IPP-SR-8: Frame-dependent effects

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The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

Today

Time dilation

Length contraction

Bell's rockets

The ladder paradox

Assessing frame-dependent effects

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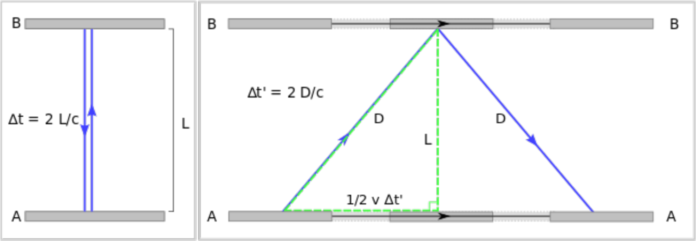
Time dilation via Langevin clocks



Lorentz transformations

$$\begin{pmatrix} c\Delta t' \\ \Delta x' \\ \Delta y' \\ \Delta z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

Lorentz transformations

$$\begin{aligned} c\Delta t' &= \gamma\left(c\Delta t - \beta\Delta x\right) \\ \Delta x' &= \gamma\left(\Delta x - \beta c\Delta t\right) \\ \Delta y' &= \Delta y \\ \Delta z' &= \Delta z \end{aligned}$$

Time dilation from the Lorentz transformations

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- Setting $\Delta x = 0$, we have $\Delta t' = \gamma\Delta t$.
- Thus, given a clock stationary in one frame, that clock will tick more slowly in a Lorentz-boosted frame.

Conventionality again

Whether or not a clock moving in a given direction runs slow relative to any given frame depends upon how distant clocks are synchronised in that frame.

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Hence, conventionalists about simultaneity should also, for consistency, be conventionalists about time-dilation.

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- We will return to this in a moment.

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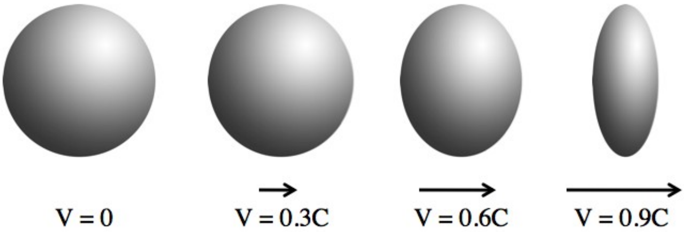
Length contraction



Length contraction from the Lorentz transformations

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- But $\gamma^{-2} = 1 - \beta^2$, so

$$\Delta x' = \frac{1}{\gamma}\Delta x.$$

Length contraction from the Lorentz transformations

$$\Delta x' = \frac{1}{\gamma}\Delta x.$$

So, given a rod stationary in one frame, the distance between the ends of that rod at a given time will be less in a Lorentz-boosted frame.

Conventionality again (again)

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Bell's rockets

Three small spaceships, A, B and C, drift freely in a region of space remote from other matter, without rotation and relative motion, with B and C equidistant from A.

On reception of a signal from A, the motors of B and C are ignited and they accelerate gently.

Let the ships B and C be identical, and have identical acceleration programmes. Then (as reckoned by the observer in A) they will have at every moment the same velocity, and so remain displaced one from the other by a fixed distance. Suppose that a fragile thread is tied initially between projections from B and C[, and that] it is just long enough to span the required distance initially. (Bell 1976, p. 67)

Question

Will the string break?

The answer

If [the rope] is just long enough to span the required distance initially, then as the rockets speed up, it will become too short, because of its need to Fitzgerald contract, and must finally break. It must break when, at a sufficiently high velocity, the artificial prevention of the natural contraction imposes intolerable stress.

The answer

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*Is it really so? This old problem came up for discussion once in the CERN canteen. A distinguished experimental physicist refused to accept that the thread would break, and regarded my assertion, that indeed it would, as a personal misinterpretation of special relativity. We decided to appeal to the CERN Theory Division for arbitration, and made a (not very systematic) canvas of opinion in it. There emerged a clear consensus that the thread would **not** break!* (Bell 1976, pp. 67-68)

Bell's rockets—diagram

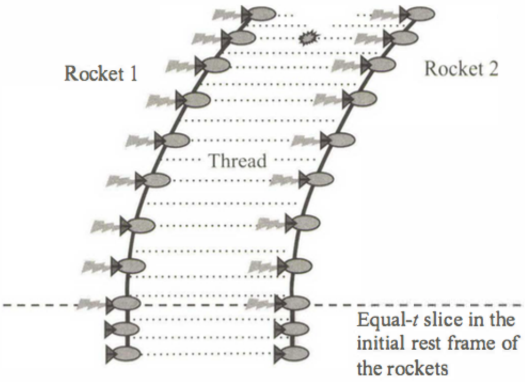

Fig. 20

(Maudlin 2012, p. 113)

