

IPP-SR-5: Spacetime structure from Aristotle to Minkowski

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HT20

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

Today

Minkowski's 1908 paper

Kleinian and Riemannian conceptions of geometry

Spacetime structure in Newtonian mechanics

Spacetime structure in special relativity

Further reflections on spacetime

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Further reflections on spacetime

The entrance of spacetime...

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- ▶ What was Einstein's reaction? He accused Minkowski's work of being “superfluous learnedness” (Pais 1982).

The world-postulate

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- ▶ ...but by expressing this notion in four-dimensional geometrical language, Minkowski felt he had shown how *the validity of the world-postulate ... now lies open in the full light of day.* (Minkowski 1909)
- ▶ **Question:** Is this the origin of a Friedman-style 'geometrical approach' to physical theories? (Cf. lecture 1.)

Today's goal

Our goal for today is to spell out the move from dynamical symmetries to spacetime structure.

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Two conceptions of geometry

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Inertial frames and spacetime structure

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- ▶ Sometimes, people also think about the inertial frames as those frames which respect spacetime's 'inertial structure' in a certain way.
- ▶ Today, we will see how this goes, from the Kleinian perspective.

Kleinian approach summarised

- Specify the class of coordinate transformations which relate the inertial frames in the theory under consideration.

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- ▶ Identify the structures and quantities which are *invariant* under those transformations.

Kleinian approach summarised

- ▶ Specify the class of coordinate transformations which relate the inertial frames in the theory under consideration.
- ▶ Identify the structures and quantities which are *invariant* under those transformations.
- ▶ Regard these structures and quantities as picking out different kinds of spacetime.

Today

Minkowski's 1908 paper

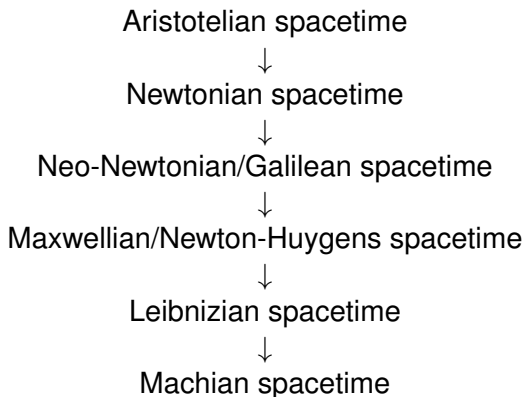
Kleinian and Riemannian conceptions of geometry

Spacetime structure in Newtonian mechanics

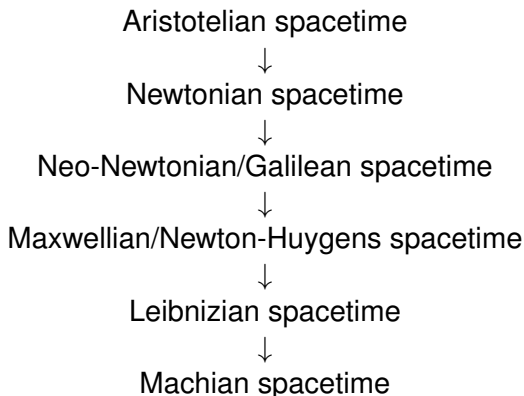
Spacetime structure in special relativity

Further reflections on spacetime

A hierarchy of structures



A hierarchy of structures



(Throughout the following, $\mathbf{R} \in SO(3)$ and any functions of t are smooth.)

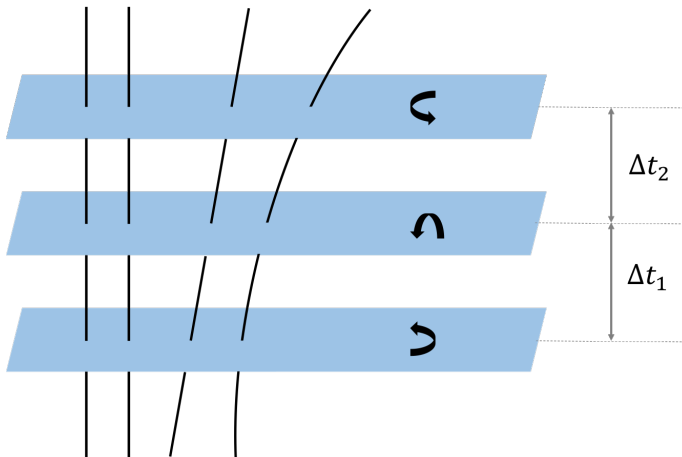
Aristotelian spacetime

$$t \mapsto \pm t + \tau$$

$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x}$$

In Aristotelian spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
3. A standard of rotation across time.
4. A notion of straightness of paths across time.
5. A preferred velocity.
6. A preferred point.



