Logic and Philosophical Logic

Précis of Gila Sher's 'The Foundational Problem of Logic' and miscellaneous remarks on the Tarski-Sher Thesis

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In section 1, I summarise Sher's paper as a refresher or in case anyone hasn't had a chance to read it; page references here are to Sher's article. Needless to say, the summary is selective: I included the points I found of most interest. Most sentences in this section are quotations, or light paraphrases, of Sher's text, though in a few places I've put things a little differently – I haven't striven for complete fidelity to the text. Section 2 contains some miscellaneous points relating to the Tarski-Sher Thesis, as a basis for class discussion.

1 Précis of Sher (2013)

1.1 Introduction (145-8)

In the introduction, Sher explains what she means by a foundation for logic. She claims that systematic attempts to construct a philosophical foundation for logic have been rare (145, 147). Frege and Russell, for example, aimed at giving a foundation of mathematics, but offered no philosophical explanation of what logic is (147). Before them, Kant took formal logic largely as given (148).

1.2 Methodology (149-157)

Sher characterises foundationalism as seeking to establish all human knowledge on (i) basic knowledge and (ii) knowledge-extending procedures (149). Foundationalism orders our system of knowledge in a tree-like fashion, in which non-basic items of knowledge are grounded in basic ones. This means that the basic units themselves cannot be grounded, thereby createing an inherent difficulty for a foundationalist approach to logic (150). Foundationalists either cannot give a foundation for logic, or cannot use knowledge-based resources to do so (150-1). Neither approach is satisfactory.

An improvement on foundationalism is to adopt a holistic or anti-hierarchical approach without abandoning the grounding project – in Shapiro's words, 'Foundations without Foundationalism' (152). Sher distinguishes her methodology from 'radical' coherentism. We can make unrestricted use of our entire system of knowledge in veridical grounding of some area (153), where 'veridical grounding' means grounding or justification centered on truth (153). She calls her preferred methodology *foundational holism*, and sees Neurath's boat as a good metaphor for it (154). The use of logic in providing a theoretical foundation for logic is unavoidable, yet is entirely compatible with the foundational project as she sees it (155). Circularity of this sort need not be paradox-laden nor does it preclude the discovery of error (156). Constructive circularity—circularity used to further epistemic goals—can be a boon and is a constituent of many philosophical methods (157). Sher's explanation of logic gives greater weight to considerations about the role logic plays in our overall system of knowledge, in particular mathematics, than to its natural-language use (157).

1.3 An Outline of a Foundation (158-196)

1.3.1 Logical Consequence and its veridicality (158-170)

We are in need of a powerful universal instrument for expanding our knowledge; a method/system of inference that applies to all fields and transmits truth with an especially strong modal force. It is logic that plays this role. It also serves to remove errors (158).

Logic is topic-neutral: it applies the same tests to inferences and sentences in all areas (159). That's compatible with logic having a subject matter of its own, namely logical inference, logical inconsistency, logical truth, etc. (158).

Logic is grounded in the world (160). A system that contains a law like *affirming the consequent* is not in sync with the work; in contrast, a system that contains a law like *affirming the antecedent* (MP) is. Just like a scientific theory, 'working in the world' is a serious constraint on the design of a logical system (160). The non-existence of the universal class and non-classical logics' claim to work better in some contexts than classical logic illustrate the point that factual considerations play a role in choosing a logic. Another example is the putative law '$\Phi(x), x \neq y \vdash \neg\Phi(y)$', which although it has a similar appearance to Leibniz's law, is objectionable because objects can and do have properties in common (161).

Consequence may be characterised generically as the transmission/preservation of truth from a set of sentences to a conclusion (162). Material, nomic and logical consequence differ in the modal force in which truth is thus transmitted. Logic requires a grounding in reality because of its inherent connection with truth: consequence relations must respect the connections or lack thereof in the world between things being as the premisses say they are and as the conclusion says they are (163). Facts about the world show that a logic which underwrites the consequence of a false sentence from a true one is incorrect (164). Similarly, but less starkly, the absence of a strong connection between a premise S_1 and a conclusion S_2 suffices to show that a logic that underwrites the consequence of S_2 from S_1 is incorrect. In sum: the world limits the options open for logical theories; logic is constrained by the world (165).

