TUTORIAL 7 – MONETARY POLICY

1. IF THE MONETARY AUTHORITIES USE INTEREST RATES TO TARGET THE MONEY SUPPLY, HOW DOES THIS COMPARE WITH A MONETARY POLICY BASED ON A TAYLOR RULE?

WE WILL COMPARE THREE DIFFERENT MONETARY POLICY REGIMES: (A) A MONETARY GROWTH RULE, WHERE THE NOMINAL MONEY SUPPLY IS GROWN AT A CONSTANT RATE EACH PERIOD. (E.G. BY 2% TO CREATE A 2% INFLATION RATE IN THE MEDIUM RUN ASSUMING THAT REAL GDP AND VELOCITY OF CIRCULATION ARE FIXED IN THE MEDIUM RUN):

$$MV = PY$$

$$\uparrow 2\% \cdot \quad \uparrow 2\% \cdot$$

$$\pi_T \qquad \pi_T$$

GENERALLY SO THE GROWTH RATE OF THE MONEY SUPPLY IS THE INFLATION TARGET π_T

(B) ACTIVE MONETARY POLICY USING INTEREST RATES TO MINIMIZE A LOSS FUNCTION OF THE FORM $\sum_{i=0}^{\infty} \delta^i \left[(Y-\bar{Y})^2 + \beta(\pi-\pi_T)^2 \right]$ WHERE δ IS THE DISCOUNT FACTOR $(0<\delta<1)$, β IS THE LEVEL OF INFLATION AVERSION, $\bar{Y}$ IS POTENTIAL OUTPUT, π_T IS THE INFLATION TARGET. (THIS IS THE 3-EQUATION MODEL.)

(C) A TAYLOR RULE, WHICH IS OF THE FORM:

$$r_t = \gamma_1 (Y_t - \bar{Y}) + \gamma_2 (\pi_t - \pi_T) + \bar{r}$$

$$\Rightarrow i_t = \pi_t + \gamma_1 (Y_t - \bar{Y}) + \gamma_2 (\pi - \pi_T) + \bar{r}$$

$$\Rightarrow i_t = \pi_t (1+\gamma_2) - \gamma_2 \pi_T + \gamma_1 (Y_t - \bar{Y}) + \bar{r}$$

($\bar{r}$ IS THE MEDIUM RUN EQUILIBRIUM REAL INTEREST RATE.)

LET US FIRST COMPARE THE ADJUSTMENT/RESPONSE TO AN AGGREGATE DEMAND SHOCK WITH A MONETARY GROWTH RULE AND THE 3-EQUATION MODEL. WE ASSUME A PHILLIPS CURVE OF THE FORM $\pi_t = \pi_t^e + \alpha(y_t - \bar{y})$ (WE CONSIDER LATER THE EFFECT OF A ONE-PERIOD LAG FROM OUTPUT GAP TO INFLATION.) WE ALSO ASSUME THAT DEMAND IS DETERMINED ACCORDING TO A KEYNESIAN MULTIPLIER MODEL:

$$y_t = k(A_0 - r_t d)$$

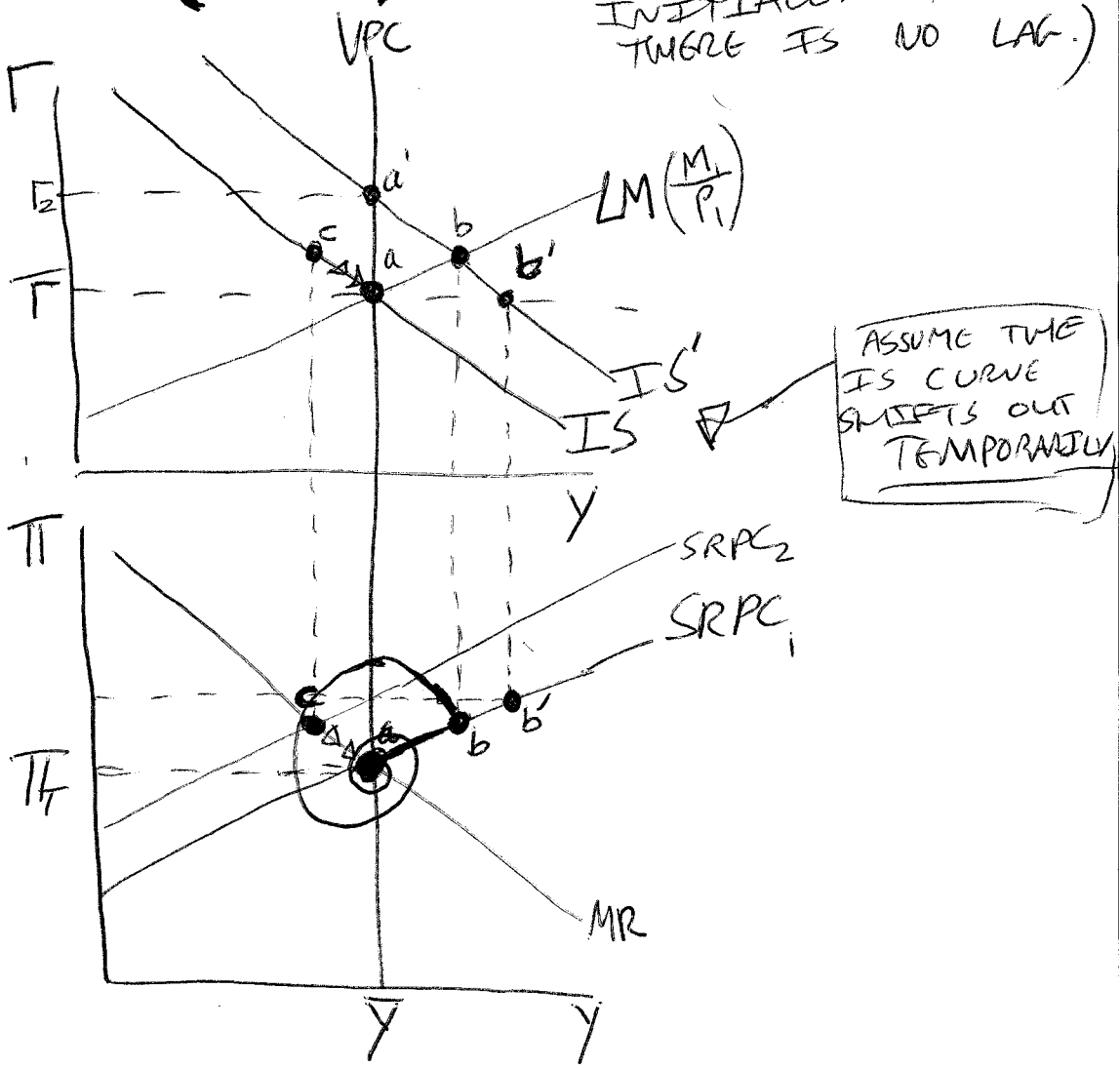
k – MULTIPLIER
A_0 – AUTONOMOUS SPENDING
d – INTEREST SENSITIVITY OF INVESTMENT

$$\bar{y} = k(A_0 - \bar{r} d)$$

$$\Rightarrow y_t - \bar{y} = kd(\bar{r} - r_t)$$

(AGAIN, WE ASSUME INITIALLY THAT THERE IS NO LAG.)

GRAPHICALLY:



WE ALSO ASSUME ADAPTIVE EXPECTATIONS SO THAT $\pi_t^e = \pi_{t-1}$ INITIALLY.

IF WE CONSIDER FIRST THE 3-EQUATION MODEL, IF THE CENTRAL BANK CAN RESPOND IMMEDIATELY THEN THE INTEREST RATE IS RAISED TO r_2 AND SO THE ECONOMY STAYS AT POINT a IN THE PHILLIPS CURVE DIAGRAM (I.E. AT THE CENTRAL BANK'S BLISS POINT). IF THERE IS A ONE-PERIOD DELAY BEFORE r CHANGES THEN WE GO TO POINT b', THEN POINT c AND THE BACK DOWN THE MR CURVE TO a.

IN THE CASE OF A MONETARY GROWTH RULE, THE NOMINAL MONEY SUPPLY AND PRICE LEVEL ARE FIXED IN THE SHORT RUN SO WE MOVE UP THE LM CURVE TO POINT b. SO, FIXING THE MONEY SUPPLY DOES DAMPEN THE INITIAL INCREASE IN Y FOR AN UNANTICIPATED IS CURVE SHOCK. HOWEVER, THE SUBSEQUENT REARJUSTMENT BACK TO MEDIUM RUN EQUILIBRIUM AT POINT a IS FAR MORE PROTRACTED AND LESS EFFICIENT BECAUSE WHILST THERE IS A POSITIVE OUTPUT GAP INFLATION IS RISING, BUT WHILE INFLATION IS ABOVE TARGET THE OUTPUT GAP SHRINKS, BUT (DUE TO THE CONTRACTION OF THE REAL MONEY SUPPLY) THEN WHEN THE OUTPUT GAP BECOMES NEGATIVE AT THE TOP OF THE SPIRAL INFLATION REMAINS ABOVE TARGET SO THE OUTPUT GAP CONTINUES TO SHRINK (DUE TO CONTINUED REAL MONEY CONTRACTION) AND ONLY STARTS TO INCREASE AGAIN ONCE INFLATION GOES BELOW TARGET. THUS WE KEEP 'OVER SHOOTING' THE INFLATION / OUTPUT TARGET IN A SPIRAL-SHAPED PATH BACK TO MEDIUM RUN EQ.

NOW WE DERIVE THE INTEREST RATE RULE DERIVED FROM THE 3-EQUATION MODEL AND COMPARE TO A TAYLOR RULE. THE CENTRAL BANK MINIMIZES THE FOLLOWING FUNCTION IN PERIOD t; BY CHOOSING Y_t

$$\min_{Y_t} \left\{ (Y_t - \bar{Y})^2 + \beta(\pi_t - \pi_T)^2 \right\}$$

π_t DEPENDS UPON Y_t SO WE SUB IN THE PHILLIPS CURVE: $\pi_t = \pi_t^e + \alpha(Y_t - \bar{Y})$

$$\Rightarrow \min_{Y_t} \left\{ (Y_t - \bar{Y})^2 + \beta\left(\pi_t^e + \alpha(Y_t - \bar{Y}) - \pi_T\right)^2 \right\}$$

THE FIRST ORDER CONDITION IS

$$2(Y_t - \bar{Y}) + 2\alpha\beta\left(\pi_t^e - \pi_T + \alpha(Y_t - \bar{Y})\right) = 0$$

$$\Rightarrow Y_t - \bar{Y} = -\alpha\beta(\pi_t - \pi_T)$$

$$\Rightarrow \pi_t - \pi_T = -\frac{1}{\alpha\beta}(Y_t - \bar{Y})$$

THIS GIVES US THE MR CURVE ALGEBRAICALLY. IT IS SHALLOW IF α AND β ARE LARGER, SHOWING THAT A STEEP PHILLIPS CURVE OR HIGH LEVEL OF INFLATION AVERSION (β) CREATES A MORE "AGGRESSIVE" CENTRAL BANK WILLING TO CREATE A LARGER OUTPUT GAP TO RETURN INFLATION TO TARGET MORE QUICKLY, CETERIS PARIBUS.

IF WE NOW PLUG IN THE IS CURVE, WE GET:

$$\pi_t - \pi_T = -\left(\frac{1}{\alpha\beta}\right)(\chi d)(\bar{r} - r_t)$$

$$\Rightarrow r_t = \left(\frac{\alpha\beta}{kd}\right)(\pi_t - \pi_T) + \bar{r}$$

SO, THE 3-EQUATION MODEL WITH NO LAGS ON THE IS CURVE AND PHILLIPS CURVE GENERATES A VERY SIMPLE INTEREST RATE RULE WHICH IS EQUIVALENT TO A TAYLOR RULE WITH $\gamma_1 = 0$ AND $\gamma_2 = \frac{\alpha\beta}{kd}$

THE HIGHER IS α AND β, AND THE SMALLER IS THE KEYNESIAN MULTIPLIER k AND THE INTEREST SENSITIVITY OF INVESTMENT d, THE MORE "AGGRESSIVE" IS THE CENTRAL BANK'S INTEREST RATE RESPONSE TO A CHANGE IN π_t (INFLATION).

