IPP-SR-4: Einstein's 1905 derivation of the Lorentz transformations

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HT24

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
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6. General covariance
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Principle and constructive theories

Einstein's 1905 paper

Einstein versus Lorentz

Einstein's misgivings about his 1905 derivation

The Ignatowski transformations

Introduction

- In 1905, Einstein published four papers in the journal *Annalen der Physik*, each of which precipitated a revolution in physics. The papers were on:
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    2. Brownian motion.
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    3. Special relativity.
    4. Mass-energy equivalence.
- The year would come to be known as Einstein's *annus mirabilis*.

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- In effect, it elevates contraction from a dynamical effect to a kinematical effect: *all physics must be conditioned such that it is invariant under Lorentz boosts*.

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3. It does not postulate a luminifrous ether, or a standard of absolute rest, at all.
4. It is a 'principle theory', rather than a 'constructive theory'.

Einstein's insight

Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. (Einstein 1905)

What of the ether?

The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place. (Einstein 1905)

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> *[Principle theories, by contrast,] employ the analytic, not the synthetic method. The elements which form their basis and starting point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes ... have to satisfy ... The theory of relativity belongs to the latter class.*

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- A *principle theory* is a theory that takes certain ‘phenomenologically well-grounded principles’, raises them to the status of *postulates*, and derives from them constraints on what the underlying detailed dynamical equations could be like, without attempting to give a fully detailed account of what those equations *are*.

Paradigm example

- Thermodynamics is a principle theory; the 'phenomenologically well-grounded postulates' in this case are the laws of thermodynamics:

  1LT: When energy passes, as work, as heat, or with matter, into or out of a system, the system's internal energy changes in accord with the law of conservation of energy.

  2LT: In a natural thermodynamic process, the sum of the entropies of the interacting thermodynamic systems increases.

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- The corresponding constructive theory would be the (statistical) kinetic theory of gases.

Motivating principle theories

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- Einstein in 1905 sees himself as being in this situation: Lorentz has been pursuing a constructive approach, but Einstein is bothered by deep suspicions that the true equations governing intermolecular forces are very far from being known.

Einstein's reservations

It is, however, worth registering Einstein's reservations about principle theories:

> *It seems to me ... that a physical theory can be satisfactory only when it builds up its structures from elementary foundations.* (Einstein 1908)
>
> *... when we say we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question.* (Einstein 1919)

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Einstein's operational understanding of coordinates

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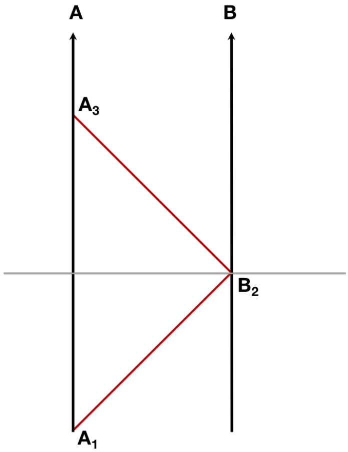
- But even to set up *one* coordinate system, we need more than this: we need to *decide* how to synchronise clocks that are spatially separated from one another.

Definition of simultaneity



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- (**Question:** Is this the *only* way of ‘spreading time through space’ in special relativity? We will seem more on this in lecture 7...)

Einstein's two postulates

RP: The laws by which the states of physical systems undergo change are not affected, whether these changes be referred to the one or the other of two systems of coordinates in uniform translatory motion.

LP: Any ray of light moves in the 'stationary' system of coordinates with the determined velocity *c*, whether the ray be emitted by a stationary or by a moving body. Hence [*sic*?]

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}},$$

where time interval is to be taken in the sense of the definition in section 1.

The constancy of the speed of light

Note that LP does *not* state that the speed of light is the same in all inertial frames—though that follows straightforwardly from RP and LP.

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1. The homogeneity of space and time. ('Every point in space and time is the same as every other.')
2. The isotropy of space. ('There is no privileged direction in space.')
3. 'Reciprocity': If two inertial coordinate systems S and S' are such that S' is moving with speed v in the positive x direction relative to S, then S is moving with speed v in the negative x direction relative to S'.

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- Main point: although Reciprocity is invoked at certain points in Einstein's derivation (as we'll see), it is not necessary to take this as an independent assumption: rather, it can be derived from Einstein's other assumptions.

Homogeneity and linearity

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- Einstein doesn't spell out how this works, but a reconstruction can be found in (Brown 2005, §2.3).

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- Homogeneity implies that the coefficients $\partial f^{\mu}/\partial x^{\nu}$ must be independent of the x^{ν} coordinates, which means that f^{μ} must be linear functions of the coordinates x^{μ}.

Back to Einstein

- Let K be a 'stationary' system, and let (t, x, y, z) be coordinates for K, determined by the conditions of surveyability-using-rods-and-clocks-that-are-stationary-in-K and the Einstein definition of simultaneity applied in K (for t).

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- Let k be a system of coordinates that is moving with speed v along the positive x-direction relative to the 'stationary' system K. Let (τ, ξ, η, ζ) be coordinates for k, determined by the conditions of surveyability-using-rods-and-clocks-that-are-stationary-in-k and the Einstein definition of simultaneity applied in k (for τ).

