INTERTEMPORAL OPTIMISATION MODEL

REPRESENTATIVE CONSUMER SOLVES:

$$\max_{C_1, C_2} \left\{ u(C_1) + \beta u(C_2) \right\} \quad \text{s.t.} \quad C_1 + \left(\frac{1}{1+r}\right) C_2 \leq M_1 + \left(\frac{1}{1+r}\right) M_2$$

$$\mathcal{L} = u(C_1) + \beta u(C_2) - \lambda \left(C_1 + \left(\frac{1}{1+r}\right) C_2 - M_1 - \left(\frac{1}{1+r}\right) M_2 \right)$$

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{du}{dC_1} - \lambda = 0 \qquad \frac{\partial \mathcal{L}}{\partial C_2} = \beta\left(\frac{du}{dC_2}\right) - \left(\frac{1}{1+r}\right)\lambda = 0$$

$\Rightarrow$ if $\beta = \left(\frac{1}{1+\rho}\right)$ then we get $\boxed{\frac{du}{dC_1} = \left(\frac{du}{dC_2}\right)\left(\frac{1+r}{1+\rho}\right)}$ EULER EQUATION

IF $u(C) = \ln(C)$ (LOGARITHMIC / COBB-DOUGLASS UTILITY)
THEN THE EULER EQUATION BECOMES:

$$\frac{1}{C_1} = \left(\frac{1}{C_2}\right)\left(\frac{1+r}{1+\rho}\right)$$

IF CONSUMPTION IF BOTH PERIODS IS <u>CERTAIN</u> THEN THIS CAN BE REARRANGED TO GIVE $C_1 = C_2\left(\frac{1+\rho}{1+r}\right)$. THIS MEANS THAT IF $r = \rho$, CONSUMPTION WILL BE EQUAL / PERFECTLY SMOOTHED OVER BOTH PERIODS. IF, HOWEVER, PERIOD 2 CONSUMPTION IS UNCERTAIN, THE EULER EQUATION BECOMES:

$$E\left[\frac{1}{C_1}\right] = E\left[\frac{1}{C_2}\left(\frac{1+r}{1+\rho}\right)\right] \qquad \left(E\left[\frac{du}{dC_1}\right] = \left(\frac{1}{1+\rho}\right) E\left[\frac{du}{dC_2}(1+r)\right] \right)$$

$$\left(\Rightarrow \frac{du}{dC_1} = \left(\frac{1}{1+\rho}\right) E\left[\frac{du}{dC_2}(1+r)\right] \right)$$

$$\Rightarrow \frac{1}{C_1} = E\left[\left(\frac{1}{C_2}\right)(1+r)\right]\left(\frac{1}{1+\rho}\right)$$

(SINCE C_1 AND ρ ARE KNOWN CONSTANTS)

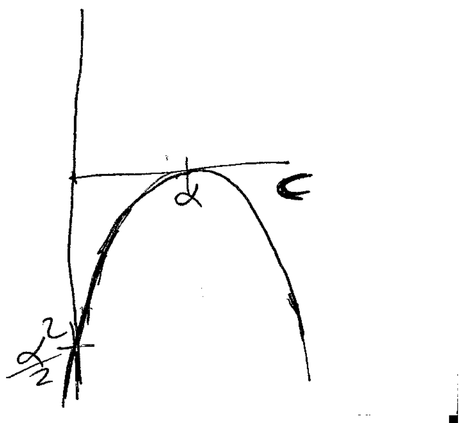
$$\Rightarrow C_1 = \frac{1+\rho}{E\left[\left(\frac{1}{C_2}\right)(1+r)\right]}$$

IF WE CONSIDER UNCERTAIN PERIOD 2 INCOME AND QUADRATIC UTILITY:

$$u(C) = -\frac{1}{2}(\alpha - C)^2$$

THEN THE EULER EQUATION BECOMES

$$(\alpha - C_1) = \left(\frac{1}{1+\rho}\right) E\left[(\alpha - C_2)(1+r)\right]$$



WITH THE FURTHER ASSUMPTION THAT r IS CERTAIN AND EQUAL TO ρ, THIS SIMPLIFIES TO GIVE:

$$\alpha - C_1 = \left(\frac{1+r}{1+\rho}\right)\left(\alpha - E[C_2]\right)$$

$$\Rightarrow \alpha - C_1 = \alpha - E[C_2] \Rightarrow \boxed{C_1 = E[C_2]}$$

THIS MEANS THAT $C_2 = C_1 + \varepsilon$ WHERE $E[\varepsilon] = 0$ AND SO GIVES US HALL'S (1978) RANDOM WALK HYPOTHESIS. ONE OF THE EMPIRICAL PREDICTIONS OF THIS MODEL IS THAT C_2 SHOULD NOT BE EXPLAINED BY ANY OTHER VARIABLE KNOWN AT TIME 1 APART FROM C_1. (THIS IS EMPIRICALLY VIOLATED — <u>EXCESS SENSITIVITY</u>.)

WE CAN ALSO USE THE EULER EQUATION SOLUTION TO DERIVE THE "SOLVED-OUT" CONSUMPTION FUNCTION

$$E\left[C_1 + \left(\frac{1}{1+r}\right)C_2\right] = E\left[M_1 + \left(\frac{1}{1+r}\right)M_2\right]$$

$$\Rightarrow C_1 + \left(\frac{1}{1+r}\right)E[C_2] = M_1 + \left(\frac{1}{1+r}\right)E[M_2]$$

(ASSUMING r IS CERTAIN)

$$\Rightarrow E[C_2]\left[1 + \left(\frac{1}{1+r}\right)\right] = M_1 + \left(\frac{1}{1+r}\right)E[M_2]$$

$$\Rightarrow C_1 = E[C_2] = \left(\frac{1+r}{2+r}\right)\left(M_1 + \left(\frac{1}{1+r}\right)E[M_2]\right)$$