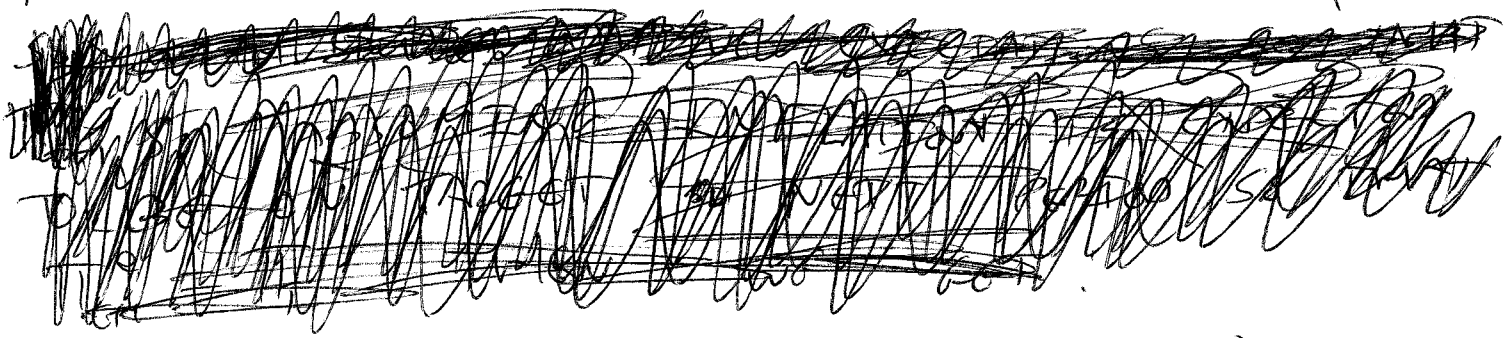
3-EQUATION MODEL WITH LAGS

SUPPOSE THERE IS NOW A LAG ON THE IS CURVE SO THAT $y_{t+1} - \bar{y} = -kd(\bar{r} - r_t)$.

TODAY'S INTEREST RATE MUST NOW BE SET ACCORDING TO TOMORROW'S MR CURVE:

$$r_t = \bar{r} + \left(\frac{\alpha\beta}{kd}\right)(\pi_{t+1} - \pi_T)$$

FROM THE PHILLIPS CURVE: $\pi_{t+1} = \pi^e_{t+1} + \alpha(y_{t+1} - \bar{y})$



$$\Rightarrow r_t = \bar{r} + \left(\frac{\alpha\beta}{kd}\right)\left(\alpha(y_{t+1} - \bar{y}) + (\pi^e_{t+1} - \pi_T)\right)$$

$$\Rightarrow r_t = \bar{r} + \left(\frac{\alpha\beta}{kd}\right)\left(\alpha kd(\bar{r} - r_t)\right) + \left(\frac{\alpha\beta}{kd}\right)(\pi^e_{t+1} - \pi_T)$$

$$\Rightarrow (r_t - \bar{r})(1 + \alpha^2\beta) = \left(\frac{\alpha\beta}{kd}\right)(\pi^e_{t+1} - \pi_T)$$

$$\Rightarrow r_t = \bar{r} + \left(\frac{\alpha\beta}{1+\alpha^2\beta}\right)\left(\frac{1}{\lambda d}\right)\left(\pi^e_{t+1} - \pi_T\right)$$

SO NOW THE CENTRAL BANK NEEDS TO ANTICIPATE TOMORROW'S EXPECTED INFLATION IN ORDER TO DETERMINE THE INTEREST RATE TODAY (WITH ADAPTIVE EXPECTATIONS, $\pi^e_{t+1} = \pi_t$.)

TO GET A TRUE TAYLOR RULE WHERE BOTH TODAY'S OUTPUT AND TODAY'S INFLATION IS USED, WE NEED A DOUBLE LAG STRUCTURE WHERE THERE IS A ONE-PERIOD DELAY ON THE PHILLIPS CURVE SO THAT.

$$\pi_{t+1} = \pi^e_{t+1} + \alpha\left(Y_t - \bar{Y}\right)$$

THE MR CURVE IS THEN DERIVED BY MINIMIZING THE CENTRAL BANK'S LOSS OVER TWO PERIODS:

$$\min_{Y_t}\left\{\left(Y_t-\bar{Y}\right)^2 + \beta\left(\pi_t - \pi_T\right)^2 + \delta\left(Y_{t+1}-\bar{Y}\right)^2 + \delta\beta\left(\pi_{t+1}-\pi_T\right)^2\right\}$$

π_{t+1} DEPENDS UPON Y_t, SO SUB IN P.C.

$$\min_{Y_t}\left\{\left(Y_t-\bar{Y}\right)^2 + \beta\left(\pi_t-\pi_T\right)^2 + \delta\left(Y_{t+1}-\bar{Y}\right)^2 + \delta\beta\left(\pi^e_{t+1} + \alpha\left(Y_t-\bar{Y}\right) - \pi_T\right)^2\right\}$$

SINCE WE ARE USING r_{t-1} TO AFFECT Y_t AND π_{t+1}, π_t AND Y_{t+1} ARE TREATED AS CONSTANT SO THE FOC IS:

$$2\left(Y_t-\bar{Y}\right) + 2\alpha\delta\beta\left(\pi^e_{t+1}-\pi_T\right) + 2\alpha^2\delta\beta\left(Y_t-\bar{Y}\right) = 0$$

IF WE ASSUME SIMPLE ADAPTIVE EXPECTATIONS SO THAT $\pi^e_{t+1} = \pi_t$ THEN THIS CAN BE REARRANGED TO GIVE:

$$\lambda d\left(\bar{r} - r_{t-1}\right)\left(1+\alpha^2\delta\beta\right) + \alpha\delta\beta\left(\pi_t - \pi_T\right) = 0$$

ALSO, FROM THE PHILLIPS CURVE $\pi_t = \pi_{t-1} + \alpha\left(Y_{t-1} - \bar{Y}\right)$

So, THIS BECOMES

$$r_{t-1} = \bar{r} + \left(\frac{1}{kd}\right)\left(\frac{\alpha\delta\beta}{1+\alpha^2\delta\beta}\right)\left((\pi_{t-1}) - \pi_T + \alpha(y_{t-1} - \bar{y})\right)$$