Nomic connections in the world justify claims of nomic consequence; a potential example might be something like Newton's laws and the entailment 'The force exerted by a on b is c; therefore, the force exerted by b on a is c' (166). More strongly, the world might be governed by laws that connect conditions/situations with a stronger than nomic force, a force appropriate for logic; an example might be the connection 'Non-empty $\mathcal{A} \cup (\mathcal{B} \cap \mathcal{C}) \Rightarrow$ Non-empty $\mathcal{A} \cup \mathcal{B}$' between properties $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ and the claim '$\exists x(Ax \vee (Bx \& Cx)) \vDash_L (\exists x)(Ax \vee Bx)$' (166). So logic is grounded in certain laws governing the world, possessing especially strong modal force (167). Although the mind can play a role in logical consequence, any logic generated by our minds must ultimately answer to the world (168).

The question now arises: what kind of law connects conditions/situations with the strong modal force logic is concerned with? Answer: a formal law (168). In the previous example, the formal law connects non-emptiness, union and intersection. So logical consequence is grounded in formal laws governing reality (169). The sentence σ is a logical consequence of Σ iff the *formal skeleton* of the situation delineated by Σ is related to the *formal skeleton* of the situation delineated by σ by a law that guarantees that if the former holds then so does the latter (169). Something similar can be said about commands, which can also be linked by logical implication (e.g. the command to answer all the questions logically implies the command to answer Q1). This account, then, explains why the model-theoretic semantics of logic is sound and how it works (169).

1.3.2 Logical constants and the nature of logicality (170-177)

The role of logical constants is to designate relevant parameters of the formal structures, the laws connecting which ground logical consequence (170). To characterise formality, we aim at an account that is general, informative as well as precise (171). This will be done in a three-step manner.

Step 1. Formal or structural properties hold in all regions. Examples of such properties and relations: being self-identical, transitive, being an intersection, etc. (172). In our system, logical constants designate formal properties or operators and logical forms designate formal structures of objects (formal skeletons of situations); rules of proof encode laws; and models represent formal possibilities.

Step 2. Logical operators are formal in the sense that they distinguish only the pattern delineated by their arguments. So if we replace an argument of a formal operator by any other argument that is its image under some 1-1 replacement of individuals by individuals of any type, the formal operator 'will not notice', i.e. the truth-value will be unchanged (172-3). Examples: the 2nd-level operator of non-emptiness; the 1st-level identity operation; and the intersection operation. The idea that the formality of logic consists in abstracting from differences has roots in Kant and Frege (173-4), or at least affinities with what they said about logic. Sentential/propositional logic is also formal because its operators do not distinguish between atomic sentences with the same truth-value (174).

Step 3. The conception of formality just delineated can be cashed out in terms of the Boolean truth-functional criterion and the invariance under isomorphisms criterion (174-5). An argument structure includes not just the extension of the operator but the underlying universe (175). Sher's notions of structure, operator, argument and argument-structure are all objectual, i.e. worldly rather than linguistic (175). Two argument-structures are isomorphic iff each is the image of the other under some bijection from the universe of one to that of the other. The formality criterion for predicative operators can then be formulated more precisely: an operator is formal iff it is invariant under all isomorphisms of its argument-structures. Identity and both the universal and existential quantifiers are formal in this sense (176). A constant—a linguistic item whose objectual correlate is an operator—is then logical iff (i) it denotes a formal operator, and (ii) it satisfies additional conditions that ensure its proper functioning in a given logical system: i.e. it is a rigid designator, its meaning is exhausted by its extensional denotation, it is semantically fixed (its denotation is determined outside rather than inside models and is built into the apparatus of models), it is defined over all models, etc. (176). The formality criterion licenses infinite cardinality quantifiers, the generalised quantifier 'Most Bs are Cs', the well-ordering quantifier and others as logical. There is some controversy as to whether the logicality condition just given—which we can call LOGICALITY—should be necessary and sufficient, or just necessary (177).

1.3.3 Red herrings and real problems (177-182)

Sher dismisses criticisms of LOGICALITY based on linguistic intuitions and the like. She takes more seriously Feferman's three criticisms (178):

A. The thesis assimilates logic to mathematics, more specifically set theory.

B. The set-theoretical notions involved in explaining [the thesis] are not robust. (Robustness here can be captured by absoluteness.)

C. No natural explanation is given by it of what constitutes the same logical operation over arbitrary basic domains.

Sher's response to A, in brief (178-9): logic may have commitments and the expressibility of CH (the Continuum Hypothesis) is merely an artifact of choosing a particular mathematical theory in Step 3 above. Furthermore, ignorance of CH's truth-value is a fact of life everywhere, logic included.

