DISCUSSION PAPER SERIES

No. 7979

A NEW KEYNESIAN OPEN ECONOMY MODEL FOR POLICY ANALYSIS

Wendy Carlin and David Soskice

INTERNATIONAL MACROECONOMICS

ISSN 0265-8003

A NEW KEYNESIAN OPEN ECONOMY MODEL FOR POLICY ANALYSIS

Wendy Carlin, CEPR and University College London
David Soskice, Duke University and Nuffield College, University of Oxford

Discussion Paper No. 7979
September 2010

Centre for Economic Policy Research
53–56 Gt Sutton St, London EC1V 0DG, UK
Tel: (44 20) 7183 8801, Fax: (44 20) 7183 8820
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 7979

September 2010

ABSTRACT

A New Keynesian Open Economy Model for Policy Analysis*

"Macroeconomics without the LM curve" has begun to move advanced undergraduate closed economy macroeconomics teaching models away from the IS/LM approach to simple versions of the New Keynesian models taught in graduate courses and used in central banks. But the equally traditional and antiquated Mundell-Fleming model still dominates undergraduate open economy macroeconomics. We develop a graphical and simplified New Keynesian model of the small open economy to replace it. The model features rational expectations in both the foreign exchange market and the central bank, and is well-suited to analyze how an inflation targeting central bank responds optimally to a variety of shocks. The graphical approach highlights how exchange rate expectations in the open economy impinge on the central bank's decision-making. The basic model assumes the central bank targets domestic inflation and we show how a CPI inflation target modifies the analysis.

JEL Classification: E1, E3, E4 and E5
Keywords: CPI inflation targeting, domestic inflation targeting, exchange rate overshooting, Mundell-Fleming model, new Keynesian open economy model and optimal monetary policy rule

Wendy Carlin
Department of Economics
University College London
Gower Street
London
WC1E 6BT

Email: w.carlin@ucl.ac.uk

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=115115

David Soskice
University of Oxford
Nuffield College
New Road
Oxford
OX1 1NF

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=109050

* We are very grateful for comments and advice from Philip Arestis, Javier Lozano, Liam Graham, David Vines, and Simon Wren-Lewis.

Submitted 15 August 2010

1. Introduction

The lively debate about how modern macroeconomics can be made accessible to non-specialists and undergraduates has been largely confined to the closed economy (Carlin and Soskice, 2005, Romer, 2000, Taylor, 2000, Walsh, 2002). The need to analyze the macroeconomic consequences of the global financial crisis has highlighted the lack of a tractable open economy macroeconomic model suitable for policy analysis. Economists have at hand the Mundell-Fleming model and although it has been referred to in the policy debate surrounding the crisis, it is unsatisfactory because the mechanisms behind the model predictions do not match those of contemporary economics or economies.

Specifically:

(i) Mundell-Fleming is *LM*-based and assumes the monetary authorities use the money supply to implement monetary policy. By contrast, modern central bank practice normally uses the interest rate.

(ii) Mundell-Fleming assumes constant prices. By contrast, the analysis of inflation is central to modern open economy macroeconomics and central bank methodology is concerned to stabilize the rate of inflation.

(iii) Mundell-Fleming sees fiscal and monetary policy as alternatives depending on the exchange rate regime, indeed fiscal policy is ineffective outside a fixed exchange rate regime. In modern central bank methodology, the exchange rate is not targeted, and fiscal policy is a substitute for monetary policy when the latter loses traction.

Graduate teaching and research as well as central bank practice meanwhile have increasingly adopted some form of New Keynesian open economy model with an inflation targeting central bank (e.g. Svensson, 2000, Clarida, Gali and Gertler, 2001, Gali and Monacelli, 2005, Gali, 2008 or an estimated model such as Smets and Wouters, 2003). The major problem of these models is not a theoretical one; it is that they are not user-friendly and are largely inaccessible other than to specialists. In practical terms, it is not feasible to make these important models as they stand available to undergraduates and non-specialists.

Our aim is to provide a simplified version of the New Keynesian model, which combines the benefits of a graphical approach as exemplified by Mundell-Fleming with the fit to contemporary inflation-targeting central banks and the forward-looking behaviour of the foreign exchange market and policy-makers of the New Keynesian models. An advantage of the model we propose is that it builds directly on the closed economy case, extending an apparatus that is already familiar.

We follow Gali and Monacelli (2005) in providing a common model for closed and open economy analysis and in centering the analysis on domestic inflation targeting. In line with Clarida, Gali and Gertler (2001), we assume that the supply-side is such that the central bank faces a short-run trade-off between inflation and output. Like Svensson (2000) we focus on the prominent role the exchange rate plays in the transmission mechanism of monetary policy. We also adopt a number of his assumptions about the lags in the model that capture timing relationships most relevant to real-world central banks. In particular, we assume there is a one period policy lag from a change in the interest rate to its effect on output. The latter reflects pass through lags in the pricing of tradeables.

It is worth emphasizing two points. The first is that New Keynesian theory has tended to downplay the role of permanent demand shocks, while undergraduate teaching, recent events as well as policy discussion more generally attach to them a greater importance. Second, this underlines a central difference between the analysis of open and closed economies. In closed economies the real interest rate (the stabilizing rate (Allsopp and Vines, 2000) or the natural rate or the Wicksellian rate (Woodford, 2003)) brings aggregate demand into line with supply. Thus a positive permanent demand shock raises the Wicksellian/ medium term real interest rate. But in a small open economy, the medium term real interest rate is fixed by the world rate. So instead it is the real exchange rate that has to appreciate. Thus as we shall see, in the small open economy it is the real exchange rate and not the real interest rate that acts as the stabilizer.

Section 2 sets out the open economy extension of the 3-equation New Keynesian model (Carlin & Soskice, 2005) assuming domestic inflation targeting. Section 3 develops the graphical analysis of the model and shows how an inflation-targeting central bank sets policy optimally in the face of a variety of shocks. In Section 4, we show how the baseline model of a central bank that targets domestic inflation can be amended to allow for CPI inflation targeting. This brings out another important channel through which the exchange rate affects the economy.

2. The Small Open Economy Model

The open economy *IS* curve is

$$y_{t+1} = A - ar_t + bq_t . \tag{1}$$

where A is the level of autonomous demand, r is the real interest rate and q is the log of the real exchange rate. This is defined as follows: $q \equiv p^* + e - p$, where p^* is the log of the world price level, e, the log of the nominal exchange rate and p the log of the domestic price level. Producers of differentiated goods in the home economy set their prices on the basis of domestic costs, and labour is the only input. A rise in e (q) is a nominal (real) depreciation: home's competitiveness improves vis-à-vis the rest of the world. We assume the Marshall-Lerner condition holds so that a real depreciation raises net exports, i.e. $b > 0$. There is a one-period lag from both the interest rate and real exchange rate to output.

Medium-run equilibrium is defined by $y = \bar{y}$ and $r = r^*$. This pins down the real exchange rate in medium-run equilibrium:

$$\bar{q} = \frac{1}{b}\left(\bar{y} - A + ar^*\right). \tag{2}$$

Note that $\bar{q}$ is a function of the level of autonomous demand, A, in the medium-run equilibrium. This parallels the dependence of the stabilizing or Wicksellian real interest rate, $\bar{r}$ on the level of autonomous demand in the closed economy model (Carlin and

Soskice, 2005). An exogenous rise in equilibrium output, a fall in autonomous demand (including world demand) or a rise in the world real interest rate implies a depreciated real exchange rate in the new medium-run equilibrium.

