The Unruh Effect in Slow Motion

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Why is direct detection hard?

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$$\text{Lorentz Factor}: \quad \gamma_{\text{thermal}} = \cosh(a\,\tau_{\text{thermal}}/c) = e^{4000} \tag{1}$$

$$\text{Distance (Lab Frame)}: \quad \Delta x = (c^2/a)(\gamma - 1) = e^{4000}(0.3\text{ mm}) \tag{2}$$

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This suggests that any feasible proposal will have $a\,\tau_{\text{thermal}}/c \lesssim 1$.

Why is it direct detection hard?

So let's consider a proposal with $a\,\tau_{\text{thermal}}/c \lesssim 1$. We then have,

$$1 \gtrsim \frac{a\,\tau_{\text{thermal}}}{c} \gg \frac{a\,\tau_{\text{H}}}{c} = (2\pi)^2 \frac{k_{\text{B}}\,T_{\text{Unruh}}}{\hbar\Omega}$$

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So we must have $k_{\text{B}}\,T_{\text{Unruh}}/\hbar\Omega \ll 1$, very few excitations in the probe.

Direct Detection Dilemma

Thus we have a dilemma:

1) $a\,\tau_{\text{thermal}}/c \gg 1$ implies that Δx and Δt are astronomical

2) $a\,\tau_{\text{thermal}}/c \lesssim 1$ implies that $k_{\text{B}}\,T_{\text{Unruh}}/\hbar\Omega \ll 1$ very few excitations

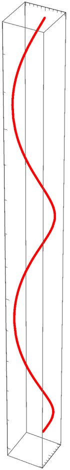
Is picking one of these options unavoidable?

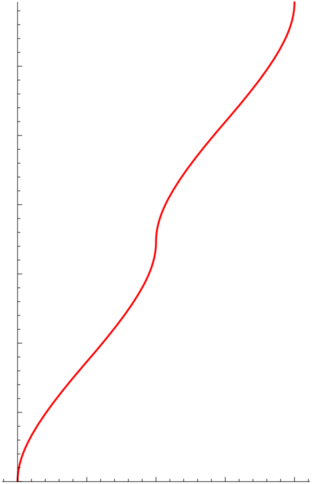
A possible escape hatch

We assumed that the probe was always accelerating in the same direction.

Let's have the acceleration change directions:
  Circular[1] (Left)
  Alternating Linear (Right)

But this introduces jerks:
  Constant Jerk (Left)
  Sudden Jerks (Right)





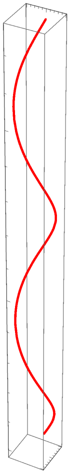
[1] S. Biermann, S. Erne, C. Gooding, J. Louko, J. Schmiedmayer, W. G. Unruh, S Weinfurtner; PhysRevD.102.085006; https://arxiv.org/abs/2007.09523

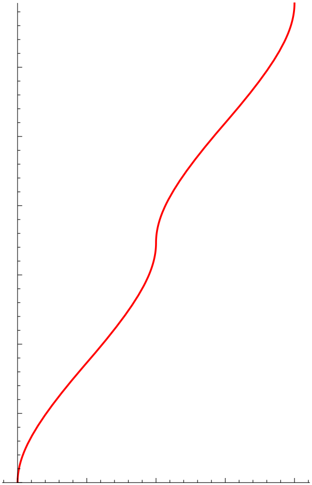
A possible issue

Will the probe still thermalize to $T \propto |a|$ on these jerky trajectories?

If so, is T independent of everything else (probe gap, orbit speed, orbit radius)?

This study[2] indicates that in the circular case, it is not in general.





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When is a temperature a temperature?

But why should we demand $T \propto |a|$ and independent of everthing else?
Isn't it enough to have $T = \kappa(\Omega, v)\,|a|$ with $\kappa(\Omega, v)$ roughly constant?
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No, it's not enough! By the zeroth law of thermodynamics: temperature is the label for equivalence classes of equilibrium systems. If the Unruh temperature is supposed to be the temperature <u>of some thing</u>, then <u>any thermometer</u> must agree on its temperature.

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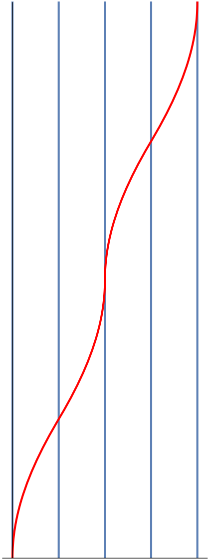
Any temperature which varies from thermometer to thermometer is not a temperature!