So, MOVING TO PERIOD t:

$$r_t = \bar{r} + \left(\frac{1}{(\alpha\delta\beta)+\alpha}\right)(\pi_t - \pi_T)\left(\frac{1}{kd}\right) + \left(\frac{\alpha}{(\alpha\delta\beta)+\alpha}\right)(y_t - \bar{y})\left(\frac{1}{kd}\right)$$

SO HERE WE HAVE A TAYLOR RULE WITH

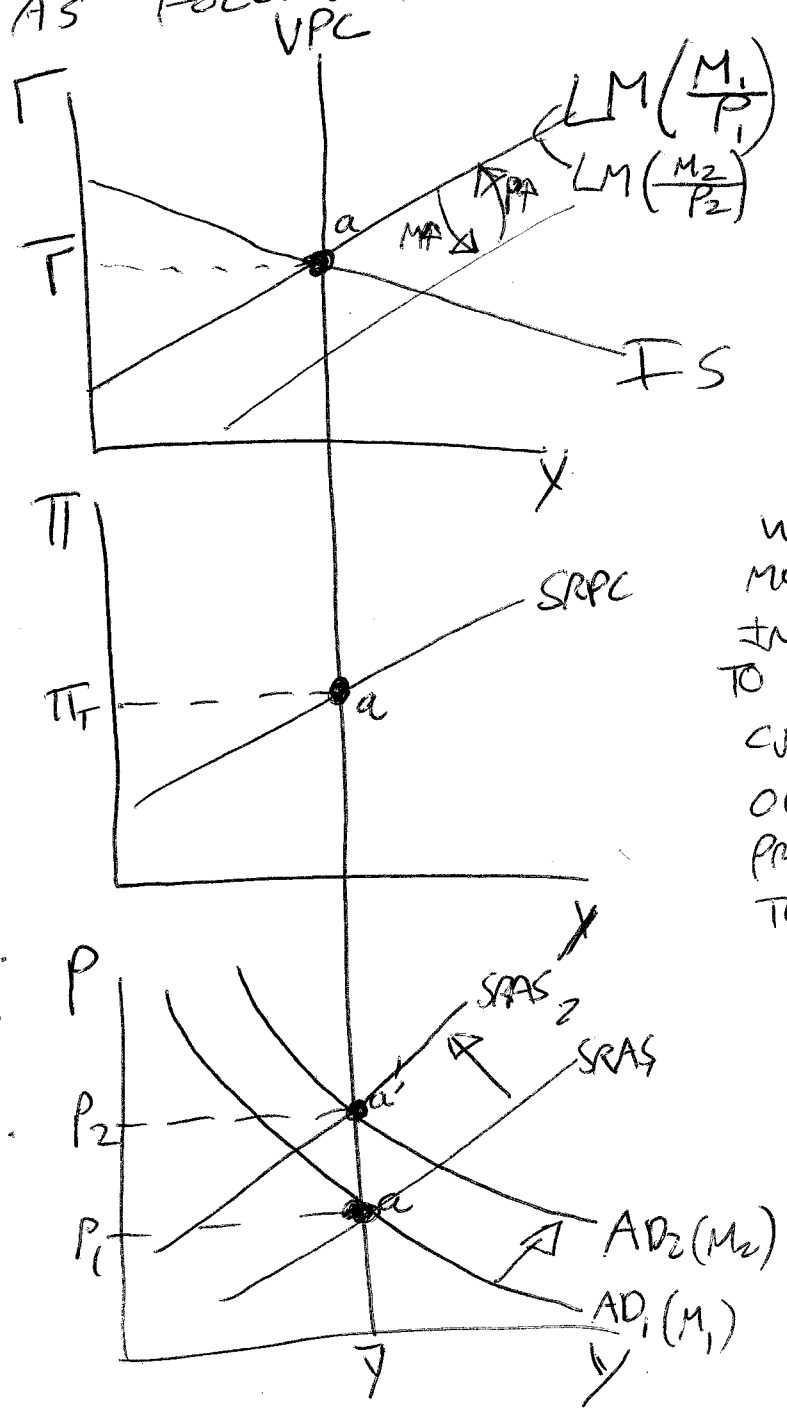
$\gamma_1 = \left(\frac{\alpha}{(\alpha\delta\beta)+\alpha}\right)\left(\frac{1}{kd}\right)$ AND $\gamma_2 = \left(\frac{1}{(\alpha\delta\beta)+\alpha}\right)\left(\frac{1}{kd}\right)$

WE CAN SEE THAT AN INCREASE IN β OR δ WILL DECREASE THE DENOMINATOR IN THE OUTER FRACTION AND THUS GENERATE A MORE AGGRESSIVE CENTRAL BANK. (ALSO, AS d WITH A NON-LAG STRUCTURE, A DECREASE IN d WILL INCREASE γ_1 AND γ_2).

SO WE HAVE SHOWN THAT UNDER A DOUBLE-LAG STRUCTURE WITH ADAPTIVE EXPECTATIONS THE 3-EQUATION MODEL YIELDS A TAYLOR RULE. THIS STILL LEAVES OPEN THE QUESTION WHETHER SUCH A RULE SHOULD BE DISCRETIONARY OR ENTRENCHED. A DISCRETIONARY RULE HAS THE ADVANTAGE OF FLEXIBILITY BUT IN A WORLD OF RATIONAL EXPECTATIONS IT COULD BE SUBJECT TO TIME INCONSISTENCY PROBLEMS. WE ALSO HAVE NOT CONSIDERED WHETHER A NOMINAL PRICE LEVEL OR NOMINAL GDP TARGET WOULD WORK BETTER. THIS WE DO IN THE EXTENDED ESSAY QUESTION.