Sher's response to B, in brief (179-80): features of the background vocabulary invoked in Step 3 like absoluteness have not been shown relevant to the foundational problem of logic. Also, absoluteness is relative to a set theory: seeing as absoluteness is not a robust notion, why should it be relevant here? Finally, Sher notes that non-robustness is a symptom of using a relatively weak logic in which to couch set theory, *viz.* first-order logic; formulating set theory in the stronger logic underwritten by LOGICALITY promises to improve matters.

Sher's response to C, in brief (180-1): even in the case of logic, Feferman accepts operators that intuitively lack unity of meaning, e.g. a 135-place truth-functional connective with a gerrymandered truth table. Logical operators receive their internal unity, Sher insists, from their characteristic trait of distinguishing only formal features of their argument-structures.

Returning to LOGICALITY, Sher claims that it demarcates a maximalist conception of logicality under a unified theme, formality. And on this conception, her criterion provides necessary and sufficient conditions on logicality (182). The resulting expanded conception of logic leads to interesting formal results, as witnessed by work on generalised quantifiers, etc.

1.3.4 A structuralist foundation for logic and its connection to mathematics (183-196)

Sher's theory of logic requires a background theory of formal structure – but which one? Given that no individuals are formal under her criterion, Sher defends—against the nominalist—the reality of formal features, though she does not defend the existence of formal individuals (184). It would be unreasonable to deny the reality of formal features, for instance it would be unreasonable to deny the reality of being self-identical, or of some students instantiating a reflexive, symmetric and non-transitive relation. Even nominalists should find it hard to deny that individuals have these formal features (184). When we look for a theory of the formal, we inevitably turn, then, to mathematics. Accounting for universal and highly necessary laws precisely and in full generality is a job that mathematics is excellently suited to. In short, logic is grounded in the formal and the formal is studied by mathematics.

Mathematics and logic stand in a systematic and fruitful relationship to one another, but differ in subject matter and the formality of their objects (185). On the first point: although logic is concerned with the world, it approaches it through language, by studying inferences, theories and sentences; the direct subject matter of mathematics in contrast is objectual (186). Mathematical notions are logical when construed as higher-level notions but non-logical when construed as lower-level notions, as Tarski observed. You might wonder why mathematics, in studying the formal, which consists of higher-level notions, turns it into a first-theory of individuals. The answer is that we humans find it easier, in discovering regularities and systematising, to work with lower-level concepts (186). Mathematics thus studies formal reality in an indirect fashion (because it treats what are higher-level notions as lower-level ones). If you connect it to reality as Sher has just described, mathematics is true (187).

Mathematics studies the formal through an ontology of structures whose individuals represent formal features of objects through their role in structures, and the laws governing these structures are the mathematical representations of the laws governing formal features of objects (188). Sher's account of mathematics is thus akin to struturalist conceptions. She offers a joint account of maths and logic, which is not logicist, since it does not reduce maths to logic, and does not reduce logic to maths ('mathematism') either. Instead, it grounds logic and maths in a third element: the structural or the formal (189).

The interplay between logic and mathematics is as follows: mathematics provides logic with a background theory of formal structure, whereas logic provides mathematics with an inferential framework for the development of theories (of formal structure and possibly other things) (190). Sher concludes with a discussion of three issues: the normativity of logic, the traits of logic, and error and revision in logic.

Normativity. On the present account, the source of logic's normativity is its truth (191). Cognitive truthfulness is a central value in the intersection of ethics and epistemology, and every discipline that upholds this value is a normative discipline. The normativity of logic, though, has a broader scope than that of physics (192). It is also grounded in a different type of truth than that of physics: formal truth. Logic carries its normativity on its sleeve because it deals with assertions, theories and inferences directly.

Traits. Logic has been traditionally characterised as formal, highly general, topic neutral, basic, modally strong, highly normative, a priori, highly certain, and analytic. Because Sher grounds logic not just in the mind or language, she rejects analyticity as a trait. But with the slight exception of a priority, she accepts all the other traits (192). Formality is logic's key trait. Because logical notions have the highest degree of invariance, they are the most general. As for topic neutrality, logic's formality ensures that it abstracts from other disciplines' subject matters. As a result, logical laws hold in a broader space of possibilities than e.g. physical laws, which means that logic has an especially strong modal force (193). For this same reason, logic is more basic than other disciplines. Also connected to its formality or its strong invariance is the fact that logic is more shielded from new results than other sciences. As for a priority: traditional apriorism requires absolute independence from empirical considerations. Sher's foundational holism, however allows only relative independence. So logic is largely, but not completely, immune to empirical considerations (194). It is quasi-apriori rather than absolutely apriori.