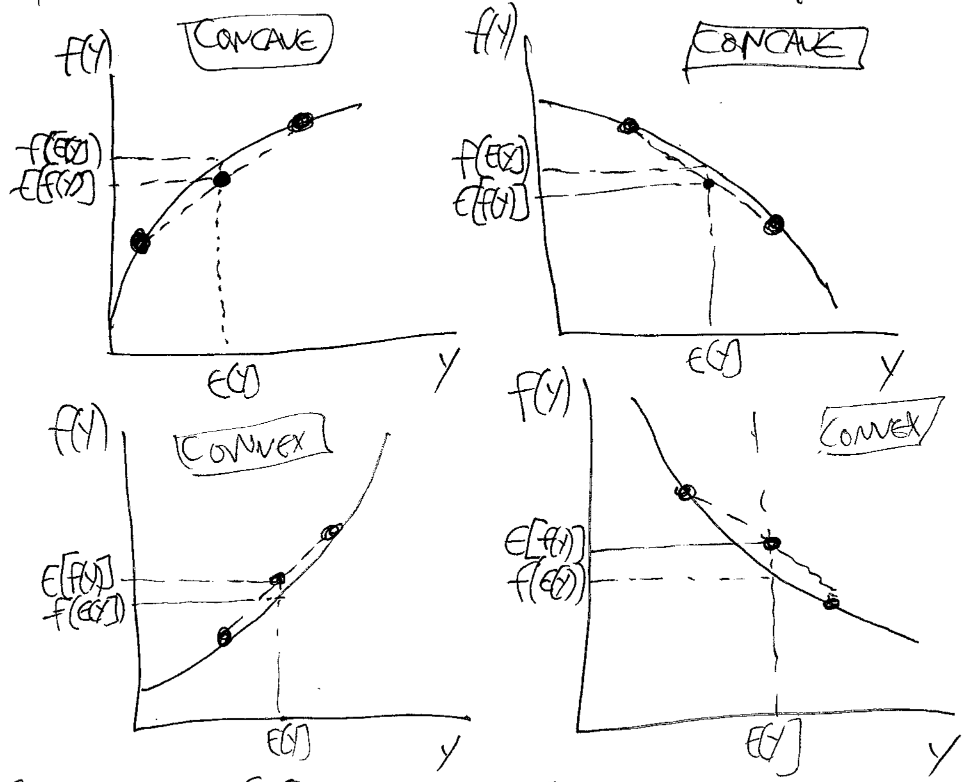
A <u>TEMPORARY</u> RISE IN INCOME BY ΔM_1 CAUSES A CHANGE IN C_1 EQUAL TO $\Delta C_1 = \left(\frac{1+r}{2+r}\right)(\Delta M_1)$. A <u>PERMANENT</u> RISE IN INCOME CAUSES A RISE OF $\Delta M_1 = \Delta E[M_2] = \Delta M_p$ AND SO $\Delta C_1 = \left(\frac{1+r}{2+r}\right)\left(1 + \frac{1}{1+r}\right)\Delta M_p$

$\Rightarrow \Delta C_1 = \Delta M_p$. SO, ANY UNEXPECTED RISE IN PERMANENT INCOME CAUSES AN EQUAL RISE IN CURRENT CONSUMPTION. EMPIRICAL EVIDENCE (DEATON, 1987), HOWEVER, SUGGESTS THAT CONSUMPTION IS SIGNIFICANTLY LESS RESPONSIVE THAN THIS TO NEW INFORMATION WHICH CHANGES PERMANENT INCOME. THIS IS <u>EXCESS SMOOTHNESS</u>.

LOGARITHMIC UTILITY (OR ANY OTHER UTILITY FUNCTION WHERE MARGINAL UTILITY IS CONVEX) CAN HELP TO EXPLAIN EXCESS SMOOTHNESS. MATHEMATICALLY, THIS IS DUE TO JENSEN'S INEQUALITY, WHICH SAYS THAT FOR A CONVEX FUNCTION

$E[f(Y)] > f(E[Y])$ AND FOR A CONCAVE FUNCTION

$E[f(Y)] < f(E[Y])$:

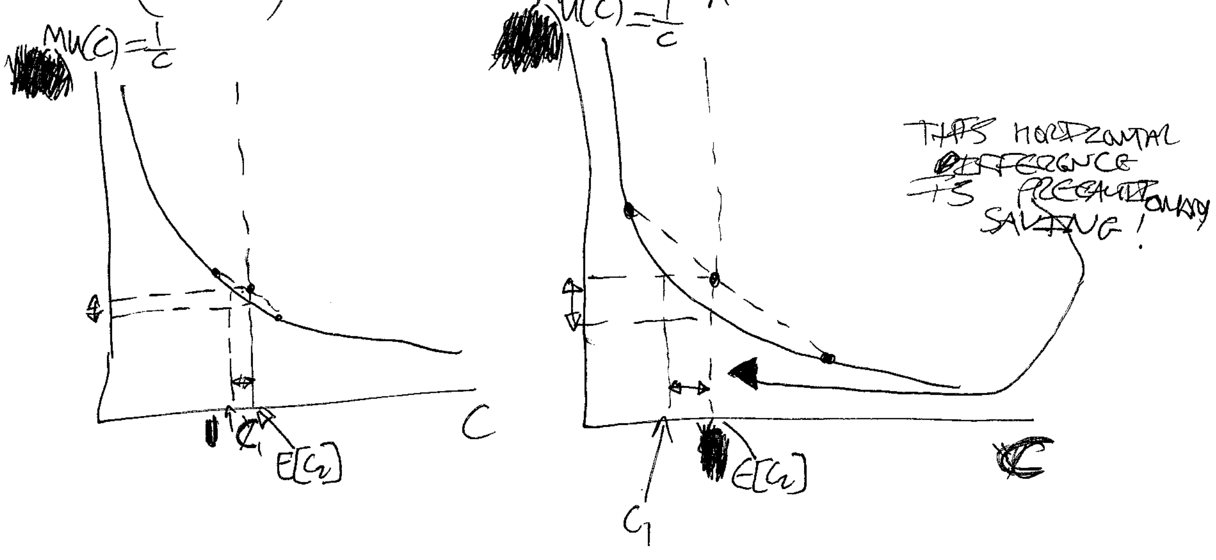


SO, SINCE THE EULER EQUATION FOR LOGARITHMIC UTILITY IS:

$$C_1 = \frac{1+\rho}{E\left[\left(\frac{1}{C_2}\right)(1+r)\right]}$$

WE CAN SEE THAT $C_1 < \left(\frac{1+\rho}{1+r}\right)E[C_2]$ SINCE

$E\left[\frac{1}{C_2}\right] > \frac{1}{E[C_2]}$. ALSO, THE DIFFERENCE BETWEEN $E[f(Y)]$ AND $f(E[Y])$ IS GREATER THE GREATER IS THE VARIANCE OF Y:



SO, IF FUTURE CONSUMPTION BECOMES MORE UNCERTAIN, PRECAUTIONARY SAVING RISES. SINCE GOOD INCOME NEWS IS LIKELY TO RAISE THE VARIANCE OF FUTURE INCOME AND CONSUMPTION, THIS CAN HELP TO EXPLAIN EXCESS SMOOTHNESS OF CONSUMPTION IN RESPONSE TO CHANGES IN ~~CURRENT~~ PERMANENT INCOME.

