IPP-SR-9:
The twin paradox

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HT20

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

Today

The clock hypothesis

The twin pardox

Frame-relative accounts

General relativity

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Clocks and the relativity principle

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- **Answer:** Yes, by the relativity principle.

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- **Question:** Will these clocks function identically in their rest frames?
- **Answer:** Not necessarily—for the relativity principle holds for systems related by *Poincaré* transformations.

The clock hypothesis

Another way to put the point is the following: given two clocks A and B, if B moves at uniform velocity with respect to A, then if A correctly reads off the Minkowski spacetime interval $\int_{\gamma_A} ds$ along its worldline γ_A, then so too will B correctly read off the interval $\int_{\gamma_B} ds$ along its worldline γ_B, by the relativity principle.

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However, if B accelerates with respect to A, then the fact that A correctly reads off the Minkowski spacetime interval $\int_{\gamma_A} ds$ along its worldline γ_A does not guarantee that B correctly reads off the interval $\int_{\gamma_B} ds$ along its worldline γ_B.

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That this is so is an additional input assumption, known as the *clock hypothesis*.

Maudlin on the clock hypothesis

Clock Hypothesis: The amount of time that an accurate clock shows to have elapsed between two events is proportional to the Interval along the clock's trajectory between those events, or, in short, clocks measure the Interval along their trajectories. (Maudlin 2012, p. 76)

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- Whether a particular clock ticks in accordance with the spacetime metric is not a matter of stipulation or luck, but depends crucially on the constitution of the clock.
- For any given clock, no matter how ideal its performance when inertial, there will in principle be an acceleration-producing external force, or even tidal effects inside the clock, such that the clock 'breaks,' in the sense of violating the clock hypothesis. Might it therefore not be more appropriate to speak of the clock *condition*?

Lesson

Regardless of what one thinks of this, what's uncontroversial is that, whenever we have accelerating clocks, the clock hypothesis/condition must be brought into consideration.

Today

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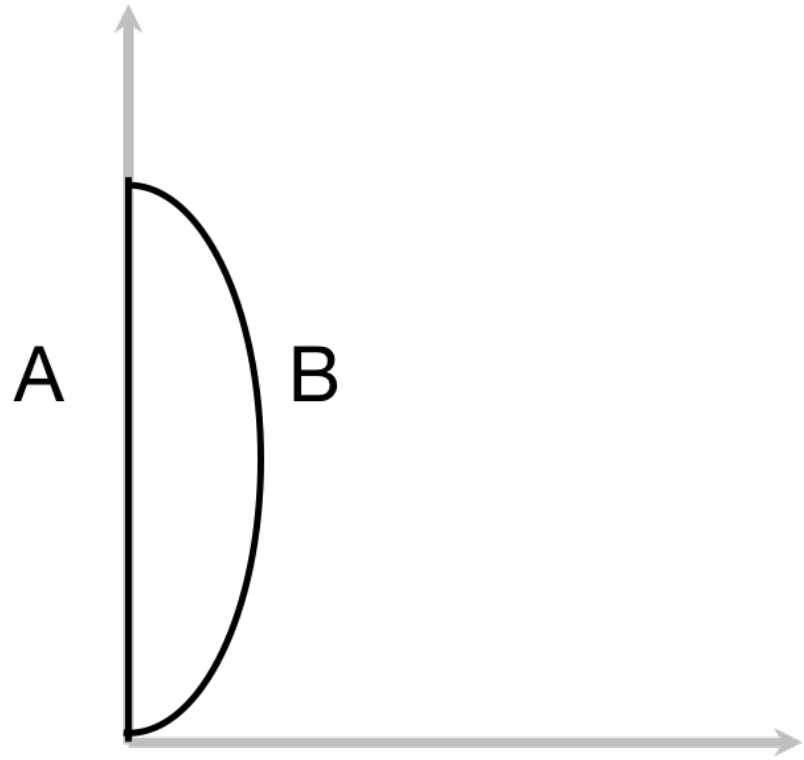
The twin pardox

Frame-relative accounts

General relativity

The twin paradox



$$T_A = \int_o^p d\tau_A$$

$$\begin{aligned} T_B &= \int_o^p d\tau_B \\ &= \int_o^p \left(1 - \left(\frac{dx}{d\tau_A}\right)^2 - \left(\frac{dy}{d\tau_A}\right)^2 - \left(\frac{dz}{d\tau_A}\right)^2\right)^{\frac{1}{2}} d\tau_A \\ &< T_A. \end{aligned}$$

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Note that the result of this computation is not relative to a particular frame.