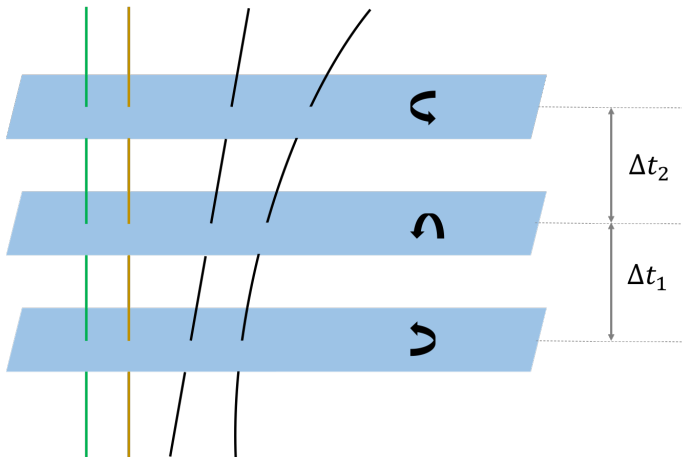
Newtonian spacetime

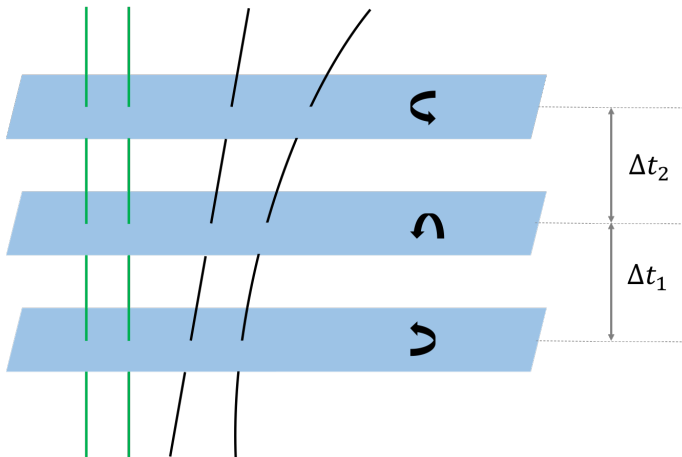
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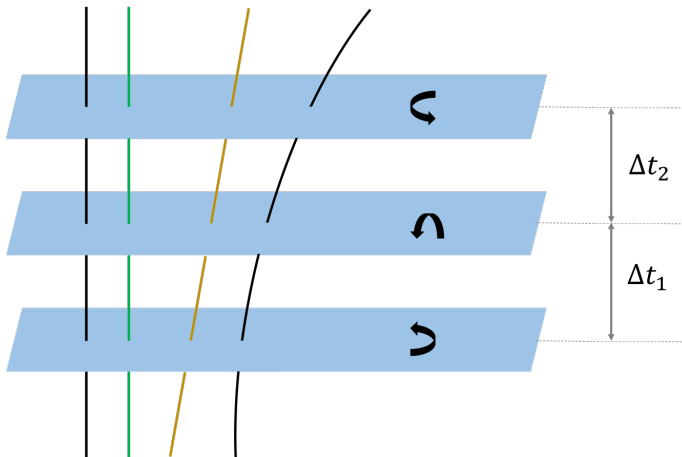


