Ramsification and Semantic Indeterminacy

Is it possible to maintain classical logic, remain close to classical semantics, and yet accept that language might be semantically indeterminate? The article gives an affirmative answer by proposing an application of Ramsification: Ramsifying classical semantics yields a new semantic theory that remains much closer to classical semantics than supervaluationism but which avoids the problematic classical presupposition of semantic determinacy. The resulting Ramsey semantics is developed in formal detail, it is shown to supply a classical concept of truth and to fully support the rules and metarules of classical logic, and it is applied to the interpretation of vague terms, the interpretation of theoretical or open-ended terms in mathematics and science, and to higher-order vagueness. The theory also demonstrates how diachronic or synchronic interpretational continuity across languages is compatible with semantic indeterminacy.

Since the publication of Ramsey's (1929) "Theories", the *Ramsification* of scientific theories has become a major tool in theory reconstruction and interpretation. It has been applied to argue for the instrumental character of theories (which is one way of reading "Theories", cf. Sahlin 1990), to determine the synthetic content of theories (Carnap 1966) or the empirical claim made by theories (Sneed 1971), to explicate structural realism about science (Worrall & Zahar 2001) and various kinds of functionalism, including functionalism about mental terms (Lewis 1972) and about truth (see, e.g., Lynch 2000, Wright 2010).

In what follows, I will argue that the Ramsification of *classical semantics* can also help us overcome problems of semantic indeterminacy that result from the vagueness of ordinary terms in natural language or from the theoreticity and open-endedness of technical terms in mathematical and scientific language. Call the result of that Ramsification *Ramsey semantics*, which, I want to show, saves all of classical logic and almost all of classical semantics, while embracing semantic indeterminacy without going down the epistemicist or supervaluationist road. The upshot will be: if one wants to be prepared for semantic indeterminacy—unlike epistemicism—and if one also aims to stay closer to classical semantics than supervaluationism, then one ought to Ramsify classical semantics.[1]

0. Introduction

Before developing the theory in full detail, let me present the idea in a nutshell by means of an example, which will also allow me to introduce the different parts of the paper.

Say, we are interested in stating the semantics of the language of the real number calculus. At some point we want to introduce the semantic interpretation of the function symbol '√' for the (principal) square root of a real number, or, alternatively, the interpretation of the predicate '*Sqrt*', where '*Sqrt*(y, x)' is meant to express that y is the (principal) square root of x. Within the calculus of real numbers, mathematicians standardly define (principal) square roots only conditionally (see Suppes 1957, Section 8.6 for a survey), that is, under the condition that x is a non-negative real:

For all x, y: if *Real*(x), *Real*(y), and $x \geq 0$, then

$$\sqrt{x} = y \text{ (that is, } Sqrt(y, x)\text{) if and only if } y^2 = x \text{ and } y \geq 0.$$

[1] This motivation for Ramsifying classical semantics is orthogonal to instrumentalist or functionalist motivations: the point of Ramsey semantics is neither to show that talk of interpretation is merely instrumental nor to convey insights into the "nature" of truth, but to deal with semantic indeterminacy. In contrast, e.g., Wright's (2010) paper on Ramsification and monism vs. pluralism about truth does not apply Ramsification for the sake of doing semantics and in fact presupposes semantic *determinacy* (see Wright 2010, p. 272).

Clearly, this leaves the square roots of negative reals undefined: nothing is said whatsoever about '$\sqrt{x}$' or '*Sqrt*(y, x)' in the case when 'x' denotes a negative real number. Accordingly, on the metalevel, "the" intended interpretation of '√' and '*Sqrt*' should only be partially determined by what might be called the "metasemantic facts" of mathematical concept formation. And although classical (meta-)semantics does not recognize the possibility of a partially determined intended interpretation (about which more in Section 1), mathematicians still manage to reason successfully about square roots in classical logic. How is that possible?

If one were to extend epistemicism about vague terms (see e.g. Williamson 1992, 1994b, Sorensen 2001) to the present case of what seems to be semantic indeterminacy without vagueness, the answer would be: there might *seem* to be a factual gap left by the conditional definition of '√' and '*Sqrt*', but really the usage of these terms by mathematicians—and possibly some further facts—somehow conspire to determine a unique fully defined function on the reals. There is no guarantee anyone might know how that gap is filled in, but nevertheless classical semantics can be applied just as usual, and hence it is no wonder that mathematicians are able to reason classically about square roots.

But how plausible is that? As far as the predicate '*Sqrt*' is concerned, one might try to adapt a suggestion by Williamson (1994) and argue that, since the conditional definition does not do enough to make *Sqrt*(y, x) true for any negative real x, "it *thereby* does enough to make it false" (Williamson 1994, p. 213): if so, for every negative real x, there is a fact of the matter correctly described by $\forall y(Real(y) \rightarrow \neg Sqrt(y, x))$ and the semantic interpretation of '*Sqrt*' is determined completely after all. But what would be the argument for this other than that it saves the classical presupposition of a unique fully determined intended interpretation? And wouldn't the proposal suggest too much? For by the same token, the same conditional definition "For all x, y: if *Real*(x), *Real*(y), and $x \geq 0$, then…" interpreted over the domain of *complex* numbers would not do enough to make *Sqrt*(y, x) true for any negative real-valued complex number x either, but thinking that thereby $\forall y(Complex(y) \rightarrow \neg Sqrt(y, x))$ would be determined true would run counter to the standard conservative extension of the conditional definition to the full definition of square roots (thought not of *principal* square roots) on the complex numbers, according to which negative reals x *do* have (complex) square roots.

When I will summarize the tenets of classical semantics in Section 1, I will reconsider its central metasemantic presupposition of the existence of a unique factually determined intended interpretation in more detail, and I will present three examples to the effect that the presupposition is likely to be false: one is about semantic indeterminacy by vagueness in natural language, while the other two examples concern semantic indeterminacy of mathematical and scientific languages without vagueness. Since it would be dangerous to rest semantics on what is likely a false presupposition, we should find an alternative way of making semantic indeterminacy compatible with a semantics that saves classical logic and its applications—which is the goal of this paper.

As I will argue from Section 2, Ramsifying classical semantics with respect to its theoretical term 'intended interpretation' is going to deliver that combination. While it is less common for semanticists to reflect on the logical form in which their semantic theory is stated, Ramsifying a theory (in this case, classical semantics/metasemantics) by replacing some of its terms by variables (in this case, 'intended interpretation' by 'F'), and existentially closing the resulting formula (in this case, by 'there is an admissible interpretation F, such that…') is a well-known procedure in the philosophy of science. The point of the paper is to put these insights from the philosophy of science to good semantic use. Sections 2 and 3 will show that the semantic consequences of the resulting Ramsey semantics resemble those of classical semantics. Sections 4, 5, and an Appendix will demonstrate how Ramsey semantics copes successfully with the semantic indeterminacy of vague terms, such as in the Sorites Paradox and in higher-order vagueness. Section 6 will demonstrate the same for cases of semantic indeterminacy that do not result from vagueness but from theory-ladenness and conceptual change in mathematics and science. The same section will also show that Ramsey semantics offers a new understanding of inter-theoretical interpretational continuity that is compatible with semantic indeterminacy. In particular, Ramsey semantics is compatible with conceptual extensions, including the extension of the interpretation of '√' and '*Sqrt*' from the reals to the complex numbers.

But maybe it would not even be necessary to beat a new path towards an "almost classical" semantics while allowing for semantic indeterminacy: for isn't supervaluationism (see e.g. Fine 1975, Keefe 2000) doing precisely that? In the square root example, supervaluationists might reconstruct the conditional definition of real-valued square roots model-theoretically with the help of a space *Adm* of semantically "admissible" classical interpretations of the language of the calculus that satisfy the conditional definition from above. Admissible interpretations serve more commonly as "precisifications" of imprecise vague terms, but they can be used to fill other kinds of metasemantic gaps, too, including those left by precise conditional definitions. On that basis, supervaluationists would proceed to re-define classical truth as super-truth: truth with respect to *all interpretations* in *Adm*. Consequently, they would count *Sqrt*(*2*, *4*) as (super-)true, *Sqrt*(*2*, *9*) as (super-)false, and (*Sqrt*(*2*, *-4*) ∨ ¬*Sqrt*(*2*, *-4*)) as (super-)true, although both *Sqrt*(*2*, *-4*) and ¬*Sqrt*(*2*, *-4*) would be neither (super-)true nor (super-)false and hence lack a truth value.

Ramsey semantics will start from the same class *Adm* of admissible interpretations as supervaluationism, but it will postulate *the existence of an interpretation in Adm*, *such that truth and falsity for all sentences of the object language are determined from that interpretation in the standard Tarskian manner*. This metalinguistic existence postulate will turn out to be the Ramsey sentence for classical semantics, as will be explained in Section 2. Section 3 will prove the resulting Ramsey semantics to remain much closer to classical semantics than supervaluationism: in contrast with supervaluationist entailment, the rules and metarules of classical logic will be validated by Ramsey semantics in precisely the same way as they are validated by classical semantics. And unlike super-truth, the concept of truth will remain classical by satisfying all T(ruth)-biconditionals, being compositional with respect to all classical logical operators, and avoiding truth value gaps. For instance, in the example above, *Sqrt*(*2*, *4*) would be evaluated as true, *Sqrt*(*2*, *9*) as false, (*Sqrt*(*2*, *-4*) ∨ ¬*Sqrt*(*2*, *-4*)) as true, *Sqrt*(*2*, *-4*) would be either true or false, and ¬*Sqrt*(*2*, *-4*) would be either true or false, too, even though neither *Sqrt*(*2*, *-4*) nor ¬*Sqrt*(*2*, *-4*) would be determined to be true by the metasemantic facts and thus would not be determinately true. *In Ramsey semantics, truth may outrun the facts*.

Where epistemicists have a classical concept of truth and maintain that the metasemantic facts fit that concept by determining a unique intended interpretation, and where supervaluationists accept that the metasemantic facts might not determine a unique intended interpretation and fit a non-classical concept of truth to these incomplete states of affairs, Ramsey semantics will occupy a position "in between": the metasemantic facts might not determine a unique intended interpretation but the concept of truth will remain classical. The details of the theory, including explanations of its terms (e.g. 'metasemantic fact'), a formalization of its postulates with the help of metalinguistic Ramsey sentences and epsilon terms, and a discussion of its consequences and extensions (in the final Section 7) will be developed from Section 1.

Along the way, there will various pointers to overlaps with the existing literature, in particular to: Carnap (1959, 1961, 1966) on the reconstruction of scientific theoretical terms by Ramsification and epsilon terms, McGee and McLaughlin (1994) on semantic determination and the combination of supervaluationism for vague terms with a disquotationalist theory of truth, and Breckenridge and Magidor (2012) on arbitrary reference and irreducible semantic facts. Although none of them actually states or advocates the metalinguistic Ramsification of classical semantics, they come close to it, and in many ways the present theory is going to continue threads of reasoning begun by them.

1. Classical Semantics and the Challenge from Semantic Indeterminacy

Let me start by summarizing classical semantics in the broad sense of the term. By that I mean classical formal semantics in the Tarskian-Carnapian-Montagovian-… model-theoretic tradition that includes both classical semantics in the narrow sense of the term (the statement of the classical semantic rules) and classical metasemantics (the explanation in virtue of what these classical semantic rules hold). Not every semanticist is going to agree with every detail of my summary, but they should at least agree with something in that ballpark.

For simplicity, I will concentrate on extensional semantics both in this section and throughout the paper, though a similar story could be told for intensional semantics.[2] For linguists, intensional semantics is much more interesting than extensional semantics, as only intensional semantics allows for the formal reconstruction of, e.g., the update of an agent's belief state or of the common ground of a conversational context by the *intension* of an asserted sentence, or for stating the truth conditions of sentences with *intensional operators*. But extensional semantics still serves as the basis of intensional semantics and continues to attract attention for its own sake in philosophical semantics (as in discussions of the concept of truth), which is why restricting our attention to it will leave us with an interesting enough case to consider. I will have to postpone topics such as intensional indeterminacy, the assertion of intensionally indeterminate sentences, and the pragmatic aim of such assertions to follow-up work on intensional Ramsey semantics.

Classical extensional semantics builds on the analysis and formalization of the syntax of some fragment of natural, mathematical or scientific language as given at some point in time; let the result of that formalization be L, which—as all other languages in this paper—is supposed to be an *interpreted* formal language. For illustration, let us take the syntax of L to be that of a standard first-order extensional language with identity, and let us assume that L is not itself concerned with semantic matters, so that the vocabulary of the object language L does not itself include semantic terms, such as 'true'.

A (classical) interpretation F of L is a function that assigns references/extensions to the members of the descriptive vocabulary of L, based on a universe $Uni(F)$ (which is a non-empty class); e.g., the extension $F(P)$ of a unary predicate P is a subclass of $Uni(F)$. A variable assignment s relative to F is a mapping that assigns values to the variables of L, where these values are taken from $Uni(F)$. (I will ignore complex singular terms.) I am going to leave open here whether the metalinguistic variable 'F' is a first-order variable for set-theoretic functions with set-sized domains, or whether it is a first-order or second-order variable for functions in a more general sense, such that the domain of a function might have the cardinality of a proper class. The same holds for '$Uni(F)$', which might denote a set or a proper class of individuals.[3]

Subsequently, the truth conditions of formulas A in L are specified by (classical) semantic rules by which the (classical) truth or satisfaction of a formula A is defined relative to an interpretation F and a corresponding variable assignment s. In particular:

(1) For all F, s:

[2] The Ramsey sentence for classical intensional semantics would begin with: 'There is an admissible *intensional* interpretation, that is, an assignment of references and *intensions* to the descriptive terms of the object language, such that…'.

[3] *Uni*(*F*) might even be the class of *all first-order individuals whatsoever*, if classical semantics is formulated in a suitable higher-order language: see Williamson (2003).

$F, s \vDash P(a)$ iff $F(a) \in F(P)$;
$F, s \vDash \neg A$ iff $F, s \nvDash A$;
$F, s \vDash C \vee D$ iff $F, s \vDash C$ or $F, s \vDash D$;
$F, s \vDash \exists x A$ iff $\exists d \in Uni(F)$, such that $F, s\frac{d}{x} \vDash A$.

(Analogously for n-ary predicates with $n>1$ and for the other classical logical connectives. $s\frac{d}{x}$ is like s except that d is assigned to x; for *sentences* A, that is, for closed formulas, reference to s may be omitted. 'iff' is short for 'if and only if'.)

The semantic rules are classical in virtue of their compositional format and the classical manner in which they treat the classical logical operators in the logical vocabulary of L. The logic of semantics itself, that is, of the semantic metatheory to which e.g. definition (1) belongs, is assumed to be classical, too, and a sufficiently strong deductive system of classical set/class theory or higher-order logic is presupposed as well.

The logical consequence relation of (classical) logic (as applied to L) results from quantifying universally over all interpretations of L and all corresponding variable assignments:

(2) $A_1,\ldots,A_n \vDash C$ iff $\forall F, s$: if $F, s \vDash A_1,\ldots,A_n$, then $F, s \vDash C$.