Removing the jerks



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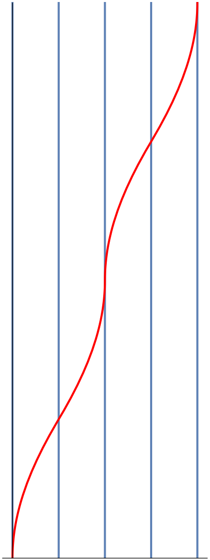

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It's the constant jerk that the probe is undergoing. By contrast the effect of the jerks can be **completely** avoided in the alternating linear setup!

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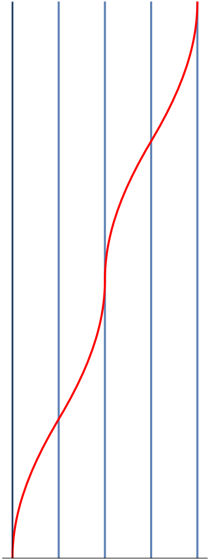

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← Spacetime diagram of alternating linear setup.
Red: Probe Trajectory, Blue: Cavity Walls

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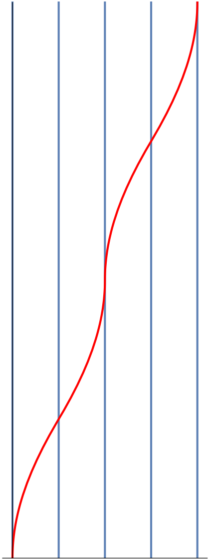

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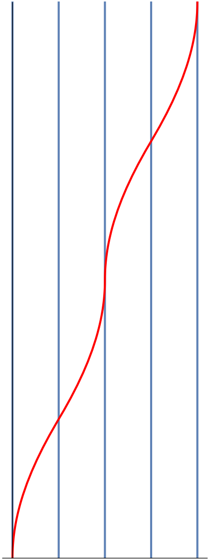
$\leftarrow$ Spacetime diagram of alternating linear setup.
Red: Probe Trajectory, Blue: Cavity Walls

By taking Dirichlet boundary conditions at the cavity walls we **completely** remove the effect of the jerks! Each time the probe experiences a sudden jerk, it is completely decoupled from the field.

Other Benefits



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What consequences can we expect to follow?

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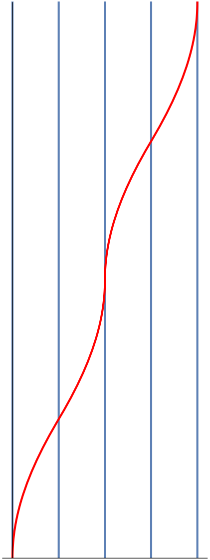

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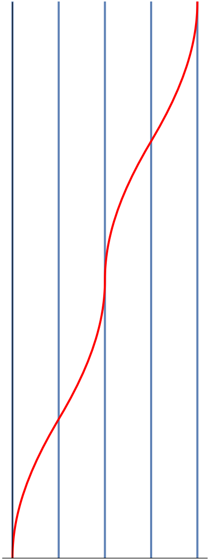
Pros (reasons to hope):

1.) We have completely removed the jerks

Other Benefits



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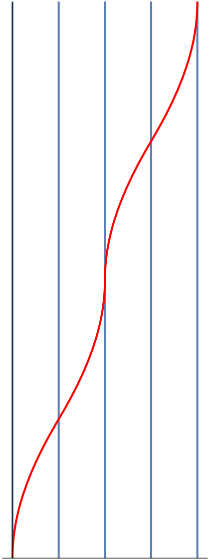
Pros (reasons to hope):

2.) Cavity modes are discrete, easier to calculate

Other Benefits



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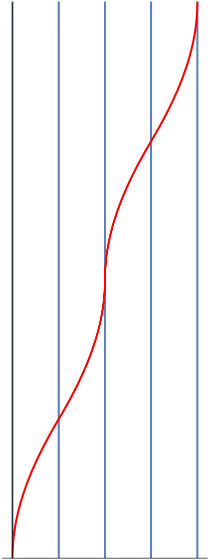
Pros (reasons to hope):

3.) Discrete Markovian dynamics: cell-by-cell we have the same update map, $\hat{\rho}(\tau_{n+1}) = \Phi_{\text{cell}}[\hat{\rho}(\tau_n)]$. The probe is shielded from the wider environment.