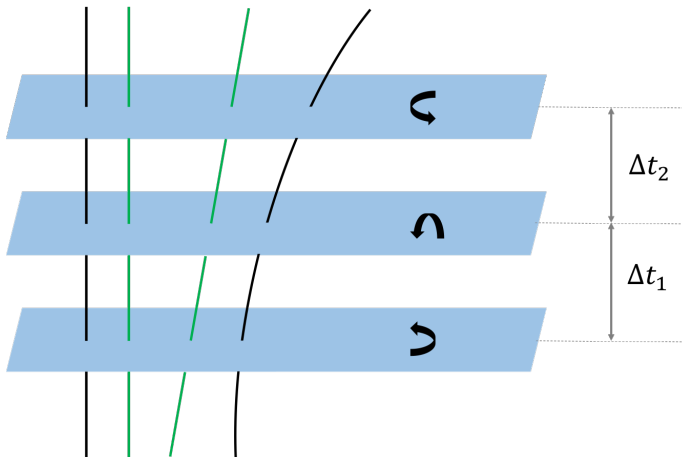
Neo-Newtonian/Galilean spacetime

$$\begin{aligned}t &\mapsto \pm t + \tau \\ \mathbf{x} &\mapsto \mathbf{R}\mathbf{x} + \mathbf{v}t + \mathbf{a}\end{aligned}$$

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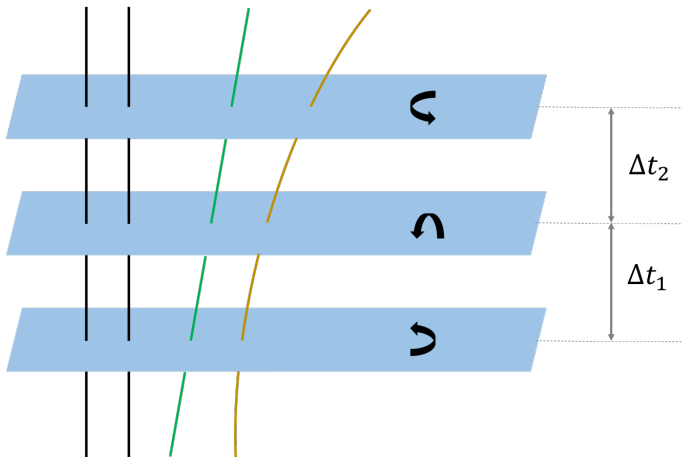


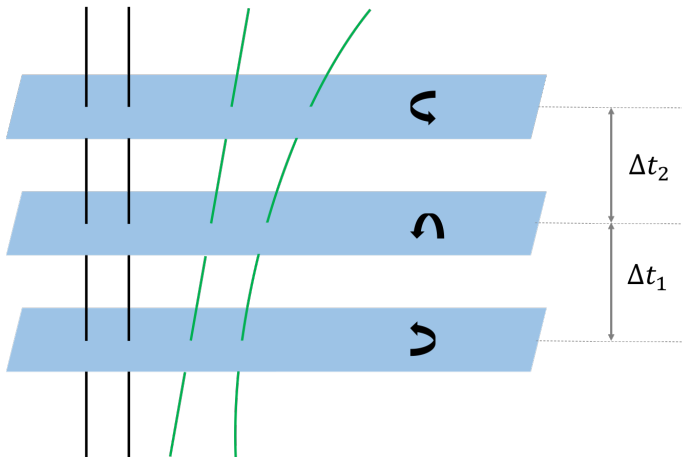
Maxwellian/Newton-Huygens spacetime

$$\begin{aligned}t &\mapsto \pm t + \tau \\ \mathbf{x} &\mapsto \mathbf{R}\mathbf{x} + \mathbf{a}(t)\end{aligned}$$

In Maxwellian/Newton-Huygens spacetime, there is:

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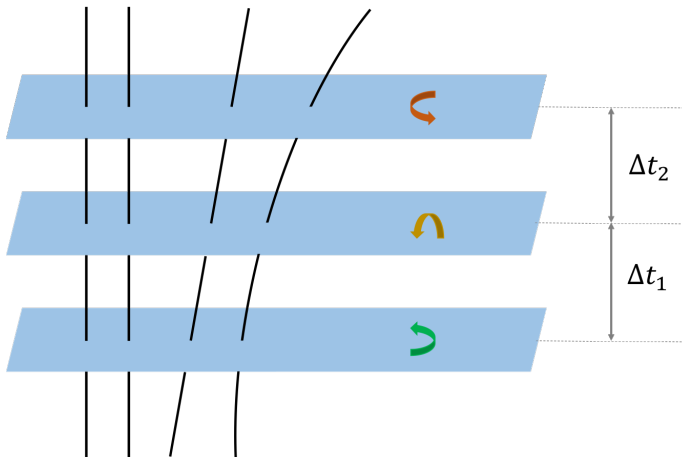


