CONSIDER A CENTRAL BANK MAKING AN OPTIMAL RESPONSE TO AN INFLATION / COST-PUSH SHOCK IN PERIOD 0 OF x. (ASSUME FOR SIMPLICITY THERE WILL BE NO FURTHER SHOCKS.) ASSUME A NEW KEYNESIAN PHILLIPS CURVE (NKPC) OF THE FORM:

$$\Pi_t = \Pi^e_{t+1} + \alpha(Y_t - \bar{Y}_t) + V_t$$

$$\Rightarrow \Pi_t = \Pi_{t+1} + \alpha \overset{\vee}{Y}_t + V_t \quad \text{WHERE } \overset{\vee}{Y}_t = Y_t - \bar{Y}_t$$

WHERE V_t IS ~~THE~~ THE ~~…~~ SHOCK TERM AND, WHERE, SINCE THERE ARE ASSUMED TO BE NO FURTHER SHOCKS, $\Pi^e_{t+1} = \Pi_{t+1}$ (SINCE THERE IS THEN NO UNCERTAINTY)

ASSUME THAT THE CENTRAL BANK HAS A QUADRATIC LOSS FUNCTION OF THE FORM:

$$L_t = \sum_{i=0}^{\infty}\left[\delta^i\left(\left(\overset{\vee}{Y}_{t+i}\right)^2 + \beta\left(\Pi_{t+i}\right)^2\right)\right]$$

(ASSUME FOR SIMPLICITY AN INFLATION TARGET OF ZERO)

IN GENERAL, THE CENTRAL BANK CAN ~~BE~~ MAKE A PLAN LASTING ANY NUMBER OF PERIODS. WE WILL CONSIDER A 3-PERIOD, THEN AN N-PERIOD AND FINALLY AN INFINITE-PERIOD CASE.

FOR A 3-PERIOD CASE, BY PERIOD 3 INFLATION WILL BE BACK ON TARGET, AS WILL OUTPUT, SO WE HAVE THE FOLLOWING SITUATION:

PERIOD	OUTPUT GAP	INFLATION
3	0	0
2	$\overset{\vee}{Y}_2$	$0 + \alpha\overset{\vee}{Y}_2 = \alpha\overset{\vee}{Y}_2$
1	$\overset{\vee}{Y}_1$	$\alpha\overset{\vee}{Y}_2 + \alpha\overset{\vee}{Y}_1$
0	$\overset{\vee}{Y}_0$	$\alpha\overset{\vee}{Y}_2 + \alpha\overset{\vee}{Y}_1 + \alpha\overset{\vee}{Y}_0 + x$

SO, THE CENTRAL BANK'S LOSS FUNCTION WILL BE:

$$L_0 = \left(\overset{\vee}{Y}_0\right)^2 + \delta\left(\overset{\vee}{Y}_1\right)^2 + \delta^2\left(\overset{\vee}{Y}_2\right)^2 + \beta\left(\left(\alpha(\overset{\vee}{Y}_2 + \overset{\vee}{Y}_1 + \overset{\vee}{Y}_0) + x\right)^2 + \delta\left(\alpha(\overset{\vee}{Y}_2 + \overset{\vee}{Y}_1)\right)^2 + \delta^2\left(\alpha(\overset{\vee}{Y}_2)\right)^2\right)$$

WHERE $0 < \delta < 1$ IS THE DISCOUNT FACTOR

WE CAN NOW FIND FIRST ORDER CONDITIONS WITH RESPECT TO $\hat{y}_0$ AND $\hat{y}_2$:

$$\frac{\partial L_0}{\partial \hat{y}_0} = 2\hat{y}_0 + 2\alpha\beta\left(\alpha(\hat{y}_2+\hat{y}_1+\hat{y}_0)+x\right) = 0$$

$$\Rightarrow \quad \hat{y}_0 = -\alpha\beta\Pi_0 \quad \boxed{1}$$

$$\frac{\partial L_0}{\partial \hat{y}_2} = \delta^2\hat{y}_2 + 2\alpha\beta\left(\Pi_0 + \delta\Pi_1 + \delta^2\Pi_2\right) = 0$$

$$\Rightarrow \quad \hat{y}_2 = -\left(\frac{\alpha\beta}{\delta^2}\right)\left(\sum_{i=0}^{2}\left[\delta^i\Pi_i\right]\right)$$