Aside

This is a nice illustration of the sense in which drawing spacetime diagrams can be misleading—for *B*'s path looks *longer* on the diagram, but is in fact *shorter*, when we do the computation.

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- What breaks the symmetry between A and B?

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- ...in which case, we should not expect the same laws of physics to apply.
- Thus, consideration of the structure of Minkowski spacetime allows us to break the symmetry between *A* and *B*, and thereby resolve the paradox.

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This is, presumably, the kind of account which an operationalist about inertial frames (cf. lecture 1) would wish to give. Here's how it might go.

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- Thus, consideration of the inertial frames allows us to break the symmetry between *A* and *B*, and thereby resolve the paradox.

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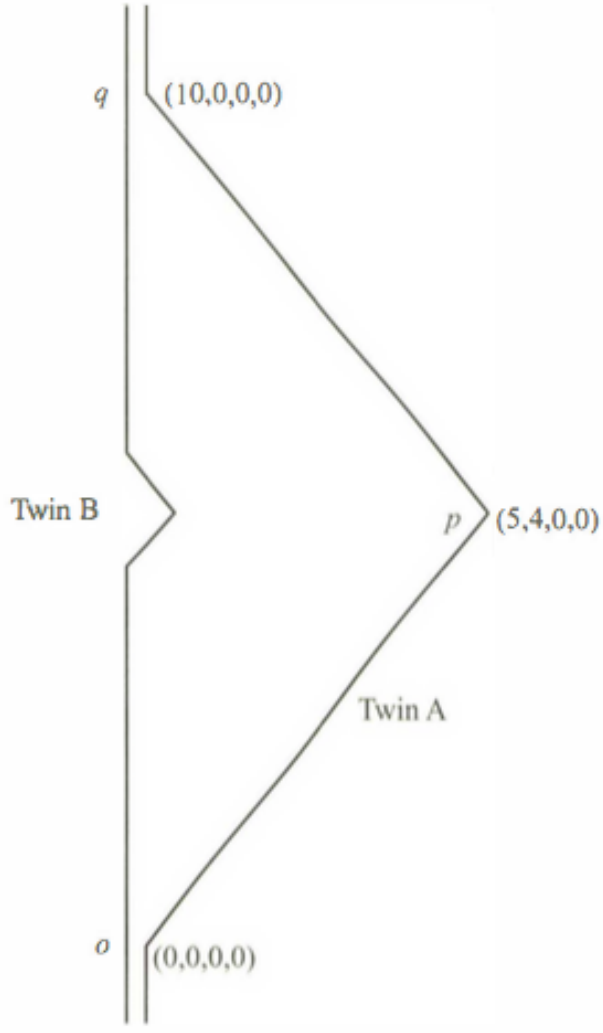
(i) equal accelerations, or

(ii) no accelerations at all!

Equal accelerations



(Maudlin 2012, p. 82)

Maudlin on the equal accelerations case

> *Both Rindler and Feynman point out that acceleration is objective in Relativity, just as it is in Newtonian absolute space and time and in Galilean space-time. This is true but irrelevant: the issue is how* long *the world-lines are, not how* bent. (Maudlin 2012, p. 83)

