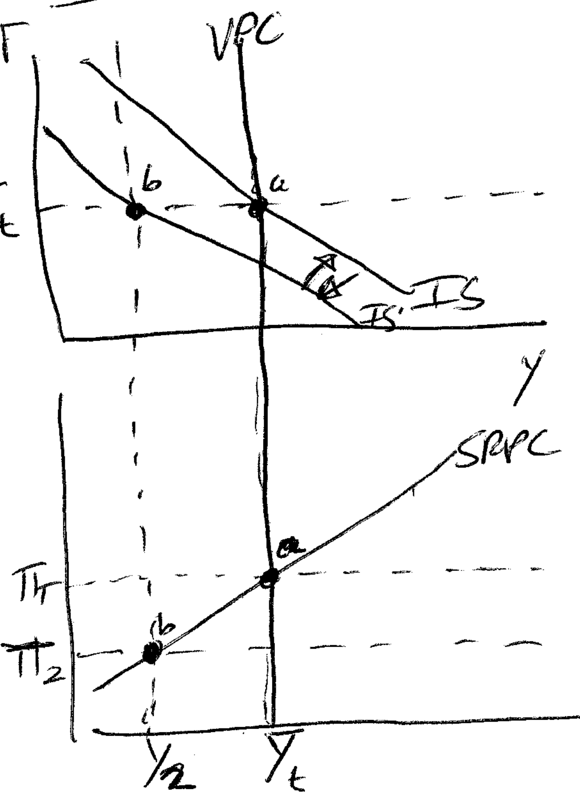
COMPARISON OF OPTIMAL STABILISATION POLICY UNDER THE TRADITIONAL ADAPTIVE EXPECTATIONS PHILLIPS CURVE AND UNDER THE NEW KEYNESIAN PHILLIPS CURVE

TPC : $\pi_t = \pi_{t-1} + \alpha(Y_t - \bar{Y}_t) + v_t$

NKPC : $\pi_t = \pi^e_{t+1} + \alpha(Y_t - \bar{Y}_t) + v_t$

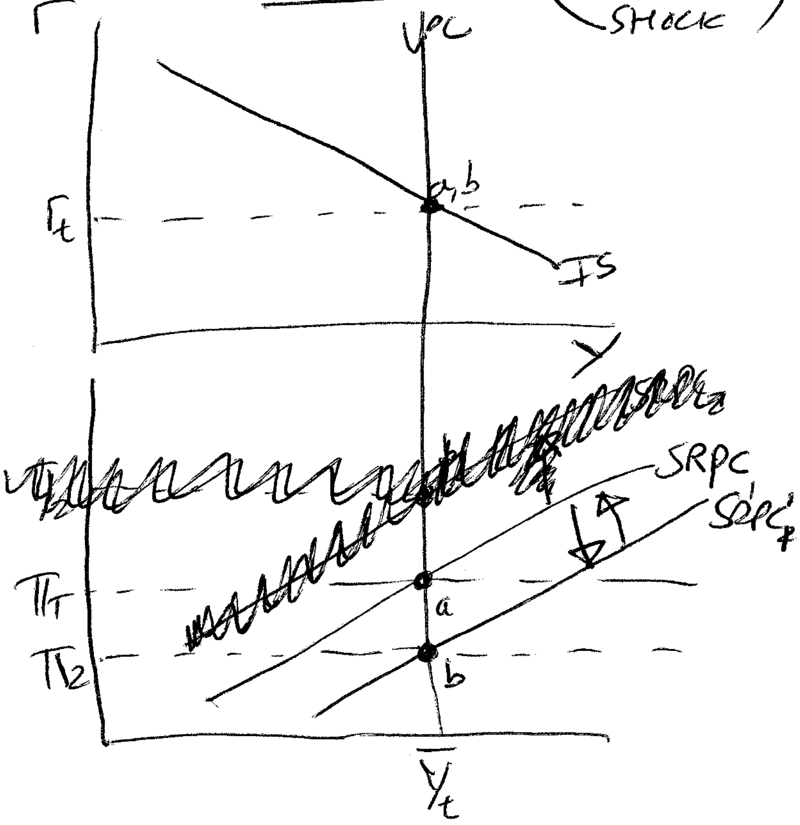
LET US FIRST COMPARE THE IMPACT OF A TEMPORARY NEGATIVE DEMAND OR SUPPLY SHOCK WHERE THE CENTRAL BANK IS COMPLETELY PASSIVE AND SO LEAVES THE REAL INTEREST RATE UNCHANGED IN THE PERIOD OF THE SHOCK AND AFTER.

