is F-field electromagnetism really non-local?
  - So can't we just hang onto this more 'standard' understanding of EM?
- Non-quantum AB effects? Non-Abelian AB effects? With trivial topologies? (Gomes 2025)

Despite its being by now an 'old chestnut', there remains much interesting to be said about the AB effect!

References I

- Frank Arntzenius, *Space, Time, and Stuff*, New York: Oxford University Press, 2012.
- Neil Dewar, “Sophistication About Symmetries”, British Journal for the Philosophy of Science 70(2), pp. 485-521, 2019.
- John Dougherty, “Sameness and Separability in Gauge Theories”, Philosophy of Science 84(5), pp. 1189–1201, 2017.
- John Earman, *A Primer on Determinism*, D. Reidel, 1986.
- Henrique Gomes, *Gauge Theory and the Geometrization of Physics*, Cambridge: Cambridge University Press, 2025.
- Richard Healey, “Nonlocality and the Aharonov-Bohm Effect”, Philosophy of Science 64, pp. 18–41, 1997.
- Richard Healey, “Quantum Analogies: a reply to Maudlin”, Philosophy of Science 66, pp. 440–47, 1999.
- Richard Healey and Henrique Gomes, “Holism and Nonseparability in Physics”, in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, 2022.
- John David Jackson, *Classical Electrodynamics*, third edition, 1998.

References II

- Caspar Jacobs, “The Metaphysics of Fibre Bundles”, Studies in History and Philosophy of Science 97, pp. 34–43, 2023.
- Matěj Krátký, “Physical Possibility of the Aharonov-Bohm Effect”, 2024.
- Tim Maudlin, “Discussion: Healey on the Aharonov-Bohm Effect”, Philosophy of Science 65, pp. 361–68, 1998.
- Michael Redhead, *Incompleteness, Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics*, Oxford: Clarendon Press, 1987.
- Carlo Rovelli, “Why Gauge?”, Foundations of Physics, 2014.
- David Wallace, “Deflating the Aharonov-Bohm effect”, 2014.
- James Owen Weatherall, “Understanding Gauge”, Philosophy of Science 83, pp. 1039–1049, 2016.