Game Theory in the Real World - Intro. to Mechanism Design

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Slides are available at:

`users.ox.ac.uk/~sedm1375/gametheoryintherealworld.pdf`

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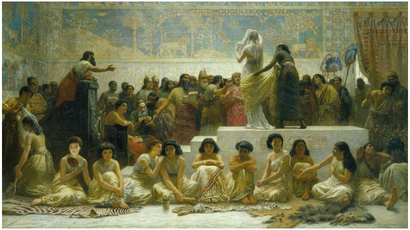
- **Public Auctions**
  - *Ancient Greece* - Unpaid loans, public lands, rights to collect taxes, goods seized from conquered enemies, criminals & debtors, prisoners of war sold as slaves.
  - *Ancient China* - Possessions of deceased Buddhist monks auctioned off as early as 7th century AD.
- **Private Auctions**
  - *Ancient Greece* - Land, crops, houses, slaves & livestock.
  - *Ancient Babylon* - Women for marriage.



Ancient Babylonian marriage auction

Christie's in 19th century London

- **Direct Democracy** - *Ancient Greece*
  - Only male citizens could vote (about 10% - 20% of inhabitants).
  - By the 5th century BC, a system of direct democracy by majority rule (a measure is passed if a simple majority votes "yes") had evolved.
- **Representative Democracy**
  - *English Parliament* - 1215 : Magna Carta
  - *French Estates General* - 1302
  - *Communes* - Paris Commune (1871), Russian Soviets (1905), Chinese National People's Congress (1954)

A marble relief showing the People of Athens being crowned by Democracy, inscribed with a law against tyranny passed by the people of Athens in 336BC

John Nash (1928-2015)

- John Nash, American mathematician and winner of the Nobel Memorial Prize in Economic Sciences (along with fellow game theorists John Harsanyi and Reinhard Selten) in 1994, is widely regarded as the creator of the discipline.
- The central concept in Game Theory, **Nash equilibrium**, is named after him.
- Nash was played by Russell Crowe in the 1998 movie "A Beautiful Mind", about his life and work.

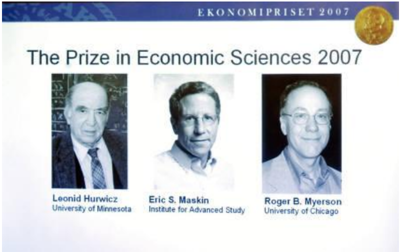
All Games have PIMPS

- **P**layers (2 or more)
- **I**nformation
- **M**oves (or actions)
- **P**ayoffs (can think of these as money, or utility)
- **S**trategies (A strategy is a rule that tells the player what action to take in every possible situation during the game)

- Nash Equilibrium - Every player's strategy is a **best response** (maximises that player's payoff) *given the strategies chosen by the other players* [Nash, 1951].
- Dominant Strategy Equilibrium - Every player's strategy is a **best response** (maximises that player's payoff) *regardless of the strategies chosen by the other players*.
- Note - all dominant strategy equilibria are Nash equilibria (but *not* vice versa).

Basic Principles of Mechanism Design Theory

1. Players receive **private information**.
   - In the case of an auction this could be their private valuation of the object(s) being offered.
   - In the case of voting, this could be their individual preference over the options/candidates up for election.
2. Players make a **move** based upon their **private information** - e.g. a **bid** in an auction or a **vote cast** in an election.
3. Based upon the moves made by all players, an **outcome** is determined, then generating a **payoff** for each player.



Leonid Hurwicz, Eric Maskin & Roger Myerson won the Nobel Memorial Prize in Economic Sciences in 2007

Basic Principles of Mechanism Design Theory

4. We typically require that mechanisms be designed to achieve **normatively desirable** outcomes via their **Nash equilibrium**:

   - **Pareto efficiency** - In an auction, the object is allocated to the agent who most values it. With voting, the outcome should not be Pareto-dominated by another outcome.
   - **Strategy-proofness / Incentive Compatibility** - Agents should be incentivized such that the move they make **reveals** their private information. (Necessary for Pareto-efficiency.)
   - **Informational efficiency** - The mechanism should reveal and aggregate as much information as possible to determine the socially optimal allocation. This can involve more than *merely* Pareto efficiency. For instance, it may require **fair** or **equitable** treatment of the preferences of individuals with different interests in society.

