

$$\mathcal{L} = u(C_t) + \beta u(C_{t+1}) - \lambda (p_t C_t + p_{t+1} C_{t+1} \left(\frac{1}{1+r}\right) - M)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \left(\frac{\partial u}{\partial C_t}\right) - \lambda p_t = 0 \quad \frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta \left(\frac{\partial u}{\partial C_{t+1}}\right) - \lambda p_{t+1} \left(\frac{1}{1+r}\right) = 0$$

$$\Rightarrow \beta \left(\frac{\partial u}{\partial C_{t+1}}\right) = \left(\frac{p_{t+1}}{p_t}\right) \left(\frac{1}{1+r}\right) \left(\frac{\partial u}{\partial C_t}\right)$$

so if there is a temporary tax cut so that $p_t = P$ and $p_{t+1} = P(1+\theta)$ then the Euler Equation becomes:

$$\frac{\beta(1+r)}{(1+\theta)} \left(\frac{\partial u}{\partial C_{t+1}}\right) = \left(\frac{\partial u}{\partial C_t}\right) \quad [1]$$

Without the tax cut, the Euler equation is:

$$\beta(1+r) \left(\frac{\partial u}{\partial C_{t+1}}\right) = \left(\frac{\partial u}{\partial C_t}\right) \quad [2]$$

These are most easily interpreted if we consider logarithmic utility as an example, so that $u(C_t) = \ln(C_t)$ and $\frac{\partial u}{\partial C_t} = \frac{1}{C_t}$. Thus [1] and [2] become

$$\frac{\beta(1+r)}{(1+\theta)} \left(\frac{1}{C_{t+1}}\right) = \frac{1}{C_t}$$

$$\Rightarrow C_t = C_{t+1} \left(\frac{1+\theta}{\beta(1+r)}\right) \quad [1]$$

$$C_t = C_{t+1} \left(\frac{1}{\beta(1+r)}\right) \quad [2]$$

So, the increase in C_t relative to C_{t+1} is, in this case $\frac{\theta}{\beta(1+r)} = \Delta \left(\frac{C_t}{C_{t+1}}\right)$. So the response of current consumption is increasing in the tax rate/cut θ and decreasing in β and r (if the consumer is more patient or the interest rate higher then it becomes more costly in terms of future utility to spend more and so save less/borrow more in the current period (i.e. today), so the consumer increases consumption by less. caused by the tax cut