Tutorial Sheet 5

1. [From CORE 4.3 Altruism and Selfishness] Consider two individuals A and B with other-regarding preferences over their own and the other's pay-offs (denoted $\{x_A, x_B\}$). In $\{x_A, x_B\}$ −space, with A's payoff on the horizontal and B's on the vertical

   (a) draw what A's indifference curves would look like if she cared about B's consumption in the same way as she cared about his own;

   (b) draw A's indifference curves assuming she derived utility only from the total of his and B's;

   (c) draw A's indifference curves assuming she derived utility only from B's consumption.

   (d) For each of these cases, provide a real world situation in which A might have these preferences, making sure to specify how A and B derive their payoffs.

2. [From CORE 4.11 Sequential Prisoner's Dilemma] Consider the following Prisoner's Dilemma

	Cooperate	Defect
Cooperate	3, 3	1, 4
Defect	4, 1	2, 2

   Suppose that the game is played sequentially like the Ultimatum Game. One player (chosen randomly) choses a strategy (the first mover), and then the second moves (the follower/second-mover).

   (a) Suppose you are the first mover and you know that the second mover has a strong preference for reciprocity, meaning the second mover will act kindly towards someone who cooperates and will act unkindly to someone who defects. What would you do?

   (b) Suppose the person who favours reciprocity is now the first mover interacting with the person she knows to be entirely self-interested. What do you think would be the outcome of the game?

3. [From Core 4.12 A Coordination game] Consider the following game in which two software developers are deciding which programming language to use.

	Java	C++
Java	4, 3	2, 2
C++	0, 0	3, 6

   What is the likely result if:

   (a) Row can choose which language she will use first, and commit to it (just as the Proposer in the ultimatum game commits to an offer, before the Responder responds)?

   (b) The two can make an agreement, including which language they use, and how much cash can be transferred from one to the other?

   (c) They have been working together for many years, and in the past they used Java on joint projects?

4. [From CORE 4.14 Nash Equilibria and Climate Change] Think of the problem of climate change as a game between two countries called China and the US, considered as if each were a single individual. Each country has two possible strategies for addressing global carbon emissions: Restrict (taking measures to reduce emissions, for example by taxing the use of fossil fuels) and BAU (the Stern report's "business as usual" scenario). The following matrix describes the actions and outcomes.

		US	
		Restrict	BAU
China	Restrict	Reductions sufficient to moderate climate change	US free-rides on Chinese emissions reductions
	BAU	China free-rides on US emissions reductions	No reduction in emissions

The following matrix describes the pay-offs when the game is modelled as a Prisoner's Dilemma with egoistic preferences.

		US	
		Restrict	BAU
China	Restrict	Good, Good	Worst,Best
	BAU	Best,Worst	Bad,Bad

The following matrix describes the pay-offs when the game is modelled with other-regarding preferences.

		US	
		Restrict	BAU
China	Restrict	Best,Best	Worst,Good
	BAU	Good,Worst	Bad,Bad

(a) Show that in the PD version both countries have a dominant strategy. What is the dominant strategy?

(b) The outcome would be better for both countries if they could negotiate a binding treaty. Why might it be difficult to achieve this?

(c) Explain how the payoffs in the other-regarding version of the game could represent the situation if both countries were inequality averse and motivated by reciprocity. Show that there are two Nash equilibria. in this version of the game. Would it be easier to negotiate a treaty in this case?

(d) Describe the changes in preferences or in some other aspect of the problem that would convert the game to one in which both countries choosing Restrict is a dominant strategy equilibrium.

5. [From Prelims Collection Paper 2021] Consider a group of k individuals deciding simultaneously whether to contribute (C) or not to contribute (N) 1 unit of their wealth to a public good (assume initial wealth of w for all individuals). The amount of public good produced is then equal to the aggregate contributions of all members of the group and produces a benefit of b per unit per person (so if all members together contribute X then every individual gains a benefit equal to bX).

   (a) Assuming $0.5 < b < 1$ and $k = 2$, represent the situation as a 2-by-2 simultaneous game and identify all Nash equilibria. Can we predict whether or not the outcome will be Pareto-efficient?

   (b) What difference would it make to your answer to part (a) if $b < 0.5$?

   (c) What about if $b > 1$?

   (d) Given a particular k, for what values of b would the k-player version of this game result in a Pareto-inefficient Nash equilibrium outcome?

   (e) Explain how altruistic preferences, where each player receives a payoff equal to the sum of the benefits (net of contribution costs) for the two players would change your answer to part (a)

   (f) What implications do the Nash equilibria of these simple games have for the policy interventions that might be needed to ensure Pareto-efficient provision of a public good?

6. Consider the Dictator Game. Player 1 has a stake of size 1 which she can divide between herself and Player 2. Player 2 is passive recipient. Let x_1 denote Player 1's retained portion and let x_2 (which equals $1 - x_1$) denote her offer to Player 2. Suppose that Player 1 has a utility function given by

$$u_1 = x_1 - \frac{\alpha}{2}\left[\sum_{j=1}^{2}(x_j - \bar{x})^2\right]$$

   where $\alpha \geq 0$ and $\bar{x}$ is the average pay-off for the two players.

   (a) What general properties does this utility function have? How is inequality-aversion displayed? What restrictions on this model would give you the *homo economicus*, egoistic, special case?

   (b) Show that this utility function can also be written as

$$u_1 = x_1 - \frac{\alpha}{4}(x_1 - x_2)^2$$

   [hint: you can think geometrically rather than algebraically, if you wish, and use the fact that in this case $\bar{x} = \frac{1}{2}$ and x_1 and x_2 must be located symmetrically around it].

   (c) Derive the offer to Player 2 (x_2) as a function of Player 1's inequality aversion parameter α.

   (d) Show that the offer to Player 2 is increasing in α.

   (e) Show that this utility function can explain any offer less than or equal to $\frac{1}{2}$, but could not explain why anyone would wish to give away more than half the stake.

   (f) Suppose that you observe

      i. someone offering $\frac{1}{4}$. What is their α?

      ii. someone offering nothing. What is their α?

ESSAYS

7. What are the principal justifications for Nash Equilibrium as a solution concept for non-cooperative games? Critically evaluate these justifications.

8. In game theory, threats that are too costly to carry out are typically judged to be non-credible. However, explicit threats are common in everyday life, and threat displays rather than fights are also a common strategy in animal populations. Explain the notion of a credible threat and discuss what might explain these observations.