DEMAND



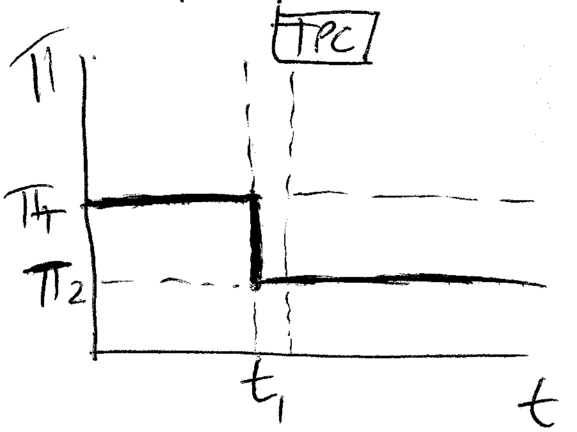
SUPPLY

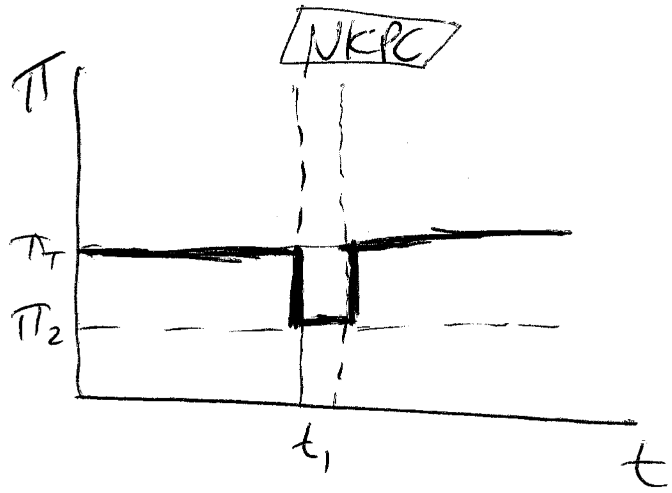
(COST-PUSH / INFLATION SHOCK)



ASSUMING AN UNANTICIPATED SHOCK, UNDER THE NKPC, AS WITH THE T.P.C, THERE ARE NO EFFECTS ON INFLATION BEFORE THE SHOCK. ASSUMING ALSO THAT THE DEMAND SHOCK IS ONLY TEMPORARY SO THAT THE STABILISING REAL INTEREST RATE IS THE SAME AFTER THE SHOCK AND THAT THE CENTRAL BANK'S INFLATION TARGET REMAINS

CREDIBLE TO THE PRIVATE SECTOR THEN UNDER THE NKPC INFLATION RETURNS STRAIGHT TO TARGET IN TIME PERIOD IMMEDIATELY AFTER THE SHOCK. UNDER THE T.P.C., ON THE OTHER HAND, THE NEW LOWER INFLATION RATE PERSISTS!