Error and revision in logic. We can go wrong in logic. The formal necessity of the logical laws does not imply that logic is infallible any more than the nomic necessity of physical laws implies that physics is infallible (194). We could, for instance, get the background theory of formal structure wrong, by misdescribing the laws governing formal configurations of properties and situations. Alternatively, we might mischaracterise the logical constants, e.g. by taking 'is taller than' or 'is a property of humans' as such. Or we might mischaracterise the formal structures that exist (195). Pragmatic or methodological considerations might also be thrown into the mix, when no veridical considerations are at stake. Empirical considerations might also be relevant, and 'new experiences of a very fundamental nature', as Tarski put it, might lead us to change our logic. It should be borne in mind, however, that due to its special nature, in logic theoretical considerations will always carry more weight than experiential considerations (195). In her conclusion, Sher mentions other forms of invariantism, owed to Feferman and Bonnay respectively.

2 Miscellaneous related technical points

I mention a few technical points, to make sure everyone is on board with the technical side of things. A more comprehensive recent textbook exposition of this material may be found in chapter 16 of Button & Walsh (2018).

2.1 Tarski's Thesis and the Tarski-Sher Thesis

Suppose D is a domain of some objects. We may construct a hierarchy of classes over this domain: subclasses of the domain, classes of ordered pairs from the domain, ..., subclasses of the previous classes, and so on. As we saw last week, Tarski in his 1966 lecture (published as Tarski 1986) proposed the following criterion for what it is to be a logical class:

> C is a logical class iff C is permutation-invariant on D.

A permutation on D, recall, is a function from D to itself that is one-one and onto (in other words, injective and surjective, i.e. a bijection). A permutation π on D is then lifted to the classes over D. For example, if F is a subclass of D then $\pi(F) = \{\pi(x) : x \in F\}$; if G is a class of subclasses of D then $\pi(G) = \{\pi(F) : F \in G\}$, where $\pi(F)$ is as just defined; and so on. To say that C is permutation-invariant is to say that $C = \pi(C)$ for all permutations π on D. There was some discussion last week as to what D should be: can it vary, or is it fixed as the class of all things?

What has come to be known as the Tarski-Sher Thesis involves a slight modification of this idea. To state it, we need a bit of terminology. Call a *quantifier* an expression whose arguments are a finite number of predicates, each with its own adicity. The extension of an n-adic quantifier Q over a model $\mathcal{M}$ with domain $D_{\mathcal{M}}$ is thus a subset of $\mathbb{P}(D_{\mathcal{M}}^{i_1}) \times \cdots \times \mathbb{P}(D_{\mathcal{M}}^{i_n})$, where i_1 is the first relation's arity, i_2 is the second relation's arity, and so on up to n. For example, the extension of the quantifier $\exists$ in a model $\mathcal{M}$, which we may write as $\exists^{\mathcal{M}}$, consists of all and only the non-empty subsets of the domain, i.e $\exists^{\mathcal{M}} = \{C : C \subseteq D_{\mathcal{M}}$ and C is non-empty$\}$. The extension $\mathsf{Most}^{\mathcal{M}}$ of the quantifier Most in a model $\mathcal{M}$ consists of all and only the ordered pairs of subsets of the model's domain whose intersection is of greater size than the first subset complement the second subset, i.e $\mathsf{Most}^{\mathcal{M}} = \{\langle B, C\rangle : B, C \subseteq D_{\mathcal{M}}$ and $B \cap C > B \setminus C\}$.

The Tarski-Sher Thesis states necessary and sufficient conditions on what it is for a quantifier Q to be logical:

> Q is logical iff Q is bijection-invariant.

To say that Q is bijection-invariant is to say that $Q^{\mathcal{N}} = \pi(Q^{\mathcal{M}})$ for all models $\mathcal{M}$, $\mathcal{N}$ and bijections $\pi : \mathcal{M} \longrightarrow \mathcal{N}$. For example, $\exists$ is logical under this definition, because $\exists^{\mathcal{N}}$ consists of all and only the non-empty subsets of $D_{\mathcal{N}}$; if π is a bijection from $D_{\mathcal{M}}$ to $D_{\mathcal{N}}$, this is precisely the image under π of the set of non-empty subsets of $D_{\mathcal{M}}$. A similar sort of argument shows that Most is logical under the criterion.