Next, amongst all possible interpretations F of L, one presupposes there to be *the intended or actual (classical) interpretation I* that involves *the intended or actual universe Uni*(I) of objects and which assigns *the intended or actual references/extensions* to the members of the descriptive vocabulary of L. The actual truth values of formulas A in L are defined with the help of I and the semantic rules from before. In particular, one defines:

(3) for all sentences A in L: A is true iff $I \vDash A$.[4]

Obviously, this does not mean that semanticists would be interested in *all* aspects of truth—e.g., *finding out* which sentences of L are true normally involves empirical or mathematical investigation far beyond semantics—but they are interested in *defining* or *axiomatizing* truth and in its *semantic properties*. It is for such purposes that I is important, at least so far as extensional semantics is concerned, since I does not just fix the intended references and extensions of terms but also the intended extensions of sentences, that is, their (actual) truth values.

I itself is supposed to be determined jointly by

(i) all linguistic facts concerning the competent usage of predicates and singular terms (individual constants, individual variables, function terms) in L, and
(ii) all non-linguistic facts that are relevant as to whether the atomic formulas in L are satisfied.

The facts in (i) are meant to determine the truth conditions of atomic formulas and the universe of discourse over which the individual variables (attached to quantifiers) range. Or in other

[4]More precisely, *for each context c*, one assumes there to be the actual or intended interpretation I_c of *L in c*, and then one uses I_c to define truth of *A in c*. For simplicity, I will suppress reference to contexts in what follows, which is of course not to say that classical semantics (or supervaluationist semantics or Ramsey semantics) could not or should not be combined with contextual parameters, such as those relevant to the extension of vague predicates, and with corresponding operations of context change. (See e.g. Shapiro 2008a for a contextualist version of supervaluationist semantics of vague terms, and see e.g. Pagin 2010 for a contextualist version of a classical semantics of vague terms.)

words: these facts might be said to determine the *intensions* of predicates and singular terms in *L* (including, possibly, rigid intensions). For instance, in the example of the principal-square-root predicate '*Sqrt*' from the introduction, the linguistic fact in question would be that mathematicians understand '*Sqrt*' to be partially defined on the set of real numbers by the conditional definition stated in Section 0.

The facts in (ii) are supposed to determine whether the truth conditions determined by (i) are actually met over the universe that is determined by (i). These non-linguistic facts yield the "worldly" contribution that, if taken together with intensions determined by the linguistic facts in (i), determines the extensions and references of predicates and singular terms in *L* at the "actual world". In the square-root example, the relevant facts would be mathematical facts concerning which real numbers y are such that $y^2 = x$ and $y \geq 0$ for a given non-negative real number x: e.g., one such fact would be described by '$2^2 = 4$ and $4 \geq 0$'. More generally, since the best present-day science and mathematics are our best present approximations of what the actual world is like, one may think of the facts in (ii) as obtaining states of affairs which, at least in principle, can be described by the best available language(s) of mathematics and science. For the same reason, it should be possible to describe the facts in (ii) in a manner that is also precise enough according to the best present mathematical or scientific standards. This does not mean that (ii) would require an absolute notion of "precise fact" (as McGee and McLaughlin 1997, p. 231 worry)—if anything, it would be the other way around: one might call the facts in (ii) 'precise (enough at this point in time)' in virtue of mathematicians and scientists describing them by the best linguistic means they have available (by their present standards).

The metasemantic determination of *I* by (i) and (ii) above is assumed to be governed by

(iii) all metasemantic laws taken together that concern the atomic formulas, and hence the predicates and singular terms, of *L*.[5]

In the square-root example, the respective metasemantic law might be expressed by: *For all objects d and d'*, *if mathematicians understand 'Sqrt' to be defined on the set of real numbers by the conditional definition* 'For all x, y: if *Real*(x), *Real*(y), and $x \geq 0$, then: *Sqrt*(y, x) if and only if $y^2 = x$ and $y \geq 0$', *and d'* (*for* 'y') *and d* (*for* 'x') *are such that the definiens of that conditional definition applies to them, then the pair* <*d'*, *d*> *is a member of the intended interpretation I*(*'Sqrt'*). E.g., for $d = 4$ and $d' = 2$ it follows: since the first conjunct of the embedded antecedent describes a fact in (i) (the fact that mathematicians use the relevant conditional definition), and since the second conjunct of the embedded antecedent describes a fact in (ii) (the fact that $2^2 = 4$ and $4 \geq 0$), the law implies that <2, 4> is a member of *I*(*'Sqrt'*), which yields the expected constraint on the intended interpretation of '*Sqrt*'.

Considering some more prominent examples, another metasemantic law in (iii) might be the following quasi-Kripkean one (cf. Kripke 1980): *for all proper names a in L, for all objects d*, *if present usage of a in L is suitably causally connected to an act of baptism in which d was named a, then* $d \in Uni(I)$ *and* $I(a) = d$. Consider any concrete instantiation of that universal claim: if the corresponding 'if'-part describes one of the linguistic facts in (i), then the law jointly with that fact determines the constraint on the intended interpretation *I* of *L* that is described by the 'then'-part. Or yet another law in (iii) might be expressed in the following

[5] This account closely resembles McGee & McLaughlin's (1994, pp. 210f) "psycholinguistic" account of "definite application/satisfaction". More about determination can be found in McGee & McLaughlin (1998). I prefer 'metasemantic determination' over their term 'semantic determination', but I will use their term 'constraint' (McGee & McLaughlin 1994, p. 225) in much the same way as they do. Most authors working on vagueness and semantic indeterminacy would at least accept that semantic meaning supervenes on, or is determined by, use (but see Kearns & Magidor 2012 for a contrary view). Use corresponds to (i) above, (ii) adds what is needed to determine *referential/extensional* semantic meaning, and the laws in (iii) govern the determination.

quasi-Putnamian way (cf. Putnam 1975a): *for all kind terms K in L*, *for all objects d* and *d’*, *if K was collectively specified in L by pointing at d while being interested in the physical structure of d, and if d’ has the same physical structure as d, then: if d’*$\in$*Uni*(*I*) *then d’*$\in$ *I*(*K*). Once again, consider any concrete instantiation of that universal statement: if the first conjunct of the corresponding ‘if’-part describes one of the linguistic facts in (i), whilst the second conjunct of the ‘if’-part describes one of the non-linguistic facts in (ii), then the law and the two facts jointly determine the constraint on *I* that is described by the ‘then’-part.

In terms of an analogy, metasemantic determination in classical semantics is supposed to be much like a physical or economic quantity *z* being determined from other such quantities *x* and *y* in the sense that *z* is a function *f* of *x* and *y*: *z* is like the intended interpretation *I*, *x* and *y* are respectively like (i) and (ii) above, and the law $z = f(x, y)$ corresponds to (iii). E.g., *x* being of some value, say, 3, and *y* being of some value, say, 1, determine that *z* is of the value $f(3, 1)$, which is analogous to (i)-(iii) determining *I*.

Let us call the facts in (i) and (ii) *metasemantic facts* (for *L*). They are metasemantic in the sense that the linguistic expressions in *L* have their intended references and extensions in virtue of them and the metasemantic laws in (iii). Sometimes I will also drop the qualification ‘metasemantic’ and just refer to *facts*.

Let us call the constraints that (i)-(iii) jointly impose on the interpretation of *L the existing metasemantic constraints* on the interpretation of *L*. One may think of these constraints as being summed up by a theory that states what *I* must be like in view of (i)-(iii). It may be possible to (at least approximately) express some of these constraints linguistically, as in the examples for (iii) above, and if one does so, one might use terms, such as ‘baptize’, ‘specify’, ‘interest’, ‘point to’, ‘describe’, ‘rule’, ‘practice’, ‘intention’, ‘intension’, ‘think’, ‘cause’, ‘historical’, ‘kind’, ‘same as’, ‘physical structure’, ‘natural’, ‘division of labor’, ‘expert’, ‘intuition’, ‘define’, ‘axiom system’, ‘observe’, ‘measure’, and more. (I am not saying all these terms are sufficiently clear or that each of them is required for that purpose, just that classical semantics is in principle compatible with all sorts of metasemantic constraints.) But there is no guarantee that every existing metasemantic constraint can be expressed easily in such terms or in others. Indeed, Williamson (1994, p. 209) worries that “Meaning may supervene on use in an unsurveyably chaotic way”, that is: while interpretation may be determined by the metasemantic facts, the ways in which this comes about—the metasemantic laws—might be extremely sensitive to how certain parameters are set (imagine a chaotic dynamic system of differential equations) and therefore also difficult to survey and to express in language.

For that reason, it is more helpful to summarize the existing metasemantic constraints taken together as a theory in the sense of the *non-statement* view of scientific theories (see e.g. Suppes 1967), that is, as a *class of interpretations* (*models*) of *L*: let us call that class ‘*Adm*’. (‘*Adm*’ is short for ‘admissible’, which I borrow from Fine 1975 and supervaluationist semantics—about which more from Section 2.) The existing metasemantic constraints on the interpretation of *L* show up in what *Adm* is like. Exploring these constraints and hence *Adm* goes beyond extensional semantics and even beyond semantics as a whole: the study of (i) from above belongs to pragmatics and intensional semantics, the study of (ii) is normally not the subject matter of linguistics at all (but, e.g., of mathematics or physics), and the study of (iii) lies at the interface between all of the previous subjects and extensional semantics.

Following a reasonable divide-and-conquer strategy, classical semantics is therefore not itself concerned with axiomatizing or testing metasemantic hypotheses on ‘*Adm*’ but merely *presupposes* that the metasemantic constraints (whatever they are like) yield a particularly strong and restrictive theory: the *singleton* set $Adm = \{I\}$, from which the intended interpretation *I* can be defined as the sole member of *Adm*. Nor is the claim that it is *known* exactly what *I* is like—one merely *presupposes* that there is a classical interpretation *I*, such that *I* conforms to the existing metasemantic constraints ($I \in Adm$), and where *I* is in fact determined uniquely by these constraints ($Adm = \{I\}$). Putting these metasemantic presuppositions of classical semantics on record:

(4) $\exists! F(F \in Adm)$ and $I \in Adm$ (where '*Adm*' is understood as explained before).

What classical extensional semanticists do is to formulate and test hypotheses about the (classical) semantic rules of *L* and about semantically salient aspects of the (classical) intended interpretation *I* of *L*, while presupposing (4) in the background. If all goes well, these hypotheses lead to empirically successful predictions and explanations of some linguistic phenomena concerning *L*. That is classical extensional semantics explained in a nutshell.[6]

So far, so good. But there is a problem[7]: while it is common to speak of *the* intended interpretation *I* (to which the function term '*I*' supposedly refers), and hence to presuppose that there is one and only one intended classical interpretation, *we lack good reasons for believing that the existing metasemantic constraints on the interpretation of a language will always determine a unique such interpretation*. That is: *Adm* might well happen not to be a singleton class. Let me illustrate this by means of three examples: one from natural language, another one from mathematical language, and a third one from scientific language.

Example 1: As is well known, *vague* terms may cause trouble: terms for which there seem to be "indeterminate" or "imprecise" borderline cases beyond the clear-cut positive and clear-cut negative ones. I will concentrate on what Alston (1967) calls *degree vagueness*, but the general conclusions should apply to all kinds of semantic vagueness.[8] Let *B* be a predicate of *L* that formalizes the English term 'bald', or rather, for simplicity, 'is the number of hairs on the head of a bald person', where I will presuppose that every two people with the same number of hairs either both count as bald or both do not, and where I will ignore the distribution of hairs. ('*Bald*(*Peter*)' may then be understood as: there exists an *x*, such that *number-of-hair-on-head-of*(*Peter*) = *x* and *B*(*x*).)

Plausibly, the following claims express some existing metasemantic constraints on the intended interpretation of *B*, that is, the actual extension of *B*: any person with 0 hairs on their scalp belongs to the extension of *B* ($0 \in I(B)$); any person with 100000 hairs does not ($100000 \notin I(B)$); if one person has more hair on their head than a second one, then, if the former

[6]More precisely, that is how classical semantics works as a descriptive empirical discipline. There is also the more normative philosophical project of semantics as the rational reconstruction of meaning with the aim of improving language and interpretation through formal languages (see Partee 2011, p. 21). The main part of the present paper deals with classical semantics in the former sense and hence belongs to the philosophy of semantics as a part of the philosophy of science. I will turn briefly to the rational reconstruction of meaning at the end of Section 6.

[7]Classical semantic has of course many additional problems. For instance: its traditional formalization of the indicative if-then from natural language by the material if-then connective of classical logic is questionable; some linguistic phenomena call for the semantic rules to take other than classical form, such as in dynamic semantics; and in view of semantic paradoxes such as the Liar paradox, it is an open question whether classical semantics is able to satisfyingly interpret a primitive type-free truth predicate for and within a language with sufficient syntactic resources. But these problems are largely orthogonal to the problem of semantic indeterminacy.

[8]I will not deal at all with vague *objects*, which would constitute a form of metaphysical vagueness that is orthogonal to the topic and aim of this paper. Assuming that objects themselves are not a source of vagueness, and assuming the same holds for predicates and singular terms as syntactic objects (e.g. the string *b-a-l-d*), mappings from predicates and singular terms to (sets of tuples of) objects will be perfectly unproblematic, too. Hence, classical interpretation mappings will not themselves be affected by vagueness, only *which interpretation is meant to be "the" intended one*. Similarly, intensions (functions from possible worlds to references/extensions) in intensional semantics are unproblematic, so long as worlds do not themselves give rise to vagueness either. If so, concepts/properties in the sense of intensions will not themselves be vague, though the predicates expressing them may be so in the sense that what is meant by *"their" intended intension* may be affected by vagueness.

belongs to the extension of B, the same holds for the latter (for all m, n, if $m>n$ and $m \in I(B)$ then $n \in I(B)$)[9]; $Uni(I)$ includes the required natural numbers (e.g., $0 \in Uni(I)$); and so on.

But even if one combines all linguistic competent-use facts concerning L (e.g. pertaining to all rules for B-usage and all competent B-assertions ever made) with all relevant non-linguistic facts (e.g. concerning the number of hairs those people have had about whom competent B-assertions have or could have been made), and if one takes these facts together with all metasemantic laws for L, the following seems *unlikely*, at least at first glance: for each number n, either the existing metasemantic constraints determine that $n \in I(B)$ (and hence, by (1), the truth of $B(n)$), or they determine that $n \notin I(B)$ (and hence, by (1) again, the truth of $\neg B(n)$). (I am using 'n' both as a metalinguistic and an object-linguistic numeral here.) For, at least *prima facie*, it is *plausible* that there are borderline cases n to which one may competently ascribe B, but to which one may also competently refrain from ascribing B and indeed ascribe $\neg B$. It does not seem to be just our knowledge of I that is incomplete or empirically underdetermined: rather, it seems that no competent speaker of L would be right in correcting or criticizing either of the two ascriptions even if they knew all metalinguistic facts whatsoever. Thus, there should be at least two different classical interpretation functions F and F' that conform to *all* existing metasemantic constraints, but which fill the gaps left by them differently: for some n, $n \in F(B)$ whereas $n \notin F'(B)$. Consequently, the corresponding class *Adm* of admissible interpretation functions includes more than just one member, and hence there isn't a uniquely factually determined intended interpretation I of L. Let us call this *semantic indeterminacy* of "the" intended interpretation of L: vagueness seems to be "semantic indecision" (Lewis 1986, p. 213; see Weatherson 2010 for more on vagueness as indeterminacy.)