Other Benefits



Pros (reasons to hope):

4.) Let τ_{max} be the proper time the probe is in one cavity. In principle, we can have $\tau_{\text{max}} \ll \tau_{\text{thermal}}$. The probe does not need to thermalize within a single cavity. Instead, it can thermalizes with them all collectively.

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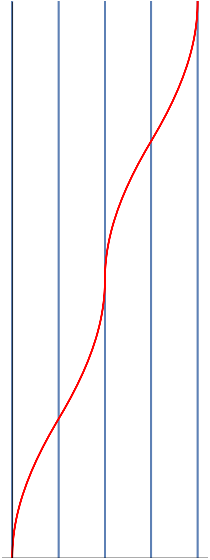

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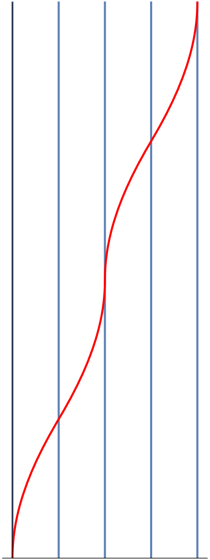
In this case, the probe does not need to become ultrarelativistic, we could have:

$$\gamma_{\text{max}} = \cosh(a\tau_{\text{max}}/c) \ll \gamma_{\text{thermal}} = \cosh(a\tau_{\text{thermal}}/c)$$

If $a\,\tau_{\text{max}}/c \sim 1$ then we can avoid the astronomical distances and lab-times.

Downsides of cavity walls



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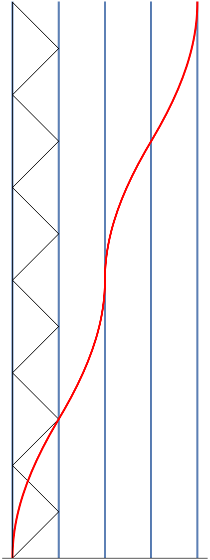
Cons (reasons to doubt):

1.) Cavity modes are discrete, we have broken Lorentz invariance. The probe might be able to resolve this discreteness and spoil the effect.

Downsides of cavity walls



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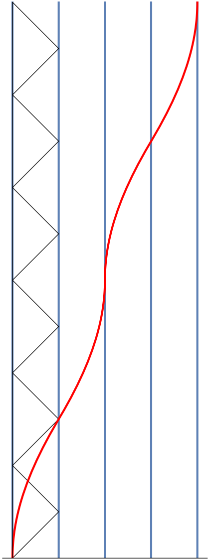

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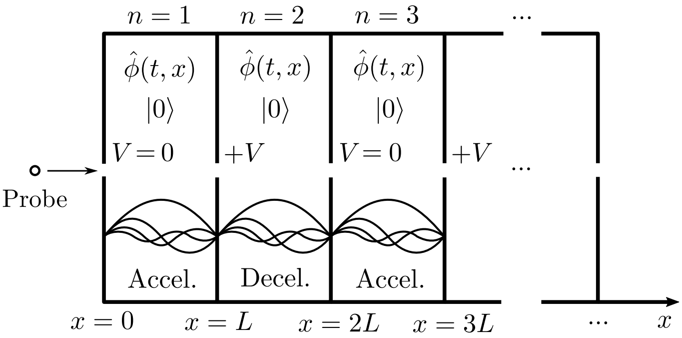
Spoiler: Luckily there are regimes where both of these concerns are avoided simultaneously.

Example Schematic Diagram

There are many ways to implement this alternating linear acceleration setup (voltages, laser pulses, etc). Here is one schematic drawing:



We can reuse old cavities once they return to their ground state!

Details of our setup

For simplicity, we consider a $1 + 1$D massless scalar field, $\hat{\phi}(t, x)$ and a harmonic oscillator probe, $\hat{q}_{\mathrm{p}}(\tau)$, with gap Ω.

[3] My PhD thesis: https://arxiv.org/abs/2009.10472

Details of our setup

For simplicity, we consider a $1+1$D massless scalar field, $\hat{\phi}(t, x)$ and a harmonic oscillator probe, $\hat{q}_\mathrm{p}(\tau)$, with gap Ω.