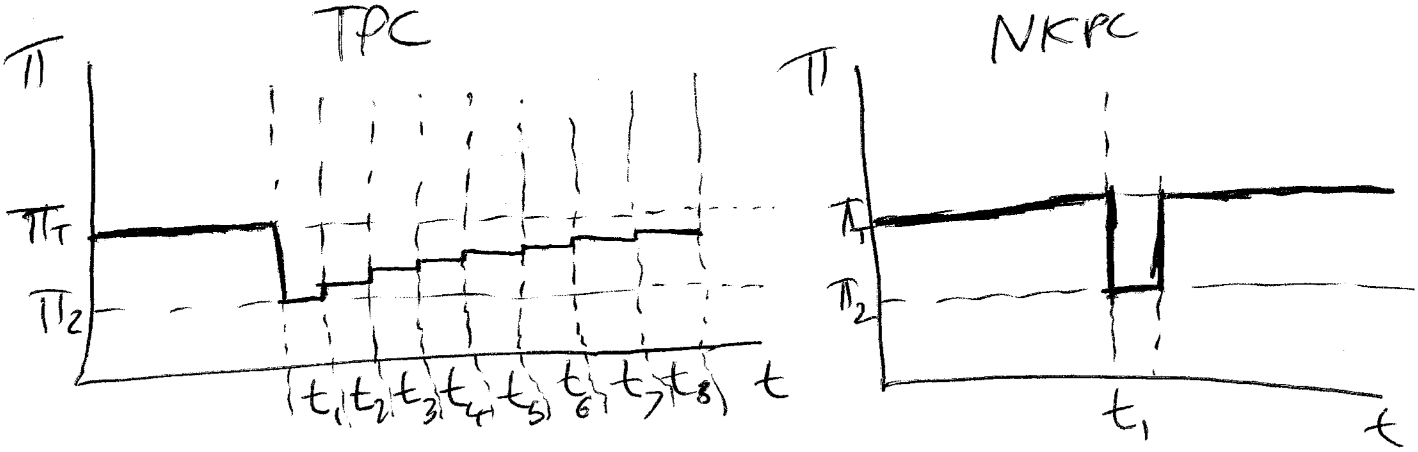


NOW IF WE ASSUME THAT THE CENTRAL BANK RESPONDS OPTIMALLY AND IN A TIME CONSISTENT MANNER UNDER DISCRETION WITH A LOSS FUNCTION OF THE FORM

$$L_t = \sum_{i=0}^{\infty}\left[\delta^i\left((Y_{t+i}-\bar{Y}_{t+i})^2+\beta(\Pi_{t+i}-\Pi_T)^2\right)\right]$$

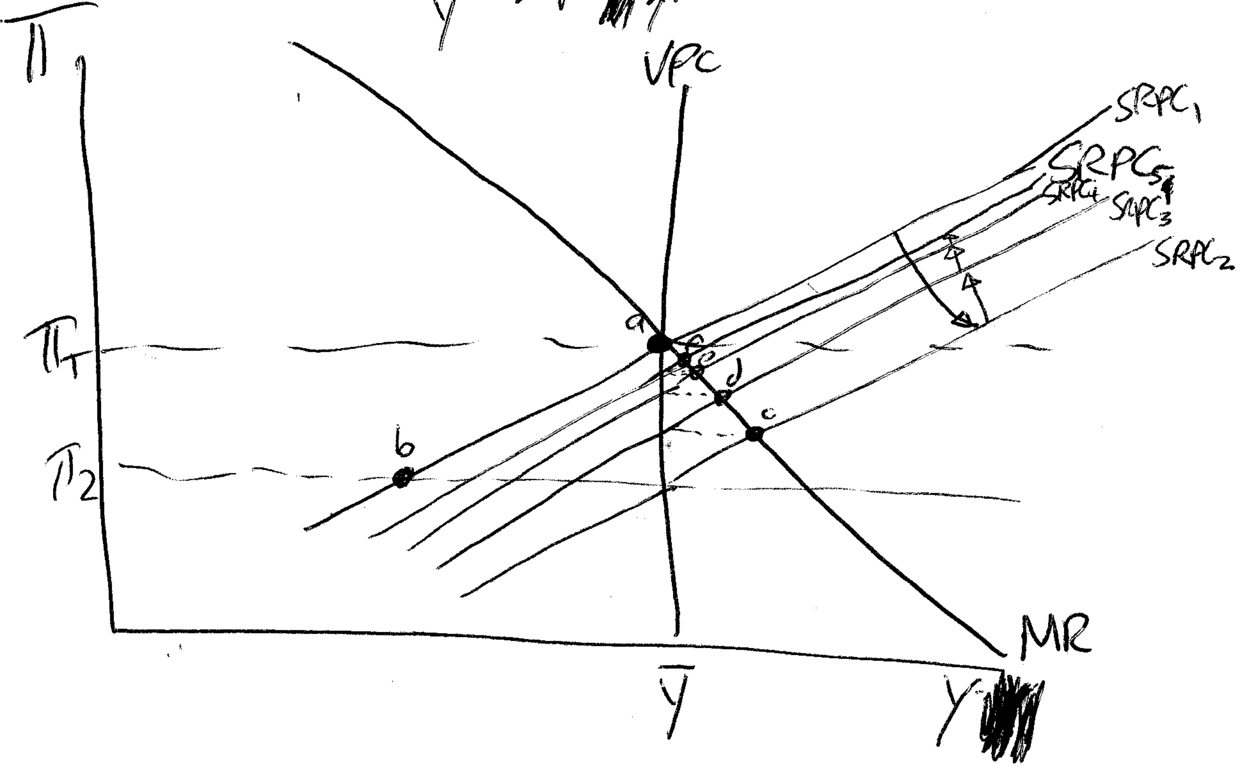
THEN UNDER ADAPTIVE EXPECTATIONS THERE WILL BE A GRADUAL RETURN TO INFLATION TARGET (THIS ASSUMES A ONE-PERIOD DELAY IN MONETARY POLICY RESPONSE SINCE IF THE CENTRAL BANK WAS ABLE TO ANTICIPATE A DEMAND SHOCK AND IMMEDIATELY RESPOND, IT WOULD BE ABLE TO ADJUST THE REAL INTEREST RATE AND PREVENT ANY OUTPUT / INFLATION GAP FROM OPENING UP.) FOR THE NEW KEYNESIAN PHILLIPS CURVE, HOWEVER, SINCE THE ONLY CREDIBLE TIME CONSISTENT POLICY IS TO RETURN TO THE INFLATION TARGET IMMEDIATELY AFTER THE SHOCK, THEN IF WE ASSUME A ONE-PERIOD DELAY IN CENTRAL BANK MONETARY