Of course, 'unlikely' is not 'impossible' and 'plausible' does not mean 'true': indeed, epistemicists about vagueness *do* believe the reference of 'I' to be determined uniquely by the metasemantic facts, and hence that vagueness does not entail semantic indeterminacy. And they might be right about this—who knows? *But what if not?* Epistemicists might also rightly complain that not enough has been said above about facts and metasemantic determination. For instance, they might ask: are the non-linguistic facts mentioned under (ii) above so that for each n, it is either a fact that $B(n)$ or it is a fact that $\neg B(n)$? For if so, these "baldness facts" would determine the interpretation of B uniquely after all. If not, which would seem more likely, as "baldness-facts" would not be countenanced to be precise enough according to the best present mathematical or scientific standards (recall our description of (ii)): what exactly counts as a fact in (i) and (ii) above, and how do these facts constrain semantic interpretation by means of the laws in (iii)? Will the explication of 'determination' and '*Adm*' possibly reveal some epistemic components in the special case of *vague* terms? (In contrast, the way in which the interpretation of the *non-vague* term '*Sqrt*' was determined based on the mathematicians' usage of a conditional definition and some mathematical facts did not seem epistemic at all.)[10]

These are reasonable questions, and more should be said indeed. *But shouldn't the previous considerations on facts about 'bald'-usage and hair-numbers, and on the metasemantic constraints they might impose on the interpretation of B, at least cast enough doubt on semantic determinacy that the burden of proof is switched to the classical semanticist?*[11] Wouldn't we simply tempt the fate of semantics by building it on presupposition (4) for which it seems

[9] In the terminology of Fine (1975), that would corresponds to a so-called "penumbral connection".

[10] For more on such worries, see Williamson (1992, 1994b) and Williamson's (2004) criticism of McGee & McLaughlin (1997). For a reply, see McGee & McLaughlin (2004). E.g., McGee & McLaughlin (2004, pp. 126-129) criticize Williamson's (2004) argument that determinate truth would have to collapse into truth and determinate falsity into falsity. One can use the example of the conditional definition of '*Sqrt*' from Section 0 to show which of the premises of Williamson's argument are likely to be false; but I will not do so here.

[11] Compare: one might think that it is not clear enough what knowledge is and what knowledge we have, and yet agree that considerations on knowledge may suffice to cast substantial doubt on certain epistemological theses, such as that that one's evidence is what one believes, to switch the burden of argument to the opponent.

difficult to cite empirical evidence and which, at least *prima facie*, sounds doubtful with respect to the interpretation of vague terms?

Instead of continuing the philosophical debate on metasemantic determination, which is not itself the topic of this paper, I will rather add to the pressure on (4) by presenting further examples of semantic indeterminacy without vagueness. Accodingly, the focus of the paper is really the general phenomenon of semantic indeterminacy and not vagueness *per se*. And I will continue to understand '*Adm*' in terms such as 'determinately' and 'metasemantic facts' rather than, say, 'definitely' and 'knowledge'.

(The present example of the language *L* that includes vague terms, such as '*B*' for "bald", will remain the central example from Section 3 with the sole exception of Section 6. In what follows, any reference by '*L*' is a reference to the language from Example 1.)

Example 2: Let L' be a second-order formalization of the language of *arithmetic* (so here I am deviating from the previous assumption of a first-order language *L*): let N, 0, s, $+$, $\cdot$ be terms in L' that formalize the corresponding arithmetical terms for 'natural number', 'zero', 'successor', 'sum', and 'product', as used by number theorists.

It is plausible that the following claim expresses an existing metasemantic constraint on "the" intended interpretation I' of L': $I'(N)$, $I'(0)$, $I'(s)$, $I'(+)$, $I'(\cdot)$ jointly satisfy the *second-order Dedekind-Peano axioms* for arithmetic. (That is: $I'(0)\in I'(N)$; for all $d\in I'(N)$ it holds that $I'(s)(d)\in I'(N)$; and the other axioms, including second-order induction.) As proven by Dedekind (1988), the axiom system of second-order Dedekind-Peano arithmetic is categorical, that is, it pins down the *structure* of the natural number sequence uniquely. If structuralists about arithmetic are right (see Hellman & Shapiro 2019 for a survey), there do not exist any other metasemantic constraints on the interpretation of arithmetical symbols than getting the structure of the natural number sequence right; hence, the second-order Dedekind-Peano axioms taken together might actually express *all* the metasemantic constraints on I' there are.

But, at the same time, these constraints do not to pin down I' uniquely: e.g., if $I'(N)$ is identified with the set of *finite von-Neumann-ordinals*, there are suitable and easily definable choices for $I'(0)$, $I'(s)$, $I'(+)$, $I'(\cdot)$, such that the axioms are satisfied; but the same is true also of the set of *finite Zermelo-ordinals* and interpretations of the arithmetical symbols that are suitable to those ordinals and as easily definable (as famously highlighted by Benacerraf 1965). The corresponding class *Adm* should therefore include at least two distinct classical interpretations *F* and *F'*—perhaps even all (infinitely many) interpretations isomorphic to the two previous ones—which is why there isn't a uniquely factually determined interpretation of L'. The intended interpretation of the language of second-order arithmetic is semantically indeterminate, and since arithmetic terms do not have borderline cases, this is a case of semantic indeterminacy without vagueness.[12]

[12]The discussion here presupposes an *eliminative set-theoretic* structuralism (see Hellman and Shapiro 2019, Chapter 3) that restricts the purely mathematical resources of the metalanguage of L' to those of set theory. *Non-eliminative* structuralists would not find that satisfying: they claim that *N does* have a uniquely determined intended interpretation, it is just that $I'(N)$ does not coincide with the set of finite von-Neumann-ordinals or with the set of finite Zermelo-ordinals or with *any* set of set-theoretic entities for that matter. Instead, the intended interpretation of arithmetical terms is given by a uniquely determined structure *sui generis*—*the* abstract structure of natural numbers—that cannot and should not be eliminated in favor of sets; and if the metalanguage of L' offers ways of talking about that non-set-theoretic structure, the alleged semantic indeterminacy of 'natural number' dissolves.

Even if one agreed with non-eliminative structuralists about that, worries about semantic indeterminacy would reiterate as far as *singular* terms for *single* objects in some abstract structures are concerned: e.g., the imaginary unit *i* in the structure of the complex field of numbers is structurally indistinguishable from its numerically distinct "sibling" $-i$, as there exists a field automorphism that maps the one to the other. Therefore, for a structuralist, there does not seem to be any fact of the matter whether the numeral '*i*' actually denotes *i* or whether it rather denotes $-i$ (see Brandom 1996, Shapiro 2008b). The debate about these matters is ongoing and some authors (see Shapiro 2006, 2008b, 2012, Leitgeb 2007, Pettigrew 2008, Schiemer and Gratzl 2016) have suggested ways of developing

On behalf of classical semantics, one might try to argue that set-theoretic structuralists about arithmetic are simply wrong—there *do* exist metasemantic constraints beyond those of second-order Dedekind-Peano arithmetic, and these *do* restrict the interpretation of arithmetical symbols, say, to the interpretation with the finite von-Neumann-ordinals. However, other than in the context of a textbook in foundational set theory, in which natural numbers are indeed often defined as the finite von-Neumann-ordinals, number theorists may not feel bound by any specification of the intended interpretation of arithmetic over and above the Dedekind-Peano axioms. Once made aware of the existence of different set-theoretic interpretations of their axioms, some of them might simply not accept that one such interpretation is taken to be intended *by fiat* while all other interpretations are rejected as inadmissible; they might regard the assumption of a uniquely factually determined intended interpretation of the arithmetical symbols to be unfaithful to the mathematical content of arithmetic. Again, these number theorists might be wrong about all of that, but the mere existence of corresponding verdicts by number theorists and structuralist philosophers of number theory (Dedekind would count as a concrete instance on both sides) should suffice at least for a *prima facie* case against semantic determinacy. For the same reason, the burden of proof is really on the classical semanticists to justify their presupposition of a unique factually determined intended interpretation of arithmetical terms (even when doing so would lead them beyond semantics).

Example 3: Let L'' formalize the language of *Newtonian mechanics*—no matter whether first-order or second-order—and let m formalize the term 'mass' as used in Newtonian physics. (m is a function symbol, but let us put all questions about the syntax of m to one side now.) Field (1973) has argued, using the language of modern relativistic mechanics on the metalevel, that there are two interpretations of m, such that there is no fact of the matter which of them delivers "the" actual or intended reference of m: according to the one, $I''(m)$ coincides with relativistic mass (total energy/c^2), according to the other, $I''(m)$ coincides with proper mass (non-kinetic energy/c^2), and relativistic mass and proper mass come apart in value and physical properties. Each of the two interpretations saves *some* of the central claims of the Newtonian theory from falsity (and *many* of Newton's empirical predictions about slowly-enough-moving objects), neither of the two interpretations saves *all* central claims of Newton's theory, and the theoretical roles played by relativistic mass and proper mass in modern relativity theory are equally salient and important. Since these properties seem to exhaust, in present terminology, the existing metasemantic constraints on the interpretation of m in Newtonian mechanics, Field's diagnosis concerning 'mass' is: "the situation is not that we don't *know* what Newton's word denoted, but that Newton's word was referentially indeterminate" (Field 1973, p. 467, his emphasis).

What is going on here may be viewed as an application of the principle of charity: the Newtonian language is to be interpreted so that the truth of the sentences asserted by the Newtonian theory gets maximized; and as often the case when one maximizes a function, there might not be a unique maximum, but two of them (such as for 'mass') or three… or even infinitely many. If so, the class *Adm* of admissible interpretation functions of L'' includes more than one member, and hence there is no unique factually determined intended interpretation of L'': "the" intended interpretation of Newtonian mechanics is semantically indeterminate.[13]

This argument for indeterminacy does not concern vagueness or structuralism but "incongruencies" between, on the one hand, the language of Newtonian mechanics, and on the other hand, what *we* think the world is like as described by present-day relativistic mechanics. As Field argues against Kuhn (1962), these "incongruencies" do not require

a structuralist semantics that resemble aspects of Ramsey semantics. (I will return to part of that literature later in Section 2.)

[13]Field (1973) continues to develop a variant of supervaluationist semantics for semantically indeterminate expressions such as m. See Chapter 2 of Button and Walsh (2018) for more on referential indeterminacy and supervaluation. I will deal with supervaluationism in Sections 2-4.

incommensurability or complete referential discontinuity between the Newtonian language and the relativistic language but rather manifest themselves as semantic indeterminacies. (I will return to the discussion of referential change and continuity in science in Section 6.)

Obviously, the classical semanticist might claim this analysis to be wrong: there *are* existing metasemantic constraints of which Field is unware, which determine the intended interpretation of *m* to coincide with, say, proper mass. If so, the defender of classical semantics should supply arguments and data in favour of this thesis (even when doing so would lead them beyond semantics): does proper mass possess some "objective naturalness" that relativistic mass lacks, and if so in what sense? Is there something in the experimental practice of Newtonian physicists that rules out relativistic mass as an interpretation of 'mass'? In view of Field's arguments to the contrary, the presupposition that 'mass' in Newtonian physics has a unique factually determined intended interpretation would seem to rest on shaky grounds unless backed up by evidence. Presupposition (4) of classical (meta-)semantics is challenged by examples like these.

2. Ramsey Semantics

In the last section I have described the gist of classical semantics. I ended up pointing out that classical semantics is risky business by building on a presupposition—the existence of a unique factually determined intended interpretation *I*—that might not be met: the metasemantic facts seem to leave 'bald', 'natural number', and 'mass (in Newtonian mechanics)' without uniquely determined intended interpretations, and each time for different reasons. The defender of classical semantics, it seems, will have to fend off each worry on separate grounds: *no*, *vague terms* are not semantically indeterminate, their vagueness can be explicated otherwise (e.g. counterfactually), and complete extensions are somehow determined by the facts even though we might not know how. *No*, *structuralism* about arithmetic is wrong, there is more to arithmetical terms than their structural content, whatever it is. *No*, *'mass'* in *Newtonian physics* does have a uniquely determined reference, even when it is hard to say what it refers to.[14]

Classical semanticists might reply that there might still be a global *abductive* argument for semantic determinacy: how else should one explain the success of classical semantics as applied to certain fragments of language (when successful there), if not by trusting its presupposition of a unique factually determined intended interpretation of that fragment? And even more so, when the alternatives of classical semantics seem to have their own problems!

In what follows, I want to demonstrate that this kind of inference to the best explanation does not go through either. For there exists a minor retreat from classical semantics that has similar theoretical virtues as classical semantics and which should therefore be similarly successful when and where classical semantics is successful: *Ramsey semantics*. At the same time, the new semantics will be less risky than classical semantics by not presupposing semantic determinacy. Ramsey semantics does not thereby claim semantic determinacy to be false, it only avoids presupposing it, and yet it keeps classical logic and truth on board. For these reasons, overall, Ramsey semantics should be preferable over classical semantics. What is more, by approximating classical semantics much more closely than supervaluationist semantics, Ramsey semantics should also win over supervaluationists who are attracted by classical semantics apart from not wanting to rule out semantic indeterminacy. While the

[14] Further arguments against semantic determinacy may be distilled from the literature, though all of them are controversial and none of them constitutes a knock-down argument: cf. Willard van Orman Quine on the indeterminacy of reference, Hilary Putnam on the Löwenheim-Skolem theorem and non-standard interpretations, Saul Kripke on rule-following, Solomon Feferman on the semantic indeterminacy (in his terms, "vagueness") of the Continuum Hypothesis and the set-theoretic membership predicate (see also Hamkins 2012 on that topic), and Wilson (2006) on the indeterminacy and open-endedness of terms from applied mathematics and science.

present section will explain the new semantics, Section 3 will draw the comparison with supervaluationist semantics and classical semantics.

Returning to the semantics of our first-order language *L* (with '*B*' for baldness) from Section 1, let us accept for now that "the" intended interpretation of *L* is subject to metasemantic constraints without necessarily being pinned down by them uniquely: *Adm* may be a singleton class, as presupposed by classical semantics, but it may also include *more than just one admissible classical interpretation mapping*. Either way, by being members of *Adm*, all *F* in *Adm* have something in common; e.g.: $0 \in F(B)$; $100000 \notin F(B)$; for all *m*, *n*, if $m>n$ and $m \in F(B)$ then $n \in F(B)$); $0 \in Uni(I)$; and so on. I will also assume, for simplicity, that all members *F* of *Adm* are based on one and the same universe *U*, which is a non-empty set (rather than a proper class): for all $F \in Adm$, $Uni(F) = U$.

In terms of the mathematical analogy from the last section: the quantity *z* (which was analogous to '*I*') might be *constrained* by *x* and *y*, but not in the sense of being a function of them ($z = f(x, y)$) but by there being an equation that is to be satisfied jointly by *z*, *x*, *y*: say, *eq*: $z^2 = x + y$. And that equation might offer more than one solution for '*z*' when given concrete values for '*x*' and '*y*': e.g., $x = 3$ and $y = 1$ constrain the value of '*z*' to the effect that *either* $z = 2$ *or* $z = -2$, since both $2^2 = 3 + 1$ and $(-2)^2 = 3 + 1$ are the case, without imposing further constraints that rule out either of the two solutions. The totality of all metasemantics laws that support the existing metasemantic constraints on the interpretation of *L* might be more like *eq* than like $z = f(x, y)$, and the resulting class *Adm* of admissible interpretations of *L* might be more like {2, -2} than like $\{f(3, 1)\}$.

