An Introduction to Game Theory

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Slides are available at: `users.ox.ac.uk/~sedm1375/gametheory.pdf`

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Overview - Purpose of this Session

- We are aiming to give you an introduction to Game Theory both as a theoretical/mathematical and as an experimental/empirical discipline.
- We will actually be playing a few games using our smartphones...
- ...Though only for imaginary payoffs!
- The material in this session is adapted from the **Microeconomics** and **Game Theory** courses taken by second and third year undergraduates studying **Philosophy, Politics and Economics** and **Economics and Management** at the **University of Oxford**.

John Nash (1928-2015)

- John Nash, American mathematician and winner of the Nobel Memorial Prize in Economic Sciences (along with fellow game theorists John Harsanyi and Reinhard Selten) in 1994, is widely regarded as the creator of the discipline.
- The central concept in Game Theory, **Nash equilibrium**, is named after him.
- Nash was played by Russell Crowe in the 1998 movie "A Beautiful Mind", about his life and work.

Overview - Types of Game

- **Simultaneous** - Solve using Nash Equilibrium
- Examples:
    - **Prisoners' Dilemma**
    - **Public Goods Game**

- **Dynamic / Sequential** - Solve using Subgame-Perfect Nash Equilibrium (concept proposed by Reinhard Selten, who shared the Nobel Prize with John Nash)
- Examples:
    - **Ultimatum Game**
    - **Entry Game**

- All finite-player, finite-move games can be represented in two alternative forms:
    - **Strategic Form** : The "Payoff Matrix" - Use it to find Nash equilibria
    - **Extensive Form** : The "Game Tree" - Use it to find subgame-perfect Nash equilibria

All Games have PIMPS

- **P**layers (2 or more)
- **I**nformation (Simultaneous $\implies$ Imperfect, Sequential $\implies$ Perfect)
- **M**oves (or actions)
- **P**ayoffs (can think of these as money, or utility)
- **S**trategies (A strategy is a rule that tells the player what action to take in every possible situation during the game)

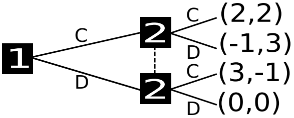
- **Nash Equilibrium** - Every player's strategy is a **best response** (maximises that player's payoff) *given the strategies chosen by the other players*.
- (When there is more than one Nash equilibrium in a finite game, usually only one of them will be subgame-perfect.)

Prisoners' Dilemma - Strategic and Extensive Form

Strategic form:

	C	D
C	2, 2	−1, 3
D	3, −1	0, 0

Extensive form:



- Note that the dotted vertical line in the extensive form indicates the imperfect information experienced by player 2, who does not know whether player 1 has played C or D.
- Both players have a **dominant strategy** to play **D**efect so (D, D) is the *unique* Nash equilibrium of the game.

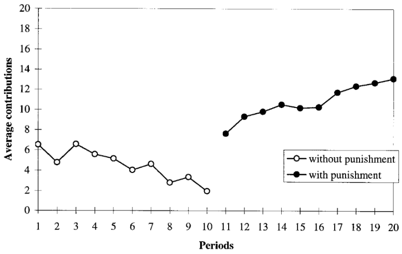
EXPERIMENT - Public Goods Game

- Players are automatically organised into group of size 5.
- Each player starts with 10 units of wealth and contributes from 0 to 10 units to the public good. The contribution represents a cost for the individual player.
- The Marginal Per Capita Return (MPCR) is 0.5, so *every player* in the group receives 0.5 when a particular player contributes 1 unit.

- Go to `https://classex.uni-passau.de/bin/`
- Select:
  - University of Oxford
  - Game Theory
  - participant
- Password: `Prospects`
- Click: `login`
- Follow the instructions on your hand-held device!

Public Goods Games Experimental Results

- Public Goods games are similar to N-player prisoners' dilemma but each player can choose contribution level, with each unit of contribution creating a benefit b which is shared over the group but at a cost $b > c > \frac{b}{N}$. (The MPCR is $\frac{b}{N}$.)
- Evidence [Dawes & Thaler, 1988] shows that for small groups average contributions are usually in the region of 40%-60% of the optimal level. When the game is repeated, the average level of contributions tends to drop over time. However, the ability to punish non-co-operators and non-punishers greatly increases the ability to sustain co-operation [Fehr & Gächter, 2000] [Fehr & Fischbacher, 2003].