EXCESS SENSITIVITY

EMPIRICAL EVIDENCE SUGGESTS THAT CHANGES IN CURRENT INCOME THAT WERE PREDICTABLE FROM PREVIOUS CHANGES IN INCOME CAUSE ABOUT 30%–40% OF THE EQUIVALENT CHANGE IN CONSUMPTION. THIS IS A PUZZLE FOR THE PERMANENT INCOME HYPOTHESIS, BUT CAN ARGUABLY BE EXPLAINED BY

1. CREDIT CONSTRAINTS / LIQUIDITY CONSTRAINTS
2. HABIT FORMATION (BOUNDED RATIONALITY)
3. ROLE OF CONSUMER DURABLES IN CAUSING DISCONTINUITIES IN CONSUMPTION RESPONSE

(SEE CARLIN + SOSKICE AND MUELLBAUER SURVEY ARTICLE) "THE ASSESSMENT: CONSUMER EXPENDITURE"

USING THE INTERTEMPORAL EULER EQUATION TO MODEL THE CURRENT ACCOUNT

LET US ASSUME A WORLD OF CERTAINTY SO THAT LOGARITHMIC / COBB-DOUGLAS UTILITY GIVES US $C_1 = C_2\left(\frac{1+\rho}{1+r}\right)$ FOR THE EULER EQUATION. NOW WE CAN SUBSTITUTE INTO THE BUDGET CONSTRAINT TO GET THE "SOLVED OUT" CONSUMPTION FUNCTION:

$$C_1 + \left(\frac{1}{1+r}\right) C_2 = M_1 + \left(\frac{1}{1+r}\right) M_2$$

$$\Rightarrow C_2\left(\frac{1+\rho}{1+r}\right) + \left(\frac{1}{1+r}\right) C_2 = M_1 + \left(\frac{1}{1+r}\right) M_2$$

$$\Rightarrow C_2\left(\frac{2+\rho}{1+r}\right) = M_1 + \left(\frac{1}{1+r}\right) M_2$$

$$\Rightarrow C_2 = \left(\frac{1+r}{2+\rho}\right)(M_1) + \left(\frac{1}{2+\rho}\right)(M_2)$$

$$\Rightarrow C_1 = \left(\frac{1+\rho}{2+\rho}\right)(M_1) + \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{1}{1+r}\right)(M_2)$$

$Y = C + I + G + NX$

$D_1 = (-NX) = C + I + G - Y$

INITIALLY ASSUME $G = I = 0 \Rightarrow D = C - Y$

NOW (ASSUMING ZERO INVESTMENT, TAXES, AND GOVERNMENT SPENDING) THE CURRENT ACCOUNT DEFICIT IN PERIOD 1 IS $D_1 = C_1 - M_1 = \left(\frac{1+\rho}{2+\rho} - \frac{2+\rho}{2+\rho}\right)(M_1) + \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{1}{1+r}\right)(M_2)$

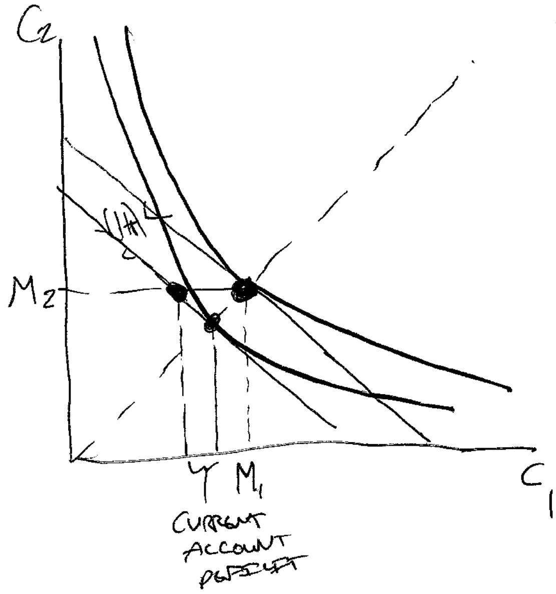
$$\Rightarrow D_1 = C_1 - M_1 = -\left(\frac{1}{2+\rho}\right)(M_1) + \left(\frac{1+\rho}{2+\rho}\right)\left(\frac{1}{1+r}\right)(M_2)$$

WE CAN NOW SEE THAT CURRENT ACCOUNT ~~DEFICIT~~ D_1 IS DECREASING IN M_1, INCREASING IN M_2 AND DECREASING IN r. THEREFORE A SWING INTO CURRENT ACCOUNT DEFICIT COULD BE EXPLAINED BY (1) A NEGATIVE TECHNOLOGY SHOCK TODAY (2) A POSITIVE TECHNOLOGY SHOCK ANTICIPATED TOMORROW. (3) AN DECREASE IN THE INTEREST RATE. WE CAN ALSO REWRITE THE ABOVE AS:

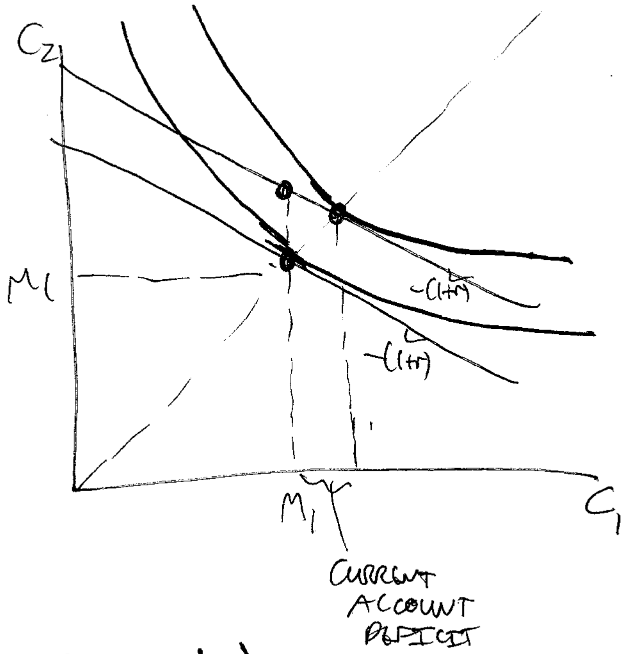
$$D_1 = (\Delta M_2)\left(\frac{1}{2+\rho}\right)\left(\frac{1+\rho}{1+r}\right) + (M_1)\left(\frac{1}{2+\rho}\right)\left(\frac{1+\rho}{1+r} - 1\right)$$

$$D_1 = (\Delta M_2)\left(\frac{1+\rho}{1+r}\right)\left(\frac{1}{2+\rho}\right) + (M_1)\left(\frac{1}{2+\rho}\right)\left(\frac{\rho - r}{1+r}\right)$$

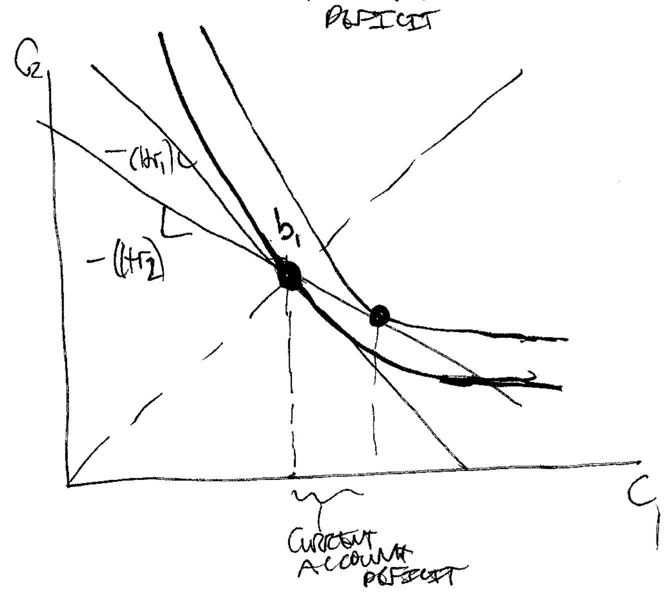
IF WE TAKE THE CASE WHERE $\rho = r$ (RANDOM WALK MODEL) THEN A CURRENT ACCOUNT DEFICIT ONLY OCCURS WHEN A RISE IN INCOME IS EXPECTED.



NEGATIVE TECHNOLOGY SHOCK TODAY – E.G. NATURAL DISASTER

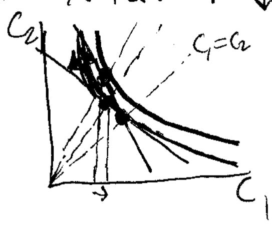


POSITIVE TECHNOLOGY SHOCK EXPECTED TOMORROW (E.G. INSTITUTIONAL REFORM TO OPEN UP ECONOMY)



DECREASE IN WORLD INTEREST RATE

NOTE – DUE TO HOMOTHETIC PREFERENCES, AN ECONOMY RUNNING A SURPLUS WILL ALSO DEFINITELY RUN A SMALLER SURPLUS AFTER $r \downarrow$



WE COULD EXTEND THIS MODEL TO INCLUDE FISCAL POLICY AND INVESTMENT BY ALTERING THE CONSUMER'S INCOME TO BE $M_1' = M_1 - T_1$ AND $M_2' = M_2 - T_2$ AND ALSO REQUIRING THAT THE GOVERNMENT BALANCE ITS BUDGET SO THAT $G_1 + \left(\frac{1}{1+r}\right)G_2 = T_1 + \left(\frac{1}{1+r}\right)T_2$

THIS WOULD CHANGE THE SOLVED OUT CONSUMPTION FUNCTION TO BE:

$$C_1 = \left(\frac{1+p}{2+p}\right)(M_1 - G_1) + \left(\frac{1+p}{2+p}\right)\left(\frac{1}{1+r}\right)(M_2 - G_2)$$

THE CURRENT ACCOUNT/TRADE DEFICIT WOULD THEN BECOME:

$$D_1 = (-NX) = -(Y_1 - C_1 - I_1 - G_1))$$

SO, LETTING $Y_1 = M_1$ FOR REPRESENTATIVE AGENT'S OPTIMISATION PROBLEM:

$$D_1 = C_1 + G_1 + I_1(r) - M_1 = G_1 + I_1(r) - \left(\frac{1}{2+p}\right)(M_1) - \left(\frac{1+p}{2+p}\right)(G_1) + \left(\frac{1+p}{2+p}\right)\left(\frac{1}{1+r}\right)(M_2 - G_2)$$

$$\Rightarrow D_1 = I_1(r) + \frac{\;}{(2+p)}G_1 - \left(\frac{1}{2+p}\right)(M_1) + \left(\frac{1+p}{2+p}\right)\left(\frac{1}{1+r}\right)(M_2) - \left(\frac{1+p}{2+p}\right)\left(\frac{1}{1+r}\right)(G_2)$$

SO NOW FISCAL CONTRACTION NOW / AN ANTICIPATED FISCAL EXPANSION IN THE FUTURE WOULD REDUCE THE TRADE DEFICIT. HOWEVER, THIS WOULD ONLY BE OPTIMAL IF THERE WERE MICROECONOMIC REASONS TO ALTER THE PATH OF GOVERNMENT SPENDING. THE CURRENT ACCOUNT DEFICIT IS ~~NOT~~ AN EFFICIENT RESPONSE TO THE PATH OF INCOME!