Instead of presupposing that all the facts taken together determine metasemantically a unique intended classical interpretation *I* and hence what is true in virtue of *I*, a less risky way of proceeding in semantics should therefore be: to merely presuppose *there exists a classical interpretation F that conforms to all existing metasemantic constraints and from which truth is defined by means of the classical semantic rules*. For that existence statement will be true both in case the intended interpretation of *L* *is* factually determined—when there is a uniquely determined *F* for which $F \in Adm$—and in the case of semantic *in*determinacy—when there is more than one *F*, such that $F \in Adm$. (When there is no *F* at all such that $F \in Adm$, classical semantics suffers from an even deeper-going problem than indeterminacy, about which I will have nothing to say here. The obvious way out of that case would be to consider a larger class of "approximately admissible" interpretations and work with it.)

But this means we enter well-trodden territory, since what is going on now may be viewed as an instance of *Ramsification*: first, one regards the terms '*I*' ("intended interpretation") and 'true' from classical semantics as theoretical terms, the meanings of which are determined by '$I \in Adm$' and the definition (3) of truth, that is, 'for all sentences *A* in *L*: *A* is true iff $I \vDash A$'. That is plausible because *I* was meant to be intended precisely in the sense of belonging to *Adm*, that is, of conforming to all existing metasemantic constraints, and at the same time delivering the domain, references, and extensions that feed into the definition of truth by means of the standard semantic rules ((1)).

Secondly, one replaces '*I*' and 'true' in the metalinguistic sentence

'$I \in Adm$ and for all sentences *A*: *A* is true iff $I \vDash A$'

by the function variable '*F*' and the class variable '*T*', respectively, which yields the open metalinguistic formula

'$F \in Adm$ and for all sentences *A*: $A \in T$ iff $F \vDash A$'.

Finally, one does *not* presuppose any longer

(4) & (3) $\exists!F(F \in Adm)$ and $I \in Adm$, and for all sentences A: A is true iff $I \vDash A$ (equivalently: $\exists!F(F \in Adm)$, and $I = \iota F(F \in Adm)$[15], and for all sentences A: A is true iff $I \vDash A$).

Instead, one merely claims the *existence* of an F and a T, such that $F \in Adm$ and where F "defines" T by means of satisfaction, that is:

(5) $\exists F \exists T(F \in Adm$ and for all sentences A: $A \in T$ iff $F \vDash A)$.

But (5) is nothing but the *Ramsey sentence* of '$I \in Adm$ and for all A: A is true iff $I \vDash A$' with respect to the theoretical terms 'I' and 'true'.[16]

The classical presupposition '$\exists!F(F \in Adm)$' from (4) is dropped rather than Ramsified, because it is exactly what Ramsey semantics intends to avoid; the remaining parts of classical semantics, that is, the definition of 'interpretation', 'variable assignment', and (1) and (2) from Section 1, are explicit model-theoretic definitions that are presupposed by classical semantics and Ramsey semantics (and supervaluationist semantics) and do not need to be Ramsified. '*Adm*' is regarded as an O(ld)-term in the terminology of Lewis (1970), which is left invariant by Ramsification: as explained in Section 1, the characterization of *Adm* does not belong to semantics *proper*, and the term '*Adm*' (or something like it, such as 'intended' or 'actual') needs to be presupposed equally by classical (meta-)semantics, supervaluationist semantics, and Ramsey semantics. (As in Lewis 1970, the distinction between *observational* and *non-observational terms* known from more traditional applications of Ramsification does not play any role in Ramsey semantics.)

Since, for given F, 'for all sentences A in L: $A \in T$ iff $F \vDash A$' uniquely characterizes T so far as its sentence members of L are concerned (as can be shown set-theoretically or in higher-order logic), one may also formulate (5) alternatively as the combined existence/unique-existence claim

(5') $\exists F \exists! T\ (F \in Adm$ and for all sentences A: $A \in T$ iff $F \vDash A)$.

Either way, the idea is:

- classical semantics consists in the definition of 'interpretation', 'variable assignment', and (1)-(4),
- while Ramsey semantics consists in the same definitions of 'interpretation' and 'variable assignment', (1), (2), *and the Ramsey sentence* (5)/(5'). (I am going to add an alternative formulation of Ramsey semantics below, which may be understood as a reformulation of the present one.)

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[15] This is an instance of Lewis' (1970) definition of a theoretical term by a definite description ($T = \iota X Th[X]$). Lewis' account builds historically on Ramsey's and Carnap's work; Lewis also cites Carnap's definition of theoretical terms by epsilon terms that will become important later in this section.

[16] I want to leave open here how (4) and (5) would be stated in a completely formalized language. One option would be to understand them as second-order formulas, such that, e.g., 'F' is a variable for functions as second-order entities and '$F \in Adm$' is really an instance of higher-order predication ($Adm(F)$). Another option is to treat (4) and (5) as first-order formulas, in which case, e.g., 'F' is a variable for functions as first-order individuals, and '$F \in Adm$' is either an instance of standard predication ($Adm(F)$), or it actually invokes the set-theoretic membership predicate ($\in(F, Adm)$), in which case '*Adm*' is a singular term.

As the term '*Adm*issible' had already suggested before, Ramsey semantics shares the assumption of a non-empty class *Adm* of admissible interpretations with supervaluationism (see Fine 1975, McGee & McLaughlin 1994, Keefe 2000), such that neither of them presupposes that *Adm* is a singleton, and where '*Adm*' is interpreted similarly in both semantics.

However, that is also where paths will diverge: in particular, Ramsey semantics does *not* introduce a supervaluationist notion of *super-truth* (van Fraassen 1966) for sentences, that is,

for all sentences A in L: A is super-true iff for all $F \in Adm$, $F \vDash A$,

to the effect that super-truth takes over some, if not all, conceptual roles that truth *simpliciter* plays in classical extensional semantics. (I am going to compare Ramsey semantics with supervaluationism in more detail in Section 3.)

Nor should the present theory be mistaken for so-called subvaluationism about vague terms (cf. Hyde 1997, Hyde & Colyvan 2008, Cobreros 2011), which defines

for all sentences A in L: A is sub-true iff $\exists F \in Adm$, such that: $F \vDash A$,

and which assigns *sub-truth* the roles that truth *simpliciter* plays in standard semantics.

Instead, Ramsey semantics maintains (reformulating (5') just a bit)

$\exists F \in Adm\ \exists! T$, such that for all sentences A in L: $A \in T$ iff $F \vDash A$,

in which 'F' is bound *existentially*, in which the existential quantifier expression '$\exists F \in Adm$' takes *wide scope* (instead of occurring on the right-hand side of the embedded equivalence), and in which T (truth relative to F) is determined uniquely from F by the semantic rules for *classical* satisfaction. The claim is: *there exists* a classical admissible interpretation in terms *of which* the truth values of all sentences of L are given in a classical manner. For instance, while there may be sentences A, such that neither A nor $\neg A$ is *super*-true, and where both A and $\neg A$ are *sub*-true, neither of this could possibly happen in Ramsey semantics, as there is no classical interpretation mapping F *of which* it would hold that $F \nvDash A$ and $F \nvDash \neg A$ (and hence $A \notin T$ and $\neg A \notin T$), or *of which* it would hold that $F \vDash A$ and $F \vDash \neg A$ (and thus $A \in T$ and $\neg A \in T$).

Ramsey semantics should not be misunderstood either as requiring that the theoretical terms 'I' and 'true' need to be eliminated—Ramsified *away*—when doing semantics: semantic statements including them merely need to be used and interpreted more cautiously than in classical semantics, so that they can always be understood as *stand-ins* for *existential* statements. There are two ways of achieving that.

One is to start from the existential statement (5) or (5'), to apply to it the elimination rule of natural deduction for existential quantifiers, thereby "picking" one of the relevant Fs and calling it 'I', calling the members of the set T that is given uniquely by I the 'true sentences (of L)', and introducing '$I \in Adm$ and for all sentences A: A is true iff $I \vDash A$' as a *temporary assumption*.

That being in place, the Ramsey semanticist is able reformulate in their own terms every metalinguistic semantic statement

(6) $S[I, \text{true}]$

with the term 'I' or '*true*' that the classical semanticist might want to put forward. This does not mean that each single statement (6) would require a new piecemeal application of Ramsification: instead one application of Ramsification (the result of which is (5) or (5')) is used to reconstruct all possible metalinguistic instances of (6) simultaneously. Of course, at some point this application of the existential elimination rule will have be to be terminated by discharging the assumption involving '*I*' and 'true' from above and by deriving a statement that does not involve the "newly introduced" terms anymore. For instance, in a case in which the classical semanticist would logically derive (6) from the classical *thesis* '$I \in Adm$ and for all sentences *A*: *A* is true iff $I \vDash A$', the Ramsey semanticist would derive (6) in the same manner from the *temporary assumption* '$I \in Adm$ and for all sentences *A*: *A* is true iff $I \vDash A$', hence derive the conjunction of (6) and that temporary assumption, derive from that conjunction that

(7) $\exists F \in Adm\ \exists T$, such that: for all sentences *A*, $A \in X$ iff $F \vDash A$, and also $S[F, T]$

by existential introduction, and finally discharge the temporary assumption and derive (7) simply from the Ramsey sentence (5) or (5') that is part of the theory of Ramsey semantics.[17]

But there is also a second, more stable, way of presenting Ramsey semantics that allows one to preserve the terms '*I*' and 'true' without having to discharge assumptions: one recycles Carnap's (1959, 1961) treatment of theoretical terms as Hilbertian epsilon terms (see Schiemer and Gratzl 2016 for a recent survey, discussion, and application) and employs it in the present semantic context. The rest of this section will be devoted to this topic.

Epsilon terms 'εF...' are indefinite descriptions, which are much like definite descriptions 'ιF...' ("*the F*, such that…"), except that the uniqueness presupposition of definite descriptions is dropped. Accordingly, one may read 'εF...' as "*an F*, such that…". The Hilbert school (see Ackermann 1924, Hilbert and Bernays 1934) introduced them as tools in metamathematics by which "ideal" mathematical entities could be denoted and with the help of which proof-theoretic reductions were meant to be carried out.

In the present context, epsilon terms are made formally precise by adding, first, the primitive epsilon operator 'ε' to the logical vocabulary of the metalanguage of *L*. (For the logicality of the ε-operator, see Woods 2014.) If '*F*' is a metalinguistic variable, then the result of replacing '$C[F]$' in '$\varepsilon F\, C[F]$' by a metalinguistic formula yields a metalinguistic term 'εF …' of the same type as '*F*'. (Abusing notation just a bit, I will denote the resulting term by '$\varepsilon F\, C[F]$' again.) E.g., just as '*F*' can occur one any side of the metalinguistic equality symbol (as in '$F = \ldots$'), the same holds for '$\varepsilon F\, C[F]$' (as in'$\varepsilon F\, C[F] = \ldots$'). If '*F*' is the *only* free variable in (the metalinguistic formula replacing) '$C[F]$', then the epsilon term '$\varepsilon F\, C[F]$' is a *closed term*, since the free variable '*F*' of '$C[F]$' is bound by 'εF' within '$\varepsilon F\, C[F]$' in the same way in which '$\exists F$' binds the free variable '*F*' of '$C[F]$' within the *closed formula* (sentence) '$\exists F C[F]$'.

Secondly, one extends the classical logic of the metalinguistic semantic theory by the axioms of the so-called (extensional) epsilon calculus, that is, all statements of the form

$\exists F C[F] \leftrightarrow C[\varepsilon F\, C[F]]$
Extensionality: $\forall F(C[F] \leftrightarrow C'[F]) \rightarrow \varepsilon F\, C[F] = \varepsilon F\, C'[F]$,

which, if added to classical first-order predicate logic or to the standard deductive (axiomatized) system of classical second-order predicate logic, is known to yield a conservative extension thereof. The left-to-right direction of the axiom scheme '$\exists F C[F] \leftrightarrow C[\varepsilon F\, C[F]]$' states that, by the logic of 'ε', if some *F* is such that *C*, then $\varepsilon F C[F]$ ("an *F*, such that *C*") is such that *C*, too. The rationale for this is that the epsilon term '$\varepsilon F C[F]$' is understood to "pick" one of the *F*s that exists according to '$\exists F C[F]$'. (If it is not the case that $\exists F C[F]$, there are no constraints

[17] As mentioned in Example 2 of Section 1, the literature on structuralism in the philosophy of mathematics discusses similar strategies of introducing terms ("parameters"), either for numerically distinct but structurally indistinguishable objects in structures, or for numerically distinct but structurally indistinguishable set-theoretic systems: see Shapiro (2008b, 2012), Pettigrew (2008).

whatsoever on what member of the universe of discourse is "picked" by '$\varepsilon F C[F]$'.) Extensionality adds that what $\varepsilon F C[F]$ "picks" only depends on the extension of '$C[F]$', that is, on the class of all Fs for which $C[F]$ is the case.

Carnap (1959, 1961) suggested to logically reconstruct theoretical scientific terms by defining them explicitly as epsilon terms, which he presented as a variation on Ramsification for theoretical terms. (Lewis 1970 suggested much the same, except that he defined theoretical terms as definite descriptions.) I will return to this in Section 6. In the present context, the idea is to apply Carnap's procedure to the theoretical terms 'I' and 'true' of semantics.

Instead of thinking of Ramsey semantics to be given by the definition of 'interpretation', (1), (2), and the Ramsey sentence (5)/(5'), one puts forward instead the "simplified" Ramsey sentence

(8) $\exists F(F \in Adm)$,

which follows logically from the Ramsey sentences (5)/(5') from before. Afterwards, one defines 'I' explicitly with the help of the *indefinite description*

(9) $I = \varepsilon F\, (F \in Adm)$,

and one conclude by defining 'true' explicitly based on the previously defined term 'I':

(10) for all sentences A in L: A is true iff $I \vDash A$ (with 'I' being understood as in (9)).

The "simplified" Ramsey sentence (8), the epsilon-term definition (9) of 'I', and the definition (10) of 'true' are stated as semantic eigenaxioms in this reformulation of Ramsey semantics. The previous definition of 'interpretation' as well as definitions (1) and (2) from Section 1 are also presupposed. Overall, this yields:

- Ramsey semantics (in its ε-operator version) consists in the definitions of 'interpretation' and 'variable assignment', (1), (2), (8), (9), (10), and the logic of the ε-operator.

If formulated this way, Ramsey semantics has the following consequences: by combining (8) with the logical law

$\exists F(F \in Adm)$ iff $\varepsilon F(F \in Adm) \in Adm$,

which is an instance of the epsilon calculus axiom scheme from above, one can derive

(8') $\varepsilon F(F \in Adm) \in Adm$.

And that statement, by definition (9), can be reformulated as

(8'') $I \in Adm$.