POLICY RESPONSE THEN FOR A ONE-PERIOD TEMPORARY SHOCK THE PATH OF INFLATION WOULD BE EXACTLY THE SAME AS UNDER A "PASSIVE" CENTRAL BANK:

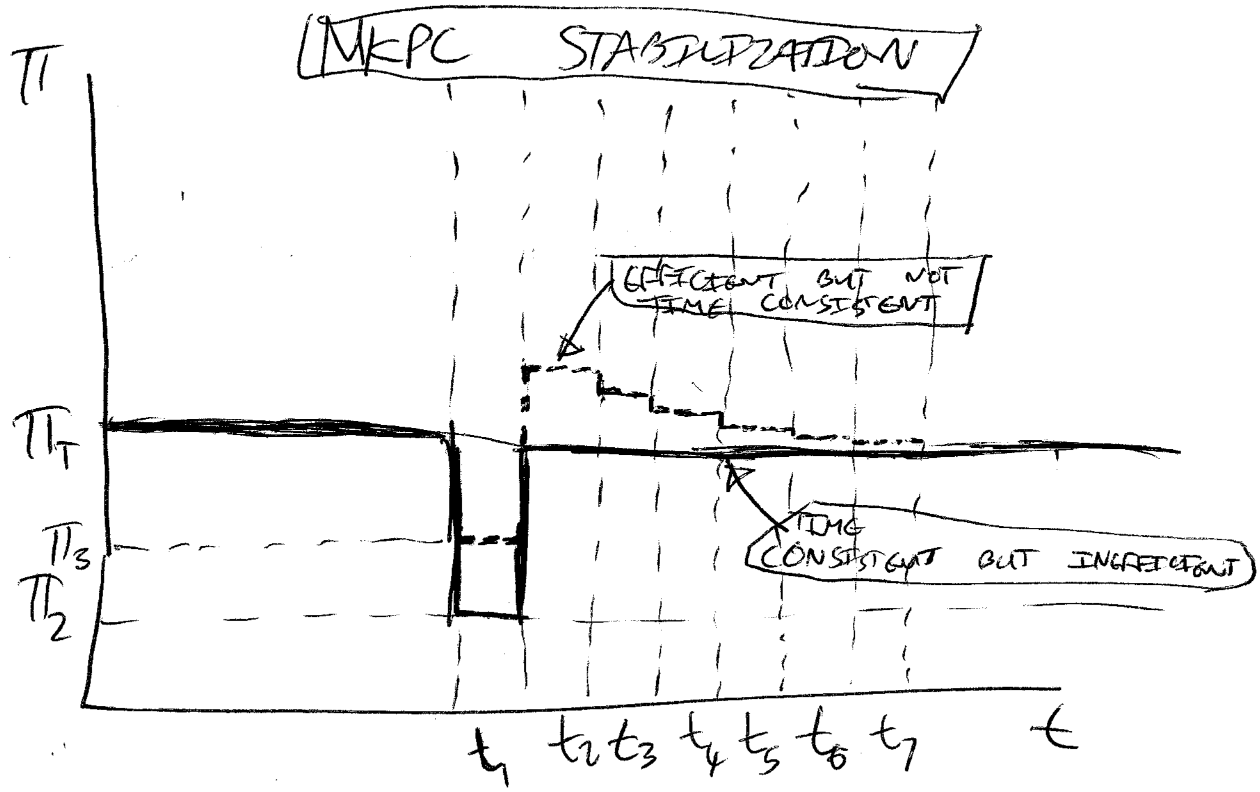


THE FOLLOWING DIAGRAM ILLUSTRATES THE ADJUSTMENT BACK TO EQUILIBRIUM UNDER THE ADAPTIVE EXPECTATIONS T.P.C:

(ASSUMING A NEGATIVE DEMAND SHOCK

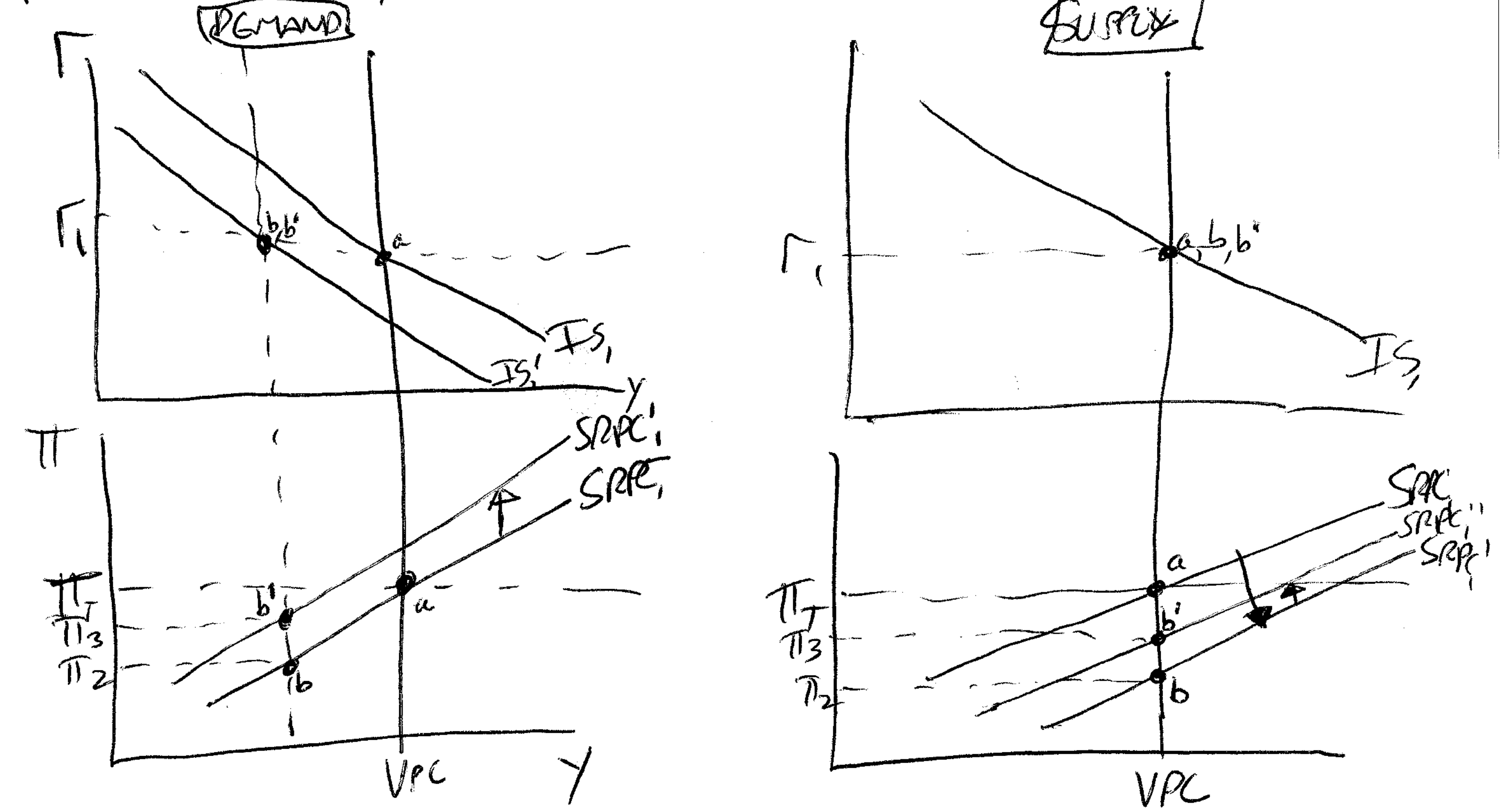


ALTHOUGH GIVEN THE PERSISTENCE OF INFLATION, THE GLOBALLY OPTIMAL POLICY WILL ALSO BE TIME CONSISTENT WITH ADAPTIVE EXPECTATIONS, THE SAME IS <u>NOT</u> TRUE UNDER THE NEW KEYNESIAN PHILLIPS CURVE AND RATIONAL EXPECTATIONS SINCE THE CENTRAL BANK'S PROMISES ABOUT THE FUTURE PATH OF INFLATION, IF CREDIBLE AFFECT THE CURRENT SRPC! IN PARTICULAR, THE CENTRAL BANK COULD LOWER THE TOTAL LOSS IF IT WAS ABLE TO CREDIBLY PROMISE INFLATION IN PERIODS <u>SUBSEQUENT</u> TO A DEFLATIONARY SHOCK. HOWEVER, SUCH A ~~POLICY~~ PROMISE WOULD NOT BE CREDIBLE AND SO SUCH OPTIMAL POLICY WOULD BE <u>TIME INCONSISTENT</u>!



BY PROMISING A PERIOD OF INFLATION <u>ABOVE TARGET</u> AND GRADUALLY RETURNING TO TARGET FOLLOWING A DEFLATIONARY SHOCK, THE SHORT RUN PHILLIPS CURVE IS <u>SHIFTED UP</u> DURING

THE SHOCK; THUS REDUCING THE INFLATION GAP.



WE CAN NOW ALSO SHOW THAT THE OPTIMAL POLICY UNDER DISCRETION (WHICH IS TIME INCONSISTENT WITH INFLATION TARGETING) IS, IF THE CENTRAL BANK'S DISCOUNT FACTOR δ IS CLOSE TO 1, APPROXIMATELY