The medium run equilibrium is characterized by

$$y=\overline{y};\ q=\overline{q};\ r=r^{*}\text{and } \pi=\pi^{T}.$$

We assume that home's inflation target is equal to world inflation, $\pi^{T}=\pi^{*}$. This is a simplifying assumption but may also be the choice of the central bank if it prefers a stable nominal exchange rate in equilibrium. If it chooses a different inflation target from world inflation, the nominal exchange rate in equilibrium will be appreciating if home's inflation target is below world inflation or depreciating if it is above in order to keep the real exchange rate constant when inflation is at target. We assume that the world inflation rate is constant.

In a flexible exchange rate small open economy there are two related differences to the closed economy. As we have seen, the real interest rate in the medium-run equilibrium is set by the exogenous world real interest rate rather than by domestic parameters such as the level of autonomous demand as in the closed economy. The real exchange rate rather than the real interest rate varies in medium-run equilibrium in response to permanent aggregate demand and supply shocks. Second, in the dynamic adjustment of the economy to shocks, the central bank chooses the deviation of home's real interest rate from the world rate. Output responds both to changes in the interest rate and to the associated changes in the real exchange rate.

We retain the assumptions from the closed economy model (Carlin and Soskice, 2005) that the optimizing central bank uses rational expectations, that there is persistence in the inflation process, which means the policy-maker faces a trade-off between inflation and the output gap in the short-run, and that there is a policy lag from central bank interest rate decisions to output. We introduce agents in the foreign exchange market who have rational expectations.

The central bank has two problems to solve in period 0 after a shock that moves inflation away from its target level to $\pi_0 \neq \pi^*$. The first problem is this: if we assume $\pi_0 > \pi^*$, the central bank has to work out the optimal future course of the deflation of output below and back up to its equilibrium level, $y_{1,} y_2, y_3, ..., \bar{y}$, which will return inflation to π^*. (Note that it cannot influence y_0 since its instrument the interest rate operates on output with a one period lag. The interest rate also operates on output indirectly via its effect on the exchange rate; and here too the exchange rate operates on output with a one period lag.) The second problem for the central bank in period 0 is to set the interest rate r_0 to implement y_1. This is straightforward in the closed economy: since the *IS* curve is $y_{t+1} = A - ar_t$, the central bank needs simply to set $r_0 = \frac{A - y_1}{a}$. But in the open economy, the IS curve is $y_{t+1} = A - ar_t + bq_t$. A change in the real exchange rate will also affect output: an appreciation in response to the central bank's interest rate hike will dampen output and reduce the required rise in the interest rate (as compared with the closed economy). The central bank must therefore take into account how the foreign exchange market will react to the shock and to the sequence of output gaps to be implemented by the central bank.

In order to pin down the interest rate the central bank needs to set once the shock has been observed, we break the problem into the following steps:

1. Find the sequence of output gaps along the path to equilibrium (using the Phillips curve and the central bank's loss function). This is exactly the same as in the closed economy.
2. Assume that the rate at which the interest rate deviation (above the world interest rate in the case of a positive shock to inflation) is reduced by the central bank is the same as the rate of decline of the output gap (from Step 1) on the path to equilibrium.
3. Given the equilibrium real exchange rate, use the uncovered interest parity (UIP) condition to calculate the initial appreciation of the real exchange rate to q_0 such that the subsequent expected exchange rate loss from holding home's bonds is

exactly offset by the interest rate gains during the adjustment to equilibrium (from Step 2).

4. Knowing y_1 from Step 1 and q_0 from Step 3, use the IS curve to work out the rise in home's interest rate above r^* in period 0, and in general r_t along the adjustment path.

Step 1. We begin with the Phillips curve and the monetary rule equations, which under the assumption of domestic inflation targeting, are the same as in the closed economy.[1] Persistence in the inflation process is reflected in the Phillips curve, which is:

(Phillips Curve, *PC*) $$\pi_t = \pi_{t-1} + \alpha(y_t - \overline{y}). \tag{3}$$

The central bank is modelled as operating under discretion. It minimizes a loss function: $L_t = (y_t - \overline{y})^2 + \beta(\pi_t - \pi^T)^2$, where $\beta > 1$ characterizes a central bank that places less weight on output fluctuations than on deviations in inflation, and vice versa.[2] The central bank optimizes by minimizing its loss function subject to the Phillips curve. This produces the monetary rule equation:

(Monetary Rule, *MR*) $$(y_t - \overline{y}) = -\alpha\beta(\pi_t - \pi^T). \tag{4}$$

It shows the relationship in period *t* between the inflation rate chosen indirectly and the level of output chosen directly by the central bank in period *t-1* to maximize its utility given its preferences and the constraints it faces.

From equations (3) and (4), we derive the rate of decline of the deviations of inflation from target and output from equilibrium:

---

[1] In Section 4, the assumption of domestic inflation targeting is replaced by CPI inflation targeting.

[2] In this simple form, the central bank's loss function implies that it cares only about the current period. However, even a central bank operating under discretion, in the sense that it cannot commit not to re-optimize each period, may care about the inflation it bequeaths to the following period's monetary policy makers. A simple way of capturing this without complicating the model is to increase the weight on inflation in the loss function.

$$
\begin{aligned}
\left(\pi_t - \pi^T\right) &= \left(\pi_{t-1} - \pi^T\right) - \alpha^2 \beta (\pi_t - \pi^T) \\
&\rightarrow \frac{\left(\pi_{t-1} - \pi^T\right)}{\left(\pi_t - \pi^T\right)} = \frac{1}{1+\alpha^2 \beta} \equiv \lambda = \frac{y_{t-1} - \bar{y}}{y_t - \bar{y}}
\end{aligned}
\tag{5}
$$

Note that the proportionate rate of decline, λ, of the output gap chosen by the central bank depends only on the parameters of the Phillips curve and of the central bank's loss function: no open economy elements are involved.

Steps 2 and 3 Assuming that the central bank reduces the interest rate deviation linearly, we can show that it does this at the same rate as the output gap falls.[3] We therefore have:

$$
r_{t+1} - r^* = \lambda (r_t - r^*) \tag{6}
$$

Hence the cumulative interest gain from holding home bonds during the adjustment process is

$$
\sum_{t=0}^{\infty} (r_t - r^*) = (r_0 - r^*)[1 + \lambda + \lambda^2 + ..] = (r_0 - r^*)/(1-\lambda).
$$

---

[3] Let y, r and q be in deviation terms. Let $r_t = \rho r_{t-1}$ and $y_t = \lambda y_{t-1}$ along the optimal adjustment path. We will show $\rho = \lambda$.

$$
\begin{aligned}
& y_{t+1} = -ar_t + bq_t \\
& -q_t = \sum_{i=0}^{\infty} r_{t+i} = r_t [1 + \rho + \rho^2 + ..] = \frac{r_t}{1-\rho} \\
& \rightarrow y_{t+1} = -\left(a + \frac{b}{1-\rho}\right) r_t \\
& \rightarrow y_t = -\left(a + \frac{b}{1-\rho}\right) r_{t-1} \\
& \rightarrow \frac{y_{t+1}}{y_t} = \frac{r_t}{r_{t-1}} \\
& \therefore \ \rho = \lambda
\end{aligned}
$$

and, by the real *UIP* condition, this must be equal to the expected real depreciation over the whole period of adjustment, $\bar{q} - q_0$, implying

$$\frac{r_0 - r^*}{1-\lambda} = \bar{q} - q_0 \tag{7}$$

(*QQ* equation) [4].