MONETARY POLICY - IN A CLASSICAL INTERTEMPORAL OPEN ECONOMY MODEL SUCH AS THIS, THE REAL INTEREST RATE IS DETERMINED BY THE INTERSECTION OF THE INVESTMENT SCHEDULE WITH THE WORLD INTEREST RATE r_w:

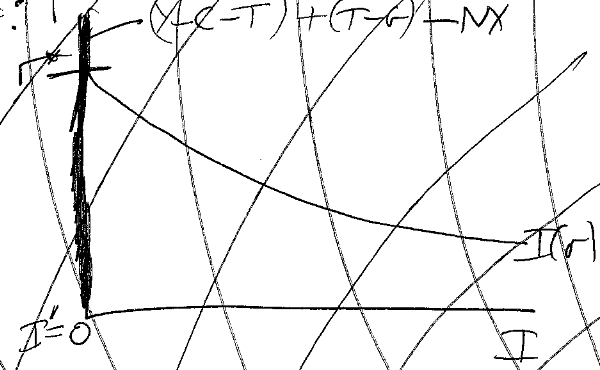
$$Y = C + I + G + NX$$

$$I(r) = (Y - C - T) + (T - G) - NX$$

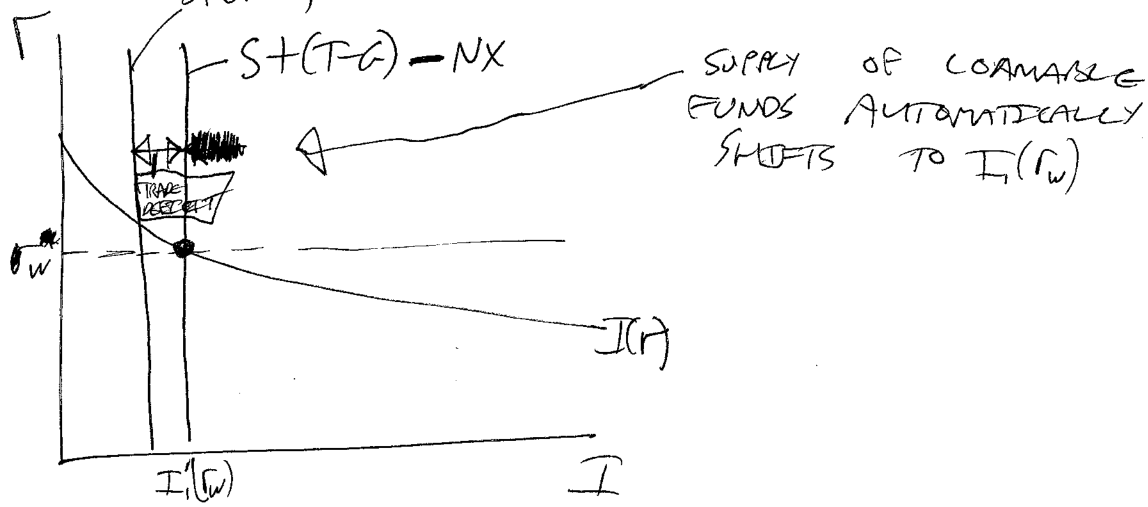
WE ALSO USE THE FACT THAT:

$$S = Y - C - T$$

~~IN OUR MODEL, WE HAVE ASSUMED THAT $I=0$ FOR SIMPLICITY, SO WE HAVE:~~



~~IN A MORE REALISTIC EXAMPLE WHERE THE APC IS LESS THAN 1 DUE TO LIFE CYCLE CONSIDERATIONS, WE WOULD HAVE:~~



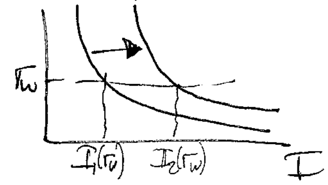
SINCE THIS IS A CLASSICAL MODEL, OUTPUT, CONSUMPTION, SAVINGS AND THE TRADE DEFICIT WOULD BE UNAFFECTED BY DOMESTIC MONETARY POLICY, SINCE THE DOMESTIC REAL INTEREST RATE IS FIXED AT THE WORLD INTEREST RATE.

SO NOW WE HAVE THE FOLLOWING EQUATION FOR THE TRADE DEFICIT. IF $r=\rho$ (THUS WE MUST ASSUME r IS FIXED) AND WITH CONSTANT $G \Rightarrow G_1=G_2$

$$-NX = I_1(r) + (\Delta M)\left(\frac{1}{2+r}\right)$$

SO IF INCOME IS EXPECTED TO RISE, A DEFICIT IS ALWAYS CREATED (WHICH IS UNREALISTIC DUE TO LIFE CYCLE CONSIDERATIONS)

ALSO, IF THERE IS AN OUTWARD SHIFT OF THE DOMESTIC INVESTMENT SCHEDULE, THIS WILL ALSO INCREASE THE TRADE DEFICIT, CETERIS PARIBUS:



TWO PERIOD MODEL OF THE INTERTEMPORAL CURRENT ACCOUNT — FULL MODEL

ASSUME $r = \rho$ AND LOGARITHMIC UTILITY WITH CERTAINTY SO THAT CONSUMPTION IS PERFECTLY SMOOTHED (I.E. $C_1 = C_2$)

B_1 — FOREIGN ASSETS OWNED IN PERIOD 1 (ASSUME $= 0$)
G_i — GOVERNMENT SPENDING I_i — INVESTMENT NX_i — NET EXPORTS
CU_1 — CURRENT ACCOUNT SURPLUS Y_i — OUTPUT/INCOME
T_i — TAXES. $\boxed{NX_i = Y_i - G_i - I_i - C_i}$

SOLVED OUT CONSUMPTION FUNCTION ①: $C_1 + \left(\frac{1}{1+r}\right) C_2 = (Y_1 - T_1) + \left(\frac{1}{1+r}\right)(Y_2 - T_2)$

GOVERNMENT BUDGET CONSTRAINT: ② $G_1 + \left(\frac{1}{1+r}\right) G_2 = T_1 + \left(\frac{1}{1+r}\right) T_2$

EXTERNAL BUDGET CONSTRAINT: ③ $NX_1 + \left(\frac{1}{1+r}\right) NX_2 = 0$

COMBINING ① AND ② WITH $C_1 = C_2 = C$:

$$C\left(\frac{1+r+1}{1+r}\right) = (Y_1 - G_1) + \left(\frac{1}{1+r}\right)(Y_2 - G_2)$$

$$\Rightarrow C = \left(\frac{1+r}{2+r}\right)\left((Y_1 - G_1) + \left(\frac{1}{1+r}\right)(Y_2 - G_2)\right)$$

