IPP-SR-12: Acceleration and gravitational redshift

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HT20

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

Acceleration in special relativity

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- Today, we will present and resolve one significant confusion involving acceleration in relativity theory—*viz.*, that regarding *gravitational redshift*.
- Before we do so, however, we must see something of Einstein's *equivalence principle*.

Today

The Einstein equivalence principle

Inertial frames

The strong equivalence principle

Gravitational redshift

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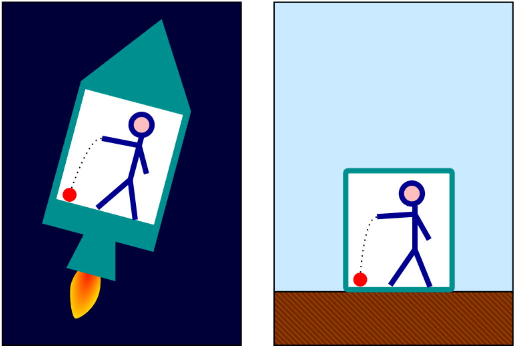
- He realised that in the immediate vicinity of such an observer, gravity would seem to disappear.
- Of course, this would have been known since Newton...
- ...but Einstein's revolutionary insight was that the gravitation field itself "has only a relative existence".

[F]or an observer falling freely from the roof of a house there exists—at least in his immediate surroundings—no gravitational field ... The observer therefore has the right to interpret his state as "at rest". (Einstein, quoted by Pais 1982, p. 178)

Einstein's elevator



Inertial effects $=$ Gravitational effects

The Einstein equivalence principle

Gravity and inertia are the same in their very essence ["wesensgleich"]. (Einstein 1918)

Pauli's take

In Einstein's theory, gravitation is just as much a ficti-tious *force as the coriolis and centrifugal forces are in Newton's theory. (However, it is equally justified to say that in Einstein's theory neither of these two forces is a fictitious force.)* (Pauli 1921, p. 709)

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- Gravitational effects just are inertial effects. (Cf. Einstein's rocket.)
- If you're in a freely-falling frame in which you don't feel any gravitational effects, that's because *there literally are no gravitational effects there for you*.

The heuristic importance of the EEP

> *Einstein himself stressed again and again the heuristic importance of the EEP in his search for what came to be GR. This role of the principle is intimately connected to Einstein thinking of it as a relativity principle. He clearly saw it as extending the special principle of relativity, that states that all inertial motions, including rest, are empirically indistinguishable and thus equivalent in an important sense.* (Lehmkuhl 2019, p. 7)

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For Einstein, seeing the presence of gravitational fields as a coordinate-dependent state of affairs was not a price to be paid but a major achievement of the theory. (Lehmkuhl 2019, pp. 13-14)

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- This will be *crucial* to a complete understanding of gravitational redshift effects.

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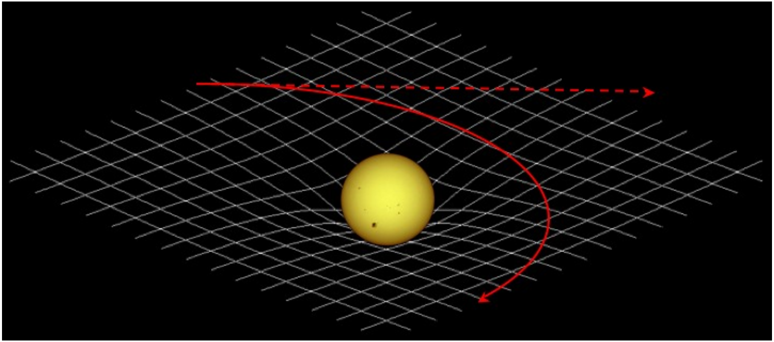
Gravitational redshift

General relativity



The theory in brief

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- g_{ab} is *dynamical*, obeying the *Einstein equation*,

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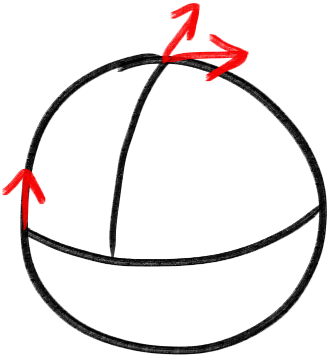
- This says that the spacetime curvature associated with g_{ab} is proportional to the amount of matter (energy-momentum) content in the relevant region.

Curvature



$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}$$

The strong equivalence principle

[L]et us now introduce the following premise: For infinitely small four-dimensional regions the theory of relativity in the restricted sense [i.e., special relativity] holds, if the coordinates are suitably chosen. (Einstein 1916, p. 777)

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(For more on the SEP, see (Brown 2005, ch. 9), (Knox 2013), (Ghins and Budden 2001), or (Read *et al.* 2018).)