A 2-Player 2-Move Public Goods Game Creates a Standard Prisoners' Dilemma

- Suppose the MPCR is 0.75 so that when each play contributes 1 unit to the public good it creates a return of 0.75 for *each* player.
- Further suppose that each player can only choose a contribution of 0 or 1 (binary move game).

	C (contribute 1)	**D (contribute 0)**
C (contribute 1)	$1.5 - 1 = 0.5$ $1.5 - 1 = 0.5$	$0.75 - 0 = \underline{0.75}$ $0.75 - 1 = -0.25$
D (contribute 0)	$0.75 - 1 = -0.25$ $0.75 - 0 = \underline{0.75}$	$\underline{0}$ $\underline{0}$

Importance of Public Goods Games in Social Science

- **Economics** - Game Theory predicts the under-provision of goods that are **non-rival** (once produced for one person, they are produced for everyone) and **non-excludable** (people cannot be individually charged for consuming them). In policy terms, this implies a role for government in compelling people to contribute towards the provision of such **public goods**.

- **Political Science** - Political action that benefits a group but incurs an individual cost (e.g. voting or lobbying) will be under-provided, particularly when the group contains many members. This helps to explain why small groups are often able to organise more effectively against the interests of larger groups. (E.g. small number of big businesses influence government to weaken the position of small business and consumers, small number of farmers influence government to provide subsidies paid for by the rest of society.)

EXPERIMENT - Ultimatum Game

- Players are randomly sorted into pairs and selected to be either the **proposer** (player 1) or **receiver** (player 2).
- There are 10 units of pie available. The proposer chooses an amount X to take and leaves $10 - X$ for the receiver.
- The receiver can then either **accept** (in which case the payoff is $10 - X$ for the receiver and X for the proposer) or **reject** (in which case the payoff is 0 for both players).

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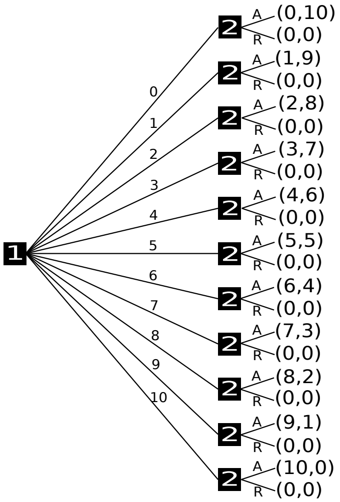
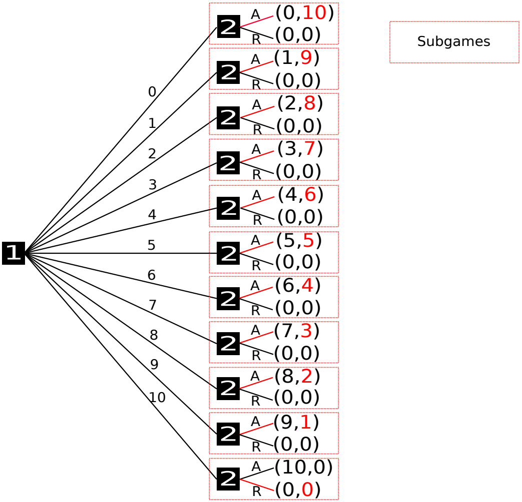
Ultimatum Game - Solving Subgame-Perfect Nash Equilibrium using Backwards Induction

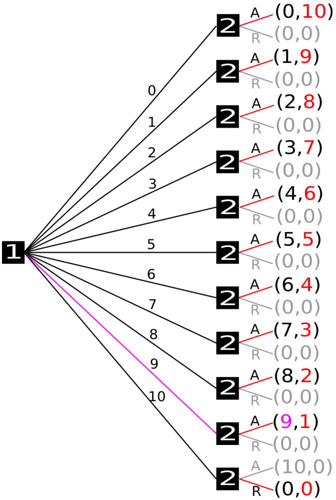


Ultimatum Game - Solving Subgame-Perfect Nash Equilibrium using Backwards Induction



"Tie-breaker" assumption - if player 2 is **indifferent** between **A**ccept or **R**eject then they choose **R**eject.

