IPP-QM-9: EPR and Bell's theorem

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The course

1. Basic quantum formalism
2. Density operators and entanglement
3. Decoherence
4. The measurement problem
5. Dynamical collapse theories
6. Bohmian mechanics
7. Everettian structure
8. Everettian probability
9. EPR and Bell's theorem
10. The Bell-CHSH inequalities and possible responses
11. Contextuality
12. The PBR theorem
13. Quantum logic
14. Pragmatism and QBism
15. Relational quantum mechanics
16. Wavefunction realism

Today

Locality and separability

The Einstein-Podolsky-Rosen argument

Bell's 1964 theorem

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Locality and separability

The first thing to get clear on today are two key terms which are ubiquitous in the foundations of quantum mechanics: *locality* and *separability*.

Locality

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Is locality the conjunction of these? Or some subset or other?

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- What about quantum mechanics?
  - Dirac/von Neumann collapse dynamics is non-local in this sense.
  - Bohmian dynamics involves action-at-a-distance, as we saw in Lecture 6.

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- Why can't we use it to send superluminal signals?
- In particular contexts (of e.g. particular collapse mechanisms), people show that this is not possible by proving 'no signalling theorems'.
- So you might have action-at-a-distance at the fundamental ontological level, but still signal locality at the empirical/operational level.

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(The first two of these are from Redhead (1987), while the third is from Maudlin (2014).)

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- A failure of EPR locality implies a failure of Bell locality and Einstein locality.
- If it is assumed there can be causal influences between events only if those events are not separated by a spacelike spacetime interval, then Bell locality reduces to Einstein locality.
- Thus, Bell locality can be distinguished from Einstein locality in that it does not explicitly assume relativity theory.

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- There is clearly, then, a strong connection between quantum entanglement and non-separability.
- That said, some authors (e.g. Ney (2021), see Lecture 16) do try to maintain that one can have entanglement without non-separability.

Today

Locality and separability

The Einstein-Podolsky-Rosen argument

Bell's 1964 theorem

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- EPR took the ‘incomplete’ horn of the dilemma.
- Let’s see how their argument goes.

The EPR dilemma

Consider the singlet two-electron spin state (NB: this is Bohm's (1951) version of the EPR setup),

$$|\psi^-\rangle_{12} = \frac{1}{\sqrt{2}} \left(|{\uparrow_z}\rangle_1 \, |{\downarrow_z}\rangle_2 - |{\downarrow_z}\rangle_1 \, |{\uparrow_z}\rangle_2 \right).$$

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(In fact, the singlet is spherically symmetric, i.e. it takes the same form for all spin directions.)

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- At time t_1 the systems 1 and 2 are far apart and no longer interacting and we consider performing measurements on one of them, say system 1.
- z-spin measurement: before measurement, neither system has a definite value of spin in the z-direction. At $t > t_1$ we obtain a definite value of spin for system 1. But not only has the property of system 1 changed, but so also have the properties of the far away system 2 ("instantaneous action at a distance"—Einstein: "spooky action").

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- z-spin measurement: before measurement, neither system has a definite value of spin in the z-direction. At $t > t_1$ we obtain a definite value of spin for system 1. But not only has the property of system 1 changed, but so also have the properties of the far away system 2 (“instantaneous action at a distance”—Einstein: “spooky action”).
- Similarly if we had performed an x-spin measurement on system 1 at time $t > t_1$. *Our choice of which measurement to perform in the region of system 1 affects at a distance the properties of system 2*.

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Either: Quantum mechanics is non-local: the effect of measurement is to cause a collapse which has an instantaneous causal effect at a distance.

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Naturally, the second of these options invites thinking about quantum mechanics in terms of hidden variable theories.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

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- Introducing hidden variables might help us with the measurement problem: the value of the hidden variable will determine which outcome of the experiment is definite, even if the *quantum state* happens to be left in a superposition of distinct measurement outcomes.
- Of course, we saw a lot of this already in Lecture 6 in the context of Bohmian mechanics.

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- Therefore, taking the incompleteness horn of the EPR dilemma does not lead us away from non-locality after all.
- Today, I'll show you how Bell's 1964 theorem works; in the next lecture, I'll present some later, arguably more powerful (and intricate) versions of Bell's theorem.

Setup for the 1964 result

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- Begin by considering a deterministic theory which ascribes definite values to the results of all experiments we might perform on a system (‘deterministic hidden variable theory’).
- Specifying the initial state leads to deterministic predictions about what the results of any measurement will be.
- Assume, furthermore, that the theory satisfies a locality condition: the values for the outcomes of experiments on a system depend only on the initial state and on the particular experiment performed on that system.