Bridge principles

> *[O]ne might look at the EEP as a bridge principle, a principle forming a bridge from GR to Newtonian theory, a bridge that allows us to see the shadows of Newtonian theory in GR. But this bridge is not just about accommodating our "physical habits of thinking" in allowing us to keep operating with the terms 'gravity' and 'inertia', it also implies that a curvature-free spacetime is just as 'gravitational' as a strongly curved spacetime.* (Lehmkuhl 2019, p. 25)

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> *While the Einstein equivalence principle can be seen as a bridge from GR to Newtonian theory, the strong equivalence principle can be seen as a bridge from GR to SR.* (Lehmkuhl 2019, p. 25)

$$\begin{tikzcd}
 & \mathrm{GR} \arrow[dl, "\mathrm{EEP}"'] \arrow[dr, "\mathrm{SEP}"] & \\
\mathrm{NG} & & \mathrm{SR}
\end{tikzcd}$$

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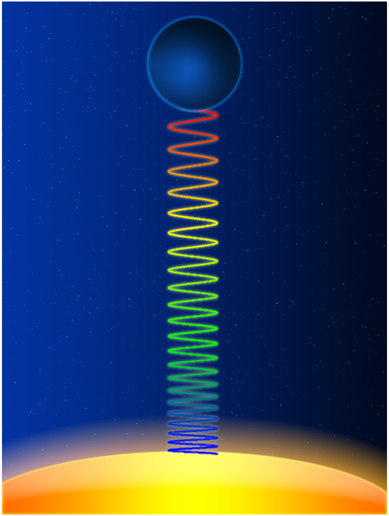

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- More specifically: the wavelength of a photon is longer when observed from further out of a gravitational well.
- Here, the ‘clock’ is the frequency of the photon and a lower frequency is the same as a longer (‘redder’) wavelength.

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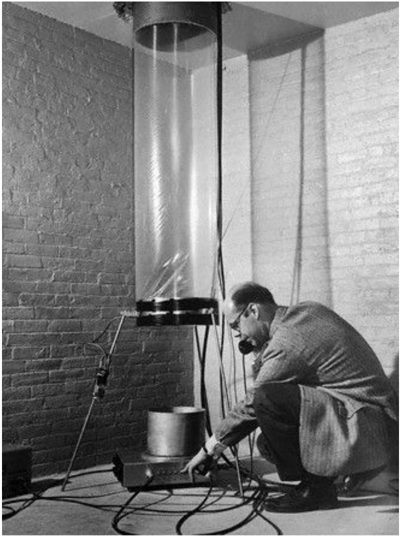
- These experiments used the *Mössbauer effect*: γ-rays in a certain narrow frequency range are emitted and absorbed by two solid samples containing radioactive Fe^{57}.
- When two such samples are placed vertically with a height difference h, the photons emitted from one sample will no longer be absorbed by the other.
- But if the absorber is put into a certain degree of vertical motion relative to the source, the resulting Doppler effect can restore absorption.



Question

Do the results of a single gravitational redshift experiment provide direct evidence for spacetime curvature?


Quantum Mechanics (an embarrassment) - Sixty Symbols

Carroll on gravitational redshift experiments

...simple geometry seems to imply that the [emission and reception intervals] must be the same. But of course they are not; the gravitational redshift implies that the elevated experimenters observe fewer wavelengths per second. ... We can interpret this roughly as 'the clock on the tower appears to run more quickly'. What went wrong? Simple geometry—the spacetime through which the photons traveled was curved.

We therefore would like to describe spacetime as a kind of mathematical structure that looks locally like Minkowski space, but may possess nontrivial curvature over extended regions. (Carroll 2004, pp. 53-54)

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- The SEP states that the local neighbourhood of a gravitational redshift experiment should look approximately special relativistic (i.e., like Minkowski spacetime).
- By the EEP, the experimental setup on the surface of the Earth is in an accelerating frame of reference.
- So considering this setup in an accelerating frame in SR should allow us to derive the correct results!—And indeed we do!

Quantitative calculation

- Quantitatively, we find

$$\Delta t_B \simeq \left(1 - \frac{gh}{c^2}\right) \Delta t_A,$$

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(For more, see (Brown and Read, 2016).)

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Spacetime curvature is *not* required to explain these results; the equivalence principles and special relativity in an accelerating frame will suffice!

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- Ultimately, this motivates the introduction of a curved 'affine connection'—*this is the true place for spacetime curvature in discussions of gravitational redshift*.

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3. Used the EEP and SEP to account for the results of a single gravitational redshift experiment of Pound-Rebka type.
4. Seen the true role of spacetime curvature when accounting for the results of gravitational redshift experiments.

Thanks for attending!

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