NO, GENERALIZING TO N-PERIODS:

$$\hat{y}_N = -\left(\frac{\alpha\beta}{\delta^N}\right)\left(\sum_{i=0}^{N}\left[\delta^i\Pi_i\right]\right) \quad \boxed{2}$$

COMBINING $\boxed{1}$ WITH THE PHILLIPS CURVE IN PERIOD 0:

$$\Pi_0 = \Pi_1 + \alpha\hat{y}_0 + x$$

$$\Rightarrow \quad \Pi_0 = \Pi_1 - \alpha^2\beta\Pi_0 + x$$

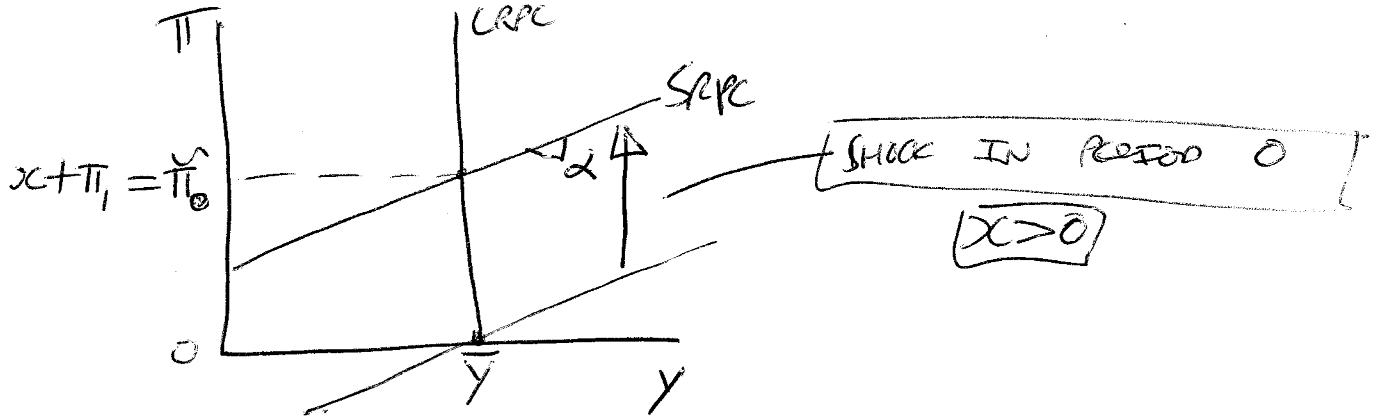
$$\Rightarrow \quad \Pi_0 = \left(\frac{1}{1+\alpha^2\beta}\right)\left(\Pi_1 + x\right) \quad \boxed{3}$$

NOW SUBBING $\boxed{3}$ INTO $\boxed{2}$:

$$\hat{y}_N = -\left(\frac{\alpha\beta}{\delta^N}\right)\left(\Pi_0 + \sum_{i=1}^{N}\left[\delta^i\Pi_i\right]\right)$$

$$\Rightarrow \hat{y}_N = -\left(\frac{\alpha\beta}{\delta^N}\right)\left(\left(\frac{1}{1+\alpha^2\beta}\right)\left(\Pi_1+x\right) + Z_N\right) \quad \boxed{4}$$

WHERE Z_N DENOTES THE PRESENT DISCOUNTED VALUE OF ALL FUTURE INFLATION GAPS COMBINED FROM PERIOD 1 UP TO PERIOD N. WE ALSO LET $\tilde{\Pi}_0$ DENOTE THE CROSSING Y-VALUE FOR THE SHORT AND LONG RUN PHILLIPS CURVES IN PERIOD 0 AS ILLUSTRATED BY THE FOLLOWING DIAGRAM:



HENCE $\boxed{4}$ CAN BE REARRANGED TO GIVE:

$$Z_N = -\left(\frac{\hat{Y}}{N}\right)\left(\frac{\delta^N}{\alpha\beta}\right) - \left(\frac{1}{1+\alpha^2\beta}\right)\left(\hat{\Pi}_0\right) \qquad \boxed{5}$$

WE CAN SEE FROM $\boxed{5}$ THAT A GREATER INITIAL SHIFT UPWARDS IN THE SRPC WILL INVOLVE A GREATER TOTAL DEFLATION (I.E. Z_N WILL BECOME MORE NEGATIVE.)