Step 4. The central bank now uses equation (7), which we call *QQ*, in the *IS* equation to substitute for q_0 :

$$\begin{aligned} y_1 - \bar{y} &= -a(r_0 - r^*) + b(q_0 - \bar{q}) \\ &= -\left(a + \frac{b}{1-\lambda}\right)(r_0 - r^*) \text{ and in general,} \\ y_t - \bar{y} &= -\left(a + \frac{b}{1-\lambda}\right)(r_{t-1} - r^*). \end{aligned} \tag{8}$$

(*RX* equation)

Equation (8), which we call *RX*, pins down the interest rate the central bank has to set to achieve its desired output gap in response to a shock, given the rational expectations behaviour of the foreign exchange market.

From the *RX* equation, we can see that the central bank's interest rate response to a shock depends on the parameters of the IS equation, the Phillips curve and on the central bank's

---

[4] The *UIP* condition is that $i_t - i* = e^E_{t+1} - e_t$ where e^E_{t+1} is the log of the nominal exchange rate expected in $t + 1$ and i is the nominal interest rate. Adding $-\pi^E_{t+1} + \pi*^E_{t+1}$ to both sides and rearranging the RHS implies the real *UIP* condition $r_t - r* = q^E_{t+1} - q_t$. Summing both sides over the whole adjustment period, letting $N \to \infty$, and noting $q^E_N \to \bar{q}$, we have:

$$\sum_{t=0}^{N}(r_t - r*) = \left(q^E_N - q^E_{N-1}\right) + \left(q^E_{N-1} - q^E_{N-2}\right) + .. + \left(q^E_2 - q^E_1\right) + \left(q^E_1 - q_0\right) = \frac{r_0 - r*}{1-\lambda} = \bar{q} - q_0.$$

preferences:[5] $y_t - \bar{y} = -\left(a + b\left(1 + \frac{1}{\alpha^2\beta}\right)\right)(r_{t-1} - r^*)$. The intuition is as follows. For a given optimal output response (e.g. to a positive inflation shock), the extent to which the interest rate must be raised above the world interest rate is reduced, the more interest sensitive is investment, a, (as in the closed economy) and the more that demand responds to the real exchange rate, b. The real exchange rate appreciation that accompanies the rise in the interest rate does ‘some of the work’ and perhaps in practice a lot of the work in achieving the central bank’s desired negative output gap. In graphical terms, higher a, b flatten the slope of the *RX* curve (in r, y space): a given desired output gap leads to a smaller interest rate response.

The second effect (via the parameter b) is modified by the Phillips curve and monetary rule parameters. For a given b, the more responsive is inflation to the output gap (higher α) and the more averse to deviations of inflation from target is the central bank (higher β), the more is the interest rate raised above the world interest rate. The reason is that higher α and or higher β produce a more rapid adjustment back to equilibrium along the central bank’s optimal path. The foreign exchange market response to the shock has to take into account not only the interest and exchange rate sensitivity of aggregate demand but also the interaction between the monetary policy rule and the Phillips curve, since this determines the adjustment path to equilibrium (i.e. the sequence of interest rate deviations and output gaps). If the desired output gap shrinks more rapidly, then the higher is the initial interest rate and real exchange rate response. In graphical terms, higher α, β steepen the slope of the *RX* and *QQ* curve (in r, y space): a given desired output gap leads to a larger interest rate response.

---

[5] By substituting for $\lambda = \frac{1}{1+\alpha^2\beta}$, we can express the *RX* equation in terms of the parameters of the Phillips curve, central bank loss function and IS curve:

$$y_t - \bar{y} = -\left(a + \frac{b}{1-\lambda}\right)(r_{t-1} - r^*) = -\left(a + b\left(1 + \frac{1}{\alpha^2\beta}\right)\right)(r_{t-1} - r^*).$$

3. Graphical Analysis of Stabilization Policy under Flexible Exchange Rates

The graphical analysis of dynamic adjustment in the 3-equation closed economy model uses the *IS* and Phillips curve diagrams. For open economy analysis, we add a third panel as shown in Fig. 1 with the real interest rate on the vertical axis and the log of the real exchange rate on the horizontal. The *UIP* equation is shown by a line with a slope of –45° through the intersection of $r = r^*$ and $q = \bar{q}$. The *QQ* and *RX* curves are also shown. The *QQ* curve goes through r^* and $\bar{q}$ and shifts with a shift in the world real interest rate or the equilibrium real exchange rate. The *RX* curve goes through r^* and $\bar{y}$ and shifts correspondingly; it is invariant to shifts in $\bar{q}$ and in *A*. The initial constant inflation equilibrium is shown in Fig. 1, where the *IS* curve is indexed by the equilibrium real exchange rate, $\bar{q}$.

Figure 1. Small Open Economy Model

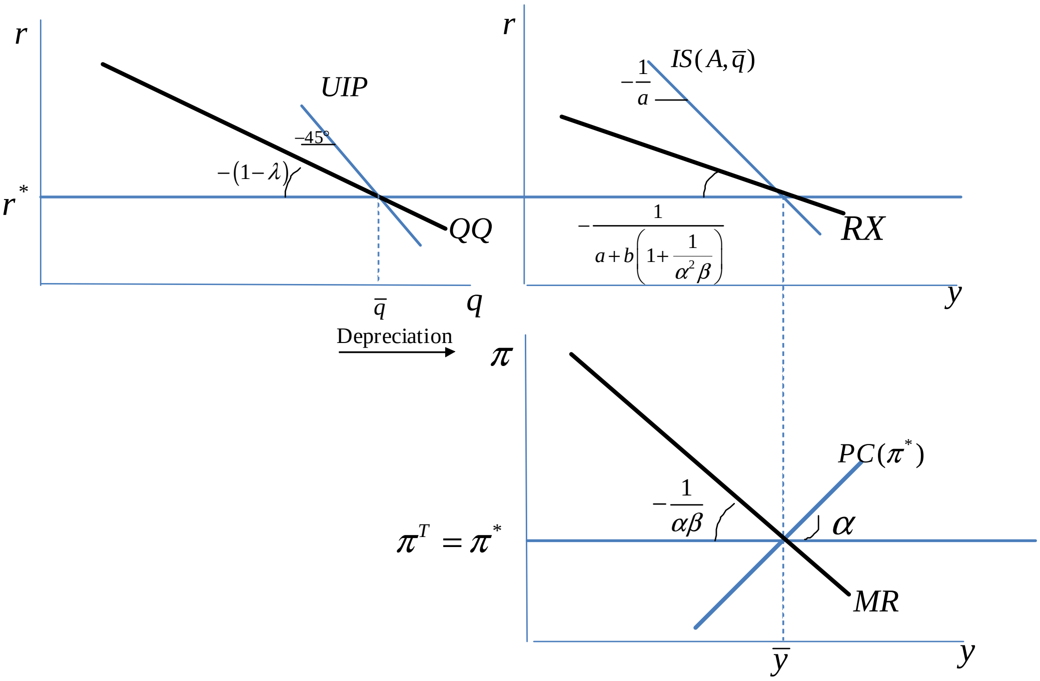
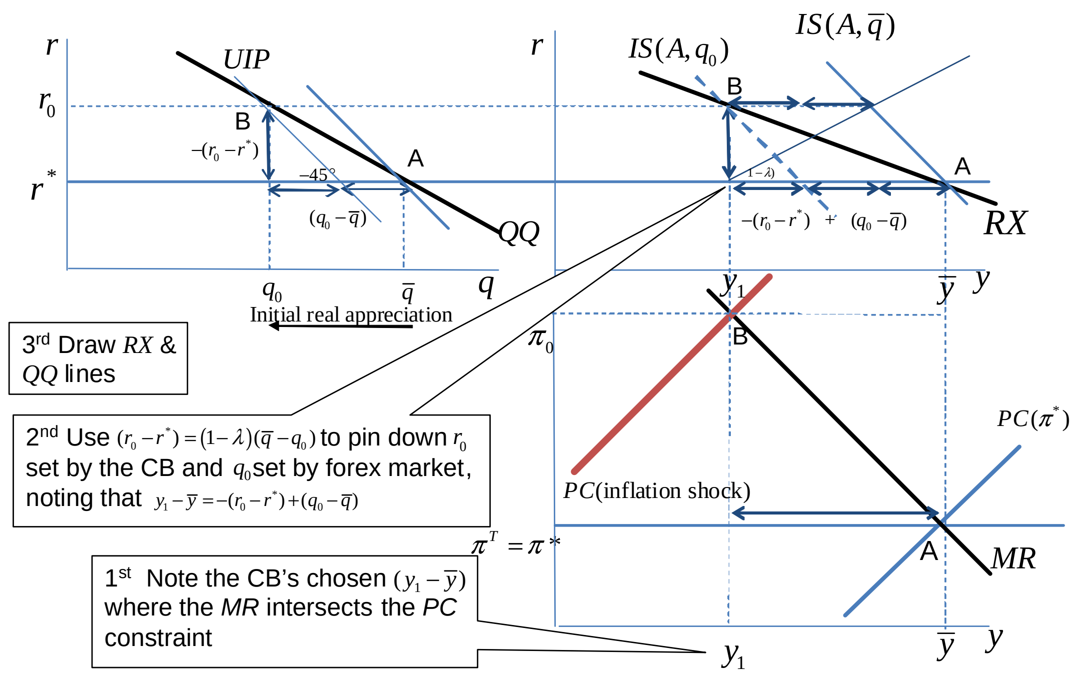