3. WHAT IS MONEY NEUTRALITY AND SUPERNEUTRALITY? WHY MIGHT THE LATTER NOT HOLD?

MONEY NEUTRLITY MEANS THAT A ONE-OFF CHANGE IN THE NOMINAL MONEY SUPPLY WILL CAUSE NO CHANGE IN REAL VARIABLES SUCH AS REAL GDP AND REAL INTEREST RATES. IT HOLDS IN THE MEDIUM-TO-LONG RUN IN NEW KEYNESIAN MODELS BUT NOT IN THE SHORT RUN WHEREAS IN STANDARD RBC MODELS IT HOLDS ALL OF THE TIME. GRAPHICALLY, THIS CAN BE ILLUSTRATED AS FOLLOWS:

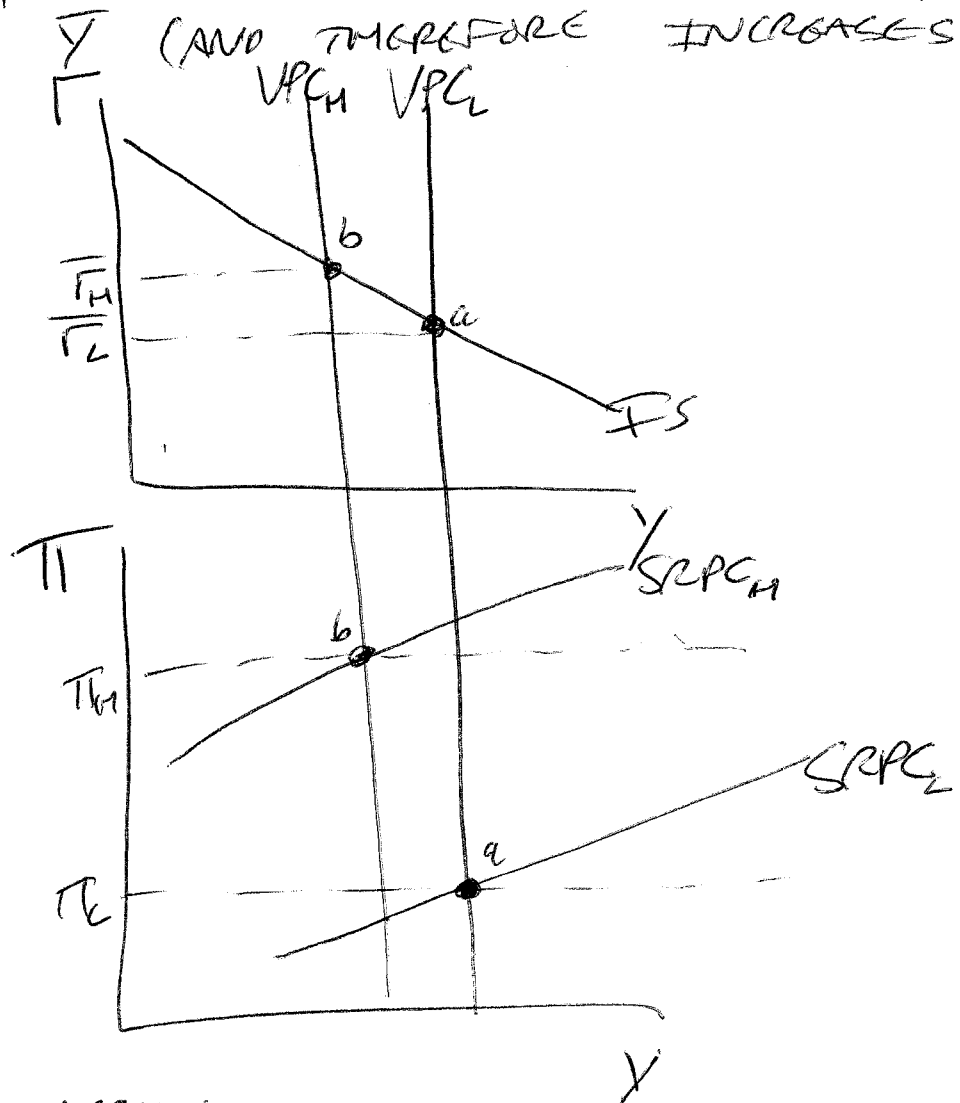


$$\frac{M_1}{P_1} = \frac{M_2}{P_2} = \bar{M}_S$$

WE ASSUME THAT THE INCREASE IN MONEY SUPPLY IS ONE-OFF SO ONCE PRICES ADJUST, EXPECTED INFLATION IS NOT AFFECTED.

WHEN THE NOMINAL MONEY SUPPLY M INCREASES FROM M_1 TO M_2 THE AD CURVE SHIFTS OUTWARDS, BUT PRICES (FROM P_1 TO P_2) RISE SO THAT REAL MONEY SUPPLY STAYS FIXED AT $\bar{M}_S$ AND SO THE LM CURVE STAYS FIXED AND THUS $\bar{Y}$ AND $\bar{r}$ ARE UNAFFECTED.