THE SAME AS THE OPTIMAL POLICY UNDER DISCRETIONARY PRICE LEVEL TARGETTING (OR NOMINAL GDP TARGETING), (BUT IS ALSO TIME CONSISTENT WITH DISCRETIONARY NOMINAL PRICE OR NOMINAL GDP TARGETING). THE INTUITION FOR THIS RESULT IS THAT UNDER NOMINAL PRICE/GDP TARGETTING IT DOES BECOME CREDIBLE TO PROMISE A PERIOD OF INFLATION ABOVE TARGET FOLLOWING A DEFLATIONARY SHOCK BECAUSE THIS WOULD BE NEEDED TO RESTORE THE PRICE LEVEL BACK TO ITS ORIGINAL TARGET LEVEL.

TO PROVE THIS, CONSIDER THE OPTIMAL RESPONSE TO AN INFLATION SHOCK IN PERIOD 1 OVER N PERIODS. FOR EXAMPLE, IN A 3-PERIOD CASE, THE LOSS FUNCTION WOULD BE: (LET x BE THE SHOCK.)

$$L = (y_1-\bar{y})^2 + \delta(y_2-\bar{y})^2 + \delta^2(y_3-\bar{y})^2 + \beta\left((\pi_1-\pi_T)^2 + \delta(\pi_2-\pi_T)^2 + \delta^2(\pi_3-\pi_T)^2\right)$$

ASSUMING THAT INFLATION WILL BE BACK ON TARGET IN PERIOD 4, WE CAN THEN USE THE NKPC AND BACKWARDS INDUCTION TO CALCULATE THE INFLATION AND OUTPUT GAPS IN EACH PERIOD:

$\boxed{\pi_t = \pi^e_{t+1} + \alpha(y_t-\bar{y}) + v_t}$ $\boxed{\begin{matrix} v_1 = x \\ v_2 = 0 \\ v_3 = 0 \end{matrix}}$