Figure 2 shows the graphical derivation of the interest rate and real exchange rate responses to the central bank's desired output gap in response, for example, to an inflation shock. From these, the *RX* and *QQ* curves are drawn. The benchmark case is shown with all parameters set equal to one $\left(a=b=\alpha=\beta=1, \text{ which implies } \lambda=0.5\right)$. The geometry of the benchmark case is very simple: since the coefficients in the *IS* curve on the interest rate and exchange rate are equal to one, the central bank's output gap is equal to the initial interest rate deviation from the world interest rate plus the initial exchange rate deviation from the equilibrium real exchange rate. With $1-\lambda=0.5$, the interest rate deviation is one-half the exchange rate deviation. This pins down the slope of the *RX* and *QQ* curves. In the appendix, we show how the graphical derivation can be used to develop the economic intuition of the effect of changes in the size of the parameters relative to the benchmark case.

Figure 2. Graphical derivation



Simple Geometry: Assume all parameters equal 1 $(a=b=\alpha=\beta=1;\ \text{hence } \lambda=0.5)$

3rd Draw *RX* & *QQ* lines

2nd Use $(r_0-r^*)=(1-\lambda)(\bar{q}-q_0)$ to pin down r_0 set by the CB and q_0 set by forex market, noting that $y_1-\bar{y}=-(r_0-r^*)+(q_0-\bar{q})$

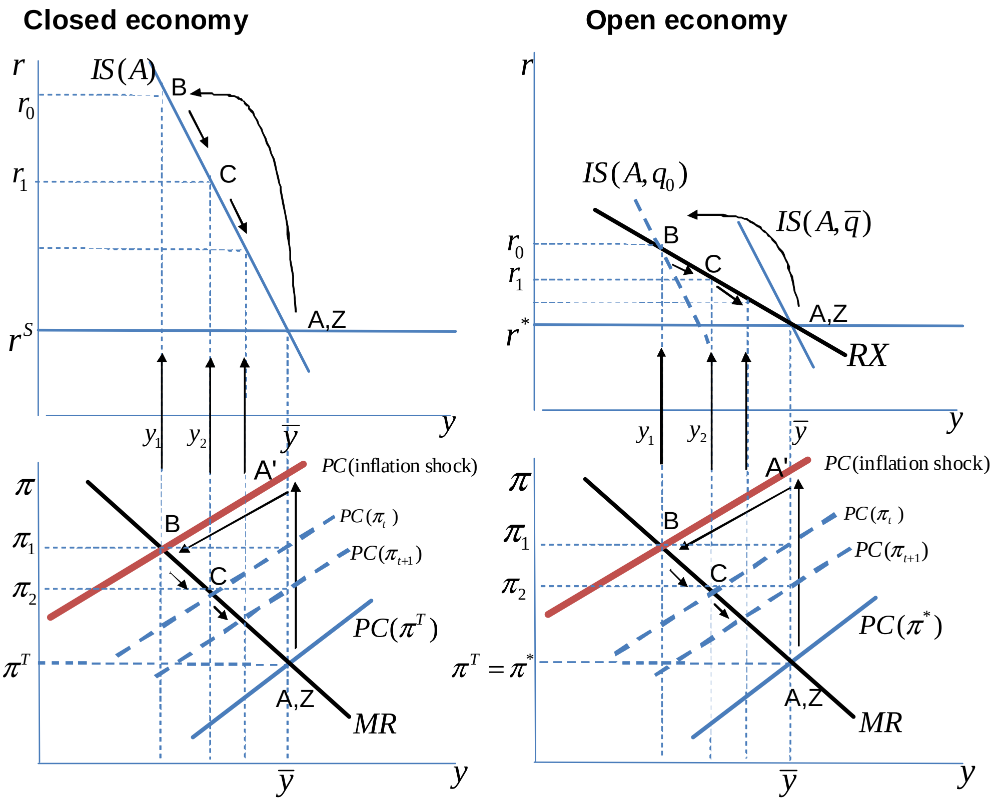
1st Note the CB's chosen $(y_1-\bar{y})$ where the *MR* intersects the *PC* constraint

Table 1 summarizes the steps in the central bank's decision-making process in the closed and open economies in response to a shock, which could be an inflation shock that shifts the Phillips curve, an aggregate demand shock (from home or abroad in the case of the open economy) that shifts the *IS* curve, a supply shock that shifts $\overline{y}$, or a shock to the world interest rate.

Table 1. Central bank decision-making: comparison between closed and open economy models