CURRENT ACCOUNT SURPLUS IN PERIOD 1: $CU_1 = NX_1 + B_1 r$

$CU_1 = NX_1$ (if $B_1 = 0$ for simplicity)

$$NX_1 + NX_2\left(\frac{1}{1+r}\right) = 0$$

$$(Y_1 - C_1 - I_1 - G_1) + (Y_2 - C_2 - I_2 - G_2)\left(\frac{1}{1+r}\right) = 0$$

$$(Y_1 - I_1 - G_1) + (Y_2 - I_2 - G_2)\left(\frac{1}{1+r}\right) = \left(1 + \frac{1}{1+r}\right)\left(\frac{1+r}{2+r}\right)\Big(Y_1 - G_1 \ldots$$

$$\ldots + \left(\frac{1}{1+r}\right)(Y_2 - G_2)\Big)$$

$$(Y_1 - I_1 - G_1) + (Y_2 - I_2 - G_2)\left(\frac{1}{1+r}\right) = \left(\frac{2+r}{1+r}\right)\left(\frac{1+r}{2+r}\right)\left(Y_1 - G_1 + \left(\frac{1}{1+r}\right)(Y_2 - G_2)\right)$$

$$-I_1 - I_2\left(\frac{1}{1+r}\right) = 0 \quad \Rightarrow \quad I_2 = -(1+r)(I_1)$$

$$\Rightarrow \Delta I = -(2+r)(I_1)$$

INTERPRETATION — SINCE PROPENSITY TO CONSUME OUT OF PERMANENT INCOME IS SIMPLY 1 IN THIS MODEL, ANY INVESTMENT MUST COME FROM ABROAD, AND IN PERIOD 2 THE GAME ENDS SO FOREIGNERS WILL LIQUIDATE THEIR ASSETS PLUS INTEREST.

$$CU_1 = NX_1 = Y_1 - C_1 - G_1 - I_1(r)$$

$$= Y_1 - \left(\frac{1+r}{2+r}\right)\left((Y_1 - G_1) + \left(\frac{1}{1+r}\right)(Y_2 - G_2)\right) - G_1 - I_1(r)$$

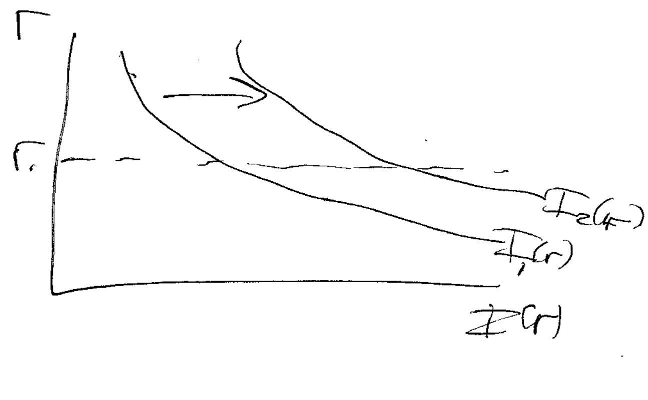
$$= Y_1\left(\frac{1}{2+r}\right) - G_1\left(\frac{1}{2+r}\right) - \left(\frac{1}{2+r}\right)Y_2 + \left(\frac{1}{2+r}\right)G_2 - I_1(r)$$

$$= \left(\frac{1}{2+r}\right)(\Delta G - \Delta Y) - I_1(r)$$

$$\boxed{\text{where } NO = Y - I - G}$$

$$= \left(\frac{1}{2+r}\right)(\Delta G - \Delta Y) + \left(\frac{1}{2+r}\right)\Delta I = -\left(\frac{1}{2+r}\right)(\Delta NO)$$

SO, AN EXPECTED DECREASE IN GOVERNMENT SPENDING OR INCREASE IN Y WILL CAUSE AN INCREASED TRADE DEFICIT, AS WILL AN INCREASE IN I.



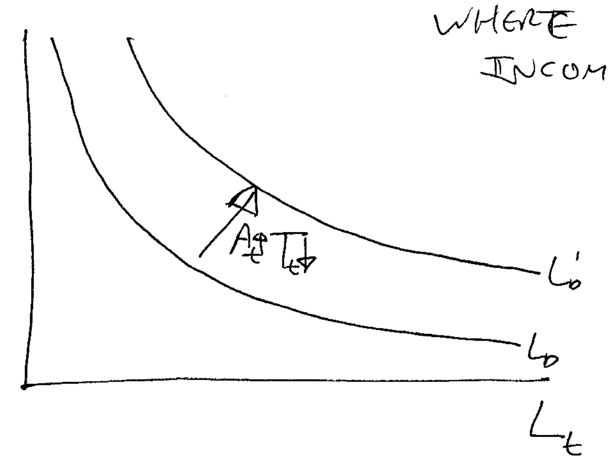
NOTE — THIS RESULT THAT THE CURRENT TRADE DEFICIT DEPENDS ONLY ON THE FUTURE ANTICIPATED CHANGE IN NET OUTPUT IS THE SHEFFRIN AND WU RESULT.

REAL BUSINESS CYCLE MODEL OF LABOUR SUPPLY – EFFECT OF TEMPORARY TAX REDUCTION

FIRSTLY, WE ASSUME A COBB-DOUGLAS PRODUCTION FUNCTION $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ AND ASSUME A FIXED CAPITAL STOCK BETWEEN TWO PERIODS 1 AND 2 SO THAT $K_1 = K_2 = \bar{K}$.

THE DEMAND FOR LABOUR IN EACH PERIOD IS THEREFORE DETERMINED BY THE $MPL = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}$ SO THAT THE POST-TAX REAL WAGE PAID TO L_t WORKERS IS $w_t = (1-\alpha) A_t \left(\frac{\bar{K}}{L_t}\right)^{\alpha} (1-T_t)$

WHERE T_t IS THE PROPORTIONAL RATE OF INCOME TAX AT TIME t.



INCREASES IN A_t / DECREASES IN T_t THEREFORE CAUSE AN OUTWARD SHIFT IN LABOUR DEMAND!