Explanations for the snap: A, B, and C

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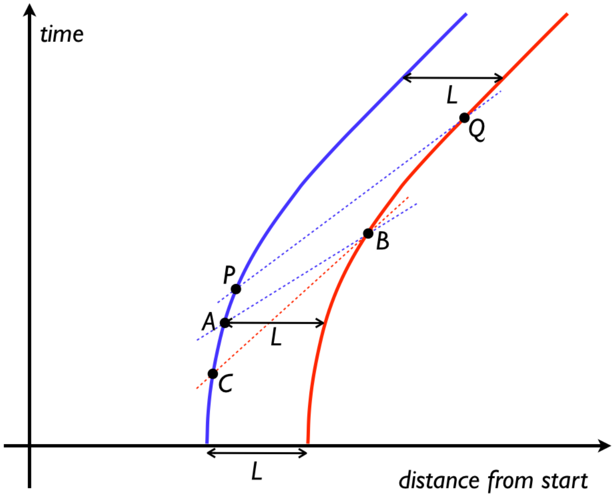
Explanations for the snap: *A*, *B*, and *C*

- From the point of view of the control tower *A*, the breakage happens as a result of length contraction.
- From the point of view of the first rocket *B*, the breakage happens as the second rocket moves progressively further away (due to the relativity of simultaneity—draw a spacetime diagram!).
- From the point of view of the second rocket *C*, the breakage happens as the first rocket lags further behind (due to the relativity of simultaneity—draw a spacetime diagram!).



(Weiss 2017)

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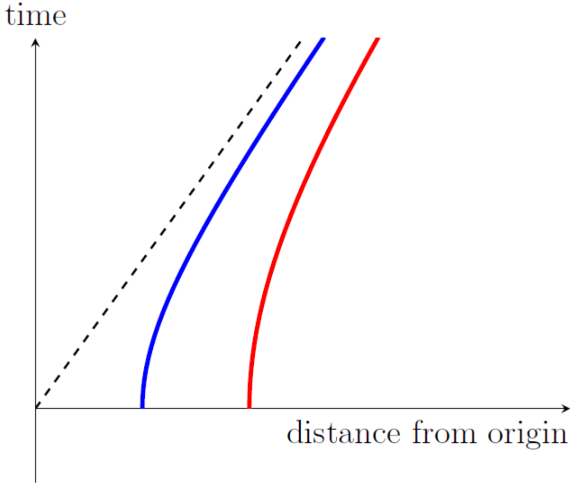
- This difference in accelerations would mean that the rockets move closer to one another as they accelerate, thereby implementing the length contraction effects.
- This does *not* happen in the Bell rocket scenario—so the rest frame of A is *not* a Rindler frame.

Rindler frames



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- In Bell's scenario, the string connecting the rockets is weak: it breaks under only a small applied force, and is unable to keep the rockets together.
- Suppose instead that the string were *infinitely strong*.
- In that case, the string would contract as the rockets accelerate, and the rockets would be pulled together: they would form a *Rindler pair*.

Maudlin on Bell's rockets

Let's return to our three frame-dependent accounts of why the string breaks, in Bell's original scenario. Maudlin repudiates such explanations:

> *The surface contradiction between these three account of why the thread breaks illustrates that frame-dependent narrations of events in Relativity can be misleading. There is one set of events, governed by laws that are indifferent to which coordinate system might be used to describe a situation. In each frame-dependent account, the interatomic forces in the thread play a role in determining exactly when the thread breaks. But how that role is described in a particular reference frame depends critically on which frame is chosen.* (Maudlin 2012, p. 120)

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Question: What, exactly, is misleading about frame-dependent accounts of phenomena?

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The ladder paradox

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- The ladder undergoes length contraction, meaning that it can fit inside the garage, at a particular time.

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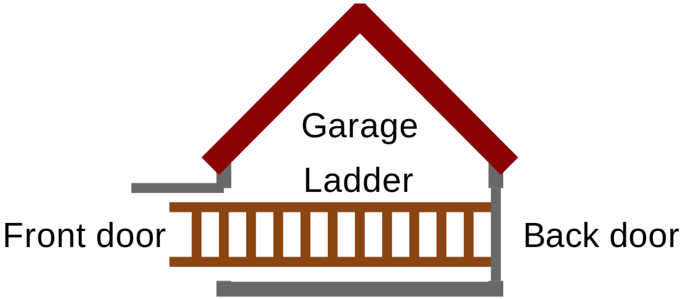
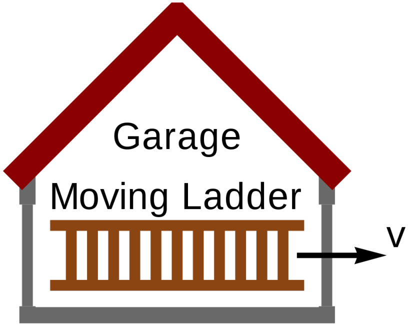
- Consider a garage with a front and back door which are open, and a ladder which, when at rest with respect to the garage, is too long to fit inside.
- Now move the ladder at a high horizontal velocity through the stationary garage.
- The ladder undergoes length contraction, meaning that it can fit inside the garage, at a particular time.
- We could, if we liked, simultaneously close both doors for a brief time, to demonstrate that the ladder fits.


