# Logic and Philosophical Logic
Formalisation

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Some formalisations are better than others. Logicians are skilled at coming up with good formalisations, or at least formalisations as good as the target logic will allow. What does this skill consist in? More precisely, what is it for a formal sentence $\sigma_1$ to be a better formalisation than $\sigma_2$ of some given natural-language sentence? Surprisingly, the literature contains relatively little by way of discussion of this question.

In today's class, we briefly introduce some potential criteria of good formalisation. We then focus on a particular one (*viz.* reflecting implicational structure) by discussing the ideas in my paper 'Capturing Consequence'. You'll find the paper on the class website alongside these notes. The notes are brief and are intended as a springboard for conversation in class. I don't claim that these really are all the relevant criteria, or that they are exclusive and exhaustive.

## 1 A global criterion

*Reflecting implicational structure* is a criterion that takes the whole language into account. Natural-language sentences stand in some implicational relations. For example, 'Fido is a dog' logically implies 'There's a dog' but does not logically imply 'There's a cat'. A good formalisation must capture these implicational facts, or at least as many as possible, or at least as many of the most important ones as possible (for some purpose). For instance, the propositional formalisation $p \therefore q$ of the argument 'Felix is a cat, therefore there's a cat' renders it invalid; in contrast, the first-order formalisation $Fa \therefore \exists x Fx$ succeeds in capturing its validity. The first-order formalisation is usually preferred to the propositional one precisely for this reason. The history of logic amply illustrates the value of capturing natural-language consequence. Augustus de Morgan, for example, pointed out that Aristotle's logic cannot capture the validity of the argument 'All dogs are animals, therefore all heads of dogs are heads of animals' whereas, as we now know, first-order logic can. That is a clear point in favour of first-order logic over Aristotelian logic. The criterion is *global* because it takes the whole language into account.

## 2 A sentential criterion

*Semantic proximity* enjoins us to formalise the natural-language sentence $s$ as a formal sentence $\sigma$ if $\sigma$ may be interpreted so as to be as close in meaning to $s$ as possible[1] On this criterion, a formalisation $\sigma_1$ of the natural-language sentence $s$ is

[1] As Benson Mates writes:

better than another formalisation $\sigma_2$ (be it in the same logic or in different ones) if some interpretation of $\sigma_1$ is closer in meaning to $s$ than any interpretation of $\sigma_2$. Note that the criterion presupposes the existence of comparative similarity facts among propositions. It is sentential because it operates at the level of sentences.

# 3 Two sub-sentential criteria

We have encountered two criteria so far: a global one, namely capturing implicational relations, and a sentential one, namely semantic proximity. Two sub-sentential criteria are *respecting grammatical form* and *respecting fixed interpretations*.

A rough gloss on the first criterion is that a formalisation should respect the grammatical form of a natural-language sentence. In other words, its syntax should reflect the original sentence's syntax. A grammatical parsing of the earlier argument 'Felix is a cat, therefore there is a cat' might be:

$$\underbrace{\text{Felix}}_{\text{Noun}}\ \underbrace{\text{is a cat}}_{\text{Predicate}},\ \underbrace{\text{therefore}}_{\text{Inference marker}}\ \underbrace{\text{there}}_{\text{Quantifier}}\ \underbrace{\text{is a cat}}_{\text{Predicate}}.$$

yielding the first-order formalisation

$$\underbrace{a}_{\text{Constant}}\ \underbrace{F}_{\text{Predicate}}\ \therefore\ \underbrace{\exists x}_{\text{Quantifier}}\ \underbrace{Fx}_{\text{Predicate}}$$

or $Fa \therefore \exists x Fx$ as it is more conventionally written. A first-order formalisation grammatically constrained in this manner respects the original English argument's validity. Contrast a propositional formalisation. A propositional parsing of the argument closest to its surface grammar is:

$$\underbrace{\text{Felix is a cat}}_{\text{Sentence 1}}\ \underbrace{\text{therefore}}_{\text{Inference marker}}\ \underbrace{\text{there is a cat}}_{\text{Sentence 2}}.$$

resulting in the propositional formalisation

$$p \therefore q,$$

which is not valid.

How exactly to spell out the grammatical criterion is a difficult question. In tackling it, particular care must be taken not to impose a parochial view of grammar engendered by familiarity with certain languages, or families of languages. Without good reason, we should not, for instance, privilege the grammar of English over other languages, or the grammar of linear-alphabet-deploying languages (e.g. languages

> ...to formulate precise and workable rules for symbolizing sentences of the natural language is a hopeless task. In the more complicated cases, at least we are reduced to giving the empty-sounding advice: ask yourself what the natural language sentence means, and then try to find a sentence of [the formal language] $\mathfrak{L}$ which, relative to the given interpretation, has as nearly as possible the same meaning. (Mates 1972, p. 84)

Another notable articulation of the criterion of semantic proximity may be found in Sainsbury (2001, pp. 52 & 372).

written using the Roman alphabet) over that of others (e.g. languages that contain ideograms, such as Egyptian hieroglyphs or Chinese characters).

Observe that, in logic, the grammatical criterion is usually of lesser importance than respecting implicational structure. Here we may contrast logical practice with that of linguistics. Linguists have for instance investigated extensions of first-order logic with the operator $\iota$, which roughly stands for 'the'; more precisely, $\iota x F x$ is a term that means 'the thing that is $F$'. This account of definite descriptions can be mimicked by Russell's classic account, which requires no resources beyond first-order logic. The main difference between a formalisation of 'The King of France is bald' as $B(\iota x K x)$ and $\neg\exists x\forall y((Ky \leftrightarrow y = x) \wedge Bx)$ is then syntactic: the former cleaves to the structure of the English sentence more closely than the latter, even if their truth-conditions are identical. For a linguist this speaks in favour of the $\iota$-formalisation; for the logician, the difference is minor, negligible even. Linguists strive to respect grammatical structure in a way that logicians do not.

The criterion of respecting fixed interpretations is also subsentential. Consider the conjunction 'and' in natural language and propositional logic's connective $\wedge$. Since the latter is always interpreted as truth-functional 'and', the formalisation of 'and' in propositional logic should be $\wedge$. Notice that this criterion is supplementary to that of respecting grammatical form, since the latter enjoins us to formalise 'and' as *some* two-placed sentential connnective but does not tell us which one (e.g. it does not rule out the formalisation of sentential 'and' as $\vee$). Similarly, the identity predicate in natural language should be interpreted as first-order logic's identity predicate, because the latter has an invariant interpretation. More generally, if a logic's vocabulary item $\alpha$ is always interpreted as the natural-language word or expression $a$ then $a$ should be formalised as $\alpha$ in that logic.

# References


[1] B. Mates (1972), *Elementary Logic* ($2^{nd}$ ed.), Oxford University Press.

[2] M.R. Sainsbury (2001), *Logical Forms* ($2^{nd}$ ed.), Blackwell.