TO ANALYSE THE RESPONSE OF LABOUR SUPPLY, WE NEED TO FORMULATE THE REPRESENTATIVE AGENT'S INTERTEMPORAL OPTIMISATION PROBLEM AS A LAGRANGEAN

$$\mathcal{L} = U(C_1, L_1) + \left(\frac{1}{1+\rho}\right) U(C_2, L_2) - \lambda \left(C_1 + \left(\frac{1}{1+r}\right) C_2 + B - w_1 L_1 - w_2 L_2 \left(\frac{1}{1+r}\right) \right)$$

NOTE THAT SINCE A PERFECTLY COMPETITIVE LABOUR MARKET IS ASSUMED, THE REPRESENTATIVE WORKER IS A "WAGE-TAKER" AND SO w_1 AND w_2 ARE TREATED AS CONSTANTS;

NOW WE FIND FIRST ORDER CONDITIONS:

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{\partial U_1}{\partial C_1} - \lambda = 0 \qquad \frac{\partial \mathcal{L}}{\partial C_2} = \left(\frac{1}{1+\rho}\right)\left(\frac{\partial U_2}{\partial C_2}\right) - \left(\frac{1}{1+r}\right)\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_1} = \frac{\partial U_1}{\partial L_1} + \lambda w_1 = 0 \qquad \frac{\partial \mathcal{L}}{\partial L_2} = \left(\frac{1}{1+\rho}\right)\left(\frac{\partial U_2}{\partial L_2}\right) + \left(\frac{1}{1+r}\right)\lambda w_2 = 0$$

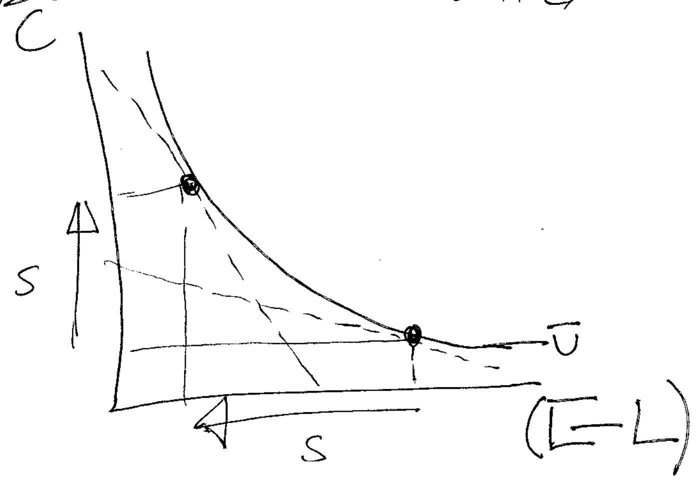
WE CAN IMMEDIATELY DERIVE THAT:

$$-\frac{\partial U_1}{\partial L_1}\left(\frac{1}{w_1}\right) = \frac{\partial U_1}{\partial C_1} \quad \text{AND} \quad -\frac{\partial U_2}{\partial L_2}\left(\frac{1}{w_2}\right) = \frac{\partial U_2}{\partial C_2}$$

SINCE THE TAX CUT IN PERIOD 1 MEANS THAT $w_1 > w_2$, THESE FIRST ORDER CONDITIONS IMPLY THAT

$$\frac{\left(-\frac{\partial U_1}{\partial L_1}\right)}{\left(\frac{\partial U_1}{\partial C_1}\right)} > \frac{\left(-\frac{\partial U_2}{\partial L_2}\right)}{\left(\frac{\partial U_2}{\partial C_2}\right)}$$

INTUITIVELY, THIS MEANS THAT THE MRS BETWEEN CONSUMPTION AND LEISURE IS GREATER IN PERIOD 1 THAN PERIOD 2, CREATING A SUBSTITUTION EFFECT TOWARDS GREATER LABOUR SUPPLY IN PERIOD 1



THERE WILL ALSO, OF COURSE BE AN INCOME EFFECT FROM THE WAGE CHANGE AS WELL. TO FULLY PIN THIS DOWN, WE WOULD NEED A SPECIFIC FUNCTIONAL FORM FOR $U(C_t, L_t)$. FOR EXAMPLE, IF UTILITY WERE ADDITIVELY SEPARABLE AND LOGARITHMIC IN CONSUMPTION, WE WOULD HAVE $U(C_t, L_t) = \ln(C_t)\ f(L_t)$ AND THE CONSUMPTION EULER EQUATION WOULD BE $\frac{1}{C_1} = \left(\frac{1+r}{1+\rho}\right)\left(\frac{1}{C_2}\right) \Rightarrow C_1 = \left(\frac{1+\rho}{1+r}\right) C_2$

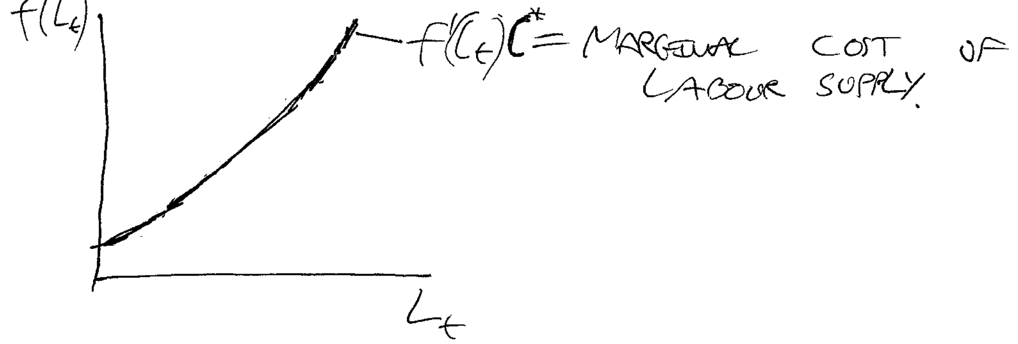
IF WE FURTHER ASSUME THAT $r = \rho$ THEN WE HAVE $C_1 = C_2 = C^*$. WE CAN THEN ALSO PIN DOWN THE LABOUR SUPPLY CURVE