Taking this together with definition (10) yields

(11) $I \in Adm$ and for all sentences A: A is true iff $I \vDash A$,

which constitutes a part of the classical (4) & (3) from above, and which thus becomes derivable in Ramsey semantics based on (8), (9), (10). Indeed, (11) may be viewed as just an analytically equivalent variant of the simplified Ramsey sentence (8) on which its derivation relies (since (9) and (10) are just definitions). For the same reason, the Ramsey sentences (5) and (5') can also be derived with the help of (8), (9), (10).

(10) (the second conjunct of (11)), which coincides syntactically with the classical definition (3) of truth based on the term '*I*' for "intended interpretation", is now short for

(10') for all sentences *A*: *A* is true iff $\varepsilon F(F{\in}Adm) \vDash A$,

in which 'ε' is tied logically to existential quantification by the logical axioms of the epsilon calculus. What (10') makes transparent is that truth according to Ramsey semantics is nothing but standard Tarskian truth relative to *an* admissible interpretation mapping that might not be determined uniquely by the existing metasemantic constraints on the interpretation of *L* (since *Adm* might include more than one member). *I* may still be said to be *unique* in the sense that $\exists!F(F = I)$, which is derivable in the classical logic of Ramsey semantics. If understood in that sense, it is even fine to speak of *the* intended interpretation *I*, so long as one does not mean by this the *by the metasemantic facts* uniquely determined interpretation: for *I* may not be the only member of *Adm*. As I have done before, I will sometimes use scare quotes for *"the"* intended interpretation to signal this ambiguity.

By deriving *I* to be a member of *Adm*, Ramsey semantics still maintains that truth *simpliciter* is preserved by classical logic; that is: if $A_1,\ldots,A_n \vDash C$ holds in classical logic, then it follows that

(12) if $A_1,\ldots,A_n$ are true, then *C* is true.

That is because: (12) may be unpacked, with (10), as

(12') if $I \vDash A_1,\ldots,A_n$, then $I \vDash C$.

And (12') holds because it can be understood, with (9), as abbreviating

(12'') if $\varepsilon F(F{\in}Adm) \vDash A_1,\ldots,A_n$, then $\varepsilon F(F{\in}Adm) \vDash C$,

which indeed follows logically from $A_1,\ldots,A_n \vDash C$, definition (2), and $\varepsilon F(F{\in}Adm)$ being a member of *Adm* (recall (8')) and therefore also being a classical interpretation function.

More generally, the role of the Ramsey sentence (8) is now to claim the existential presupposition of the indefinite description in definition (9) to be satisfied, since (8) claims *Adm* is to be non-empty, which implies (8'), that is, $\varepsilon F(F{\in}Adm) \in Adm$.

The role of (9) and (10) is to make sure that any instance of (6) involving '*I*' and/or 'true' can be understood as being short for

(6') $S[\varepsilon F(F{\in}Adm), \varepsilon F(F{\in}Adm){\vDash}\ldots]$,

which from now will be the official way of rendering statements *S*[*I*, true] from classical semantics in the terms of Ramsey semantics.

For instance, consider the metalinguistic sentence *S*[*I*, true] to be '*B*(*n*) is true', where *B*(*n*) is a sentence in *L* from Example 1 of Section 1: according to (10), *B*(*n*) is true iff $I \vDash B(n)$; by (9), $I \vDash B(n)$ iff $\varepsilon F(F{\in}Adm) \vDash B(n)$; and (1) entails that $\varepsilon F(F{\in}Adm) \vDash B(n)$ iff $(\varepsilon F(F{\in}Adm))(n){\in}(\varepsilon F(F{\in}Adm))(B)$, or more briefly, $\varepsilon F(F{\in}Adm)(x){\in}\varepsilon F(F{\in}Adm)(B)$. So we have that

‘$B(n)$ is true’ ($S[I$, true])

may be understood to be short for

$\varepsilon F(F \in Adm) \vDash B(n)$ ($S[\varepsilon F(F \in Adm), \varepsilon F(F \in Adm) \vDash \ldots]$)

which in turn reduces to

$\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$.

Analogously, using (1) again, $\neg B(n)$ is true iff $\varepsilon F(F \in Adm)(n) \notin \varepsilon F(F \in Adm)(B)$; $B(n) \wedge \neg B(n+1)$ is true iff $\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$ and $\varepsilon F(F \in Adm)(n) \notin \varepsilon F(F \in Adm)(B)$. Etc.

Such metalinguistic claims including ‘$\varepsilon F(F \in Adm)$’ are logically entailed by various sentences without epsilon terms, and they entail various sentences without epsilon terms. For instance, if *for all* $F \in Adm$ it holds that $F(n) \in F(B)$, then this logically implies that $\varepsilon F(F \in Adm)(n) \in \varepsilon F(F \in Adm)(B)$, that is, $B(n)$ is true. More generally, when an arbitrary sentence A is true at *all F* in *Adm*—when A is (super-)true in supervaluationism—it follows that A is true (simpliciter) in Ramsey semantics, and when A is false at *all F* in *Adm*, then A is also false (simpliciter) in Ramsey semantics. Furthermore, when e.g. $B(n)$ is true in Ramsey semantics, this logically implies that *there is* an $F \in Adm$, such that $F(n) \in F(B)$. In these ways and more, statements involving ‘*I*’ and ‘true’ as understood by Ramsey semantics can be predicted by, and can predict, statements without ‘*I*’ and ‘true’ (but perhaps including ‘*Adm*’). In fact, already the Ramsey sentence (5) logically implies the same sentences without the “theoretical” terms ‘*I*’ and ‘*true*’ as the classical thesis ‘$I \in Adm$ and for all sentences A: A is true iff $I \vDash A$’ does (as is well-known from general work on Ramsification; see Carnap 1966).

In all of that, usage of the epsilon operator ‘ε’ should not signal anything beyond the *existential* interpretation of ‘ε’ that is required by the logic of epsilon terms itself: in particular, usage of ‘$\varepsilon F(F \in Adm)$’ is not meant to suggest the existence of metasemantic facts in virtue of which ‘$\varepsilon F(F \in Adm)$’ would be able to “pick” a *factually* uniquely determined member of *Adm* after all—unless *Adm* is factually determined to be a singleton class, of course, in which case ‘$\varepsilon F(F \in Adm)$’ must denote that member (by the epsilon calculus again). But if *Adm* is *not* a singleton class, there is no metasemantic fact of the matter what member of *Adm* is being “chosen” by the epsilon term ‘$\varepsilon F(F \in Adm)$’.[18]

In the terminology of Breckenridge and Magidor (2012), one might also say that the reference of ‘$\varepsilon F(F \in Adm)$’ is *arbitrary*, where “we do not know and cannot know” (Breckenridge and Magidor 2012, p. 377) what a linguistic expression refers to when its reference has been fixed arbitrarily. But it is not merely that we do not know or could not know the particular interpretation function that is “picked” by ‘$\varepsilon F(F \in Adm)$’: there is simply no metasemantic fact to be known in that case—it is in that sense that “choice” is indeterminate.

[18] The choice semantics of epsilon terms (Leisenring 1969) interprets epsilon terms by choice functions, such that each model includes a unique choice function. Thinking of such a choice function to be *uniquely determined by the facts* is precisely how one should *not* think of epsilon terms in Ramsey semantics: it is *not* presupposed that each epsilon term of the language of Ramsey semantics possesses a unique intended interpretation that is given by a factually uniquely determined choice function. Schiemer and Gratzl (2016) use choice semantics when they state two truth definitions by means of epsilon terms that are interpreted by choice functions. But neither of these definitions corresponds to the definition of truth in Ramsey semantics ((10) & (9)): their notion of plain *truth* involves an existential quantification over choice functions and thereby yields a variant of sub-truth as in subvaluationism, while their notion of *universal truth* involves a universal quantification over choice functions, which yields a variant of super-truth as in supervaluationism. (Compare the previous discussion of how sub-truth and super-truth differ from truth in Ramsey semantics.)

Breckenridge and Magidor (2012) would describe this as follows: if an expression with arbitrary reference refers to x, then this is not determined by any non-semantic facts but only by the semantic fact that the expression refers to x, where that semantic fact does not supervene on non-semantic use-facts (see Kearns and Magidor 2012 for more on this). Whereas I would say: there is no metasemantic fact of the matter which admissible interpretation '$\varepsilon F(F \in Adm)$' refers to, and *there is no other kind of fact what* '$\varepsilon F(F \in Adm)$' *refers to either*, *not even a semantic one*. The disadvantage of my way of expressing oneself will be that I will have to acknowledge the possibility of true sentences that are not determinately true and hence (by my understanding of 'determinate') not true in virtue of any facts. (See Section 4.) The disadvantage of their way of talking is that they have to acknowledge the possibility of semantic facts that do not obtain in virtue of any non-semantic facts. In the final Section 3 of Breckenridge and Magidor (2012), they also add that there is a "determinate fact of the matter" what a term with arbitrary reference refers to—which contradicts the way in which 'determinate' is used in the present paper in which it is reserved for a sentential operator or a semantic predicate that characterizes truth at all admissible interpretations (see Sections 3-5 and the Appendix). But these might be merely terminological differences.[19]

What we do know is: if *Adm* is non-empty (which is claim (8) of Ramsey semantics), then $\varepsilon F(F \in Adm)$ is one of the members of *Adm*, by the logic of epsilon terms.[20] Whereas if *Adm* is empty after all, the epsilon calculus does not tell us anything about the reference of '$\varepsilon F(F \in Adm)$', except that, obviously, it could not be a member of the (then empty) class *Adm*. For the same reason, if *Adm* is not a singleton, the metalanguage of L in which Ramsey semantics is formulated will itself include semantically indeterminate linguistic expressions, such as '$\varepsilon F(F \in Adm)$'. (The Appendix will explain how the indeterminacy of the metalinguistic term '$\varepsilon F(F \in Adm)$' can itself be expressed formally at the metametalevel.)

(8), (9), (10) taken together should only be read as requiring that

> $\exists F \in Adm$, such that for all sentences A, A is true iff $F \vDash A$,
> and 'I' refers to *that very* F in all metalinguistic contexts:
> '$S[I, \text{true}]$' expresses that $S[F, \text{true}]$,
> '$S'[I, \text{true}]$' expresses that $S'[F, \text{true}]$,
> '$S''[I, \text{true}]$' expresses that $S''[F, \text{true}]$,…,

in which

> $S[I, \text{true}]$, $S'[I, \text{true}]$, $S''[I, \text{true}]$,…

[19] Breckenridge and Magidor (2012) do no develop their semantic account of arbitrary reference in formal terms, which makes it difficult to compare it to Ramsification and epsilon terms in Ramsey semantics in more detail. On the one hand, they acknowledge in their Footnote 36 the parallels between their account and Hilbertian epsilon terms in the epsilon calculus, and they also mention in their final Section 3 potential applications to structuralism, structurally indistinguishable objects, and to vagueness, which is perfectly in line with Examples 1 and 2 from Section 1 of the present paper. On the other hand, Breckenridge and Magidor (2012) claim at the very end of their Section 3 that the arbitrary choice of the boundary of the vague term 'tall' "is consistent with the epistemicist claim that 'tall' determinately refers to a particular sharp property". If they meant by that there is a boundary point x, such that it is (by the metasemantic facts) determinate that being x cm is not tall while being x+1 cm is tall, then this would *not* correspond to what Ramsey semantics has to say about the Sorites Paradox in case there is more than one admissible interpretation for 'tall': see Section 4.

[20] Woods (2014, p. 290) conveys precisely that understanding of epsilon terms, which is also how Carnap (1959) used and understood epsilon terms in his reconstruction of theoretical terms as epsilon terms. Carnap gives the example of the term $\varepsilon n(n = 1 \text{ or } n = 2 \text{ or } n = 3)$ about which one might wonder whether it is the case that $\varepsilon n(n = 1 \text{ or } n = 2 \text{ or } n = 3) = 1$ or not: "there is no way of finding out the truth of this. Not because of lack of factual knowledge… Its meaning… has been specified by the ε-operator only up to a certain point", that point being that "it is not any of those numbers which are outside of the class consisting of 1, 2, 3" (Carnap 1959, p. 171).

enumerate all metalinguistic statements whatsoever involving '*I*' (and possibly also 'true'). Carnap's ε-method is nothing but a variant of Ramsification (about which Carnap 1959, 1961 is perfectly explicit) that allows the Ramsey semanticist to understand the theoretical term '*I*' to be Ramsified *uniformly in all metalinguistic contexts*. When it was said before that '$\varepsilon F(F \in Adm)$' "picks an interpretation", this should only be understood as a more picturesque way of expressing the metalinguistic existential quantification from above.

With the help of the metalinguistic epsilon term for '*I*', Ramsey semanticists may develop their semantics while using the term '*I*' for "intended interpretation" and the defined term 'true' just as much as their classical colleagues do. I will take that existential epsilon-term understanding of '*I*' and 'true' for granted now when comparing the semantics in the next section with its immediate rivals: supervaluationist semantics and classical semantics.

3. Comparison with Supervaluationist Semantics and Classical Semantics

Ramsey semantics may not only be seen to maintain the key merits of supervaluationism—validating the laws of classical logic, allowing for semantic indeterminacy, and offering a conception of semantic indeterminacy as an incomplete constraint on classical admissible interpretations—it also avoids those features of supervaluationism that should count as shortcomings at least for the classically minded. I will argue for this by considering first the concept of truth and secondly the concept of logical consequence.[21]

First of all, there is just one concept of *truth* in Ramsey semantics and it is still classical, in the sense that: Ramsey semantics derives T-biconditionals (truth-biconditionals) for all sentences of *L*; truth is compositional; and for every sentence, either it is true or its negation is true. None of this applies generally to (super-)truth in supervaluationist semantics. Of course, supervaluationists might add a disquotational truth predicate to their metalanguage by which they would also be able to express a classical concept of truth over and above their non-classical concept of super-truth (see McGee, V. and B. McLaughlin 1994). But then the single concept of truth from classical semantics would have to bifurcate into two concepts, and it would need to be determined for which purposes which of them is to be applied and how the two concepts relate to each other. (For instance, adding a theory of disquotational-truth to supervaluationism and super-truth does not just by itself suffice for the derivation of "bridge laws" such as 'what is (super-)true is (disquotationally-)true'.)[22]

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[21]Smith (2008) suggests to distinguish supervaluationism and plurivaluationism: both rely on a class *Adm* of admissible interpretations, but supervaluationism invokes three-valued evaluations based on *Adm* whereas plurivaluationism does not. In any case, since neither of the two approaches puts forward metalinguistic Ramsey sentences, both of them differ from Ramsey semantics.