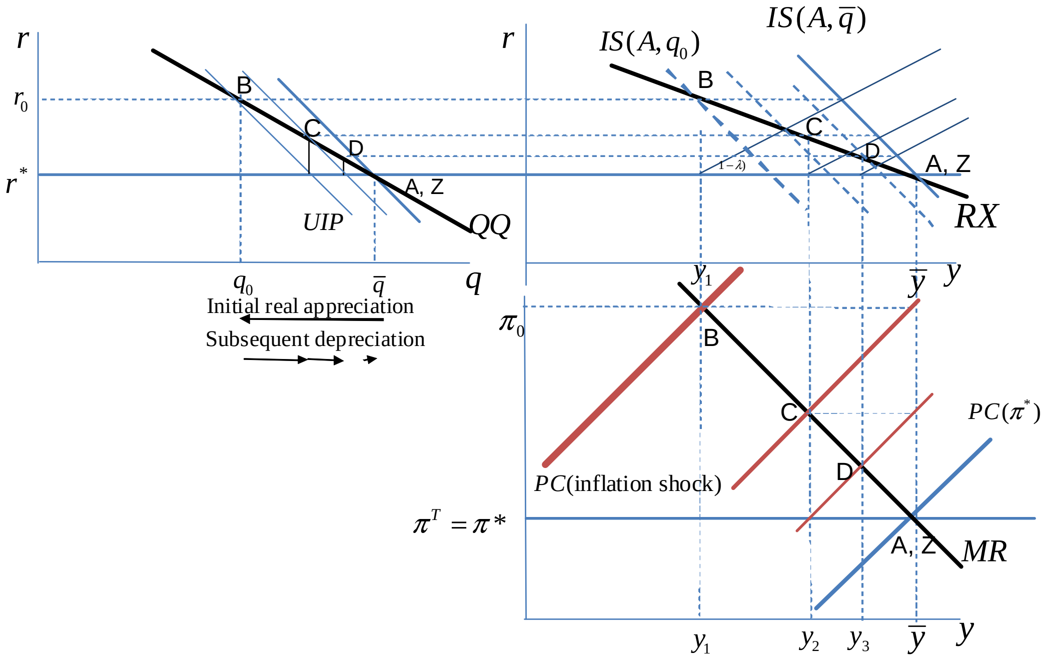
	Closed economy	Small open economy (flexible exchange rates)
1. Shock occurs at the start of period zero	The central bank works out the stabilizing real interest rate in the new equilibrium, r^S.	The central bank and the foreign exchange (forex) market work out the real exchange rate in the new equilibrium, $\overline{q}$. (The real interest rate is equal to r^* in the new equilibrium.)
2.	The central bank works out the implications of the shock for the Phillips curve and for its choice of output in period one (on the *MR* curve), y_1. The parameters of the Phillips curve (α) and the central bank's loss function β determine y_1 (as shown for the case of an inflation shock in the lower panels of Fig. 3, which are identical for the open and closed economy).	
2a.		The central bank and the forex market work out the path over time of the central bank's desired output gaps along the *MR* curve to the new equilibrium (as illustrated in Fig. 3, lower right panel).
3.	The central bank sets the real interest rate in period zero to achieve y_1. To do this, it uses the *IS* curve (parameter *a*) (Fig. 3, upper left panel).	The central bank sets the real interest rate in period zero to achieve y_1. To do this, it uses the *RX* curve (parameters α, β, a and b) (Fig. 3 upper right panel) The *QQ* curve shows explicitly the initial jump in the real exchange rate to q_0 associated with r_0 (Fig. 4 upper left panel).
4.	Step 3 is repeated and the economy moves along the *MR* and *IS* curves to the new equilibrium.	Step 3 is repeated and the economy moves along the *MR*, *RX* and *QQ* curves to the new equilibrium.

Figure 3. Inflation Shock: Closed and Open Economies



We use the analysis of an inflation shock to illustrate how the graphical model of dynamic adjustment works. Figure 3 compares the adjustment of the economy to an inflation shock under an inflation targeting central bank in a closed with an open economy. Following the steps in Table 1, we note that an inflation shock does not change any of the medium run equilibrium values. The inflation shock shifts the Phillips curve up (the economy moves from A to A′ in the lower panel of Fig. 3) and the central bank uses its monetary rule to decide on its desired output level in period 1. This places the economy on the path (the *MR* curve) at point B to return to equilibrium with inflation falling toward the target. The graphical analysis to this point is the same for the closed (left panel) and open (right panel) economies.

Figure 4. An inflation shock: flexible exchange rate economy with inflation-targeting central bank



As noted in Steps 2a and 3 in Table 1, in the open economy, the behaviour of the real exchange rate influences the way the central bank implements its chosen output level, y_1. The *RX* curve in the *IS* diagram shows how y_1 is mapped into the real interest rate deviation. The contrast between the initial interest rate adjustments made by the central bank in the open as compared with the closed economy highlights the presence in the open economy of the exchange rate channel in addition to the interest rate channel. Since agents in the forex market anticipate that the central bank will raise the real interest rate to $r_0 > r^*$ and that it will remain above the world interest rate for a temporary period along the adjustment path back to equilibrium as shown by the *RX* curve, there will be an immediate real exchange rate appreciation. The real appreciation dampens demand and the *IS* curve shifts to the left: the new *IS* curve intersects the downward sloping *RX* curve in the *IS* diagram at y_1 (point B in the upper right hand panel). As a consequence, the central bank raises the interest rate less than in the closed economy.

The central bank guides the economy back down the monetary rule curve to equilibrium (from B via C etc. to Z in the bottom panel in Fig. 3) in the identical fashion in the closed and open economies. In the open economy, this is accompanied by a gradual real depreciation. This adjustment can be observed in the *IS* diagram as the economy moves down the *RX* curve tracking the falling real interest rate and the rightward shifts of the *IS* curve, as the real exchange rate depreciates.

Figure 4 shows explicitly the behaviour of the exchange rate. The *QQ* curve indicates the initial real appreciation associated with the Central Bank's chosen level of output, y_1 (point B in the left panel). The (single period) *UIP* curve with the -45 slope shows the expected (and actual) depreciation in the following period. The *UIP* curve shifts each period to reflect changes in the expected exchange rate along the adjustment path. The initial leftward shift of the *IS* curve due to the appreciation to q_0 is shown in the top right hand panel; as are the subsequent rightward shifts of *IS* as the real exchange rate depreciates back to equilibrium.

The nominal exchange rate

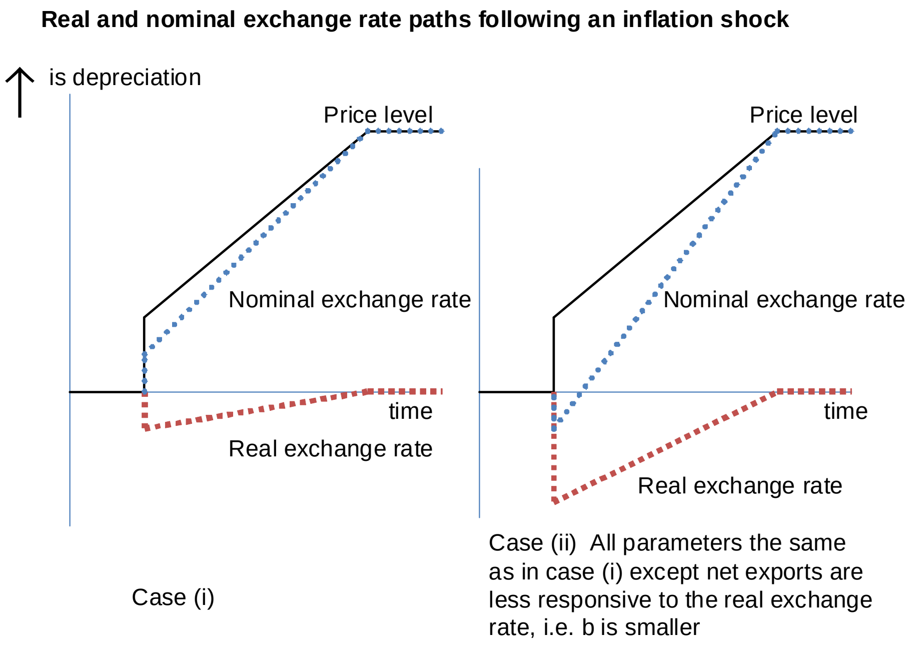
The behaviour of domestic inflation is determined entirely by the Phillips curve and monetary rule mechanisms. Since the model pins down the behaviour of the real exchange rate, the behaviour of the *nominal* exchange rate is a residual. Two features of the response of the nominal exchange are worth pointing out. The first is that the initial nominal appreciation in period zero is less than the real appreciation because part of the required real appreciation takes place through the initial inflation shock. The second relates to the path back to equilibrium (from point B to Z). In this phase, it is clear from the lower panel that home's inflation (although falling) is above world inflation, which by itself implies an appreciating real exchange rate. However, the real exchange rate must be depreciating for two reasons:

1. in order to push the *IS* curve back to equilibrium
2. because home's real interest rate is above the world's, i.e. $r > r^*$.