Garage
Ladder
Front door
Back door


Garage
Moving Ladder
v

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- Now consider an observer co-moving with the ladder.
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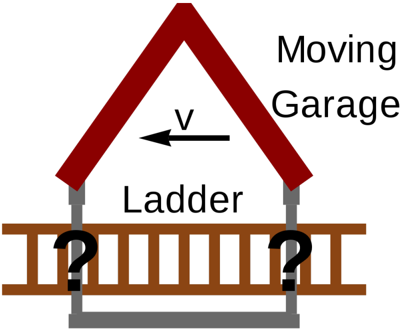
- So the garage is now length contracted—so how can the ladder fit inside the garage, and how can the doors close to contain the ladder???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A resolution

We need to properly take into account the relativity of simultaneity in the ladder's rest frame: the doors of the garage no longer close at the same time!

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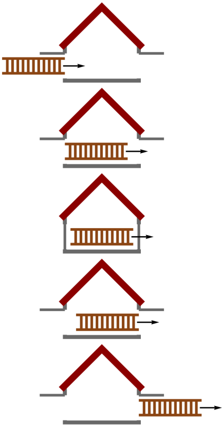
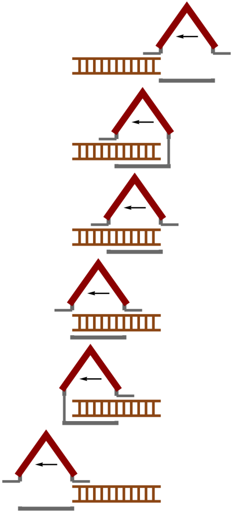
This can be brought out by considering again the situation in the barn rest frame versus ladder rest frame.

Scenario: Garage rest frame



Scenario: Ladder rest frame



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- Presumably, Maudlin would deride these frame-dependent explanations of the ladder paradox.
- But don't they seem perfectly legitimate, and physically illuminating?
- What kind of explanation would satisfy Maudlin here? And is it as physically insightful as these frame-dependent explanations?

Today

Two questions

1. Are frame-dependent explanations of physical phenomena legitimate?

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2. Are frame-dependent effects—e.g., length contraction and time dilation—‘physical’?

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The legitimacy of frame-dependent explanations

- As we have seen, Maudlin disavows frame-dependent explanations (of e.g. the Bell rocket result), for one has a different explanation in each frame.
- But what exactly is wrong with this? Why does a lack of univocity imply illegitimacy?
- Maudlin instead prefers *geometrical* explanations...

Maudlin on geometrical explanations

The presentation of space-time theory found here has slowly evolved over many classes. At first I followed standard presentations, making extensive use of coordinates and coordinate transformations. Bit by bit, class after class, reference to coordinates dropped away, leaving the fundamental geometry open to inspection. (Maudlin 2012, p. ix)

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Maudlin is here:

1. Committing to a geometrical understanding of special relativity.
2. Disavowing frame-dependent explanations.

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- Whatever one makes of this, it is clear that it is going to be anathema to e.g. Brown, for whom such invariant spacetime structures are just a codification of the symmetry properties of the dynamical equations governing matter, written in coordinate bases. (See lecture 10.)

The physicality of coordinate effects

- Phenomena such as length contraction and time dilation are associated with certain coordinate transformations.

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- Phenomena such as length contraction and time dilation are associated with certain coordinate transformations.
- Are they, therefore, 'physical' phenomena?

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- *Local* Galilean boosts are physical in Galilean spacetime. (Consider Galileo's ship.)

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- *Local* Galilean boosts are physical in Galilean spacetime. (Consider Galileo's ship.)
- *Local* Lorentz boosts are physical in Minkowski spacetime. (Consider a constant-velocity-transformation version of Bell's rockets—this is what Maudlin calls 'physical length contraction'.)

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(a) the amount of spacetime structure presupposed, and
(b) whether the associated coordinate transformations are applied globally (i.e., to the whole universe) or locally (i.e., to subsystems of the universe).

Local transformations can effect genuine physical change, even if the particular *mode of description* of that change is frame-dependent (cf. again Bell’s rockets, or the ladder).

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1. Introduced frame-dependent phenomena (time dilation, length contraction) and explanations of phenomena (as in the Bell rocket case and the ladder paradox).
2. Presented different takes on the legitimacy of frame-dependent explanations.
3. Considered senses in which effects related to coordinate transformations—again, e.g. length contraction and time dilation—are 'physical'.

References

- John S. Bell, "How to Teach Special Relativity", in *Speakable and Unspeakable in Quantum Mechanics*, second edition, Cambridge: Cambridge University Press, pp. 67-80, 2004.
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- Michael Weiss, "Bell's Spaceship Paradox". Available at: `http://math.ucr.edu/home/baez/physics/Relativity/SR/BellSpaceships/spaceship_puzzle.html`.