[22]McGee & McLaughlin's (1994) combined theory of disquotational-truth and super-truth comes close to Ramsey semantics, even though they only deal with vagueness while Ramsey semantics is devoted to all kinds of semantic indeterminacy. What their account lacks if compared to Ramsey semantics is just: Ramsification. They distinguish between determinate (or definite) truth and truth *simpliciter*, as Ramsey semantics does, the difference being that they think of 'determinately/definitely true' as a second type of truth predicate, whilst Ramsey semantics does not. That might be more a terminological difference, although in their review of Williamson (1994) it seems at first that McGee & McLaughlin (1997, pp. 224-7) reject the T-biconditionals for 'true' for standard super-valuationist reasons, which would indeed conflict with Ramsey semantics. (It is only afterwards that they acknowledge that the T-biconditionals hold for disquotational truth.) More importantly, returning to McGee & McLaughlin (1994), their theory assumes that determinate/definite truth entails truth *simpliciter*, just as in Ramsey semantics, which is why one would expect truth to given by *some* admissible interpretation. However, McGee & McLaughlin do not say much about this (they only discuss this in an appendix), and in particular, they do not tell us *which* admissible interpretation/model is meant to define truth. The closest they get to Ramsey semantics is when they state "there is no model that is distinguished as the actual model" (p. 240) and when they prove at the end of their appendix

Here is why truth is classical in Ramsey semantics. So far as *atomic* formulas in L are concerned, Ramsey semantics yields T-biconditionals for them by combining (10') with (1); e.g.,

$$P(a) \text{ is true iff } \varepsilon F(F{\in}Adm) \vDash P(a)$$
$$\text{iff } \varepsilon F(F{\in}Adm)(a) \in \varepsilon F(F{\in}Adm)(P),$$

in which '$(\varepsilon F\ F{\in}Adm)(a){\in}(\varepsilon F\ F{\in}Adm)(P)$' may be regarded as the metalinguistic translation of the object-linguistic formula $P(a)$.

If a and P are in fact semantically *determinate*, that is, when there are d and X, such that for all $F{\in}Adm$ it holds that $F(a) = d$ and $F(P) = X$, and (talking *meta*metalinguistically now) when d is denoted by the semantically determinate metalinguistic term 'a' and X is the extension of the semantically determinate metalinguistic predicate 'P', the biconditional from above can be replaced *salva veritate* by

$$P(a) \text{ is true iff } \mathrm{a}{\in}\mathrm{P},$$

which constitutes the more common form of a T-biconditional. (The same would also hold in supervaluationist semantics in that case.) But for atomic object-linguistic statements with semantically *in*determinate terms, their metalinguistic translations in Ramsey semantics involve epsilon-terms that *could not* be replaced by semantically determinate terms that would determinately have the same interpretation as the epsilon terms: if $P(a)$ is semantically indeterminate, then so is '$\varepsilon F(F{\in}Adm)(a) \in \varepsilon F(F{\in}Adm)(P)$' on the metalevel, since the value of '$\varepsilon F(F{\in}Adm)$' is not factually determined. It is just that the metalinguistic epsilon terms make the potential indeterminacy of P and a explicit. Ramsey semantics is meant to clarify semantic indeterminacy and to reconcile it with classical logic and classical truth, not to eliminate it.

With respect to *complex* formulas in L, truth in Ramsey semantics is still compositional, by (1) and (10') again: e.g.,

$C{\vee}D$ is true iff (by (10'))
$\varepsilon F(F{\in}Adm) \vDash C{\vee}D$ iff (by (1))
$\varepsilon F(F{\in}Adm) \vDash C$ or $\varepsilon F(F{\in}Adm) \vDash D$ iff (by (10'))
C is true or D is true.

Taking the two previous points together, it becomes clear that Ramsey semantics also proves T-biconditionals for all complex formulas in L, such as

$$P(a) \vee \neg P(a) \text{ is true iff}$$
$$\varepsilon F(F{\in}Adm)(a) \in \varepsilon F(F{\in}Adm)(P) \text{ or } \varepsilon F(F{\in}Adm)(a) \notin \varepsilon F(F{\in}Adm)(P),$$

and

$$\exists x P(x) \text{ is true iff } \exists d{\in}U, \text{ such that } d \in \varepsilon F(F{\in}Adm)(P).$$

Why is that important? Tarski (1933) intended all T-biconditionals for the object language (L) to be derivable from his theory of truth(-in-L) (the so-called criterion of *material adequacy*) in order to make sure the theory got the extension of 'true(-in-L)' right. The same may be said in the present context, except that, as mentioned above, some of the T-sentences include epsilon

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that there must be *some* admissible interpretation/model, such that truth in that model corresponds to truth *simpliciter* (p. 242). Ramsey semantics *starts* from that statement and works it out on systematic grounds.

terms that cannot be eliminated in favor of metalinguistic expressions with a unique factually determined intended interpretation. In partial compensation, Ramsey semantics may still derive *semantic constraints* on "the" intended truth conditions of such sentences based on descriptions of the *existing metasemantic constraints* on the interpretation of L, so long as these descriptions are derivable metatheoretically as constraints on *Adm*.

For instance, assume that one can derive in one's metatheory that for all $F \in Adm$: $F(0)=0$; $F(100000)=100000$; $0 \in F(B)$; $100000 \notin F(B)$; for all m, n, if $m>n$ and $m \in F(B)$ then $n \in F(B)$); 0, $100000 \in Uni(I) = U$. Then, from this, jointly with the T-biconditional

$B(0)$ is true iff $\varepsilon F(F \in Adm)(0) \in \varepsilon F(F \in Adm)(B)$

and $\varepsilon F(F \in Adm) \in Adm$ ((8'), as derived before), Ramsey semantics proves that

$B(0)$ is true;

analogously, it proves that $\neg B(100000)$ is true, and also the semantic constraint:

for all x, y, if $x>y$ and $B(x)$ is true, then $B(y)$ is true.

All these statements would be derivable in a supervaluationist metatheory, too, if based on the same statements concerning all $F \in Adm$.

However, Ramsey semantics proves additional semantic theorems for truth that supervaluationist semantics can*not* derive for (super-)truth: e.g., in virtue of its compositional semantic rules, Ramsey semantics proves the semantic law

for all x, either $B(x)$ is true or $\neg B(x)$ is true,

by which '$B(x)$ is true' is provably equivalent to '$\neg B(x)$ is not true'. Hence, one may replace the former by the latter in the previously derived constraint and prove that

for all x, y, if $x>y$ and $\neg B(x)$ is not true, then $B(y)$ is true.

None of these additional theorems is derivable in a supervaluationist semantics in which 'true' expresses super-truth.

For the same reason, no sentence A of L lacks a classical truth value in Ramsey semantics, since one can derive from (10') and classical logic the classical semantic law

for all sentences A in L: either A is true or $\neg A$ is true,

which cannot be derived in supervaluationist semantics either, as supervaluationism countenances (super-)truth value gaps.

In a nutshell, truth in Ramsey semantics is classical, whilst (super)-truth in supervaluationist semantics is not: super-truth is *not* compositional (e.g. the super-truth of $P(a) \vee \neg P(a)$ does *not* entail that $P(a)$ is super-true or that $\neg P(a)$ is super-true), it does *not* support (super-)T-biconditionals for all sentences (in the sense that A may be true at some F in *Adm* without being super-true), and it may well leave a sentence A *without* (super-)truth/(super-)falsity values (when neither A nor $\neg A$ is super-true).

Secondly, logical consequence is defined as *truth preservation* in Ramsey semantics and validates *classical logic* completely: not just the theorems and rules of classical logic (such as the Excluded Middle or Modus Ponens) but also its metarules (such as Conditional Proof, Reductio ad Absurdum, Proof by Cases, and metarules for sequents with multiple conclusions). In contrast, either supervaluationism does not define logical consequence as (super-)truth preservation, or if it does, logical consequence does not fully validate classical logic.

Here is why logical consequence and truth relate classically in Ramsey semantics: as explained in Section 2, logical consequence is still defined classically ((2)) as preservation of truth relative to all (classical) interpretations: $A_1,\ldots,A_n \vDash C$ iff $\forall F, s$, if $F, s \vDash A_1,\ldots,A_n$, then $F, s \vDash C$. What Ramsey semantics adds to this is to define I as $\varepsilon F(F\in Adm)$ ((9)), where Adm is a non-empty class of classical interpretations ((8)), from which it follows that $I\in Adm$ (recall Section 2). Thus, I or $\varepsilon F(F\in Adm)$ is one of the interpretations F over which the universal quantifier in (2) ranges and by which both the formulas to the left and to the right of the logical consequence symbol '$\vDash$' are interpreted. Since truth is defined by '$I \vDash \ldots$' in Ramsey semantics (see (10)), it is itself an instance of '$F \vDash \ldots$' in (2), which is why truth and logical consequence "fit together" in Ramsey semantics in precisely the same way in which they do in classical semantics. In contrast, super-truth does not "fit together" with so-called *local* supervaluationist definitions of logical consequence that are not based on the preservation of supervaluationist (super-)truth but which are instead defined, for each admissible interpretation, to preserve truth-at-*it*: see e.g. Williamson (1994, p. 148) for a criticism.[23]

What is more, the classical metarules also remain logically valid in Ramsey semantics if one expands the logical vocabulary of L by a new sentential operator *Det* (for "determinately"): in order to do so, change '$F, s \vDash\ldots$' everywhere into an '$F, s, X \vDash\ldots$' format (where 'X' denotes a non-empty class of interpretations) and augment the semantic rules by

$$F, s, X \vDash Det(A) \text{ iff } \forall F'\in X\text{: } F', s, X \vDash A.$$

(If A is a sentence, 's' may be dropped again.) The original definition (10) of truth can be adapted to the presence of the new logical operator *Det* by means of: A is true iff $I, Adm \vDash A$.

The operator *Det* is familiar from supervaluationist theories in which it expresses the metalinguistic concept of (super-)truth by object-linguistic means. But adding it to the object language is known to *undermine* the logical validity of some classical metarules if logical consequence is defined *globally* as preservation of super-truth: e.g., $A \vDash Det(A)$ holds in supervaluationist semantics with global logical consequence, while $\vDash A \rightarrow Det(A)$ fails, thus invalidating Conditional Proof (see Fine 1975, p. 290, Williamson 1994, pp. 151-3).

In contrast, $Det(A)$ does not express in Ramsey semantics that A is true but rather that competent-usage facts and relevant non-linguistic facts, governed by metasemantic laws, determine A to be true: the truth of A is entailed by what the existing metasemantic constraints are like. Indeed, for all sentences A in L it holds: if $Det(A)$ is true, then A is true,[24] but not necessarily the other way around. When n is a borderline case of the vague term B, and thus it is neither the case that for all $F\in Adm$ $n \in F(B)$ nor that for all $F\in Adm$ $n \notin F(B)$, it follows that the conjunction $\neg Det(B(n))$ & $\neg Det(\neg B(n))$ is true, while either $B(n)$ or $\neg B(n)$ must be true according to Ramsey semantics. But what is most important for present logical purposes: the presence of *Det* in the object language does *not* affect the logical validity of any of the classical metarules. E.g., the metalinguistic proof of 'if $A \vDash B$, then $\vDash A \rightarrow B$' goes through in Ramsey semantics in precisely the same way in which it does in classical semantics, independently of whether object language (and hence A or B) includes *Det* or not. Such are the logical advantages of Ramsey semantics over supervaluationist semantics. (But see Keefe 2000 and Varzi 2007

[23] Field (2008) has argued recently that logical consequence does *not* necessarily preserve truth, but that was in the special context of a semantically closed language with paradoxical sentences, such as the Liar sentence.

[24] That is because $Det(A)$ is true just in case $I, Adm \vDash Det(A)$, which is equivalent to $\forall F'\in Adm$: $F', Adm \vDash A$, which entails by universal instantiation that $I, Adm \vDash A$ and hence by (10) from Section 2 that A is true.

for defenses of supervaluationism against these challenges, and see Varzi 2007 and Cobreros 2008 for further supervaluationist notions of consequence beyond the local and the global one.)

By preserving classical truth, classical logic, and their classical relationship, Ramsey semantics is clearly much closer to classical semantics than supervaluationism, and perhaps as close as possible without committing oneself to the unique satisfiability of '$F \in Adm$'. This should put Ramsey semantics ahead of supervaluationism in the eyes of anyone who is attracted by classical semantics, who at the same time wants to grant the possibility of semantic indeterminacy, and who therefore aims to approximate classical semantics while at the same time being prepared for semantic indeterminacy.

So far as the comparison with classical semantics itself is concerned, it is easy to see what will be gained by switching from classical semantics to Ramsey semantics: the problematic metasemantic presupposition of semantic determinacy ($\exists! F(F \in Adm)$) will be avoided. It is much more difficult to see what is lost by moving to Ramsey semantics, since it preserves so many of the central theoretical features of classical semantics. Indeed, other than '$\exists! F(F \in Adm)$', the only apparent difference between the two semantics concerns the presentation and structure of their axioms and the appearance of epsilon terms in Ramsey semantics on the right-hand side of T-biconditionals for atomic formulas. Since (8), (9), (10), jointly with the epsilon calculus, prove (11), that is, '$I \in Adm$ and for all sentences A: A is true iff $I \vDash A$' (recall Section 2), every statement that a classical semanticist logically derives from that thesis is also derivable in Ramsey semantics. (Though these consequences may not always have the same meaning, since 'I' is understood differently in Ramsey semantics than in classical semantics.). In other words: on purely deductive grounds, one is never going to notice the difference between classical semantics and Ramsey semantics so long as the classical semanticist does not explicitly invoke its only additional presupposition '$\exists! F(F \in Adm)$', that is, the thesis of unique determination of the intended interpretation of L. It is precisely that assumption by which classical semantics exceeds Ramsey semantics in deductive strength: but semanticists do not normally exploit that assumption deductively in their semantic work but merely presuppose it, and Ramsey semantics happily abandons that presumption in order to accommodate semantic indeterminacy if and when necessary.

In the remaining sections, I will describe the ramifications that Ramsey semantics has for the Sorites, higher-order vagueness, and interpretational change and continuity. This, I hope, will add to the attractiveness of the semantics. Taking everything into account, Ramsey semantics should therefore cross the finishing line as the top contender amongst theories that aim to meet the challenge from semantic indeterminacy on (quasi-)classical grounds.

4. The Sorites Paradox

What does Ramsey semantics predict concerning the infamous Sorites paradox?

If formulated for the predicate B (for baldness) in L from Example 1, and with quantifiers ranging over the natural numbers, the paradoxical argument is:

(13) $B(0)$
(14) $\forall x(B(x) \rightarrow B(x+1))$
(15) Therefore, $B(100000)$,

in which 0 and 100000 are numerals for 0 and 100000, respectively (that is, $I(0) = 0$ and $I(100000) = 100000$). Whilst (13) is obvious and (14) sounds plausible at least at first glance, (15) is absurd, and yet the argument is logically valid in classical logic—hence the paradox.

Indeed, in Ramsey semantics, (13) is true ($I \vDash B(0)$) and (15) is false ($I \vDash \neg B(100000)$): recall the description of the metasemantic constraints at the beginning of Section 2 which hold for all members F of Adm, and use $I = \varepsilon F(F \in Adm)$ ((9)) and $\varepsilon F(F \in Adm) \in Adm$ ((8')). Since Ramsey semantics also preserves the classical logical validity of the argument (see Section 3), (14) must be the culprit: the Sorites is not sound since (14) is false, that is,

$$\text{(16) } \varepsilon F(F \in Adm) \nvDash \forall x(B(x) \rightarrow B(x+1)),$$

which by (1) is equivalent to

$$\text{(17) } \varepsilon F(F \in Adm) \vDash \neg \forall x(B(x) \rightarrow B(x+1)),$$

or equivalently again:

$$\text{(18) } \varepsilon F(F \in Adm) \vDash \exists x(B(x) \ \& \ \neg B(x+1)).$$

That is like the supervaluationist diagnosis, except that supervaluationists regard the existential object-linguistic claim from (18) as (super-)true without necessarily regarding any of its *instances* as (super-)true, which conflicts with the pre-theoretically plausible principle that "a true existential generalization has a true instance" (Williamson 1994, p. 154).

