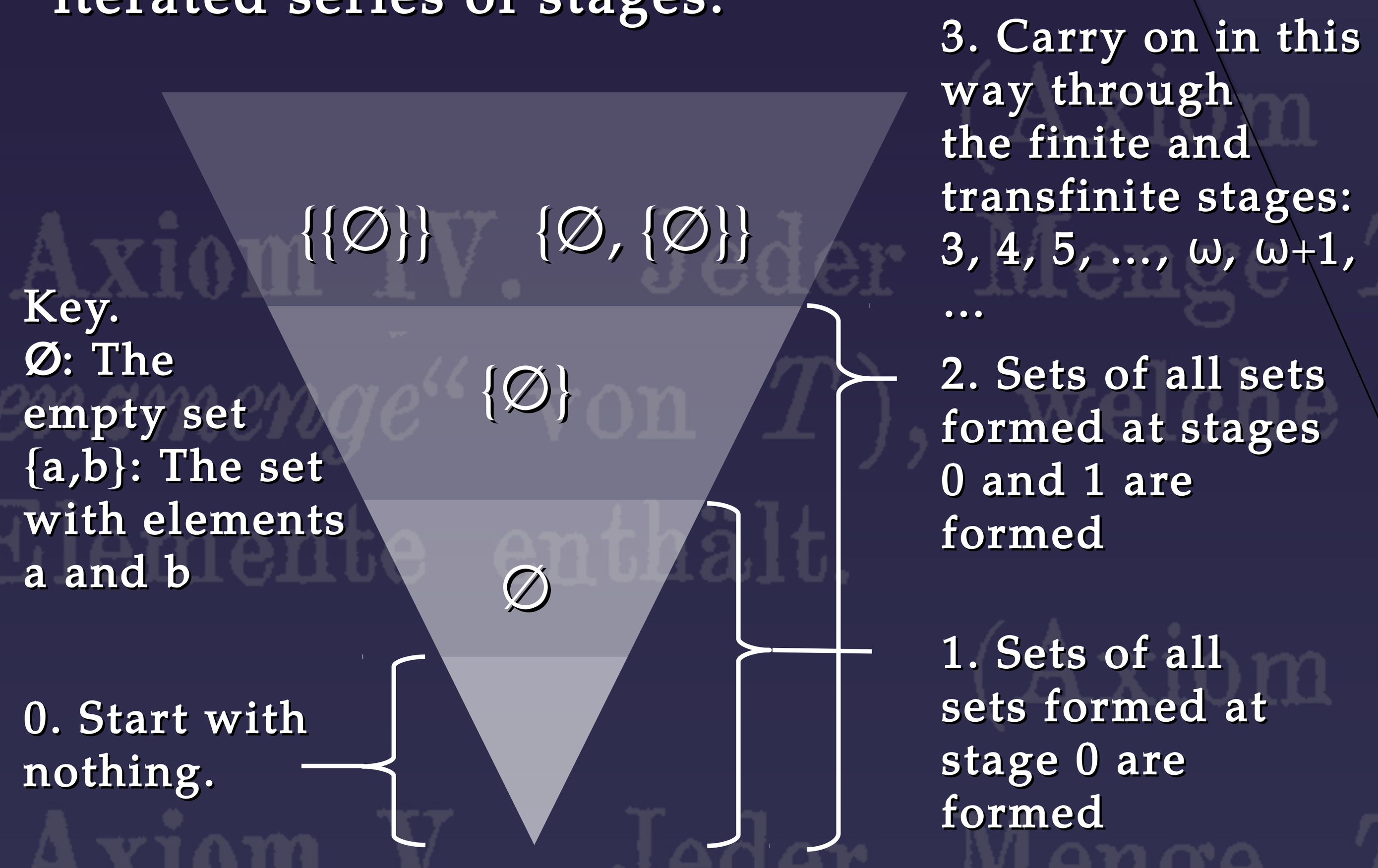
The Iterative Conception of Set

Set theory is the branch of mathematics concerned with collections. It is also a principal subject in the philosophy of mathematics due to its unique position as the "foundation of mathematics". The philosophical status of "set" is a central issue of debate, and we focus on one major view, the iterative conception of set.

In his *Grundgesetze,* the inventor of modern logic, Gottlob Frege, advanced as his 'Basic Law V' a principle incorporating the compelling idea that for any things, some set has them as its elements.

Unfortunately, one of the sets postulated by the theory, the collection of all 'non-self-membered' sets, was shown to generate an inconsistency by the British logician Bertrand Russell. The dream of a safe foundation for the 'queen of sciences' received a hard blow. Set theory itself was contradictory.

An influential picture of the set theoretic universe describes sets as 'formed' in an iterated series of stages.



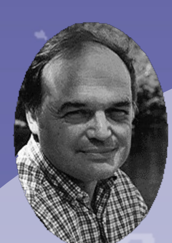
Due to the paradoxes, we are concerned to demonstrate the validity of our intuitive mathematical reasoning. We need to be explicit about the principles and rules we use.

Suitably specified principles and rules of mathematics are called an axiomatic system. The standard axiomatic system for set theory is known as Zermelo-Fraenkel set theory, or ZF. This theory has the power to recover all of the mathematics that had been developed in the centuries before its discovery.

Our project aims to enrich the language of set theory with new expressive resources in order to reconcile our pre-theoretic conception of set with the power of modern set theory.

With suitable modality, we can faithfully formalise the guiding idea in the naïve picture that any sets *can* form a set. This allows for a natural but consistent axiomatisation of the iterative conception, which allows us to recover a great deal of ZF.

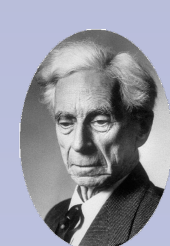
The path to the iterative conception...



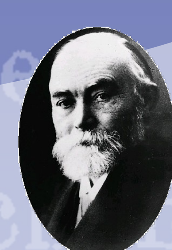
1971. George Boolos publishes 'The Iterative Conception of Set'

1922. Abraham Fraenkel extends Zermelo's axioms to create ZF

1908. Ernst Zermelo provides the first axiomatisation of set theory



1902. Bertrand Russell writes to Frege, outlining the discovery of the paradox in Frege's system



1893. Gottlob Frege publishes the ill-fated *Grundgesetze der Arithmetik*, vol. I

1874. Georg Cantor discovers infinity comes in different sizes. Set theory is born

Background image: selections from Ernst Zermelo's 'Untersuchungen uber die Grundlagen der Mengenlehre'. Mathematische Annalen, 65 (1908), pp. 261-281.

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