Types of Auction

- **Ascending/English auction** - Prices gradually rise until only a single bidder remains.
- **Descending/Japanese auction** - Prices gradually fall until a single bidder accepts.
- **First-price sealed-bid auction** - All bidders simultaneously submit a sealed bid. The highest bidder receives the object at the price they bid.
- **Second-price sealed-bid auction** - All bidders simultaneously submit a sealed bid. The highest bidder receives the object at the price bid by the second-highest bidder.

- If players' valuations of the object are **independent** then an ascending auction is **strategically equivalent** to a second-price auction (players move by choosing at what price to "drop out") and a descending auction to a first-price auction.

The Nash Equilibrium of a Second-Price Auction

- It is fairly straightforward to show that it is a (weakly) dominant strategy to bid your true valuation of the object in a second-price auction. This then implies that it is a Nash equilibrium for every bidder to truthfully reveal their private information. This Nash equilibrium is Pareto-efficient since the object will end up allocated to the agent who values it most.

- Suppose without loss of generality that you are player 1 and value the object at v_1. You make a **bid** b_1 whilst expecting the highest bid among other players to be b_2. You therefore know that you will win the object if $\boxed{b_1 > b_2}$ and if so your surplus will be $v_1 - b_2$.

- Since you *want* to win the object if and only if $v_1 - b_2 > 0 \implies \boxed{v_1 > b_2}$ then you can ensure that you win if and only if you want to by setting $b_1 = v_1$. If $v_1 = b_1 \leq b_2$ then you get 0 surplus (whether you win, lose or share in a "tie"). Hence it is a weakly dominant strategy to reveal your private information by setting $\boxed{b_1 = v_1}$ (you can never gain by "lying", and may harm yourself by doing so).

The Revenue Equivalence Theorem

- The expected revenue for the auctioneer in a second-price sealed-bid auction is the expected value of the second-highest valuation. The famous **Revenue Equivalence Theorem** [Vickrey, 1961] [Myerson, 1981] shows that this result in fact generalises to *any* Pareto-efficient auction mechanism.

Theorem

If bidders are risk-neutral *and have* independent valuations *then any auction mechanism which always allocates the object to the bidder with the highest valuation must raise expected revenue equal to the expected value of the second-highest valuation.*

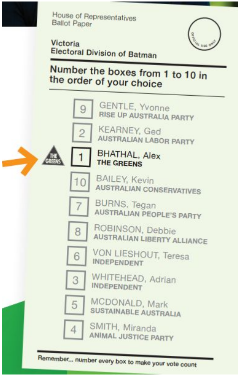
- A key implication of this result is that, if certain conditions hold, then it is logically impossible to create a "better" auction mechanism than the second-price sealed-bid (or ascending/English) auction.

- Another important implication is that there is a limit to how much a private auctioneer can expect to make, *unless Pareto-efficiency is sacrificed*.

Voting Systems

We can identify a number of different types of voting system used in the real world (both for direct and representative democracy). For simplicity, let's from here assume an odd number of voters (an even number creates the possibility of a tie where we would need to "toss a coin"). (Some more detailed examples of these will be given shortly.)

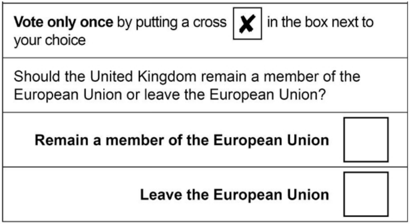
1. **"First Past the Post" (FPTP)** : (Used in most UK elections) - Every voter votes for one single candidate/option and then the candidate/option with the most votes wins.
2. **Alternative Vote Method (AV)** (Rank Order) : Sequentially eliminate least-suppored candidates/options, reallocating votes to next choice
3. **Condorcet Method** (Rank Order) : If there is a single option preferred by a simple majority to every other option then that option should win.
4. **Borda Count Method** (Rank Order) : Add up the ranking numbers for individual voters and use these to produce a social ordering.



Australian alternative vote system ballot paper

Simple Majority Rule with 2 Options

- To explore the efficiency properties of voting systems, it is most instructive to start with simple majority rule and just 2 options to choose from.
- We could either allow voters to place a single cross by their preferred option/candidate or to rank 1st and 2nd choice. The Borda Count, Condorcet Method, alternative vote and "first past the post" are **all strategically equivalent**.



Vote only once by putting a cross ✗ in the box next to your choice

Should the United Kingdom remain a member of the European Union or leave the European Union?