Non-accelerating versions of the paradox

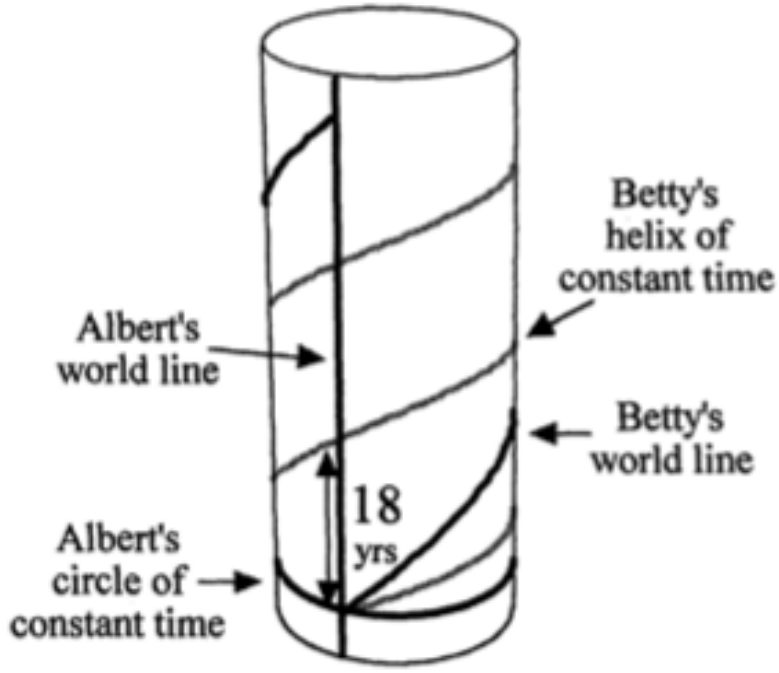


Figure 3. In a cylindrical spacetime, each twin stays in a single inertial frame.

(Weeks 2001, p. 587.)

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- **Exercise:** Assess this argument.

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- Both the equal-acceleration and cylindrical twin paradoxes show that the effect can't be accounted for solely in terms of the accelerations of the twins.
- The equal-acceleration case also shows that the effect can't always be accounted for by appeal to inertial frames—although perhaps there is room to offer this explanation in the cylindrical case.

Maudlin on the twin paradox

The Twins "Paradox" has inspired more confusion about Relativity than any other effect. The explanation of the phenomenon, in terms of the intrinsic geometry of Minkowski space-time and the Clock Hypothesis is exquisitely simple: clocks measure the Interval along their world-lines, and B's world-line between o and q is longer than A's. Period. There is nothing more to say. (Maudlin 2012, p. 79)

Maudlin versus Brown in the cylindrical case

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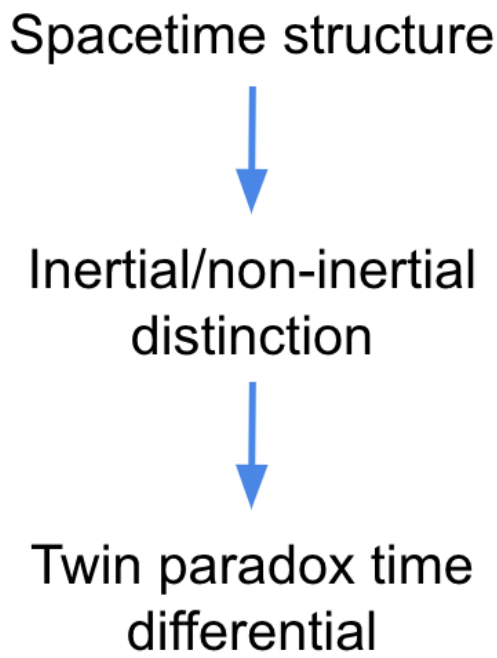
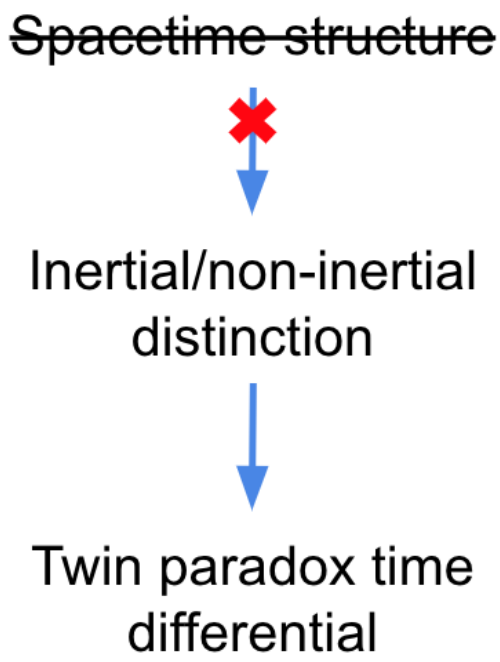
- Presumably, an empiricist/operationalist (e.g. Brown) would find the spacetime explanation of the cylindrical twin paradox (and the equal-acceleration twin paradox) similarly otiose...
- ...and would say that, even if it's not an (operationalised) notion of inertial frames which accounts for the time differential, it's still *facts about the matter out of which the twins are built*, more generally construed, which account for the difference.