In order to make a depreciating real exchange rate consistent with domestic inflation that is higher than world inflation, the nominal exchange rate must depreciate by more than the real exchange rate on the path back to equilibrium. Over the course of the adjustment to the shock, the behaviour of the nominal exchange rate exactly offsets that of domestic inflation (relative to unchanged world inflation) leaving the real exchange rate at its initial level, $\overline{q}$, in the new equilibrium.

Figure 5 shows two examples of the stylized paths of the domestic price level and the nominal and real exchange rates following an inflation shock. The paths are linearized to bring out the comparisons most clearly. It highlights that the behaviour of the nominal exchange rate depends on the parameters of the *IS* equation. For simplicity, we assume the inflation target and world inflation are both zero. The price level is therefore constant in medium-run equilibrium. Following the inflation shock, the price level jumps and continues to rise until the medium-run equilibrium is reached. The real exchange rate appreciates immediately and depreciates back to its initial level along the path to the medium-run equilibrium. In the left hand panel, the modest real appreciation implies that the nominal exchange rate *depreciates* immediately. By contrast, in the second example in the right hand panel, we assume a smaller *b* coefficient in the *IS* equation. This means that net exports are less responsive to the real exchange rate. With all other parameters unchanged, this implies a larger initial real appreciation in order to deliver the same cut in output as in the example in the left panel. As a consequence, in this case the nominal exchange rate *appreciates* immediately. In both cases, the nominal exchange rate depreciates more rapidly than the real exchange rate along the path to the medium-run equilibrium.

Figure 5. Stylized time paths for domestic price level, nominal and real exchange rates following an inflation shock



Aggregate demand shock

For an aggregate demand shock to induce a protracted period of adjustment in the presence of an inflation targeting central bank there must be a lag between the shock being observed and the effect on economic activity of the central bank’s interest rate response. If activity responded immediately to changes in the interest rate and exchange rate, then the central bank could restore aggregate demand to the level consistent with the medium-run equilibrium, there would be no consequences of the shock for inflation and the output gap would remain at zero. However with a policy (or recognition) lag and when the inflation process is persistent, the initial change in inflation caused by the demand shock is ‘built-in’ and the optimal response has to ensure the economy gets on to the *MR* curve the following period.

In the open economy, we need to modify the analysis of an aggregate demand shock in two ways. First, to deliver the same optimal output gap response, the required change in the interest rate is less than would be the case in a closed economy because the exchange rate channel operates during the adjustment process. Second, in the case of a permanent aggregate demand shock, in the closed economy a new stabilizing real interest rate achieves the crowding out or crowding in of expenditure required to offset the shock. By contrast, in a small open economy, the real interest rate in medium-run equilibrium is tied down by the world real interest rate. It is therefore the real exchange rate that adjusts to secure the required crowding out or crowding in of expenditure.

Recent events have drawn attention to the need to model aggregate demand shocks, and we take as our example the case of a permanent negative aggregate demand shock. Following a negative demand shock to a small open economy, the new medium-run equilibrium is at higher competitiveness because a depreciated real exchange rate crowds in net exports to offset the demand shock (the demand shock is a shock to *A* in equation (1)). The *IS* relationship for $r = r^*$ is labeled *AD* and the supply-side equilibrium at $y = \bar{y}$ is labeled *AS* in Fig. 6.

Figure 6. Medium-run equilibria in the open economy

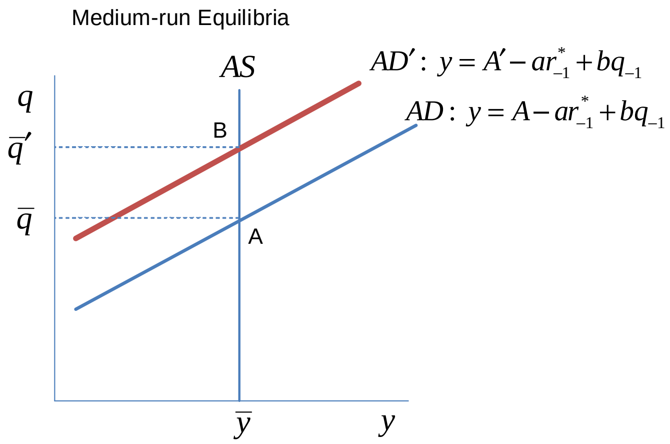
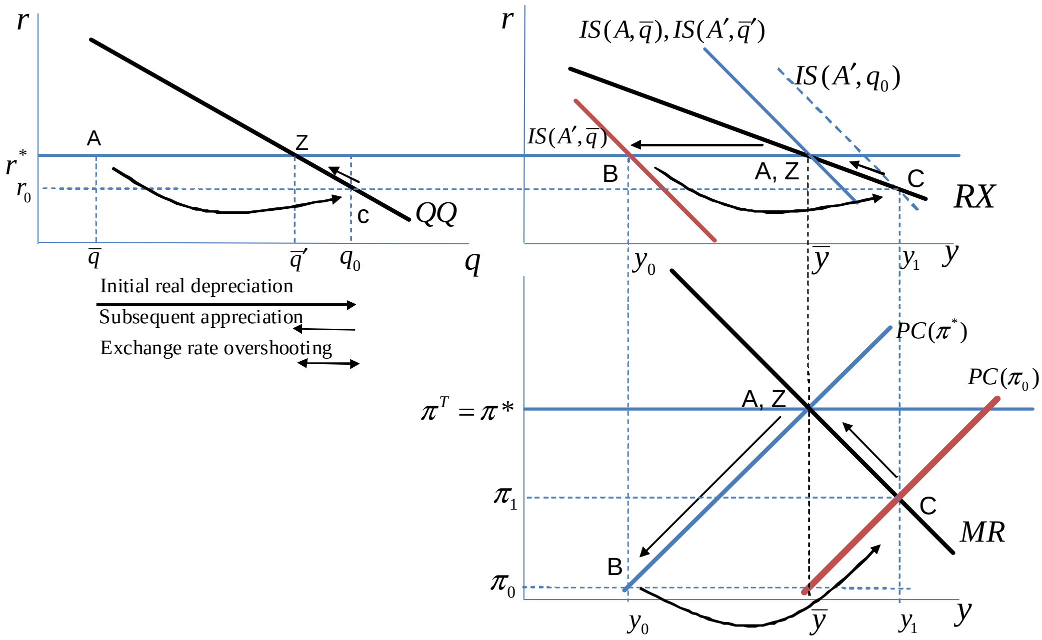


Fig. 7 summarizes the response of the central bank to the shock. In order to track the response of the exchange rate, we note that the *QQ* curve goes through the new equilibrium real exchange rate, $\bar{q}'$. Given the anticipated interest rate cut, the exchange rate depreciates immediately to q_0 in response to the shock, and as shown in Fig. 7, it *overshoots* the new medium-run equilibrium real exchange rate. The combined effect of the lower real interest rate and depreciated real exchange rate raise output above equilibrium and adjustment along the RX and MR curves restores inflation back up to the target level.