They are coupled linearly with a point-like smearing function:

$$\hat{H}_I(\tau) = \lambda \; \hat{q}_\mathrm{p}(\tau) \; \hat{\phi}(t(\tau), x(\tau)) \tag{4}$$

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The dynamics given by repeated application of some CPTP map Φ_cell as:

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The Gaussianity of our setup and the repeated update dynamics let us use the Gaussian Interpolate Collision Model formalism[3].

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Results:

Free parameters:

cavity length, L,
probe acceleration, a,
probe frequency, Ω,
coupling strength, λ.

Dimensionless variables:

N/A
$a_0 = a\,L/c$,
$\Omega_0 = \Omega\,L/c$,
$\lambda_0 = \lambda\,L/\sqrt{\hbar c} = 0.01$.

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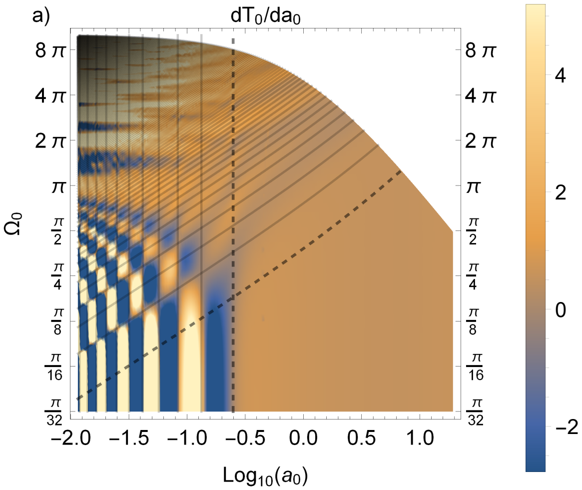
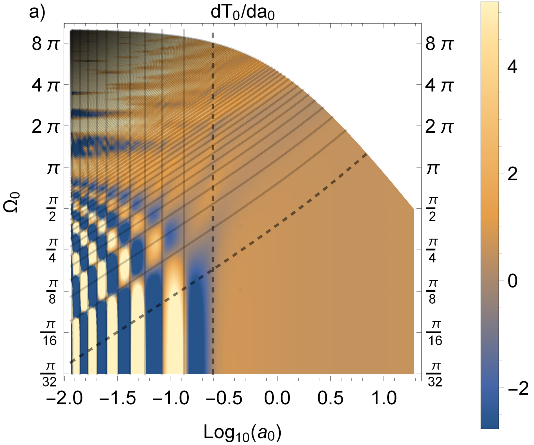
As such we can talk about the probe's temperature, T which can be made dimensionless as $T_0 = k_B T\, L/\hbar c$. The Unruh effect would have $T \propto a$ and so $T_0 \propto a_0$ and so $dT_0/da_0 =$ constant.

The dT/da Figure

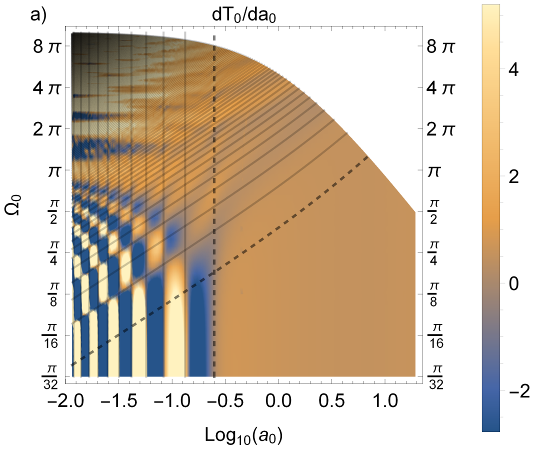
Note that $dT_0/da_0 =$ constant in the bottom right corner.





Left of the dashed vertical line:
The probe sees its own reflection.
Unruh effect spoiled.