PERIOD	INFLATION GAP	OUTPUT GAP
4	0	0
3	$0+\alpha(y_3-\bar{y})$	$(y_3-\bar{y})$
2	$0+\alpha(y_3-\bar{y})+\alpha(y_2-\bar{y})$	$(y_2-\bar{y})$
1	$0+\alpha(y_3-\bar{y})+\alpha(y_2-\bar{y})+\alpha(y_1-\bar{y})+x$	$(y_1-\bar{y})$

LET US NOW CONSIDER THE FIRST ORDER CONDITION FOR MINIMIZING THE LOSS FUNCTION WITH RESPECT TO THE OUTPUT ~~GAP~~ IN PERIOD 3, y_3. SINCE y_3 DOES NOT AFFECT THE OUTPUT GAPS IN PERIODS 1 AND 2, WE SUPPRESS THESE FOR BREVITY:

$$L = \delta^2(y_3-\bar{y})^2 + \alpha^2\beta\left(\left((y_1-\bar{y})+(y_2-\bar{y})+(y_3-\bar{y})+x\right)^2 + \delta\left((y_2-\bar{y})+(y_3-\bar{y})\right)^2 + \delta^2(y_3-\bar{y})^2\right)$$

$$\frac{\partial L}{\partial y_3} = 2\delta^2\hat{y}_3 + 2\alpha\beta\left(\hat{\pi}_1 + \delta\hat{\pi}_2 + \delta^2\hat{\pi}_3\right) = 0$$

WHERE $\hat{y}_i$ IS THE OUTPUT GAP IN PERIOD i AND $\hat{\pi}_i$ IS THE INFLATION GAP IN PERIOD i

GENERALISING TO AN N-PERIOD CASE, WE GET:

$$\delta^{N-i}\hat{y}_N = -\alpha\beta Z_N \quad \boxed{1}$$

WHERE $$Z_N = \sum_{i=0}^{N}\left[\delta^i \hat{\pi}_i\right]$$

IF WE TAKE THE LIMIT OF $\boxed{1}$ AS $N \to \infty$ THEN WE GET (SINCE $\delta < 1$)

$$0 = -\alpha\beta(Z_\infty)$$

$$\Rightarrow \sum_{i=0}^{\infty}\left[\delta^i \hat{\pi}_i\right] = 0 \quad \boxed{2}$$

IF WE THEN TAKE THE LIMIT OF $\boxed{2}$ AS $\delta \to 1$ THEN WE GET A RESULT WHICH SAYS THAT THE SUM OF ALL CURRENT AND FUTURE INFLATION GAPS IS ZERO

THIS MEANS THAT NOMINAL PRICES WILL, OVER AN INFINITE TIME HORIZON, BE RETURNED TO THEIR ORIGINAL LEVEL FOLLOWING A SHOCK. HENCE NOMINAL PRICE LEVEL / GDP TARGETING WOULD BE OPTIMAL

<u>COMPARISON BETWEEN PRICE LEVEL AND GDP TARGETING</u>

NOMINAL GDP $= (P)(y)$

WITH A PRICE LEVEL AND OUTPUT TARGET IN LOG [redacted] FORM THE LOSS FUNCTION WILL BE:

$$L_t = \sum_{i=0}^{\infty}\left[\delta^i\left(\left(\ln(y_{t+i}) - \ln(\bar{y}_{t+i})\right)^2 + \beta\left(\ln(P_{t+i}) - \ln(P_T)\right)^2\right)\right]$$

[redacted]

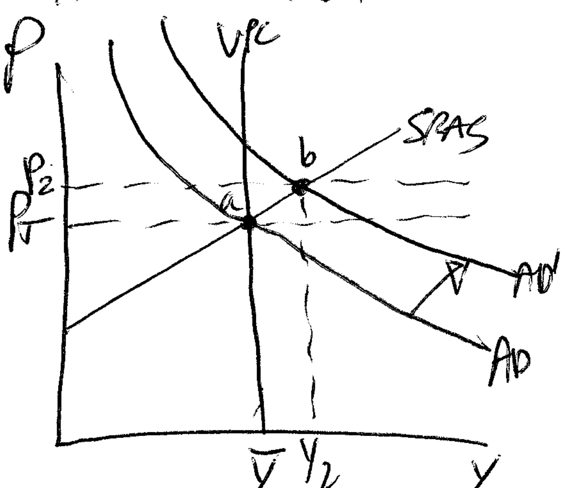
CONSIDER INSTEAD A NOMINAL GDP TARGET IN LOG FORM:

$$L_t = \sum_{i=0}^{\infty}\left[\delta^i\left(\ln(Y_{t+i}P_{t+i}) - \ln(\bar{Y}_{t+i}P_T)\right)^2\right]$$