Lorentz transformations, up to $\phi(v)$

- Using Einstein synchrony in k and the linearity of the coordinate transformations, one derives

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Lorentz transformations, up to $\phi(v)$

- Using Einstein synchrony in k and the linearity of the coordinate transformations, one derives

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- Consider a light ray emitted from the origin in the positive ξ direction. Use RP and LP to write down expressions for the relationship between ξ and τ that holds on the path of this ray, and similarly (using RP alone) for the relationship between x and t that holds on the path of this ray. From this, one derives

$$\xi = \phi(v)\,\gamma\,(x - vt).$$

Lorentz transformations, up to $\phi(v)$

- Using Einstein synchrony in k and the linearity of the coordinate transformations, one derives
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$$\xi = \phi(v)\,\gamma\,(x - vt).$$
- Similarly, by considering rays of light emitted in the η and ζ directions from the perspectives of both K and k, one obtains
$$\eta = \phi(v)\,y,$$
$$\xi = \phi(v)\,z.$$

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- We argue somehow against the rogue possibility that $\phi(v) = -1$ (using continuity and $\phi(0) = +1$?)
- It follows that $\phi(v) = 1$. This yields the Lorentz transformations!

Back to Michelson-Morley

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- Einstein's 1905 result predicts the null result of ether wind experiments, such as that of Michelson and Morley, once and for all. Indeed, it does so *trivially*—just by insisting upon the RP.
- One way to understand Einstein is as insisting that *mechanics* should also be Poincaré invariant—he is making Poincaré invariance universal, as a *kinematical constraint*.

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Einstein versus the trailblazers

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- Actually, Lorentz followed this statement with a concession:

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- As Brown writes, "The full meaning of relativistic kinematics was simply not properly understood before Einstein." (Brown 2005, p. 68)

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- This, indeed, was achieved by Albert Keinstein in 1705. (See Brown 2005, §3.3).
- In this sense, was Einstein, in deriving a *different* kinematical constraint (*viz*., Poincaré invariance, rather than Gailean invariance), really being any more radical than Newton?

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1. The treatment of rods and clocks as primitive bodies, not “moving atomic configurations”. (1921, 1949)
2. The special role of light. (1935, 1949)

Einstein's misgivings: rods and clocks

"One is struck [by the fact] that the theory [of special relativity] ... introduces two kinds of physical things, i.e. (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities. However, the procedure justifies itself because it was clear from the very beginning that the postulates of the theory are not strong enough to deduce from them sufficiently complete equations ... in order to base upon such a foundation a theory of measuring rods and clocks. ... But one must not legalize the mentioned sin so far as to imagine that intervals are physical entities of a special type, intrinsically different from other variables ('reducing physics to geometry', etc.). (Einstein 1969)

Einstein’s misgivings: the role of light

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[T]he Lorentz transformation transcended its connection with Maxwell's equations and had to do with the nature of space and time in general.
(Einstein 1955)

The mature Einstein

The content of the restricted relativity theory can accordingly be summarised in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations. (Einstein 1954)

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- Ignatowski seeks to derive the Lorentz transformations using the RP, but *without the LP*.
- This claim should elicit suspicion: which of the remaining assumptions is violated by Newtonian physics (complete with Galilean transformation—cf. again the fable of Keinstein)?

The Ignatowski transformations

The Ignatowski transformations read as follows, where K is some hitherto-unspecified universal constant:

$$
\begin{aligned}
t' &= \left(1 - Kv^2\right)^{-1/2} (t - Kvx)\,, \\
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Note now three special cases:

1. Setting $K = 0$ yields a Galilean transformation.
2. Setting $K = 1$ yields a Lorentz transformation.
3. Setting $K = -1$ yields a Euclidean transformation.

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- So dropping the LP is not sufficient to derive the Lorentz transformations.
- Sometimes, authors rule out $K = -1$ as "unphysical" (see e.g. Pelissetto and Testa 2015)—to this one should also object, for there are plenty of physical applications of theories with Euclidean symmetries—e.g., any theory which uses the Poisson equation.

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3. Discussed Lorentz's response to that derivation, and questioned the radicality of Einstein's manoeuvres.
4. Discussed Einstein's misgivings about his 1905 derivation.

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1. Articulated the distintion between principle and constructive theories.
2. Witnessed Einstein's 1905 derivation of the Lorentz transformations.
3. Discussed Lorentz's response to that derivation, and questioned the radicality of Einstein's manoeuvres.
4. Discussed Einstein's misgivings about his 1905 derivation.
5. Considered, following Ignatowski, the enriched class of transformations derivable once the LP is dropped.

Summary

In this lecture, we have:

1. Articulated the distintion between principle and constructive theories.
2. Witnessed Einstein's 1905 derivation of the Lorentz transformations.
3. Discussed Lorentz's response to that derivation, and questioned the radicality of Einstein's manoeuvres.
4. Discussed Einstein's misgivings about his 1905 derivation.
5. Considered, following Ignatowski, the enriched class of transformations derivable once the LP is dropped.

Next week: *spacetime structure*.

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