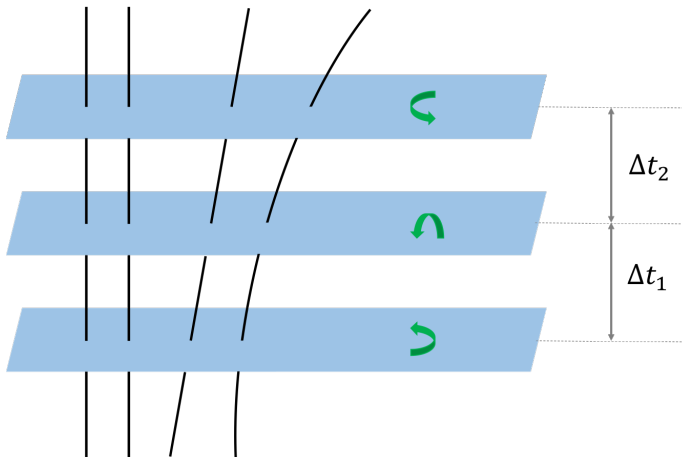
Leibnizian spacetime

$$\begin{aligned}t &\mapsto \pm t + \tau \\ \mathbf{x} &\mapsto \mathbf{R}(t) \mathbf{x} + \mathbf{a}(t)\end{aligned}$$

In Leibnizian spacetime, there is:

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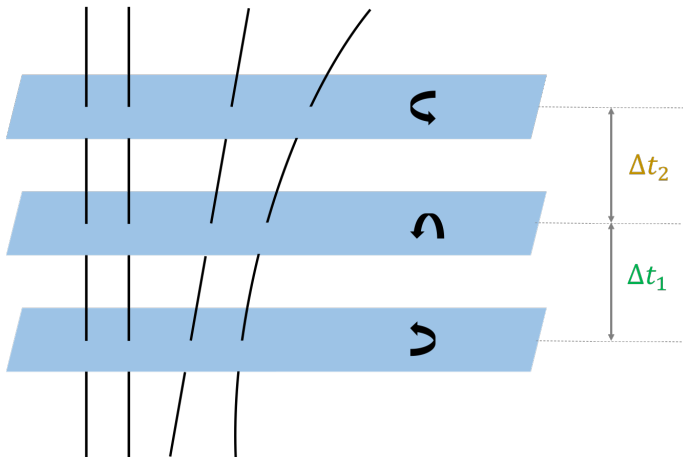


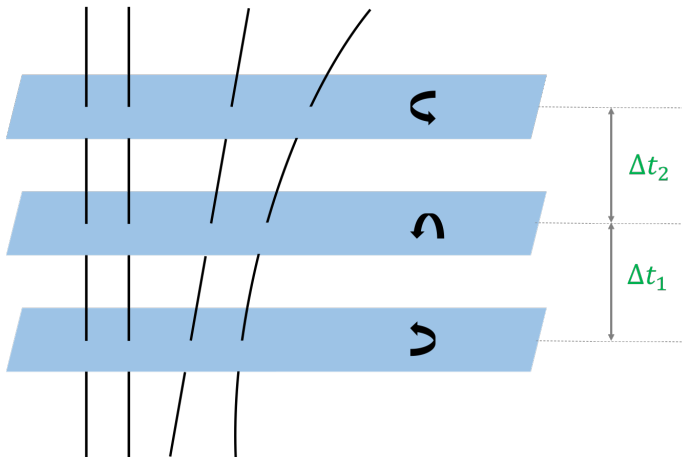
Machian spacetime

$$\begin{aligned} t &\mapsto f(t) && (f \text{ monotonic}) \\ \mathbf{x} &\mapsto \mathbf{R}(t) + \mathbf{a}(t) \end{aligned}$$

In Machian spacetime, there is:

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Question: How does the spacetime structure of special relativity compare with that of the spacetimes we have just seen?