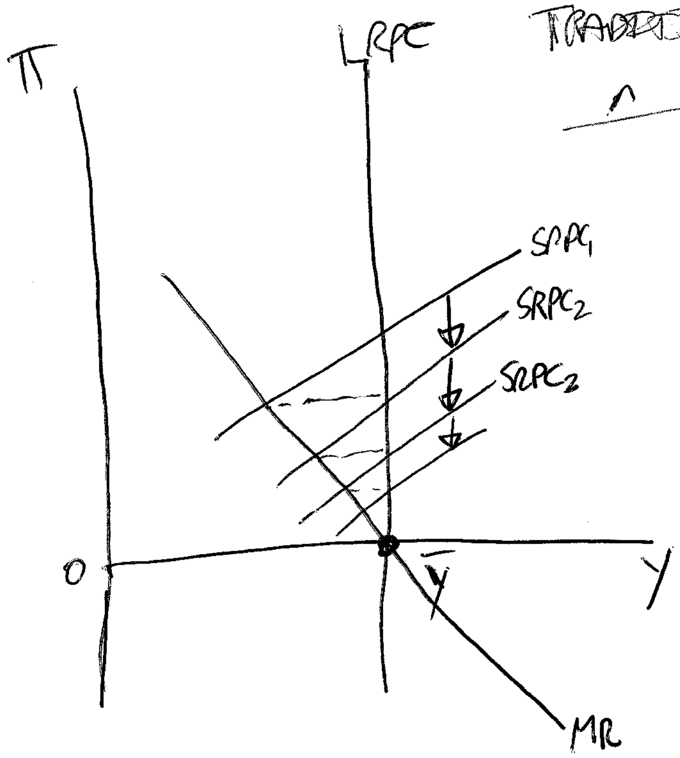
IF WE NOW LET $N \to \infty$ SO THAT THE PLAN LASTS AN INFINITE NUMBER OF PERIODS, THEN $\delta^N \to 0$ AND SO WE GET

$$Z_\infty = -\left(\frac{1}{1+\alpha^2\beta}\right)\left(\hat{\Pi}_0\right) \qquad \boxed{6}$$

AS $\delta \to 1$ SO THERE IS MINIMAL DISCOUNTING, Z_∞ APPROXIMATES THE TOTAL SUM OF THE INFLATION GAPS IN ALL PERIODS FROM 1 ONWARDS. THUS $\boxed{6}$ SHOWS THAT THE TOTAL SUM OF INFLATION GAPS IS NEGATIVELY RELATED TO THE INITIAL SHOCK.

MOST IMPORTANTLY, HOWEVER, IF EITHER THE PHILLIPS CURVE IS VERY FLAT ($\alpha \to 0$) OR THE CENTRAL BANK HAS VERY LOW INFLATION AVERSION ($\beta \to 0$) THEN THE SUM OF TOTAL DEFLATION IS APPROXIMATED BY THE INITIAL POSITIVE SHOCK AND SO THE OPTIMAL PATH OF THE PRICE LEVEL CHOSEN BY AN INFLATION-TARGETING CENTRAL BANK WOULD BE THE SAME AS WOULD BE CHOSEN WITH A PRICE LEVEL TARGET. THIS IS VERY DIFFERENT TO THE PATH CHOSEN UNDER A TRADITIONAL PHILLIPS CURVE, AS ILLUSTRATED BELOW:

TRADITIONAL PC WITH ADAPTIVE EXPECTATIONS:



HERE INFLATION IS GRADUALLY BROUGHT BACK DOWN TO TARGET, BUT REMAINS ABOVE TARGET THROUGHOUT, SO THAT THE PRICE LEVEL ENDS UP MUCH HIGHER THAN BEFORE THE SHOCK.

AS WE HAVE SHOWN, HOWEVER, WITH A FLAT NEW KEYNESIAN PHILLIPS CURVE, THE TOTAL DEFLATION WILL BRING THE PRICE LEVEL BACK DOWN TO ITS ORIGINAL LEVEL IN THE LONG RUN OVER AN INFINITE TIME HORIZON.

CRUCIALLY, ALSO, WHEREAS WITH AN INFLATION TARGET THE OPTIMAL PLAN WOULD NOT BE TIME CONSISTENT, IT WOULD BE WITH A PRICE LEVEL TARGET, THUS ENABLING EFFICIENT FORWARD GUIDANCE OF PRIVATE SECTOR INFLATION EXPECTATIONS UNDER THE N.K.P.C.

(THE SAME ANALYSIS APPLIES IN REVERSE OF COURSE FOR A DEFLATIONARY SHOCK!)