$$\frac{f'(L_1)}{\left(\frac{1}{C^*}\right)} = w_1$$

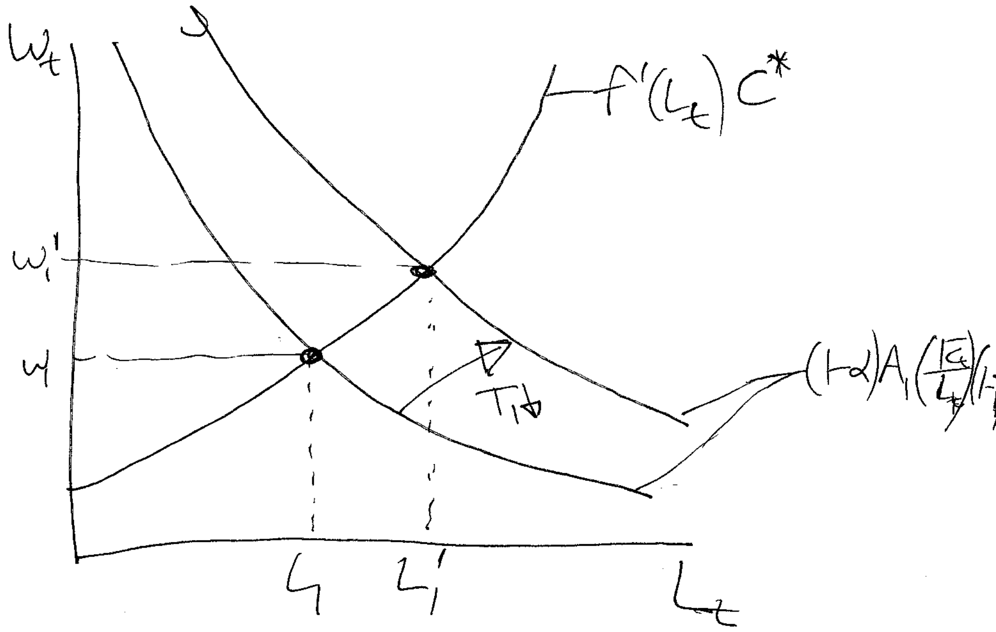
$\Longrightarrow$

$$\frac{f'(L_2)}{\left(\frac{1}{C^*}\right)} = w_2$$

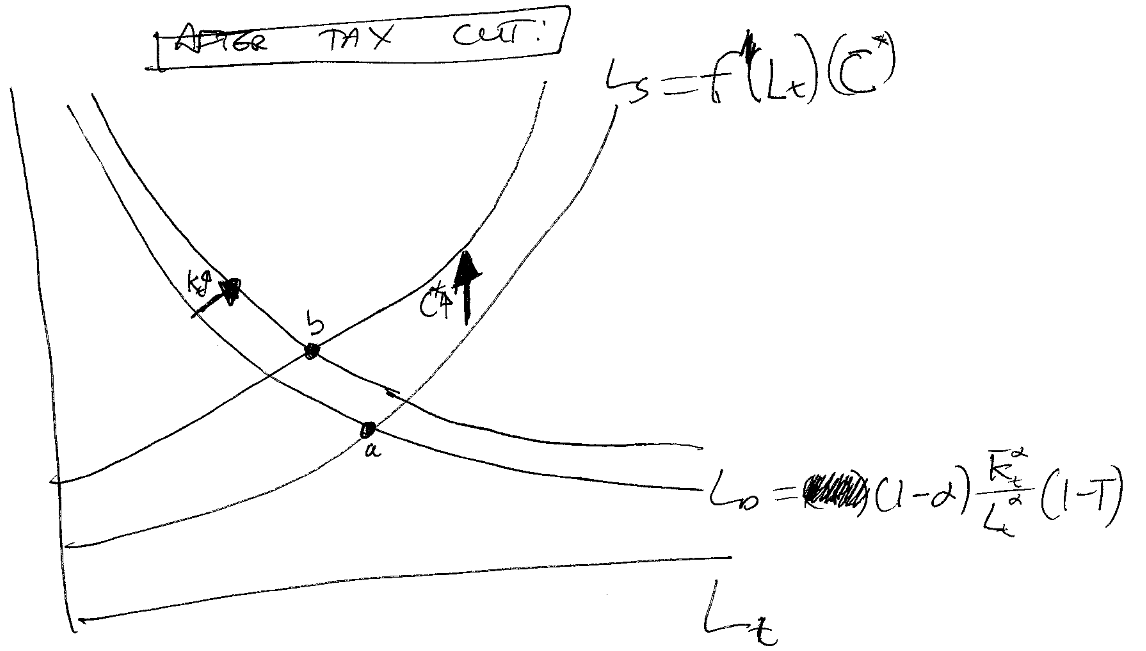
SO, LABOUR SUPPLY WILL BE THE INVERSE OF THIS FUNTION $L_t = (f')^{-1}(w_t)$ AND THE RELEVANT FUNCTION WOULD LOOK LIKE THIS:



NOW WE CAN SHOW DIAGRAMMATICALLY THE EFFECT OF THE TAX REDUCTION IN PERIOD 1 :



SINCE THE TAX CUT WILL RAISE C^* AND THE EXTRA SAVING DURING THE PERIOD OF THE TAX CUT WILL RAISE THE CAPITAL STOCK K AND HENCE THE MPL, THEN THERE WILL BE AN INCREASE IN w^* THAT WILL PERSIST EVEN AFTER INCOME TAXES HAVE RISEN AGAIN!



EFFECT ON LABOUR SUPPLY IS AMBIGUOUS. OVER TIME, DUE TO DEPRECIATION OF CAPITAL, THE LABOUR MARKET WILL RETURN TO ITS ORIGINAL EQUILIBRIUM.

THE SUBSTITUTION EFFECT CAN ALSO BE UNDERSTOOD IN TERMS OF THE INTERTEMPORAL EULER LABOUR SUPPLY EQUATION WHICH CAN BE DERIVED FROM THE LAGRANGEAN FIRST ORDER CONDITIONS:

$$\left(-\frac{\partial U_1}{\partial L_1}\right) = \left(-\frac{\partial U_2}{\partial L_2}\right)\left(\frac{W_1}{W_2}\right)\left(\frac{1+r}{1+\rho}\right)$$

THIS SHOWS THAT A SMALLER TAX CUT WILL PRODUCE A SMALLER LABOUR SUPPLY RESPONS (SINCE $\frac{W_1}{W_2}$ WILL BE SMALLER.

A LONGER DELAY BEFORE THE TAX IS REINSTATED HAS AN AMBIGUOUS EFFECT ON THE PER PERIOD LABOUR SUPPLY RESPONSE. IF $\left(\frac{1+r}{1+\rho}\right) > 1 \iff r > \rho$ THEN A LONGER DELAY INCREASES THE LABOUR SUPPLY RESPONSE BECAUSE THE REPRESENTATIVE CONSUMER IS RELATIVELY PATIENT SO THAT ANTICIPATED FUTURE WAGE RISES HAVE A ~~BIG~~ EFFECT ON TODAY'S LABOUR SUPPLY.