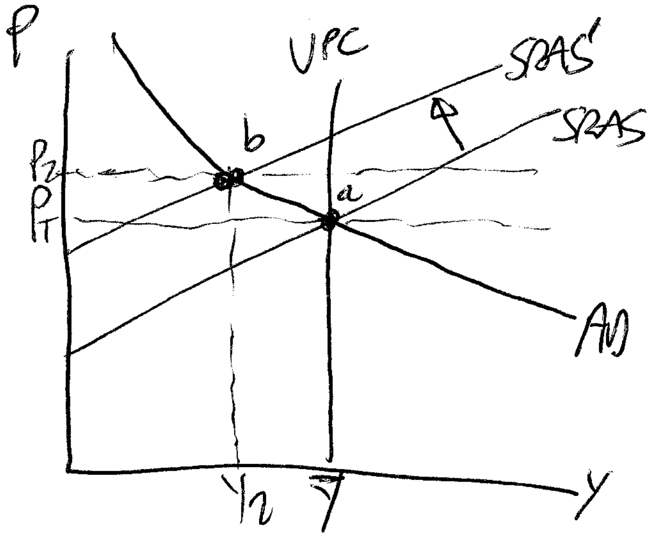
MULTIPLYING OUT, WE GET

$$L_t = \sum_{i=0}^{\infty}\left[\delta^i\left(\left(\ln(Y_{t+i}) - \ln(\bar{Y}_{t+i})\right)^2 + 2\left(\ln(Y_{t+i}) - \ln(\bar{Y}_{t+i})\right)\left(\ln(P_{t+i}) - \ln(P_T)\right) \dots\right.\right.$$
$$\left.\left. \dots + \left(\ln(P_{t+i}) - \ln(P_T)\right)^2\right)\right]$$

SO WE CAN SEE THAT IN LOG FORM A NOMINAL GDP TARGET IS SIMILAR TO A PRICE LEVEL TARGET PLUS OUTPUT TARGET WITH $\beta = 1$ EXCEPT FOR THE ADDITIONAL TERM $2\left(\ln(Y_{t+i}) - \ln(\bar{Y}_{t+i})\right)\left(\ln(P_{t+i}) - \ln(P_T)\right)$ WHICH IS POSITIVE WHEN OUTPUT GAPS AND PRICE LEVEL GAPS ARE POSITIVELY CORRELATED AND NEGATIVE WHEN THEY ARE NEGATIVELY CORRELATED. THIS MEANS THAT A NOMINAL GDP TARGET WILL BE EXTRA TOUGH / RESTRICTIVE WHEN RESPONDING TO A POSITIVE DEMAND SHOCK BUT MORE WILLING TO ALLOW A SUPPLY SHOCK:



DEMAND SHOCK



SUPPLY SHOCK

IF THERE ARE HYSTERESIS EFFECTS ON THE VERTICAL PHILLIPS CURVE THEN THIS COULD BE

BENEFICIAL SINCE A NOMINAL GDP TARGET WILL GIVE AN "EXTRA MONETARY BOOST" FOLLOWING A NEGATIVE DEMAND SHOCK (IE. ONE WHICH REDUCES OUTPUT) BUT WILL BE MORE WILLING TO ALLOW OUTPUT TO FALL IF IT IS DUE TO A SUPPLY SHOCK, THUS STABILIZING LONG RUN INFLATION EXPECTATIONS.

THE OTHER APPEAL OF A NOMINAL GDP TARGET IS THAT IT HAS A "BUILT-IN" OUTPUT TARGET AND SO THE CENTRAL BANK ONLY HAS TO TARGET ONE SINGLE VARIABLE.

ON THE OTHER HAND, BY TARGETTING OUTPUT AND THE PRICE LEVEL SEPARATELY, IT IS POSSIBLE TO CHOOSE ~~THE~~ ANY VALUE OF β (THE "INFLATION AVERSION" PARAMETER, WHICH DETERMINES HOW "AGGRESSIVE" THE CENTRAL BANK IS IN RETURNING THE PRICE LEVEL TO TARGET.