In contradistinction, *there is a true instance of the true existential claim* $\exists x(B(x) \ \& \ \neg B(x+1))$ according to Ramsey semantics, since definition (1) implies that (18) is equivalent to

$$\text{(19) } \exists n \in U, \text{ such that: } \varepsilon F(F \in Adm), s\frac{n}{x} \vDash B(x) \ \& \ \neg B(x+1)$$

and also to

$$\text{(20) } \exists n \in U, \text{ such that: } n \in \varepsilon F(F \in Adm)(B) \text{ and } n+1 \notin \varepsilon F(F \in Adm)(B),$$

both of which state the existence of a true instance of the true existential claim from (18).

On the negative side: doesn't (20) say that there is a sharp cut-off point for B, which would seem counterintuitive again? This depends on what is meant by 'sharp cut-off point': Ramsey semantics is certainly not committed to the existence of a sharp *factually determined* cut-off point, for (20) does not mean that the *existing metasemantic constraints on the interpretation of L would determine* a particular instance of the existential claim in (18) to be true. That is, using the *Det* operator from Section 4: Ramsey semantics does *not* assume that there is an $n \in U$, such that $\varepsilon F(F \in Adm)$, $s\frac{n}{x}$, $Adm \vDash Det(B(x) \ \& \ \neg B(x+1))$—for the boundary of B may be semantically indeterminate. Whereas supervaluationists interpret *Det* as expressing (super-)truth and hence regard metalinguistic statements about *Det* as statements about (super-)truth, Ramsey semanticists keep 'determined to be true by the facts' and 'true' separate: the former is about metasemantic determination, only the latter is about *truth*. While an existential statement is true just in case one of its instances is, the existing metasemantic constraints might determine *Adm* to be non-empty—here: determine the existence of various admissible cut-off points for B—without determining which admissible cut-off point is "the" intended one. Or in more logical terms: 'true' commutes with the existential quantifier, whereas *Det* does not necessarily do so (as pointed out by McLaughlin and McGee 1994, p. 212): if 'B' is semantically indeterminate, $Det(\exists x(B(x) \,\&\, \neg B(x+1)))$ will be true in Ramsey semantics while $\exists x(Det(B(x) \,\&\, \neg B(x+1))$ will be false.

But doesn't (20) at least say that there must be a sharp cut-off point for *B* in the following weaker sense that does not involve the '*Det*' operator: there is a natural number *n*, such that every number of hairs less than or equal to *n* counts as bald according to the intended interpretation of *B*, and every number of hairs greater than *n* counts as non-bald?

Now the answer is of course *yes*, since this is just a restatement of (20) (for $I = \varepsilon F\ (F \in Adm)$), to which Ramsey semantics is committed. But one should add that Ramsey semantics uses '*I*' ('intended interpretation') as a short-hand for '$\varepsilon F\ (F \in Adm)$', and statements involving '$\varepsilon F\ (F \in Adm)$' are themselves just stand-ins for existential claims. (Recall Section 2.) In particular, deriving a statement such as (20) should merely be regarded as a short-hand for deriving the existential claim

(21) $\exists F \in Adm$, such that for all sentences *A*, *A* is true iff $F \vDash A$,
and $\exists n \in U$, such that: $n \in F(B)$ and $n+1 \notin F(B)$,

from (8), (9), (10), the epsilon calculus, and the fact that for all $F \in Adm$ it holds that $\exists n \in U$, such that $n \in F(B)$ and $n+1 \notin F(B)$ (which follows from the assumptions on *Adm* that were described at the beginning of Section 2).

Jointly with the semantic determinacy of *B*, that is, when the existing metasemantic constraints determine the extension $I(B)$ uniquely—when all members *F* of Adm assign the same extension to *B*—the plain existential statement (21) would indeed commit the Ramsey semanticist to the existence of a sharp cut-off point for *B* in every sense of the term: which is fine, since in that case '$I(B)$' would refer to a uniquely factually determined set-theoretic extension with a precise cut-off. In that case, Ramsey semantics for *B* would simply collapse into epistemicism about *B*, assuming epistemicists would still be right that we would not know exactly what that extension $I(B)$ of *B* would be like.

However, if the existing metasemantic constraints do *not* determine $I(B)$ uniquely, and hence *Adm* includes more than one *F*, (21) is still claimed true by Ramsey semantics but is *only* committed to the *existence of an admissible interpretation with a sharp cut-off point for B*. And that reduces to a mere commitment to the existence of an admissible interpretation, as *all* interpretations in *Adm* assign—by their classical nature—a sharp cut-off point to *B*. In this way, Ramsey semantics takes the bite out of (21) and hence (20). Statement (20) is accepted by Ramsey semanticists but only expresses the Ramsification of what classical semanticists would mean by 'the intended interpretation involves a sharp boundary for baldness'.

Perhaps the tolerance intuition (cf. Wright 1975) that lends support to the major premise (14) of the Sorites is itself but a reflection of there being no fact of the matter about which member of *Adm* is denoted by '$\varepsilon F(F \in Adm)$' in the case of semantic indeterminacy. For assume again the interpretation of *B* is semantically indeterminate, that is, there exist at least two members of *Adm* that assign different extensions to *B*: in that case, there must also be interpretation functions *F* and F^+ in *Adm*, such that

for all *n*, if $n \in F(B)$ then $n+1 \in F^+(B)$.

(This follows easily from semantic indeterminacy and from 'for all *m*, *n*, if $m>n$ and $m \in F(B)$ then $n \in F(B)$' expressing an existing metasemantic constraint on all $F \in Adm$.) Since both *F* and F^+ are members of *Adm*, each of them could serve as the referent of '$\varepsilon F(F \in Adm)$', that is: the existential statements for which statements of the form '$S[\varepsilon F(F \in Adm, \text{true}]$' serve as stand-ins quantify both over *F* and F^+ (and all other members of *Adm*). It is therefore not far off the truth to summarize the situation by (14), even though strictly speaking the B in the antecedent of the embedded conditional of (14) needs to be interpreted by a different member of Adm (say, F) than the B in the consequent (which should be interpreted by F^+).

One may summarize the different verdicts by classical semantics, supervaluationist semantics, and Ramsey semantics concerning the existence of borderline cases of B ("bald") as follows:

- Classical semantics takes the relevant borderline facts to be complete ('$\exists x Det(B(x) \& \neg B(x+1))$' is *true*) and employs a *classical* concept of truth ('$\exists x(B(x) \& \neg B(x+1)$' is true, hence $\exists n$, s.t. '$B(x) \& \neg B(x+1)$' is true of n).
- Supervaluationist semantics takes the borderline facts to be incomplete ('$\exists x Det(B(x) \& \neg B(x+1))$' is super-*false*) and invokes a *non-classical* concept of truth ('$\exists x(B(x) \& \neg B(x+1)$' is super-true, but *not* $\exists n$, s.t. '$B(x)\&\neg B(x+1)$' is super-true of n).
- Ramsey semantics takes the borderline facts to be incomplete ('$\exists x Det(B(x) \& \neg B(x+1))$' is *false*) but uses a *classical* concept of truth ('$\exists x(B(x) \& \neg B(x+1)$' is true, hence $\exists n$, s.t. '$B(x) \& \neg B(x+1)$' is true of n).

The position taken by Ramsey semantics vis-à-vis the borderline cases of vague terms is therefore "half way in between" classical semantics and supervaluationism: it agrees with supervaluationism that the existing metasemantic constraints are likely to leave vague terms with *factual* borderline gaps, while agreeing with classical semantics that the *concept* of truth is classical, such that, e.g., it obeys the standard compositional clauses. Classical semantics presupposes that there are no metasemantic gaps, supervaluationism acknowledges metasemantic gaps and translates them into semantic ones, while Ramsey semantics acknowledges metasemantic gaps but avoids translating them into semantic gaps.

The Ramsey semanticist would therefore summarize the situation concerning the vague term B as follows: there are sentences involving B, such as $B(0)$, which the metasemantic facts determine to be true and which are thus true. There are sentences involving B, such as $B(100000)$, which the metasemantic facts determine to be false and which are hence false. The Sorites argument is logically valid but not sound: its major premise is false. Since it is likely that the metasemantic facts do not determine the intended interpretation of B uniquely, it is likely that there is a sentence of the form $B(n)$ for some number n in between 0 and 1000000, which is not determined to be true by the metasemantic facts but which is still true, while $B(n + 1)$ is not determined to be false by the metasemantic facts but still false. Both n and n+1 would be available as admissible cut-off points for baldness (and maybe others) and there is no metasemantic fact of the matter which of them (if either of them) is "picked" by the epsilon term '$\varepsilon F(F \in Adm)$' by which the intended interpretation I is defined in Ramsey semantics. That is also why the major premise of the Sorites sounded so plausible initially.

These should be reasonable verdicts. Whether or not the existing metasemantic constraints determine "the" intended interpretation of L uniquely, Ramsey semantics stays on the safe side and yet points to a way out of the Sorites that should be plausible for anyone who is inclined towards classical semantics at all.

5. Higher-Order Vagueness

If the first-order extensional object language L from Example 1 in Section 2 is indeed semantically indeterminate in virtue of terms such as B being vague: does that mean that there must also be higher-order vagueness?

I have already explained in Section 2 that if Adm is not a singleton set, '$\varepsilon F(F \in Adm)$' is meant to be semantically indeterminate, so that there is no fact of the matter which member of

Adm is "picked" by that epsilon term. Thus, in that case, there will certainly be semantically indeterminate linguistic expressions in the very metalanguage of *L* in which Ramsey semantics is developed. I will deal with the formalization of that kind of metalinguistic indeterminacy in the Appendix.

For now I only want to deal with the potential *indeterminacy of the indeterminacy of the object language L with vague terms*, which corresponds to what is more usually understood by higher-order vagueness. In other words: consider metalinguistic semantic predicates such as 'is determinate(-in-*L*)' and 'is an (*L*-)borderline case of', which express semantic properties/relations of sentences of the vague object language *L*, as in: '*B*(*0*) is determinate', '¬*B*(*100000*) is determinate', and '*n* is a *borderline case* of *B*', that is, '*B*(*n*) is not determinate and ¬*B*(*n*) is not determinate'. Unlike the previous sections, let us make such claims precise by a metalinguistic determinacy *predicate* '*Det*' of sentences instead of the previous sentential determinacy *operator*, in order not to leave the realm of standard first-order extensional languages. (Otherwise we would first have to extend Ramsey semantics for the strictly extensional language *L* from Section 2 to Ramsey semantics for a language with nested applications of non-extensional sentential operators.) The question is thus: are semantic predicates such as '*Det*' *themselves semantically indeterminate*? (See Wright 2010 for arguments to the contrary.) And if so, how would Ramsey semantics deal with that?

In the present context, in which such a metalinguistic '*Det*' predicate for sentences in *L* will be defined metalinguistically in analogy with the determinacy operator from previous sections, that is, by

(22) for all sentences *A* in *L*: $Det(A)$ iff for all $F \in Adm$, $F \vDash A$,

the only way for '*Det*' to be vague would be for '*Adm*' to be vague, too. And since '*Adm*' was characterized as the class of classical interpretations *F* that *conform to the existing metasemantic constraints on the interpretation of* L, '*Adm*' is vague only if the metalinguistic term 'conforms to the existing metasemantic constraints on the interpretation of *L*' is vague. *So does that term have a factually uniquely determined intended interpretation?*[25]

If *yes*, '*Adm*' would have a factually determined unique intended interpretation, the same would hold for (8), (9), (10), and for '*Det*' as defined in (22); hence, Ramsey semantics would steer clear of higher-order vagueness. None of this would affect any of the considerations from previous sections: for the semantic determinacy of the metalinguistic term '*Adm*' would still allow for the interpretation of the *object* language *L* to be semantically *in*determinate in the sense that *Adm* includes more than one interpretation mapping *F*. If so, the relevant metalinguistic epsilon term '$\varepsilon F(F \in Adm)$' of Ramsey semantics will still be semantically indeterminate, in virtue of it being indeterminate what it "chooses" from *Adm* (which is to be distinguished from '*Adm*' itself being indeterminate). In terms of our previous analogy: even when the equation '$z^2 = x + y$' has a uniquely determined reading over the real numbers, this still permits the existence of more than one real-valued solution for '*z*' given $x = 3$ and $y = 1$.

But if the answer to the question above is *no*, the metalinguistic term 'conforms to the existing metasemantic constraints on the interpretation of *L*' is itself semantically indeterminate: more than just one interpretation can be assigned to it without invalidating any of the existing *meta*metasemantics constraints on *its* interpretation. If so, '*Adm*' is semantically indeterminate, the same is presumably true for (8), (9), and (10), and Ramsey semantics will have to cope with higher-order vagueness. In our analogy again: '$z^2 = x + y$' might tacitly

[25] The question is not whether the existing metasemantic constraints themselves are vague, which would be yet another form of metaphysical vagueness: a vagueness of metasemantic facts or metasemantic laws. The question is rather whether the way in which the metalanguage of *L describes* these constraints is vague.

include some additional parameters that need to be set appropriately before the equation can be applied, which is why '$z^2 = x + y$' actually represents a whole *family* of equations each of which comes with its own set of solutions.

So which answer is it? This depends on the metasemantic properties of the object language in question.

For instance, consider the purely mathematical language L' from Example 2 in Section 1, even when strictly speaking the example did not concern vagueness: it is quite possible that the corresponding class *Adm′* can be characterized completely in purely class-theoretic terms, e.g., as the class of *all* interpretations that satisfy the second-order Dedekind-Peano axioms, and that any potentially remaining indeterminacy concerning the class-theoretic membership predicate '$\in$' can be safely ignored when dealing with the countably infinite matters of arithmetic (rather than, say, large-cardinal issues). If so, whether for systematic reasons or for "all practical purposes", there should not be much room left for higher-order indeterminacy. In contrast, if *Adm′* were characterized as the class of all those interpretations satisfying the second-order Dedekind-Peano axioms that are additionally *easily definable* set-theoretically, then 'easily' would be likely to be vague, and there should be higher-order vagueness. Ramsey semantics should be prepared for such possibilities.

More urgently, as far as the formalization of some fragment of *natural* language with vague terms is concerned, such as language L from Example 1, it is not even clear how one could tell which of the two answers from above applies, and there might be principal reasons for why we *could not* tell which of them applies. Therefore, once again, Ramsey semantics should at least be prepared for higher-order vagueness.

And it is not hard to see that it is: for the Ramsification "scheme" may simply be iterated at higher levels in the Tarskian hierarchy, such that: a *metameta*linguistic epsilon term is used to pick "the" intended interpretation of '*Adm*' from a class Adm_2 of interpretations of the *meta*language of L; a *metametameta*linguistic epsilon term is used to pick "the" intended interpretation of 'Adm_2' from a class Adm_3 of interpretations of the *metameta*language of L; and so forth. Or in other words: I is *a* member of *Adm*, which in turn is *a* member of Adm_2, which in turn is *a* member of Adm_3, and so on. Picture "simultaneous choices" being carried out on all levels of the Tarskian hierarchy, such that at each level metameta…-semantic facts constrain the process. At the same time, the outcomes of the process might not be determined uniquely at any level. As McGee & McLaughlin (1994, p. 230) point out, it is at least possible that "As we ascend the hierarchy of metalanguage, we find vagueness all the way up"—though in the present case in which such a hierarchy is erected on top of a first-order language L with ordinary vague terms and a fixed first-order universe of discourse, one might also expect Adm_α to become a singleton set from some ordinal level α and to remain so at all levels $\beta > \alpha$. (But what that "stabilization ordinal" α is like might itself be indeterminate.)