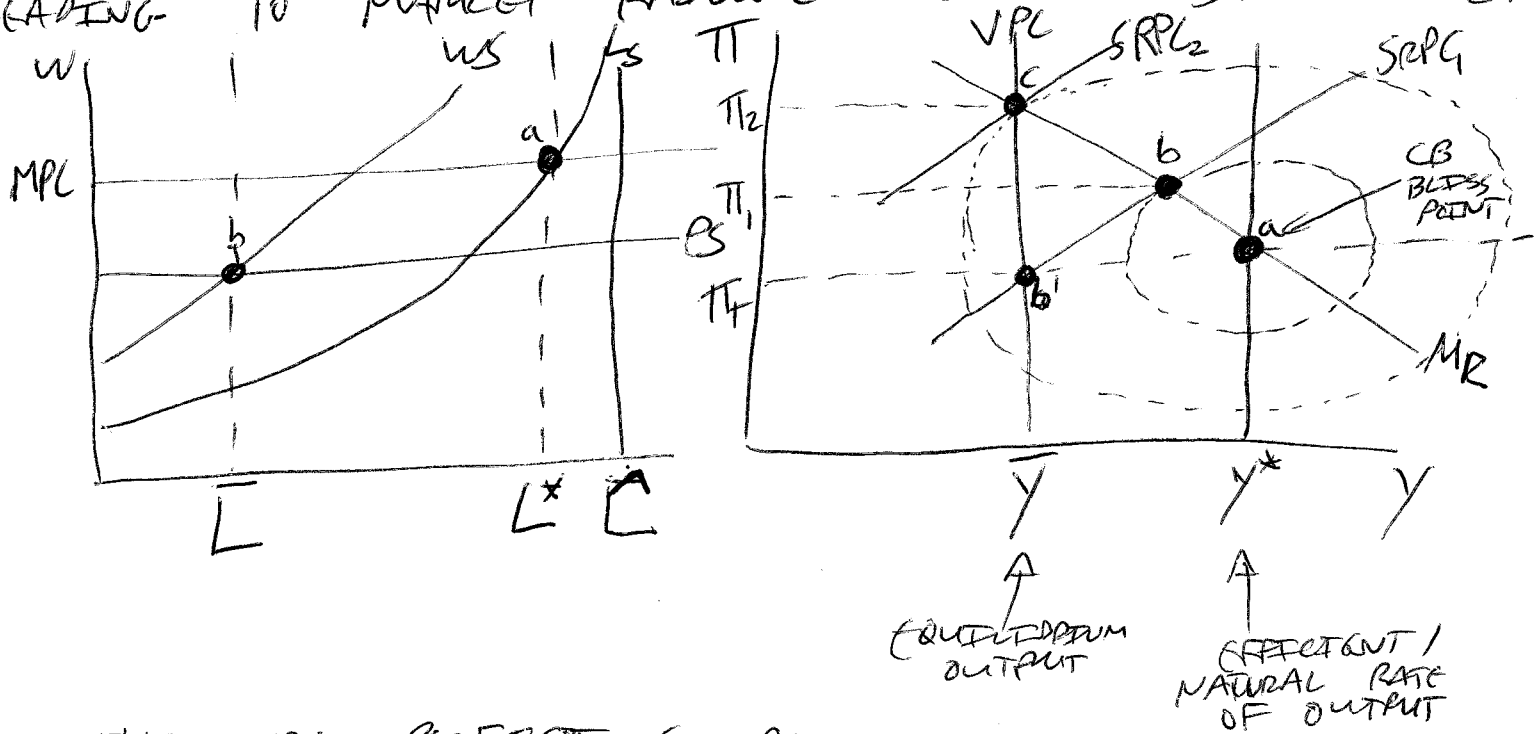
SUPERNEUTRALITY IS THE STRONGER CONDITION THAT THE MEDIUM-LONG RUN GROWTH RATE OF THE NOMINAL MONEY SUPPLY ($\hat{M}$) (WHICH, WITH FIXED VELOCITY AND REAL GDP WILL BE THE MEDIUM-LONG RUN INFLATION RATE) HAS NO EFFECT ON REAL VARIABLES. ALTHOUGH IT ALWAYS HOLDS IN BASIC RBC MODELS (WHICH ARE MONEY NEUTRAL AND SUPERNEUTRAL), IT DOES NOT HOLD EVEN IN THE LONG RUN IN NEW KEYNESIAN MODELS BECAUSE TOO HIGH OR TOO LOW A RATE OF INFLATION WILL HAVE SUPPLY SIDE EFFECTS, SHIFTING BOTH THE IS CURVE (DUE TO REAL BALANCE WEALTH EFFECTS) AND THE VPC CURVE. FOR EXAMPLE, SUPPOSE THAT TOO HIGH A LEVEL OF INFLATION REDUCES $\bar{Y}$ (AND THEREFORE INCREASES $\bar{r}$.)



SO MORE MONEY NEUTRALITY MIGHT HOLD BUT SUPERNEUTRALITY WOULD NOT.

"THE TIME INCONSISTENCY PROBLEM IN MACROECONOMICS ONLY ARISES IF THE MONETARY AUTHORITY HAS AN UNREALISTIC OUTPUT TARGET" DISCUSS

INFLATION BIAS IS CAUSED BY AN UNREALISTIC OUTPUT TARGET DUE TO IMPERFECT COMPETITION LEADING TO MARKET FAILURE IN THE SUPPLY SIDE:



IF THERE WERE PERFECT COMPETITION IN THE LABOUR/GOODS MARKET THEN POINT a WOULD BE THE EQUILIBRIUM IN BOTH THE WS-PS AND PHILLIPS CURVE DIAGRAMS AND SO EQUILIBRIUM IN INFLATION WOULD BE THE TARGET Π_T. HOWEVER WITH IMPERFECT COMPETITION IN LABOUR/GOODS MARKETS, THE EQUILIBRIUM IS POINT c WHERE INFLATION IS Π_2. HENCE THERE IS POSITIVE INFLATION BIAS. THIS IS BECAUSE b' CANNOT BE AN EQUILIBRIUM IN THE P.C. DIAGRAM BECAUSE THE CENTRAL BANK WOULD HAVE AN INCENTIVE TO CHOOSE INFLATION Π_1 AT POINT b. THE ONLY POINT WHERE PRIVATE SECTOR INFLATION EXPECTATIONS ARE FULFILLED IS AT POINT c WHERE THE LRPC CROSSES THE MR CURVE. WITH RATIONAL EXPECTATIONS, INFLATION BIAS

DUE TO A DISTORTED OUTPUT TARGET IS A KEY CAUSE OF TIME INCONSISTENCY IN MONETARY POLICY SINCE THE CENTRAL BANK COULD ONLY REACH b' INSTEAD OF c BY BEING ABLE TO PRECOMMIT TO HITTING THE INFLATION TARGET π_T. HOWEVER INFLATION BIAS IS ARGUABLY NOT THE MOST IMPORTANT FORM OF TIME INCONSISTENCY IN MODERN MONETARY POLICY SINCE IT CAN LARGELY BE SOLVED THROUGH ENSURING AN INSTITUTIONAL FRAMEWORK FOR MONETARY POLICY WHERE THERE IS NO INCENTIVE FOR THE CENTRAL BANK TO PURSUE AN UNREALISTIC OUTPUT LEVEL (E.G. THROUGH AN EXPLICIT PRECISE INFLATION TARGET ALONG WITH AN IMPLICIT OUTPUT STABILITY MANDATE, AS WITH THE BANK OF ENGLAND.)

