Does nonclassical truth impair mathematical reasoning?

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Philosophy of Mathematics Seminar
14 May 2018

Weakening the principles of classical logic in order to retain desirable properties of intensional notions such as truth has been widely embraced as a response to the intensional paradoxes. Advocates of classical logic who resist logical revision have argued that our standard for reasoning with intensional notions should not be different from that employed in our best scientific and mathematical theories. A specific version of this argument, due to Halbach, uses a proof-theoretic analysis of two classical and nonclassical theories of Kripkean truth (known as KF and PKF) to show that when we give up classical logic, we must in the process give up important non-semantic patterns of reasoning, in particular in mathematics. The nonclassical logician has a natural response to Halbach?s argument: they can bite the bullet and argue that even if one must give up the universality of these patterns of reasoning, one does not thereby lose too much in the way of genuine mathematics. However, despite first appearances, one does give up substantial mathematics by accepting nonclassical logic in this context. Drawing on recent work in reverse mathematics by Montalbán and Neeman, I show that an ordinary mathematical theorem concerning indecomposable linear orderings is proof-theoretically reducible to the classical theory of Kripkean truth KF, but not to the weaker nonclassical theory PKF.