Remain a member of the European Union ☐

Leave the European Union ☐

Ballot paper for 2016 EU Referendum in the UK

1. **Pareto efficient** outcome (though trivially, unless there is unanimous agreement, BOTH outcomes are Pareto efficient!)
2. **Utilitarian argument** : Fairest outcome since it minimises number of disappointed agents - *"greatest happiness of greatest number"*
3. **Epistemic argument (informational efficiency)** : Cordorcet's **Jury Theorem** - Using *everyone's* information maximises the probability of choosing correctly
4. **Strategy-proof** : Incentivizes truthful voting

Majority Voting 3 Or More Options - Condorcet Cycle

Rank	**A**	**B**	**C**
1	X	Z	Y
2	Y	X	Z
3	Z	Y	X

EU = European Union

- *X* *Leave the EU with no deal*
- *Y* *Remain in the EU*
- *Z* *Leave the EU with a deal*

- Consider a situation with 3 voters with preferences as shown. Here none of the voting systems mentioned so far works:
  - With **FPTP**, each option receives 1 vote.
  - With **AV**, there is no candidate with least first-choice votes to eliminate.
  - There is **no Condorcet winner** since 2 voters (**A** & **B**) prefer X to Y, 2 voters (**A** & **C**) prefer Y to Z and 2 voters (**B** & **C**) prefer Z to X.
  - All 3 options have a Borda Count of $1 + 2 + 3 = 6$, so we don't have a winner.
- One solution is assign an **agenda setter**. For example, the government might require we must first decide whether to remain or leave the EU (X versus Y) and *then* decide whether to leave with a deal (winner of stage *1* versus Z).
- If voting is "honest" then X beats Y in stage *1* but is then defeated by Z in stage 2 (so the UK leaves with a deal!)

Majority Voting 3 Or More Options - Agenda Setting

Rank	**A**	**B**	**C**
1	X	Z	Y
2	Y	X	Z
3	Z	Y	X

EU = European Union

1. *Leave the EU with no deal* (X)
2. *Remain in the EU* (Y)
3. *Leave the EU with a deal* (Z)

- **Agenda setting** creates a number of problems however:
  1. Incentives for **strategic voting** : Condider voter **A**, who under the proposed agenda gets their least preferred outcome Z (leaving with a deal). If they were to *anticipate* this and to *lie* by strategically voting for Y instead of X at stage 1 then they could achieve outcome Y, which is preferable for them.
  2. The agenda-setter may have **arbitrary power** to determine the outcome. For example, suppose the government instead set the agenda as X versus Z, then the winner versus Y. In that case Z wins at stage *1* but is then defeated by Y at stage *2* (so the UK remains in the EU!)
  3. Restricting the agenda to just 2 options solves the strategic voting problem and gives a consistent result, but **reduces voters' choice**.

Majority Voting 3 Or More Options - Borda Count

- If there *does* exist a **Condorcet winner**, then **pairwise majority voting with open agenda** (where anyone can propose amendments to the status quo) will result in this being selected. This is the **Condorcet method**, which can be thought of as a model of Athenian **direct democracy**.

Rank	**A**	**B**	**C**	**D**	**E**
1	X	X	X	W	W
2	W	W	W	Y	Y
3	Y	Y	Y	Z	Z
4	Z	Z	Z	X	X

- In the example above, X is the Condorcet winner since a majority (**A**, **B** and **C**) prefer it to every other option.
- However, the **Borda count** numbers are as follows:
  - W: 2+2+2+1+1=8
  - X: 1+1+1+4+4=11
  - Y: 3+3+3+2+2=13
  - Z: 4+4+4+3+3=18
- So the **Borda count** would produce W instead of X as the winner.

Majority Voting 3 Or More Options - Borda Count

- Now consider modifying the preferences of voters D and E:

Rank	**A**	**B**	**C**	**D**	**E**
1	X	X	X	W	W
2	W	W	W	X	X
3	Y	Y	Y	Y	Y
4	Z	Z	Z	Z	Z

- As in the previous example, X is the Condorcet winner since a majority (**A**, **B** and **C**) prefer it to every other option.
- Now the **Borda count** numbers are as follows:
  - W: 2+2+2+1+1=8
  - X: 1+1+1+2+2=7
  - Y: 3+3+3+3+3=15
  - Z: 4+4+4+4+4=20
- So now the **Borda count** would produce X instead of W as the winner, agreeing with the **Condorcet method**.
- This may be considered undesirable since the rank orderings of options X, Y and Z are (or should be) **irrelevant** to the social ordering of W and X.