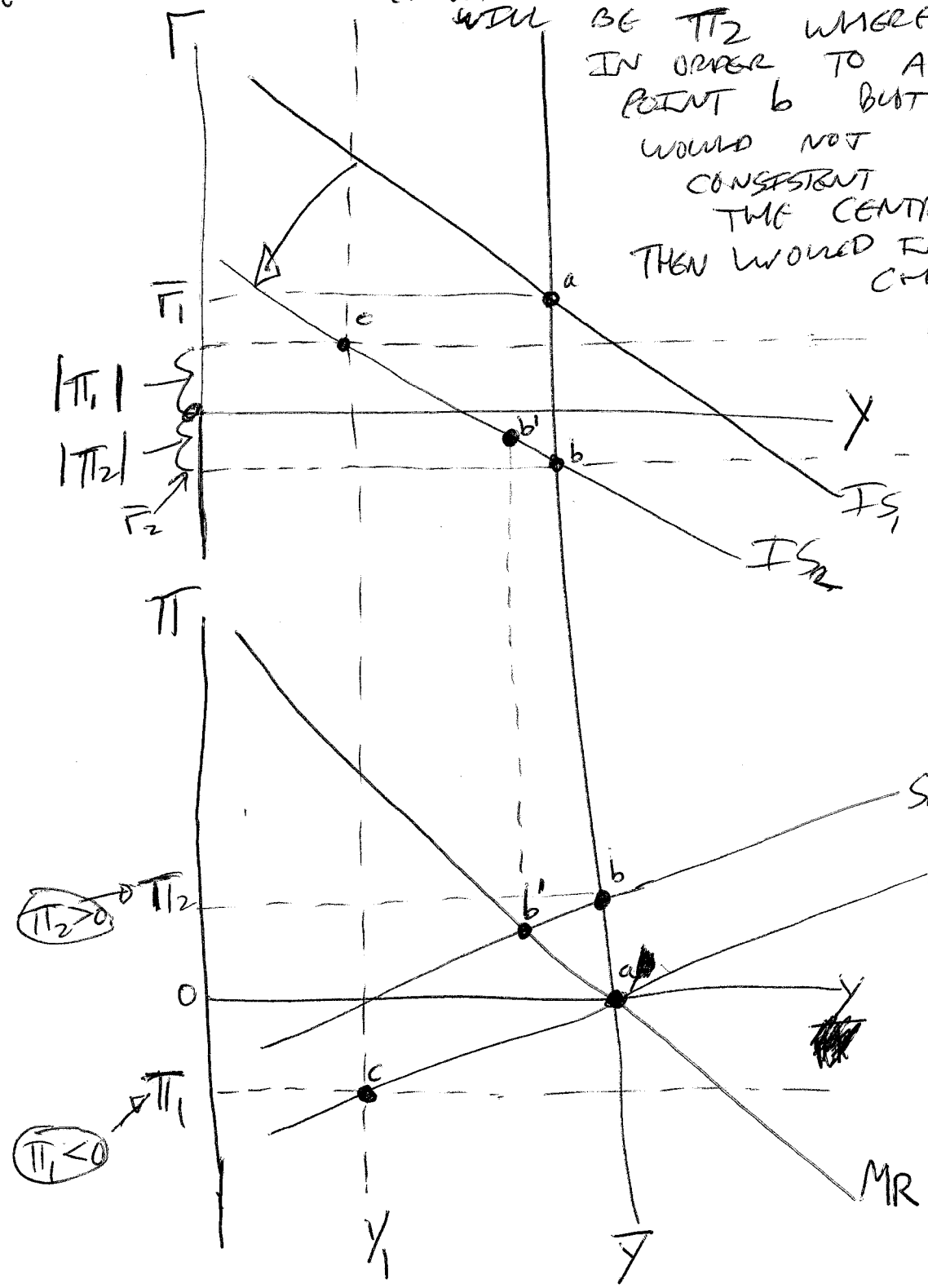
A SECOND KEY FORM OF TIME INCONSISTENCY IS STABILISATION BIAS. THIS DOES NOT REQUIRE AN UNREALISTIC OUTPUT TARGET AND WILL GENERALLY OCCUR WHENEVER THERE ARE RATIONAL EXPECTATIONS IN COMBINATION WITH NOMINAL RIGIDITIES (E.G. STICKY PRICES, WAGES OR INFORMATION) AND MULTIPLE TARGETS IN THE CENTRAL BANK'S LOSS FUNCTION (E.G. INFLATION AND OUTPUT). TWO IMPORTANT EXAMPLES ARE THE LIQUIDITY TRAP AND THE NEW KEYNESIAN PHILLIPS CURVE. THE SIMILARITY IN BOTH CASES IS THAT OPTIMAL CENTRAL BANK POLICY WILL INVOLVE MAKING PROMISES ABOUT INFLATION TOMORROW WHICH ARE NOT CREDIBLE TODAY

BECAUSE PRIVATE SECTOR AGENTS KNOW THAT THE CENTRAL BANK WILL IN FACT RENEGE ON ITS PROMISES.

LIQUIDITY TRAP _ SUPPOSE THAT THERE IS A NEGATIVE IS SHOCK SO THAT THE STABILIZING REAL INTEREST RATE $\bar{r}_2$ FALLS BELOW ZERO ($\bar{r}_2 < 0$). WITH A ZERO INFLATION TARGET THE ECONOMY WILL FALL INTO DEFLATION AT POINT C DUE TO THE ZERO LOWER BOUND ON THE NOMINAL INTEREST RATE: $i \geq 0 \Rightarrow r + \pi^e \geq 0 \Rightarrow r \geq -\pi^e$

THE CENTRAL BANK WOULD LIKE TO PERSUADE THE PRIVATE SECTOR THAT INFLATION WILL BE π_2 WHERE $\pi_2 = |\bar{r}_2|$ IN ORDER TO ACHIEVE POINT b BUT THIS WOULD NOT BE TIME CONSISTENT SINCE THE CENTRAL BANK THEN WOULD IN FACT CHOOSE b'.

(A SIMILAR ARGUMENT CAN BE MADE FOR ANY OTHER $\pi > 0$.)