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- Such an inequality will be violated by the predictions of quantum mechanics.
- Conclusion: The results predicted by quantum mechanics cannot be modelled by any *local* deterministic hidden variable theory.

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- Measurements of spin will be made in the direction $\mathbf{a}$ or $\mathbf{a}'$ on system 1; and in the direction $\mathbf{b}$ or $\mathbf{b}'$ on system 2.
- The hidden variable theory will assign definite values to all four spin quantities at the same time, i.e. $\mathbf{a}$ and $\mathbf{a}'$ for system 1 possess a definite value just before measurement, as do $\mathbf{b}$ and $\mathbf{b}'$.

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- On each run of the experiment, all four spin quantities possess a definite value. For run n, denote these quantities by a_n, a'_n, b_n, b'_n.
- Although the initial *quantum* state of the joint system is the same for each run, the hidden variable takes different values and therefore a spread of outcomes is observed.

Average values, correlation coefficients, and γ_n

Average values and correlation coefficients are given by, respectively,

$$\bar{a} = \frac{1}{N}\sum_{n=1}^{N} a_n,$$

$$c\left(\mathbf{a},\mathbf{b}\right) = \frac{1}{N}\sum_{n=1}^{N} a_n b_n.$$

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Now consider the following function:

$$\begin{aligned}\gamma_n &:= a_n b_n + a_n b'_n + a'_n b_n - a'_n b'_n \\ &= a_n\left(b_n + b'_n\right) + a'_n\left(b_n - b'_n\right).\end{aligned}$$

Since each of a, a', b, b' can be equal to ± 1, γ_n must be equal to ± 2.

Deriving the Bell inequality

Now calculate the average value of γ_n for a large number of runs of the experiment,

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Given that on each run $\gamma_n = \pm 2$, this average must lie between ± 2, so its modulus will be less than or equal to 2:

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In terms of correlation coefficients, this gives us the *Bell inequality*:

$$\left|c\left(\mathbf{a},\mathbf{b}\right) + c\left(\mathbf{a},\mathbf{b}'\right) + c\left(\mathbf{a}',\mathbf{b}\right) - c\left(\mathbf{a}',\mathbf{b}'\right)\right| \leq 2.$$

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That is, the quantum mechanical correlations are *stronger* than any correlations that can be obtained in a local hidden variable model.

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2. Measurements of **a** and **b**′ or **b** and **a**′ are at an angle θ.
3. Measurements of **a**′ and **b**′ are at an angle 2θ.

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3. Measurements of $\mathbf{a}'$ and $\mathbf{b}'$ are at an angle 2θ.

Quantum mechanics has its own predictions of correlation coefficients. If ϕ is the angle between directions $\mathbf{a}$ and $\mathbf{b}$, then $c(\mathbf{a}, \mathbf{b}) = \cos\phi$.

Illustration of the quantum mechanical inequality

Using the above assumptions, we therefore have

$$\begin{aligned}
\left|c(\mathbf{a},\mathbf{b}) + c(\mathbf{a},\mathbf{b}') + c(\mathbf{a}',\mathbf{b}) - c(\mathbf{a}',\mathbf{b}')\right| &= \left|\cos(0) + 2\cos(\theta) - \cos(2\theta)\right| \\
&= \left|2\left(1 + \cos\theta - \cos^2\theta\right)\right| \\
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$$\begin{aligned}|c(\mathbf{a},\mathbf{b}) + c(\mathbf{a},\mathbf{b}') + c(\mathbf{a}',\mathbf{b}) - c(\mathbf{a}',\mathbf{b}')| &= |\cos(0) + 2\cos(\theta) - \cos(2\theta)| \\ &= \left|2\left(1 + \cos\theta - \cos^2\theta\right)\right| \\ &> 2\end{aligned}$$

(Note: It's not that standard quantum mechanics predicts different *inequalities* to local hidden variable theories. It's that some of its specific predictions involve correlations that violate the Bell inequalities.)

> *So no local hidden variable model can be empirically equivalent to quantum mechanics!*

Empirical evidence for standard quantum mechanics

- Which is right—quantum mechanics, or local hidden variable theories?

Empirical evidence for standard quantum mechanics

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So no local hidden variable model is empirically adequate!

Conclusions so far

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NB: Bell's theorem does not force us to accept "the bare formalism of quantum mechanics" *sans* interpretation, as some (physics texts) claim. We still need to solve the measurement problem!

Prospectus

Next time, I’ll:

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2. Think about what different interpretations of quantum mechanics have to say about the EPR scenario.

References

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