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Further reflections on spacetime

Aside: index notation

- Consider again the coordinate transformations associated with Galilean spacetime. So far, I've written these in vector notation, as

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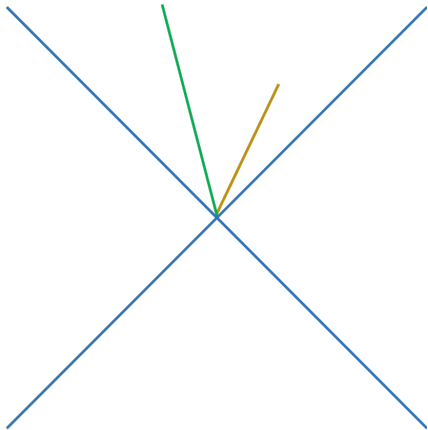
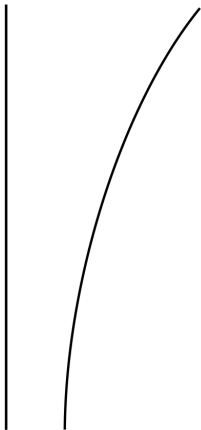
- Note that all terms must have the same free indices, and the Einstein summation convention is used.
- By convention, we use Latin indices ($i, j, \dots = 1, 2, 3$) for spatial indices, and Greek indices ($\mu, \nu, \dots = 0, 1, 2, 3$) for *spacetime* indices.

Minkowski spacetime

$$x^\mu \mapsto \Lambda^\mu{}_\nu x^\nu + a^\mu \quad (\Lambda^\mu{}_\nu \in SO(1,3))$$

In Minkowski spacetime, there is:

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7. A notion of *spacetime* distance.



The interval

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- ▶ I is preserved in all inertial frames in special relativity—i.e., in all frames related by Poincaré transformations.

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- ▶ But these are the transformations associated with Galilean spacetime, as we have seen above.
- ▶ It is natural, therefore, to regard Newtonian mechanics as being *set in Galilean spacetime*.

Earman's adequacy conditions

In (Earman 1989, ch. 3), Earman makes it a very general principle that the spacetime and dynamical symmetries of a theory should match, by laying down two conditions:

- SP1: Any dynamical symmetry of T is a spacetime symmetry of T .
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(Some have gone further, by saying that these principles are *analytically true*—see e.g. (Myrvold 2017).)

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- ▶ With hindsight, Newton violated this requirement: Newtonian physics can be formulated in (merely) *Galilean* spacetime, not *Newtonian* spacetime (as Newton maintained). Occam's razor thus advises against postulating a standard of absolute rest in addition.

Correct spacetime setting for Newtonian mechanics

If we follow the methodology of moving from Newtonian to Galilean spacetime as the correct spacetime setting for Newtonian mechanics, then (it seems) the discovery of *further* invariances of the Newtonian laws would similarly motivate moving to a different spacetime setting again, with even less structure than Galilean spacetime.

Newton's 'Corollary VI'

- ▶ Consider Newton's 'Corollary VI' in the *Principia*:
If bodies moved in any manner among themselves are urged, in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner as if they had not been urged by those forces. (Cajori 1934, p. 21)

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- ▶ This is an ongoing matter of some controversy—see (Knox 2013) and (Wallace 2016) for further discussion.

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- ▶ So...:

There is a precise sense in which Newtonian spacetime has more structure than both Galilean spacetime and Minkowski spacetime. But in this same sense, Galilean and Minkowski spacetime have incomparable amounts of structure; neither spacetime has less structure than the other. The progression towards a less structured spacetime therefore does not continue into the relativistic setting. (Barrett 2015, p. 37)

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1. Distinguished between Kleinian and Riemannian conceptions of geometry.
2. Witnessed the tower of classical spacetime structures.
3. Compared these classical spacetime structures with the structure of Minkowski spacetime.
4. Discussed the correct spacetime setting for Newtonian mechanics.

References



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