THE NEW KEYNESIAN PHILLIPS CURVE

A SIMPLE NKPC CAN BE WRITTEN AS

$$\pi_t = \pi^e_{t+1} + (y_t - \bar{y}) + v_t$$

(SEE NOTES ON WAGE AND PRICE SETTING FOR DETAILS ON HOW TO DERIVE THIS.) THE CRUCIAL FEATURE IS THAT INFLATION TODAY DEPENDS UPON EXPECTED INFLATION NEXT PERIOD. SO IF THERE IS A POSITIVE INFLATION SHOCK ($v_t > 0$) THEN BY PROMISING LOWER INFLATION TOMORROW THEN TODAY'S INFLATION CAN BE REDUCED. THIS CREATES THE POTENTIAL FOR TIME INCONSISTENCY BECAUSE THE OPTIMAL POLICY TODAY WILL GENERALLY NOT BE OPTIMAL ONCE THE TIME COMES TO IMPLEMENT THE PLAN TOMORROW. WE CAN ILLUSTRATE WITH A SIMPLE EXAMPLE WHERE WE ASSUME A ZERO INFLATION TARGET AND AN INFLATION / COST-PUSH SHOCK v_1 IN PERIOD 1. WE FIRST DERIVE THE OPTIMAL MONETARY RESPONSE OVER 2 PERIODS SO THAT INFLATION IS BACK ON TARGET BY PERIOD 3.

$v_2 = 0$, $v_3 = 0$ — FOR SIMPLICITY

$$\pi_3 = 0 \qquad y_3 = 0$$

$$\pi_2 = \pi_3 + (y_2 - \bar{y}) + 0 \implies \pi_2 = y_2 - \bar{y}$$

$$\pi_1 = \pi_2 + (y_1 - \bar{y}) + v_1 \implies \pi_1 = (y_2 - \bar{y}) + (y_1 - \bar{y}) + v_1$$

THE CENTRAL BANK NEEDS TO CHOOSE π_1 AND π_2 SO AS TO MINIMIZE:

$$(y_1 - \bar{y})^2 + \beta(\pi_1 - 0)^2 + \delta(y_2 - \bar{y})^2 + \delta\beta(\pi_2 - 0)^2$$

SO, SUBBING IN THE $\hat{}$ PHILLIPS CURVES ABOVE WE GET:

$$(y_1-\bar{y})^2+\beta\left((y_1-\bar{y})^2+(y_2-\bar{y})^2+v_1\right)^2+\delta(y_2-\bar{y})^2+\beta\delta(y_2-\bar{y})^2$$

NOW WE PARTIALLY DIFFERENTIATE WITH RESPECT TO y_1 AND y_2 TO GET TWO F.O.C.S:

$\frac{\partial}{\partial y_1}$: $2(y_1-\bar{y})+2\beta\left((y_1-\bar{y})+(y_2-\bar{y})+v_1\right)=0$

$$\Rightarrow (y_1-\bar{y})=-(\beta)(\pi_1)$$

$\frac{\partial}{\partial y_2}$: $2\beta\left((y_1-\bar{y})+(y_2-\bar{y})+v_1\right)+2\delta(1+\beta)(y_2-\bar{y})=0$

$$\Rightarrow (y_2-\bar{y})=-\left(\frac{\beta}{\delta(1+\beta)}\right)(\pi_1)$$

SINCE THE INITIAL SHOCK v_1 WILL MEAN THAT IF $v_1>0$ THEN $\pi_1>0$ THEN WE CAN SEE THAT A POSITIVE INFLATION SHOCK WILL INVOLVE TWO PERIODS OF NEGATIVE OUTPUT GAP. HOWEVER, THIS WILL NOT BE TIME CONSISTENT BECAUSE IN PERIOD 2 SINCE $\pi_2=0$ THE OPTIMAL OUTPUT GAP WILL THEN BE ZERO, NOT $-\left(\frac{\beta}{\delta(1+\beta)}\right)(\pi_1)$.

THIS TYPE OF TIME CONSISTENCY PROBLEM (STABILIZATION BIAS) IS HIGHLY PERVASIVE. A NUMBER OF SOLUTIONS HAVE BEEN PROPOSED:

(1) RULES- BASED RATHER THAN DISCRETIONARY MONETARY POLICY (E.G. MONETARY GROWTH OR TAYLOR RULE). THE PROBLEM WITH THIS IS THAT FLEXIBILITY TO RESPOND TO CHANGES IN THE STRUCTURE OF THE ECONOMY COULD BE LOST.

(2) DELEGATED INTEGRATION BETWEEN CENTRAL BANK AND PRIVATE SECTOR COULD ALLEVIATE THE PROBLEM BY ENABLING RATIONAL REPUTATION

- BUILDING. THE PROBLEM, HOWEVER, IS THAT GIVEN IMPERFECT INFORMATION THE PRIVATE SECTOR CAN NEVER BE SURE THAT THE CENTRAL BANK HAS 'CHEATED', SO IT WILL NOT BE ABLE TO "PUNISH" PERFECTLY.

(3) REPLACE INFLATION TARGET WITH NOMINAL GDP OR PRICE LEVEL TARGETS.

THESE HELP TO REDUCE STABILIZATION BIAS FOLLOWING AN INFLATION SHOCK BY ENABLING A CREDIBLE PROMISE OF MULTIPLE PERIODS OF INFLATION / DEFLATION TO BE MADE. FOR EXAMPLE, IF THERE WAS A POSITIVE INFLATION SHOCK THEN THERE WOULD THEN OPTIMALLY BE MULTIPLE PERIODS OF DEFLATION TO BRING THE PRICE LEVEL / NOMINAL GDP BACK TO TARGET. EVEN ONCE THE PRIVATE SECTOR EXPECTATIONS HAVE RESPONDED, THE CENTRAL BANK WOULD STILL CHOOSE TO GO THROUGH WITH THE DEFLATION IN FUTURE PERIODS. THE PROBLEM HERE IS THAT GENERALLY LARGER TOTAL OUTPUT GAPS WILL BE CREATED (DEFINITELY SO WITH ADAPTIVE EXPECTATIONS) SO NOMINAL GDP / PRICE TARGETS WOULD ONLY BE PREFERABLE TO INFLATION TARGETTING UNDER THE NEW KEYNESIAN PHILLIPS CURVE "SET-UP" WITH RATIONAL EXPECTATIONS AND NOMINAL RIGIDITIES SUCH AS STAGGERED PRICE SETTING. WHETHER THIS IS A GOOD IDEA THUS DEPENDS ON HOW FORWARD-LOOKING INFLATION EXPECTATIONS ARE IN THE REAL-WORLD ECONOMY.