The dashed lines



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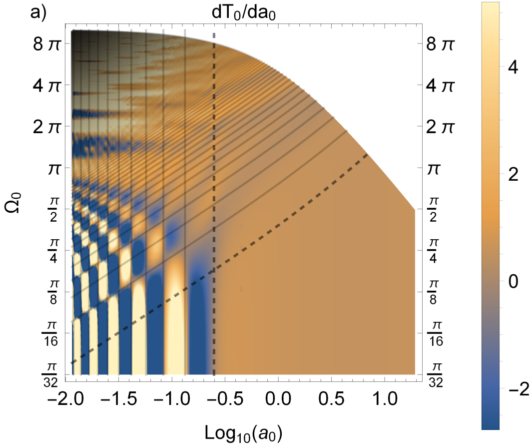

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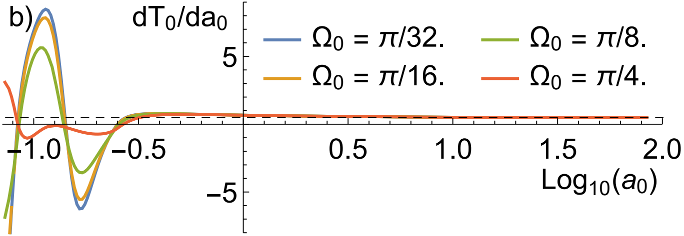
Otherwise we have $T \propto a$ and **independent** of Ω and L and λ.

The dT/da Independent of Ω Figure

So what is the slope, dT/da? Horizontal slices of the previous figure:



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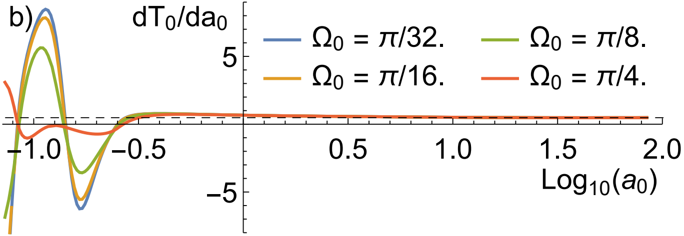


In the Unruh-effect region we have $dT_0/da_0 = 1/2$ such that

$$T = \frac{1}{2}\frac{\hbar\, a}{k_B\, c}$$

We are just missing a factor of π! (Why? See the paper or ask me later)

Conclusion

1.) The difficulty of direct detection can be cast as a dilemma: Astronomical distances/times or very low excitation numbers.

2.) This can be avoided by changing the direction of the acceleration (circular or alternating linear).

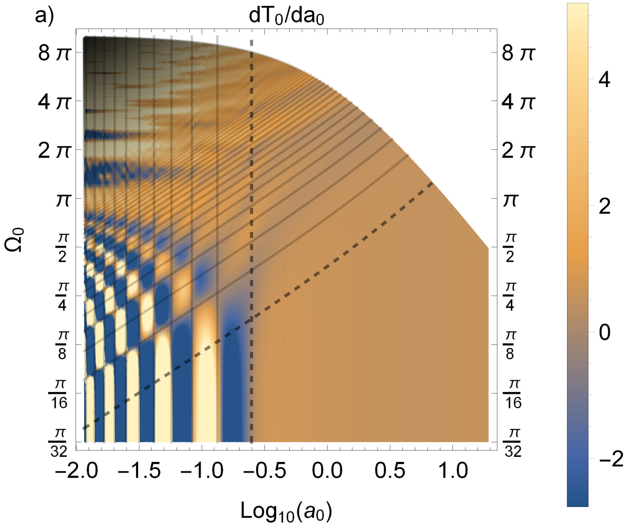
3.) However, this introduces jerks which can distort/muddy the temperature - acceleration relationship, esp. for circular setups.

4.) We show a cavity-based alternating linear setup where the effect of the jerks can be completely removed. Moreover, there is a regime in which the cavity-induced effects are absent.

Bonus: Crunching numbers, this proposal is experimentally feasible!
(I have more slides on feasibility, if anyone wants more info)

Thanks for Your Attention

Any Questions?



Experimental Feasibility

Our proposal works for accelerations as low as $a_0 = a\,L/c^2 = 1/4$

Table top: $L = 1$ m implies $a = 2.3 \times 10^{15}$ g.

LIGO-sized: $L = 4$ km implies $a = 5.7 \times 10^{11}$ g.

In either case the maximum Lorentz factor is only $\gamma_{\text{max}} = 5/4$.

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We have avoided the astronomical distances and time, at least for each cavity. But how many cavities do we need?

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These are the times to thermalize one probe, but how many probes need to be thermalized for a confident detection?

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We have successfully avoided both horns of the dilemma.