A subtlety in the statement of the Tarski-Sher Thesis is whether we formulate it in terms of bijection-invariance or isomorphism-invariance. Actually, this distinction makes no difference: the same quantifiers turn out logical either way. So we can use the notions of bijection-invariance and isomorphism-invariance interchangeably.

2.2 McGee's result

The main technical result in McGee (1996) answers the question: exactly which quantifiers are logical? Let us say that an expression $\phi(R_1, \cdots, R_n)$ defines the n-adic quantifier Q just when:

> in any model $\mathcal{M}$, the n-tuple $\langle R_1^{\mathcal{M}}, \cdots, R_n^{\mathcal{M}}\rangle$ is an element of $Q^{\mathcal{M}}$ iff $\mathcal{M}$ satisfies $\phi(R_1, \cdots, R_n)$

where predicate R_k is interpreted in the model $\mathcal{M}$ as $R_k^{\mathcal{M}}$ (for $k = 1$ to n). For example, the quantifier $\exists$, which as we saw above is logical according to Tarski-Sher, is definable in first-order logic by the formula $\exists x Rx$ (here $n = 1$): in any model $\mathcal{M}$, $\mathcal{M}$ satisfies the formula $\exists x Rx$ iff $R^{\mathcal{M}}$ is non-empty. (This is hardly surprising, of course, since first-order logic contains the existential quantifier.)

McGee's result may then be stated as follows:

$$\begin{gathered}\text{the quantifier } Q \text{ is isomorphism-invariant}\\ \Updownarrow \\ \text{for all cardinals } \kappa\text{, there is a formula } \phi_\kappa \text{ of } \mathcal{L}_{\infty\infty} \text{ such that for all}\\ \text{models } \mathcal{M} \text{ of cardinality } \kappa \text{ (i.e. } |D_{\mathcal{M}}| = \kappa\text{)}, Q^{\mathcal{M}} = (\phi_\kappa)^{\mathcal{M}}\end{gathered}$$

Notice that the expression ϕ_κ that defines Q on models of size κ (where κ is any cardinal, i.e. its range is $1, 2, \cdots, n, \cdots, \aleph_0, \aleph_1, \cdots$) need not be the same for different κ. So for instance an expression that acts as the universal quantifier on some domains and as the existential quantifier on other domains counts as logical so long as it acts in the same way on all domains of the same size.

2.3 Feferman's invariantism

In a series of publications, Solomon Feferman presented an invariantist account of logical constants rival to the Tarski-Sher thesis. Restricting attention to quantifiers as above, Feferman proposed a *strong-homomorphism-invariance* criterion of logicality, which I now sketch.[1]

A *strong homomorphism* h between models $\mathcal{M} = \langle D_{\mathcal{M}}, R_1^{\mathcal{M}}, \cdots R_n^{\mathcal{M}}\rangle$ and $\mathcal{M}^* = \langle D_{\mathcal{M}^*}, R_1^{\mathcal{M}^*}, \cdots R_n^{\mathcal{M}^*}\rangle$ is a map from $D_{\mathcal{M}}$ to $D_{\mathcal{M}^*}$ that, in Feferman's application, is stipulated to be onto and which is such that, for each i from 1 to n and any sequence $x_1, \cdots x_k$ of individuals drawn from $D_{\mathcal{M}}$, $R_i^{\mathcal{M}}(x_1, \cdots x_k)$ iff $R_i^{\mathcal{M}^*}(h(x_1), \cdots h(x_k))$.[2]

Given these definitions, the quantifier Q is strong-homomorphism-invariant just when, for any such homomorphism h, $Q^{\mathcal{M}}$ holds of the n-tuple $R_1^{\mathcal{M}}, \cdots R_n^{\mathcal{M}}$ in the first model (with domain $D_{\mathcal{M}}$) iff $Q^{\mathcal{M}^*}$ holds of the n-tuple $R_1^{\mathcal{M}^*}, \cdots R_n^{\mathcal{M}^*}$ in the second model (with domain $D_{\mathcal{M}^*}$).[3]

[1] See Feferman (1999, 2010). Sher (2013, p. 196 fn. 50) reports that in 2011, five years before he died, Feferman had given up this alternative form of invariantism, though still stood by his criticisms of isomorphism invariance, reported in Sher (2013).