Initial geometrical thought



Initial empiricist/operationalist thought



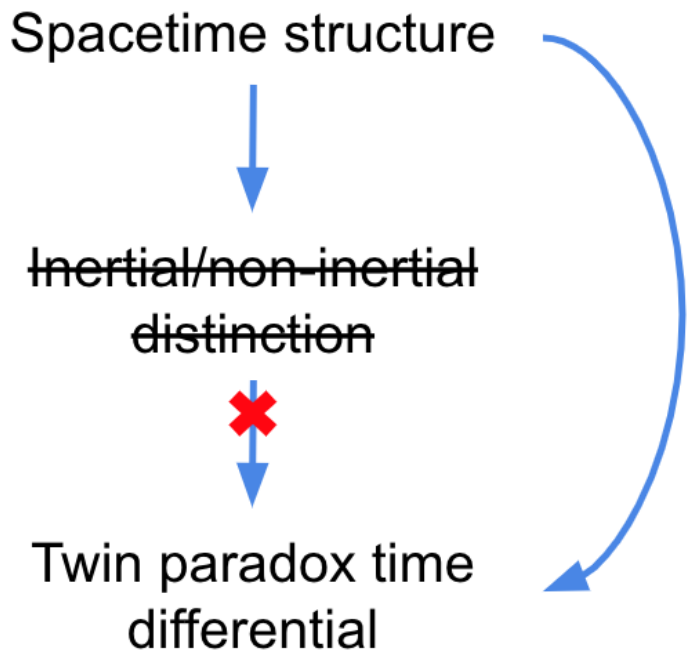
In light of our problem cases...

~~Spacetime structure~~

~~Inertial/non-inertial distinction~~

Twin paradox time differential

Revised geometrical thought



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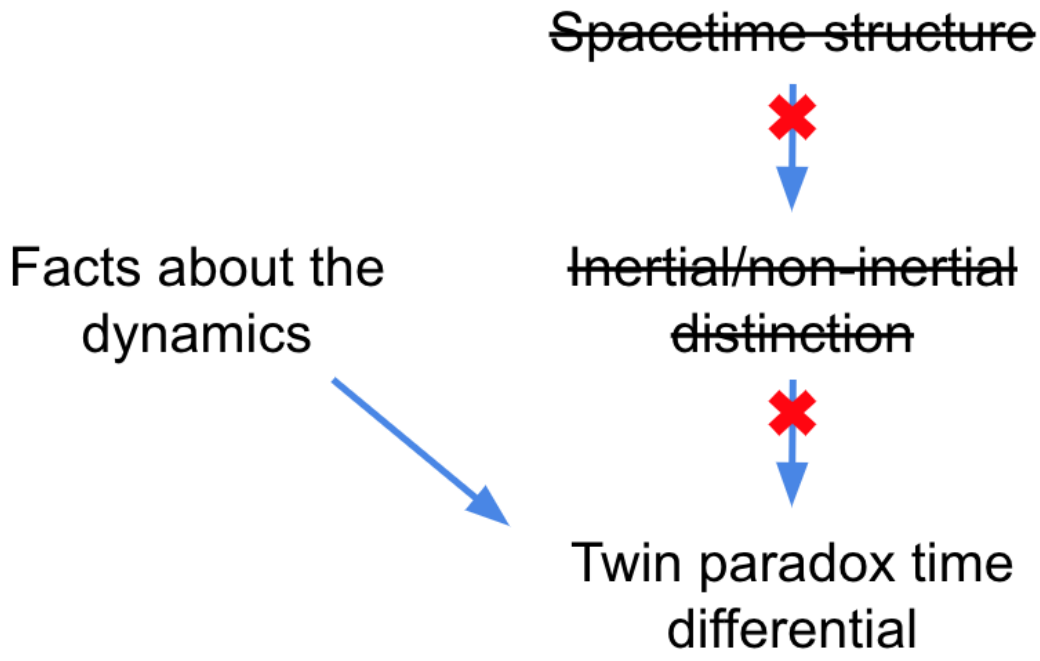

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- We will investigate one of these, which appeals to simultaneity hypersurfaces in *B*’s rest frame.
- We will then consider the more general question of the legitimacy of these accounts.