Figure 7. Adjustment in the Small Open Economy to a Permanent Aggregate Demand Shock



The optimal Taylor Rule in the small open economy model, and world interest rate shocks

In Carlin and Soskice (2005), we derive the optimal Taylor rule for the closed economy 3-equation model. In the model with a single lag from the interest rate to output, the closed economy optimal Taylor Rule is:

$$r_t = r^S + \frac{1}{a\left(\alpha + \frac{1}{\alpha\beta}\right)}\left(\pi_t - \pi^T\right) \qquad (9)$$

In the small open economy model, the optimal Taylor Rule has the same form as in the closed economy and is derived using the *RX* equation.[6] The stabilizing interest rate is replaced by the world real interest rate and the rule is modified by the factor ρ from the *RX* equation:

$$r_t = r_t^* + \frac{1}{\rho\left(\alpha + \frac{1}{\alpha\beta}\right)}\left(\pi_t - \pi^T\right), \text{ where } \rho = \left(a + \frac{b}{1-\lambda}\right) \text{ and } \lambda = \frac{1}{1+\alpha^2\beta}. \quad (10)$$

The optimal Taylor Rule says that given a rise in inflation above target (due for example to an inflation or positive aggregate demand shock), the central bank in the small open economy should raise home's real interest rate above the world real interest rate by an amount that reflects its inflation-aversion (β), the slope of the Phillips curve (α), the slope of the *IS* (a) and the sensitivity of aggregate demand to the real exchange rate (b). The higher is b, the lower must be the adjustment in the interest rate for a given deviation of inflation from target.

The Taylor Rule highlights the fact that the central bank should react immediately to a shock to the world real interest rate by changing the home real interest rate in line. If the central bank does this, then the real exchange rate jumps immediately to its new medium-run equilibrium value and there is no change in output or inflation. For example, following a fall in the world real interest rate, if the home central bank cuts its rate straight away, the real exchange rate jumps to the new equilibrium value at an appreciated real exchange rate. Since the real interest rate is lower, an appreciated real exchange rate is necessary in order to offset the boost to interest-sensitive expenditure in the new equilibrium.

---

[6] From the *MR* equation, $y_{t+1} - \overline{y} = -\left(\frac{\alpha\beta}{1+\alpha^2\beta}\right)\left(\pi_t - \pi^T\right)$ and from the *RX* equation, we have $y_{t+1} - \overline{y} = -\rho(r_t - r^*)$. Putting these together, gives $r_t = r^* + \frac{1}{\rho}\left(\frac{\alpha\beta}{1+\alpha^2\beta}\right)(\pi_t - \pi^T)$. As in the closed economy case, introducing a second lag (i.e. in the Phillips curve in addition to the IS curve) produces an optimal Taylor rule with the output deviation as well as the inflation deviation.

4. CPI inflation targeting

In the basic model, we assumed domestic inflation targeting both because of its analytical convenience and because there are theoretical arguments justifying its choice (e.g. Gali and Monacelli, 2005, Kirsanova, Leith and Wren-Lewis, 2006). However, most inflation targeting central banks set their target in terms of the consumer price index rather than using a domestic inflation target. Moreover, it may be more realistic to model wage-setters as formulating their money wage claims in terms of the expected CPI rather than in terms of the domestic price index, as we have assumed to this point. It is reasonably straightforward to extend the model to the case of a CPI inflation target with CPI based wage-setting and to allow for this direct exchange rate channel (Svensson, 2000).

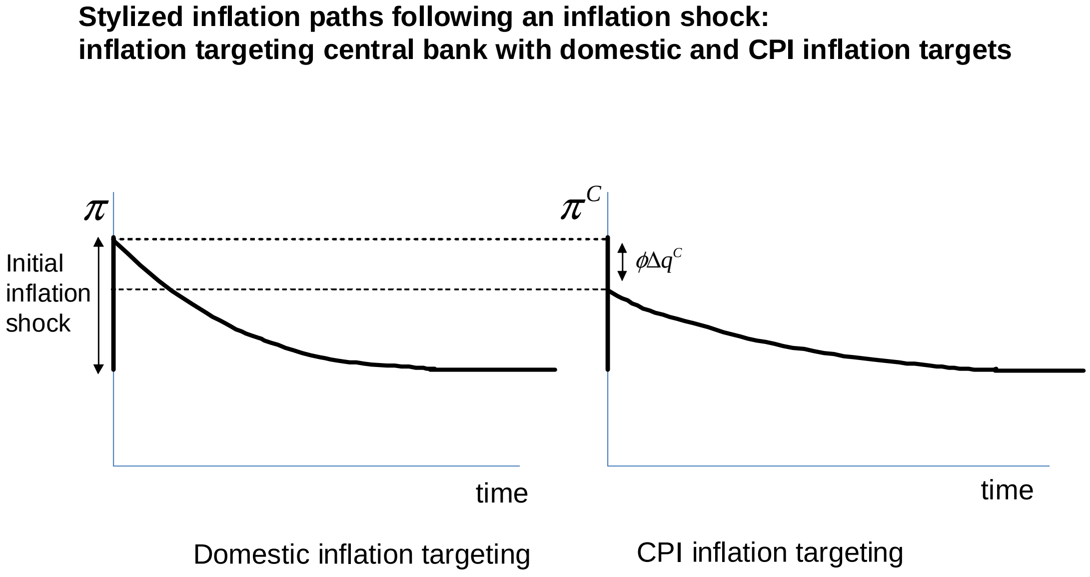
Two consequences flow from CPI inflation targeting: the first is that the central bank needs to respond less strongly to a given shock (e.g. a positive inflation shock) because achieving its inflation target is helped by the direct effect of the exchange rate appreciation on CPI inflation (Svensson, 2000). However, just as the initial exchange rate appreciation dampens the impact of the inflation shock, the subsequent depreciation (along the *QQ* curve) slows down the reduction in inflation since the depreciating exchange rate raises the imported component of inflation. Both implications can be presented graphically: the initial size of the inflation shock is reduced so that the Phillips curve jumps by less and second, the slope of the Phillips curve is flattened for the period of adjustment back to equilibrium, slowing down that process. The derivation of these results is shown in the appendix.

Figure 6 shows stylized paths of inflation following an inflation shock for domestic inflation and CPI inflation targeting. It highlights the initially faster fall in inflation in the CPI inflation target case followed by a slower return to target inflation.[7]

---

[7] This effect is noted in Clarida, Gali and Gertler (2001) p. 252.

Figure 6. CPI Inflation Targeting versus Domestic Inflation Targeting



Stylized inflation paths following an inflation shock:
inflation targeting central bank with domestic and CPI inflation targets

5. Conclusion

In this paper, we have shown how the Mundell-Fleming model can be replaced by a graphical open economy model that offers the same facility in analyzing a wide range of shocks as the graphical 3-equation closed economy model (Carlin and Soskice, 2005). Like Mundell-Fleming, the model can be taught to undergraduates and can illuminate the policy debate for non-specialists. However, in contrast to Mundell-Fleming the model matches contemporary policy settings and is related to the New Keynesian models taught in graduate courses and used in central banks. The model highlights the role of the real exchange rate and emphasizes that taking account of the forward-looking behaviour of the foreign exchange market is essential in monetary policy making. The assumption of domestic inflation targeting makes for a simpler presentation but we extend the model to CPI inflation targeting, which highlights an additional channel through which exchange rate expectations affect the central bank's policy-making problem.