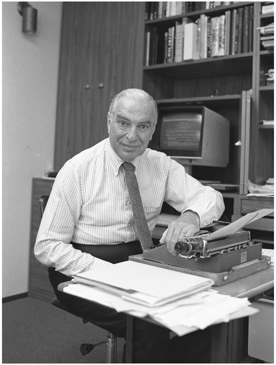
Arrow's Impossibility Theorem

- The most famous result in social choice theory by Kenneth Arrow [Arrow, 1951] shows that issues with preference cycles, agenda-setting power & irrelevent information are unavoidable.

Theorem

If there are 2 or more individuals and 3 or more options to choose from, then no social choice rule can exist which satisfies: (U) unrestricted domain, *(P) the* Pareto Principle, *(I)* independence of irrelevant alternatives *and (D)* non-dictatorship.

- (U) - All possible preferences are permitted.
- (P) - If *everyone* strictly prefers X to Y, then society must prefer X to Y.
- (I) - *Only* the individual orderings of X and Y should affect the social ordering of X and Y.
- (D) - No single individual should unilaterally determine the social orderings over all options.



Kenneth Arrow at Stanford University

The Gibbard-Satterthwaite Theorem

- A 2nd key result [Gibbard, 1973] [Satterthwaite, 1975] shows that the problems of strategic voting are also unavoidable:

Theorem

If there are 2 or more individuals and 3 or more options to choose from, then under (U) unrestricted domain *and (D)* non-dictatorship *no* strategy-proof *social choice rule can exist.*

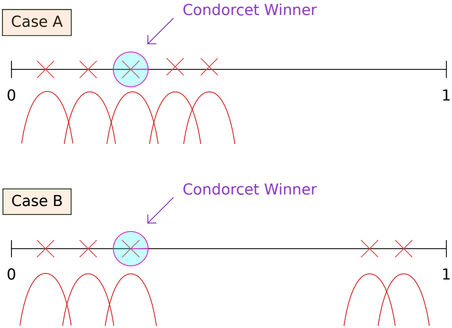
- Taken together, **Arrow's Impossibility Theorem** and the **Gibbard-Satterthwaite Theorem** tell us that no *perfect* democracy can exist. *Any* system of voting and decision-making will *necessarily* involve *at least 1* (and usually *more then 1*) of the following limitations:

1. Preference cycles resulting in **inconsistent** outcomes
2. Limitations placed upon permissible preferences (**domain restriction**)
3. Allocation of **agenda-setting** power (e.g. to government)
4. Use of information which *cannot* be obtained *just* from rank-order preferences (e.g. agents' utility inferred using cost-benefit analysis)
5. **Strategic manipulability**

Single-Peaked Preferences

There are situations where it may be reasonable to assume that preferences of voters are **single-peaked**, meaning that each voter has an “ideal” outcome and loses utility the further away from this the policy is set.

- In these cases, the **median voter** is always the **Condorcet winner**, so would always “get their way” under pairwise majority voting.
- Consider cases A and B where the decision is what proportion of local government spending should be on care for the elderly. If 60% of voters are non-elderly then the median voter is a non-elderly person. If voters are self-interested, this could mean that the median voter takes little or no account of the needs of the elderly.



Information about “strength of preference” of non-median voter is ignored by pairwise majority voting

Conclusion

- **Auctions** and **voting** are institutions that have been utilised in varying degrees in most societies since antiquity.
- There are many different types of auction mechanism and voting systems, but the **second-price sealed-bid** (or ascending) auction & **pairwise majority voting** are common.
- **Mechanism Design Theory**, an important branch of **Game Theory**, which is in turn a branch of **Mathematics** which has developed and become very influential in the social sciences since the 1950s.
- I have explored in this lecture how certain key theoretical results in this field provide a good justification and rationale for these mechanisms, in terms of their achieving socially desirable outcomes.
- However, we also see that *certain assumptions* must hold in order for these mechanisms to work well.

Conclusion

- For instance, there are situations in which other types of auction will *raise more revenue* than the second-price auction and there are situations in which pairwise majority voting produces *inconsistent outcomes* without an agenda-setter.
- Even when the outcome from majority voting is consistent, we might in some cases consider the outcome unfair to minority interests. (The **"tyranny of the majority"**.)
- The presentations you will be doing later on today and the quiz questions at the end of the week will enable you to explore these issues further...
- *Warning : Some, but not all, of the quiz questions can be answered directly from the lecture. You will* **also** *need to look at the reading list!*

THANK YOU!

References I

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