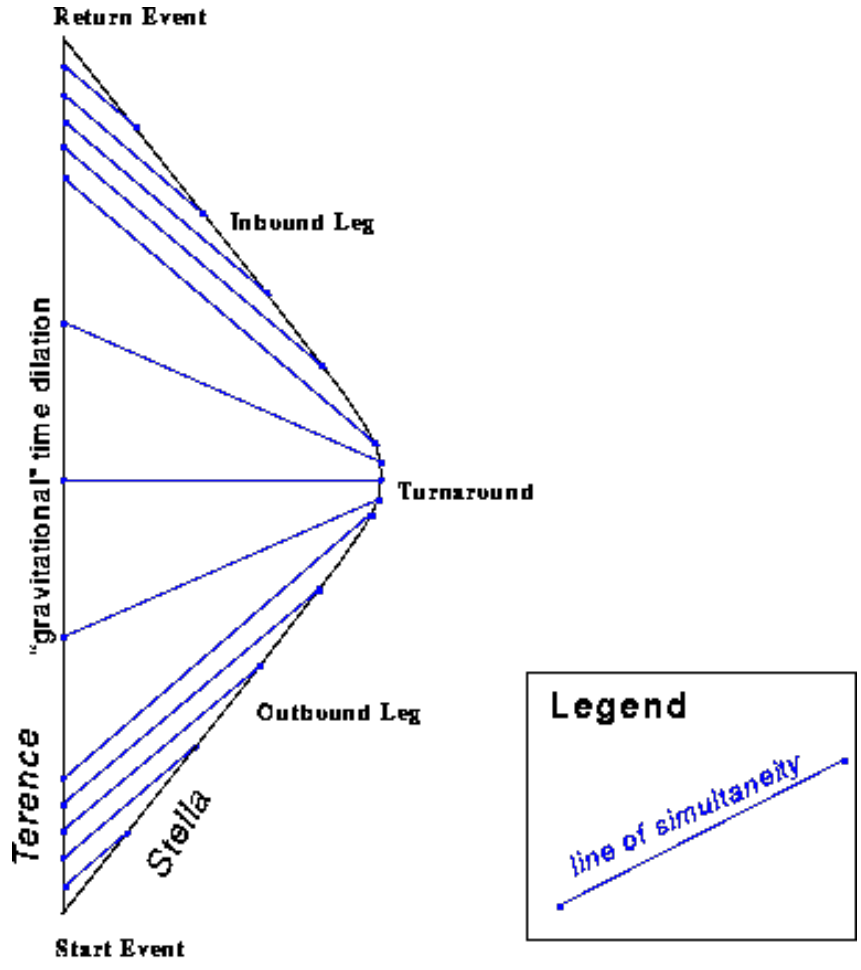
Return Event
Inbound Leg
"gravitational" time dilation
Turnaround
Outbound Leg
Terence
Stella
Start Event
Legend
line of simultaneity

Figure 3

The relativity of simultaneity account

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- Claim: This accounts for the time differential between *A* and *B*.
- This seems fine... but is the account a fundamental one?

The fundamentality of frame-relative accounts

> [E]xplanations of synchrony-independent phenomena in SR that rely crucially on the relativity of simultaneity are not fundamental. *(A common example concerns the clock retardation effect, or 'twins paradox', where it is claimed that at the point of turn-around of the travelling clock, the hyperplanes of simultaneity suddenly change orientation and the resulting 'lost time' accounts for the fact that the clocks when reunited are out of phase. It is worth bearing in mind that the clock retardation effect, like any other synchrony-independent phenomenon in SR, is perfectly consistent with all the non-standard transformations ..., including those which eliminate relativity of simultaneity.)* (Brown 2005, p. 105)

Three reasons for non-fundamentality

1. They are frame-relative.
2. They are convention-relative. (Debs and Redhead 1996.)
3. They only apply to certain versions of the paradox—e.g., not to the cylindrical case.