Appendix

1. Geometry of the *RX* and *QQ* curves: varying the parameters a, b, α, β

One way of developing an intuitive understanding of the effect of parameter changes on the central bank's optimal response to shocks in the open economy is to conduct experiments of varying the parameters away from the benchmark case shown in Figure 2. Variations in a and b, the coefficients in the *IS* curve have simple graphical representations: if $a>1$, interest sensitive expenditure responds more to changes in the interest rate, and the *IS* curve is flatter. This has the effect of reducing the optimal interest rate and exchange rate responses, and a larger share of the optimal output gap is accounted for by the interest rate component (which increases from $\left(r_0 - r^*\right)$ to $a\left(r_0 - r^*\right)$). The *RX* curve is flatter.

If $b>1$, we have $y_1 - \bar{y} = \left(r_0 - r^*\right) + b\left(\bar{q} - q_0\right)$ and since $\left(r_0 - r^*\right) = (1-\lambda)\left(\bar{q} - q_0\right)$, we use $\left(r_0 - r^*\right) = \left(\frac{1-\lambda}{b}\right) b\left(\bar{q} - q_0\right)$ to find r_0 and q_0. Geometrically, the line with slope $(1-\lambda)$ is replaced by one with slope $\frac{1-\lambda}{b}$. This lowers the initial interest rate chosen by the central bank and the associated exchange rate appreciation, and increases the share of the optimal output gap accounted for by the exchange rate component (from $\left(\bar{q} - q_0\right)$ to $b\left(\bar{q} - q_0\right)$). The *RX* curve is flatter.

If α or $\beta>1$, then the *MR* curve is flatter. This reflects a lower λ , i.e. a faster rate of decline of the output gaps on the path back to equilibrium. This means that the optimal output gap in response to the inflation shock is larger. In terms of the diagram, the slope of the line $(1-\lambda)$ increases in absolute value. Hence the initial interest rate and exchange rate responses are larger. The faster rate of decline of the output gap steepens the *QQ* line: a given interest rate deviation is associated with a smaller exchange rate deviation.

2. CPI inflation targeting

The log of the consumer price index, p^C, is

$$p_t^C = (1-\phi) p_t + \phi\left(p_t^* + e_t \right), \qquad (11)$$

where ϕ is the share of imported goods in the consumption bundle. Subtracting this equation lagged one period, we have

$$\begin{aligned} \Delta p_t^C &= \left(1-\phi\right) \Delta p_t + \phi\left(\Delta p_t^* + \Delta e_t\right) \\ \text{and } \pi_t^C &= \left(1-\phi\right)\pi_t + \phi\left(\pi^* + \Delta e_t\right) \\ &= \pi_t + \phi\left(q_t - q_{t-1}\right) \end{aligned}$$

where we assume world inflation is constant and suppress the time subscript. Domestic inflation is $\pi_t = \Delta w_t$, where w is the log of the nominal wage, labour is the only input in production and we assume that labour productivity is constant. On the assumption that wages are set relative to CPI inflation, we have

$$\begin{aligned} \pi_t &= \Delta w = \pi_{t-1}^C + \alpha\left(y_t - \overline{y}\right), \text{ and therefore,} \\ \pi_t^C &= \pi_t + \phi\left(q_t - q_{t-1}\right) \\ &= \pi_{t-1}^C + \alpha\left(y_t - \overline{y}\right) + \phi\left(q_t - q_{t-1}\right) \\ &= \pi_{t-1}^C + \alpha\left(y_t - \overline{y}\right) + \phi\Delta q_t. \end{aligned}$$

The Phillips curve in period 1 is therefore

$$\pi_1^C = \overline{\Delta}_0 + \alpha(y_1 - \overline{y}) + \phi\Delta q_1, \qquad (12)$$

where the shift in the Phillips curve in period 0, $\overline{\Delta}_0$, is equal to the inflation shock ξ modified by the effect of the jump in the exchange rate (i.e. $\pi_0 - \pi^T = \xi + \phi\Delta q_0 \equiv \overline{\Delta}_0$). From the *RX* and *QQ* equations, we can write

$$\Delta q_1 = (r_0 - r^*) = -\left(a + \frac{b}{1-\lambda}\right)^{-1} (y_1 - \overline{y}).$$

Returning to the Phillips curve for period 1,

$$\begin{aligned}\pi_1^C &= \bar{\Delta}_0 + \alpha(y_1 - \bar{y}) - \phi\left(a + \frac{b}{1-\lambda}\right)^{-1}(y_1 - \bar{y}) \\ &= \bar{\Delta}_0 + \left[\alpha - \phi\left(a + \frac{b}{1-\lambda}\right)^{-1}\right](y_1 - \bar{y}) \\ &= \bar{\Delta}_0 + \alpha(y_1 - \bar{y}),\end{aligned}$$

where $\alpha < \alpha$.

In general for $t \geq 1$,

$$\pi_t^C = \pi_{t-1}^C + \alpha(y_t - \bar{y}).$$

To summarize, in period 0, (following the inflation shock), the upward Phillips curve shift is dampened by the full appreciation of the real exchange rate, multiplied by the openness parameter, $\phi \Delta q_0$, which is negative. For period 1 onwards, we have the depreciation effect, unwinding the initial appreciation until *r* moves back down to equilibrium *r**. Using rational expectations the *QQ* and the *RX* tell us that this depreciation effect is exactly equal to $\Delta q_t = (r_{t-1} - r^*) = -\left(a + \frac{b}{1-\lambda}\right)^{-1}(y_t - y^e)$. Hence this can be summarized in the Phillips curve *during the adjustment period* being less steep with a slope of $\left[\alpha - \phi\left(a + \frac{b}{1-\lambda}\right)^{-1}\right] \equiv \alpha < \alpha$ instead of just α.

When we introduce the CPI inflation target, the initial effect of an inflation shock on period zero inflation is damped, reflecting the immediate appreciation of the exchange rate; but in subsequent periods the inflation rate moves more slowly back to target, reflecting the gradual unwinding of the overshoot.

References

Allsopp, C. and D. Vines. (2000). ‘The Assessment: Macroeconomic policy’. *Oxford Review of Economic Policy* . 16(4). 1-32.

Carlin, W. and D. Soskice (2005). ‘The 3-equation New Keynesian model – a graphical exposition’ Contributions to Macroeconomics 5(2) Article 13 http://www.bepress.com/cgi/viewcontent.cgi?article=1299&context=bejm

Clarida, R., Gali, J. and M. Gertler (2001). ‘Optimal monetary policy in open versus closed economies: an integrated approach’. *American Economic Review Papers and Proceedings* 91(2): 248-252.

Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle* Princeton: Princeton University Press

Gali, J. and T. Monacelli (2005). ‘Monetary policy and exchange rate volatility in a small open economy’. *Review of Economic Studies* 72: 707-734.

Kirsanova, T., Leith, C. and S. Wren-Lewis (2006). ‘Should central banks target consumer prices or the exchange rate?’ *Economic Journal* 116 (512): F208-231.

Romer, D. (2000). ‘Keynesian macroeconomics without the *LM* curve’ *Journal of Economic Perspectives* 14(2): 149-169

Smets, F. and R. Wouters (2003) ‘An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area’ Journal of the European Economic Association 1(5): 1123-1175.

Svensson, L.E.O. (2000). ‘Open-economy inflation targeting’. *Journal of International Economics* 50: 155-183.

Taylor, J.B. (2000). ‘Teaching modern macroeconomics at the principles level’. *American Economic Review P&P* 90(2): 90-94.

Walsh, C. E. (2002). ‘Teaching inflation targeting: An analysis for intermediate macro’. *Journal of Economic Education* 33(4): 333-347

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy* Princeton: Princeton University Press