Ultimatum Game - Solving Subgame-Perfect Nash Equilibrium using Backwards Induction



So, the unique subgame-perfect Nash equilibrium is that the Proposer offers 1 to the Receiver and keeps 9, and the Receiver accepts any offer greater than 0 (and so accepts the offer of 1).

Explanations for Experimental Game Results

- **Classical Game Theory**, based upon the assumption of **rational self-interest**, often does not accurately predict how real people play games in experiments.

- This does *not* mean that the concept of Nash equilibrium is *invalid*, however. Rather it might need to be extended or modified to take into account behavioural factors:

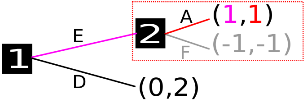
- **Learning / Evolution** - It may take a number of repetitions before players fully understand the game they are playing. Or players may play according to ingrained "rules of thumb" (**phenotypes** in biological terminology) that only evolve as the success or failure of these unfolds over time.

Explanations for Experimental Game Results

- **Altruism** - Players may have preferences for fairness (they care about the monetary outcomes for other players as well as their own), which they are willing to enforce even when this creates a private cost.
- For example, in the **Ultimatum Game** an altruistic Proposer may offer the Receiver more than the minimum amount that the Receiver would accept. An altruistic Receiver might reject a low but positive offer in order to enforce a norm of fairness (hence altruistically punishing the Proposer for making an unfair offer).
- **Incomplete Information** - In particular, a small amount of doubt about whether *other players* are rational can change the Nash equilibrium outcome drastically.

Entry Game - Strategic and Extensive Form

	A	F
E	1 1	−1 −1
D	2 0	2 0



Nash equilibria exist at (E, A) and (D, F) but backwards induction shows that only (E, A) is subgame-perfect. This is because it is *not* a **credible threat** to **F**ight in the subgame following **E**ntry.

Entry Game with Strategic Precommitment - Problem

- Suppose that player 2 (the incumbent) can make an investment which costs c_0 but which reduces the cost of fighting to c_F (instead of 2).
- For what range of values of c_F does the threat to fight entry become credible?
- For what range of values of c_0 would the incumbent choose to make the investment?

Without investment:

	A	**F**
E	1 1	-1 -1
D	2 0	2 0

With investment:

	A	**F**
E	$1 - c_0$ 1	$1 - c_F$ -1
D	$2 - c_0$ 0	$2 - c_0$ 0

Entry Game with Strategic Precommitment - Answer

- For what range of values of c_F does the threat to fight entry become credible?
    - We would need $1 - c_0 < 1 - c_F$ and so $c_0 > c_F$.
- For what range of values of c_0 would the incumbent choose to make the investment?
    - We would need $2 - c_0 > 1$ and so $c_0 < 1$.
- **Conclusion:**
    - If $c_F < c_0 < 1$ then the investment will be made and the subgame-perfect Nash equilibrium will be (Invest, D, F).

Without investment:

	A	**F**
E	1, 1	-1, -1
D	0, 2	0, 2

With investment:

	A	**F**
E	1, $1 - c_0$	-1, $1 - c_F$
D	0, $2 - c_0$	0, $2 - c_0$

Applications of Entry Game in Economics

- Entry-game implies that incumbent firms may find it optimal to invest in **excess capacity** in order to be able to credibly fight a potential entrant if they enter.

- This **strategic entry deterrence** may or may not be good for consumers, depending on whether or not the additional capacity of the incumbent takes prices below what they would have been following successful entry and accommodation.
- This depends in the market in question, and should be investigated by competition authorities on a case-by-case basis.

Concluding Remarks

- **Game Theory** is central to modern economics and political science (as well as evolutionary biology).
- **Game Theory** is vital both to **positive** and **normative** economics.
- **Positive Economics** - Explaining the way the world is and making predictions.
  - **Neoclassical economics**
  - **Behavioural economics**
  - **Analytical Marxism**
- **Normative Economics** - Providing recommendations for optimal economic and social policy by answering questions about the way the world could and should be.

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