[2] Here k is R_i's adicity. The obvious clauses for function symbols and constants may be added.

[3] See Bonnay (2008) for some results relating to Feferman's proposal. I have skated over some exegetical nuances that do not affect the philosophical thrust of the discussion. In particular,

It is an immediate consequence of Feferman's invariantist criterion that cardinality quantifiers of size ≥ 2 are not logical, as Feferman himself and many others have noted. As an illustration, let the domain $D_{\mathcal{M}}$ be $\{a, b\}$, where $a \neq b$, and the domain $D_{\mathcal{M}^*}$ be $\{c\}$. There is of course only one strong homomorphism from $D_{\mathcal{M}}$ to $D_{\mathcal{M}^*}$, which maps each of a and b to c. Now suppose we consider the models $\mathcal{M} = \langle D_{\mathcal{M}}, R^{\mathcal{M}} \rangle$ and $\mathcal{M}^* = \langle D_{\mathcal{M}^*}, R^{\mathcal{M}^*} \rangle$ in which $R^{\mathcal{M}}$ and $R^{\mathcal{M}^*}$ are unary properties that hold of all the entities in their respective domains, i.e. $R^{\mathcal{M}} = D_{\mathcal{M}}$ and $R^{\mathcal{M}^*} = D_{\mathcal{M}^*}$. Let $\exists_2$ be the property of type $((i))$ which holds of a property of type (i) iff the latter holds of at least two individuals in the domain. It's easy to see that $R^{\mathcal{M}} \in \exists_2^{\mathcal{M}}$ although $R^{\mathcal{M}^*} \notin \exists_2^{\mathcal{M}^*}$. A similar argument shows that $\exists_\kappa$ is not strong-homomorphism-invariant, where κ is any cardinal ≥ 2 and $\exists_\kappa$ applies to a subset of the domain $D_{\mathcal{M}}$ iff $R^{\mathcal{M}}$ holds of at least κ individuals in the domain.

Equally noteworthy is the fact that identity is not strong-homomorphism-invariant. Now since identity is a relation of type (i, i) and is therefore not a quantifier as defined above, the criterion of logicality under discussion does not apply to it. But we can concoct a homorphism-invariance criterion for entities of identity's type entirely in the spirit of Feferman's original one. We may stipulate that $\mathcal{M}$ is strong-homomorphism-invariant to $\mathcal{M}^*$ just when there is a map from $\mathcal{M}$'s domain to $\mathcal{M}^*$'s that is onto and respects the interpretation of any constant and function symbols. Then it is easy to see, using the same example as in the previous paragraph, that the interpretation of identity under the homomorphism $h : \{a, b\} \to \{c\}$ is not preserved: it is false that all pairs $\langle x, y \rangle$ with $x, y \in D_{\mathcal{M}}$ stand in the identity relation in the first model iff $\langle h(x), h(y) \rangle$ stand in the identity relation in the second model, since $a \neq b$ yet $h(a) = h(b)$. Thus identity is not logical on the strong-homomorphism-invariance account. Feferman is entirely clear about this consequence and accepts it.

Feferman first proposed strong-homomorphism invariance as a criterion for operations of unary type, i.e. of type $((i))$, and considered their closure under λ-definability. It seems to me, as it did to Feferman later, that there is no good reason to restrict invariantism to operations of this particular type and no other. See Bonnay (2008, pp. 43-4) for detailed criticism of Feferman's earlier proposal.

References

[1] D. Bonnay (2008), 'Logicality and Invariance', *The Bulletin of Symbolic Logic* 14, pp. 29-68.

[2] T. Button & S. Walsh (2018), *Philosophy and Model Theory*, Oxford University Press.

[3] S. Feferman (1999), 'Logic, logics and logicism', *Notre Dame Journal of Formal Logic* 40, pp. 31-54.

[4] S. Feferman (2010), 'Set-theoretical invariance criteria for logicality', *Notre Dame Journal of Formal Logic* 51, pp. 3-20.

[5] V. McGee (1996), 'Logical Operations', *Journal of Philosophical Logic* 25, pp. 567-80.

[6] G. Sher (2013), 'The Foundational Problem of Logic', *The Bulletin of Symbolic Logic* 19, pp. 145-98.

[7] A. Tarski (1986), 'What are Logical Notions?', J. Corcoran (ed.), *History and Philosophy of Logic* 7, pp. 143-154.