The Implicit Commitments Thesis Meets the Tarski Boundary.

Mateusz Łełyk

February 4, 2019

In a general formulation, the Implicit Commitments Thesis (henceforth abbreviated as ICT), states that[1]

in accepting a formal systems S one is also committed to additional resources not available in the starting theory S but whose acceptance is implicit in the acceptance of S.

Let us call the above the *weak* ICT. It should be contrasted with its stronger form, namely:

Anyone who accepts the axioms of a mathematical theory S is thereby also committed to accepting various additional statements Δ which are expressible in the language of S but which are formally independent of its axioms.

The above version (taken from [1]; let us call it the *strong* ICT) was criticised in [1]. The author observed that accepting some theories (e.g. the Primitive Recursive Arithmetic, PRA, and Peano Arithmetic, PA) from a particular foundational standpoint (e.g. finitism, in the context of PRA or first-orderism, in the context of PA) is connected with casting doubts on the legitimacy of any statements not provable in these theories. As a solution to this problem, in [3], the weak version of ICT was proposed, where the minimal additional resources which we are committed to when accepting a theory S are given by the compositional truth theory $\mathrm{CT}^-(S)$ [2] and the sentence expressing "All axioms of S are true". Since this theory is *usually* proof-theoretically conservative over S, one can defend the weak ICT at the same time rejecting the strong one.

This is where the concept of the Tarski Boundary appears: the boundary separates the truth theories that are conservative over S from the ones which

[1] We borrow the formulation from [3].

[2] The theory is known also as $\mathrm{CT}\restriction(S)$, see [2].

prove sentences in the language of S which are not provable in S (i.e. are not conservative over S). Primarily its was introduced for extensions of $\mathrm{CT}^{-}(\mathrm{PA})$, but it makes perfect sense to consider it for different base theories S and various truth theories over S. The main problem concerning the Tarski Boundary is to determine which axioms for the truth predicate cause the transition from one side of the boundary to the other one.

We start our talk by a reevaluation of the critique from [1] and argue that at least the example of finitism and PRA does not contradict the strong ICT. Then, we show that under one additional assumption, which we find very plausible, the version of weak ICT as offered in [3] actually *implies* its stronger counterpart. The additional assumption we rely on is that proof-theoretically equivalent theories represent the same foundational standpoint and, therefore, have the same implicit commitments. Last but not least, we introduce a family of extensions of $\mathrm{CT}^{-}(\mathrm{PA})$ that helps us to study the contour of the Tarski Boundary. Every member of this family extends $\mathrm{CT}^{-}(\mathrm{PA})$ with a sentence expressing "All sentences from δ are true", where δ represents a set of arithmetical sentences which is proof-theoretically equivalent to (the standard axiomatization of) PA.

References

[1] Walter Dean. Arithmetical reflection and the provability of soundness. *Philosophia Mathematica*, 23(1):31–64, 2015.

[2] Volker Halbach. *Axiomatic Theories of Truth*. Cambridge University Press, 2011.

[3] Carlo Nicolai and Mario Piazza. The implicit commitment of arithmetical theories and its semantic core. *Erkenntnis*, pages 1–25, 2018.