Two views on frame-relative accounts

Brown-style: They are legitimate, but non-fundamental.

Maudlin-style: They are illegitimate and non-fundamental.

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- This is *grossly confused*, for:
  1. Accelerations are *not* an essential feature of the twin paradox—as we have seen above.
  2. Special relativity *has the resources to distinguish accelerating from non-accelerating trajectories*. (Recall lecture 5.)

Einstein's elevator

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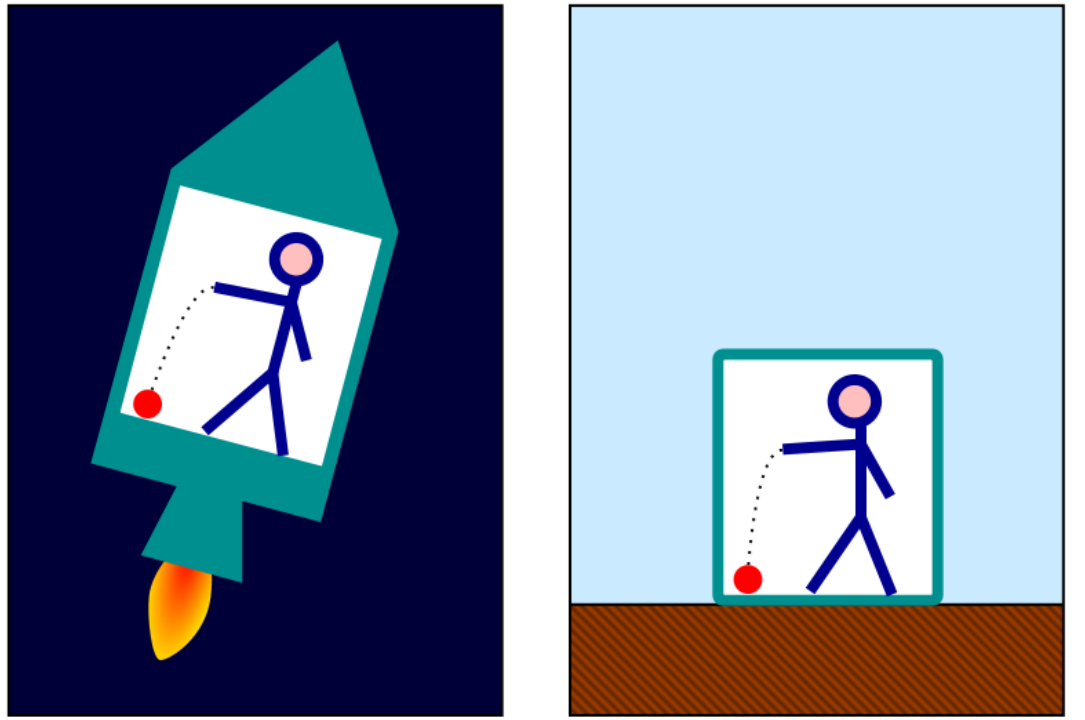

Inertial effects $=$ Gravitational effects

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- One *could* appeal to the equivalence principle to explain (accelerating versions of) the twin paradox: the accelerating twin is subject to a gravitational force.
- But this is really no better than the original (bad!) appeal to accelerations!
- This approach is also in tension with a widespread methodology in the philosophy of physics: *try to understand effects which arise in a given theory* ***in terms of that theory itself****—i.e., without introducing notions which transcend that theory*.

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2. on a Maudlin-style view, accounts of the twin paradox time differential should be given via appeal to the structure of the spacetime in which the twins are embedded.
3. while on a Brown-style view, frame-relative approaches to the twin paradox are legitimate but non-fundamental, on a Maudlin-style view, such approaches are simply illegitimate.
4. appeals to general relativity to account for twin paradox differentials are misguided.

References

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