## Étale cohomology reading seminar

## Exercises for Week 4

**Exercise 1.** Let  $n \ge 1$ . Consider the presheaf on the small etale site of Spec  $\mathbb{Q}$  that assigns to the étale  $\mathbb{Q}$ -algebra A the quotient

$$\frac{\operatorname{SL}_n(A)}{\mu_n(A)\cdot \operatorname{Id}_n}$$

of the special linear group quotiented by the roots of unity (viewed as diagonal matrices). Show that this presheaf is a sheaf if and only if n = 1. Its sheafification is denoted  $PSL_n$ . Argue that it could equally be denoted  $PGL_n$ . (This is really asking you to correctly define  $PGL_n$ .)

**Exercise 2.** Let  $n \ge 1$  and let *X* be a scheme on which *n* is invertible. Show that the Kummer sequence

$$0 \to \mu_n \to \mathbb{G}_m \xrightarrow{\cdot n} \mathbb{G}_m \to 0 \tag{1}$$

is:

- (i) not exact in the category of Zariski sheaves on *X* (in general);
- (ii) not exact in the category of Zariski presheaves on X (in general);
- (iii) exact in the category of etale sheaves on X.