In any case, on each language level a corresponding determinacy predicate may be introduced again in the expected manner; e.g., for all sentences A of the *meta*language of L:

$$Det_2(A) \text{ iff for all } F_2 \in Adm_2,\ F_2 \vDash A.$$

Previously, '$Det(A)$' expressed metalinguistically that the sentence A of L comes out true in whatever way a classical interpretation F satisfies all existing metasemantic constraints on the interpretation of L. Similarly, one may now employ 'Det_2('$Det(A)$')' to express *meta*metalinguistically that '$Det(A)$' comes out true in whatever way a classical interpretation F_2 satisfies all existing *meta*metasemantic constraints on the interpretation of the *meta*language of L, including the *meta*linguistic term '*Adm*'. Using determinacy predicates and truth predicates, one may express multiply iterated determinacy claims at one and the same language level and prove in Ramsey semantics how they interact.

E.g., at the metametalevel level, one can prove that the determinacy of the determinacy of $B(n)$ implies the determinacy of $B(n)$, but one cannot in general prove the converse. (See the Appendix for technical details.) Thus, the usual pattern of determinacy emerges for B—the determinacy area for B is a superset of the determinacy-determinacy area for B, and each area exhibits indeterminate boundaries:

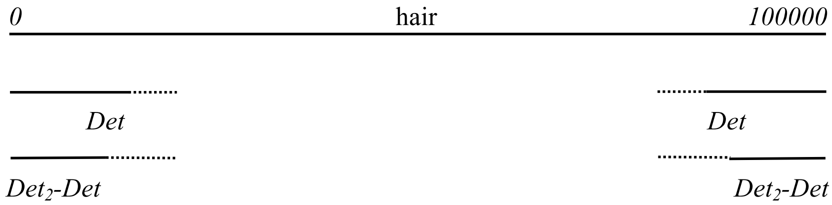

Fig. 1: Determinacy Pattern for B (baldness)

If, on each language level α, the Ramsey semanticist can rely on sufficiently many metameta…-semantic findings concerning the respective extensions of ‘Adm_i’—whether regarding these findings as likely or firmly accepting them as true or even adding them as axioms to the respective metameta…-theories—the resulting theoretical package should allow her to draw non-trivial conclusions on higher-order vagueness. If so, at each level, the Ramsification of classical semantics will be informative, and the Ramsification at the level will be consistent if classical semantics is: thus, no “infinite regress” in any bad sense of the term will emerge. No strong assumptions will be required other than postulating the existence of an admissible interpretation at each level of the Tarskian hierarchy. Nor will there be any novel phenomena over and above those covered by previous sections. Higher-order vagueness does not constitute a greater challenge to Ramsey semantics than first-order vagueness.[26]

6. Interpretational Change and Continuity

Let me return now to the mathematical and scientific Examples 2 and 3 from Section 1: by now, it should be clear what their Ramsey semantics will have to look like. This will prompt more general considerations on diachronic and synchronic referential/extensional continuity and how they are compatible with Ramsey semantics.

Example 2 (reconsidered): Let L' again be a second-order formalization of the language of arithmetic. Let us assume the class Adm' of its admissible interpretations is identical to the class of all classical interpretations that satisfy the second-order Dedekind-Peano axioms (and which are thus pairwise isomorphic):

$$Adm' = \{F: F \vDash \mathrm{PA}_2\}.$$

What Ramsey semantics for L' adds to this is:

[26] In their argument against supervaluationism, Fodor and Lepore (1996) worry that admissible interpretations are bound to invalidate certain conceptual truths, such as ‘Someone with hair of number and configuration so-and-so is a borderline case of baldness’. As explained in this section, such a statement belongs to the *meta*language of L, since ‘borderline’ is a metalinguistic semantic predicate, and I have assumed the object language L not to include any semantic terms itself. Requiring such a conceptual truth to be satisfied is therefore a *metameta*semantic constraint that ought to be respected by all admissible interpretations of the *meta*language of L. The methods from this section and from the Appendix demonstrate how such constraints can be implemented in Ramsey semantics.

$\exists F'$ ($F' \in Adm'$). (In fact, this would not have to be added, since it follows set-theoretically from the equality above.)

$I' =_{df} \varepsilon F'$ ($F' \in Adm'$).

For all sentences A in L': A is true(-in-L') iff $I' \vDash A$.

The resulting semantics takes the structuralist conception of arithmetic seriously, according to which only the structure of the interpretation of the arithmetical symbols is determined factually. And it combines this with a unique intended (though not factually uniquely determined) set $Uni(I')$ of natural numbers and the classical concept of truth for the language of arithmetic that supports classical logic as used by number theorists. Acknowledging semantic indeterminacy in that way does not spell any trouble whatsoever for the Ramsey semantics of the language of second-order arithmetic.

On the contrary, the semantics has some noteworthy consequences: the epsilon calculus and the definitions of 'I'' and 'true' above just by themselves entail (recall Section 2) that

the second-order Dedekind-Peano axioms are true iff $\exists F'(F' \in Adm')$,

which, since Adm' is the class of *all* classical interpretations that satisfy the second-order Dedekind-Peano axioms, can be reformulated as:

the second-order Dedekind-Peano axioms are true iff they are satisfiable.

In more traditional terminology: by the structuralist Ramsey semantics for L', the truth of the second-order Dedekind-Peano axioms is tied analytically to their satisfiability—which resembles in some way (though only in *some* way) how Hilbert (1899) famously characterized mathematical truth in his exchange with Frege.

Example 3 (reconsidered): Let L'' formalize again the language of Newtonian mechanics. The class Adm'' of admissible interpretation of L'' is likely to include at least two members, where one member F of Adm'' interprets 'mass' as relativistic mass, while another member F' of Adm'' interprets it as proper mass. Let us assume that these are the only members of Adm'':

$Adm'' = \{F, F'\}$.

Ramsey semantics for L'' postulates:

$\exists F''$ ($F'' \in Adm''$). (Once again, this would not actually have to be stated, since it follows set-theoretically from the equality above.)

$I'' =_{df} \varepsilon F''$ ($F'' \in Adm''$).

For all sentences A in L'': A is true(-in-L'') iff $I'' \vDash A$.

In this way, Ramsey semantics applies the classical concept of truth to the language of Newtonian mechanics and interprets the logic of Newtonian arguments and proofs as classical, while granting that, to the best of our present physical knowledge, it is not possible to ascribe the Newtonian term 'mass' a unique factually determined intended reference. In the terminology of Field (1973), Newtonian 'mass' only *refers partially*, which Field himself makes precise in a supervaluationist manner, while the Ramsey semantics from above does so by Ramsifying classical semantics. For instance, Ramsey semantics for L'' derives from the postulates above that

$I''(m) = F(m)$ or $I''(m) = F'(m)$,

without deriving either of the disjuncts (recall Section 2), which seems like the right prediction to make.

Example 3 concerned *revolutionary* language change in the sense that one is giving a semantics for the language of a scientific theory that belongs to an earlier scientific paradigm while using terms from the language of a scientific theory that belongs to a contemporary scientific paradigm (using the terminology of Kuhn 1962).

More generally, Ramsey semantics offers semanticists the resources to study interpretational change and diachronic continuity in all cases of scientific development, including *normal* ones in which the central theoretical terms remain to be used in similar ways. In either case, scientific realists would expect referential/extensional continuity between empirically successful successor theories: the references/extensions of theoretical terms in successful theories are preserved in the transition to their successful successor theories (see Putnam 1975b).

Carnap's (1959, 1961) proposal to view theoretical terms as definable by epsilon terms (which he employed object-linguistically while Ramsey semantics uses them metalinguistically) was meant to support similar continuity considerations, but without the presupposition that a unique intended interpretation has been determined factually. Instead, he meant to capture the open-ended, increasing, and yet partial specification of theoretical terms in scientific progress (as Ramsey 1929 had done before):

> this definition [gives] just so much specification as we can give, and not more. We do not want to give more, because the meaning should be left unspecified in some respect, because otherwise the physicist could not—as he wants to—add tomorrow more and more postulates, and even more and more correspondence postulates, and thereby make the meaning of the same term more specific than [it is] today. So, it seems to me that the ε-operator is just exactly the tailor-made tool that we needed, in order to give an explicit definition, that, in spite of being explicit, does not determine the meaning completely, but just to that extent that it is needed. (Carnap 1959, pp. 171-2)

Of course, present-day scientific realists would not buy into Carnap's overly descriptivist conception of scientific theoretical terms here. But so long as the linguistic metasemantic facts underlying the metasemantics of theoretical terms involve at least *some* theoretical component—as acknowledged, e.g., by combined causal-theoretical accounts of theoretical terms (see Psillos 1999, Chaper 12)—and hence some scientific theory is partially constitutive of these terms, it is plausible that the interpretation of such terms may remain partial in virtue of the theory being deductively incomplete or vague or both. What is more, the causal or ostensive components of the metasemantics of theoretical terms may themselves give rise to semantic indeterminacy: reference-fixing phrases such as 'whatever causes phenomenon so-and-so shall be called…' may be semantically indeterminate by leaving "causal distance" and the relevant type of causality open; and an act of ostension may be directed simultaneously towards a plurality of objects, kinds, and physical structures, such that neither intentions nor interests may suffice to rule out all but one potential target. For the same reason, it should be possible to extend the metasemantic constraints on the interpretation of a theoretical term by extending or "precisifying" the theory that partially characterizes it, or by diambiguating its reference-fixing description, or by excluding relevant causal pathways as semantically unintended: the class of admissible references/extensions of theoretical terms may change.

If so, Ramsey semantics (as, to some extent, supervaluationist semantics) may capture the respective semantic changes as follows: let L_1 be the language of a scientific theory at a time, let L_2 be the language of its immediate successor theory, and let *Terms* be a set of theoretical terms that belong simultaneously to both languages. Let Adm_1 be the class of all restrictions of L_1-admissible interpretations to *Terms* (while leaving the domain of the interpretations the same), such that only the members of *Terms* are interpreted by the restricted interpretation functions F_1 in Adm_1; and let Adm_2 be the corresponding class of all interpretation functions F_2 that result from restricting L_2-admissible interpretations to *Terms*.

Then one can define:

> The L_2-interpretation of the members of *Terms* is a (proper) *specification* of their L_1-interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_2 \subseteq Adm_1$, that is, Adm_2 is a (proper) subclass of Adm_1.

Specification is what Carnap describes in the quote from before: the interpretation of the members of *Term* become more specific in the transition from L_1 to L_2. In the language of supervaluationism, one might also speak of increased "precisification" or "sharpening".

But specification is just one kind of interpretational change next to others:

> The L_2-interpretation of the members of *Terms* is a (proper) *diversification* of their L_1-interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_1 \subseteq Adm_2$, that is, Adm_2 is a (proper) superclass of Adm_1.
>
> The L_2-interpretation of the members of *Terms* is a (proper) *compatible modification* of their L_1-interpretation just in case ($Adm_1 \neq Adm_2$ and) $Adm_1 \cap Adm_2 \neq \varnothing$, that is, Adm_1 (differs from and) has non-empty overlap with Adm_2.
>
> The L_2-interpretation of the members of *Terms* is a *complete revision* of their L_1-interpretation just in case $Adm_1 \cap Adm_2 = \varnothing$, that is, Adm_1 does not overlap with Adm_2.[27]

Proper diversification is the "opposite" of proper specification: it adds new ways of interpretation and thereby makes interpretation *less* specific.

(Proper) specification and (proper) diversification of interpretation are both special cases of (proper) compatible modification, but compatible modification is broader: it also covers cases in which the previous metasemantic constraints on the interpretation of the members of *Terms* are compatible with newly introduced ones ($Adm_1 \cap Adm_2 \neq \varnothing$) while the interpretation of the terms does not become more or less specific.

Finally, in cases of complete revision, the new metasemantic constraints rule out the previous ones completely (that is, $Adm_1 \cap Adm_2 = \varnothing$). For the same reason, in the case of complete revision, Ramsey semantics *rules out* perfect referential/extensional continuity between L_1 and L_2 with respect to all theoretical terms in *Terms*: for the logical laws of the epsilon calculus imply in that case that

$$\varepsilon F_1(F_1 \in Adm_1) \neq \varepsilon F_2(F_2 \in Adm_2),$$

that is, there must be some P in *Terms*, such that $(\varepsilon F_1(F_1 \in Adm_1))(P) \neq (\varepsilon F_2(F_2 \in Adm_2))(P)$.

However, in the case of compatible modification ($Adm_1 \cap Adm_2 \neq \varnothing$), Ramsey semantics *allows* one to regard all terms in *Terms* as preserving their references/extensions in the transition from L_1 to L_2: for it is easy to see that the postulates of Ramsey semantics may always be expanded consistently in that case by

$$\varepsilon F_1(F_1 \in Adm_1) = \varepsilon F_2(F_2 \in Adm_2),$$

[27] More fine-grained classifications might also take into account how the domain(s) of interpretation may change in the transition from L_1 to L_2. But I will put this to one side here for the sake of simplicity.

that is, by

$$I_1 = I_2.$$

In such case, let us call '$I_1 = I_2$' the statement of *perfect interpretational* (*referential/extensional) continuity* (holding between L_1 and L_2 and with respect to *Terms*).

Cases of compatible interpretational modification include the *extreme* case of improper specification/diversification in which Adm_1 is identical to Adm_2 and where they are *singleton* classes, that is, $Adm_1 = \{I_1\} = \{I_2\} = Adm_2$: in that extreme case, Ramsey semantics even *entails* perfect interpretational continuity, by the logic of epsilon terms. Scientific realists, who are normally wedded to classical semantics, either mean that extreme case when they are speaking of referential/extensional continuity between scientific successor languages, or they mean (more realistically) that Adm_1 and Adm_2 are singletons while their members are merely similar or approximately identical: $Adm_1 = \{I_1\}$, $Adm_2 = \{I_2\}$, and $I_1 \approx I_2$. Call '$I_1 \approx I_2$' a statement of *approximate interpretational* (*referential/extensional) continuity* (again between L_1 and L_2 and with respect to *Terms*). Cases of approximate continuity that are not cases of perfect continuity would be cases of complete (but still approximate) revision.[28]

But Ramsey semantics also permits perfect and approximate continuity in cases of compatible modification in which Adm_1 and Adm_2 are *not* both singleton classes, that is, when the members of *Terms* are semantically indeterminate either *qua* L_1-terms or *qua* L_2-terms or both. For instance, consider the case of specification again ($Adm_2 \subseteq Adm_1$), that is, when *more* metasemantic constraints on the interpretation of the members of *Terms* get introduced in the transition from L_1 to L_2: if so, the metasemantic facts determine that every admissible L_2-interpretation on *Terms* is also an L_1-interpretation, and hence every L_1-determinate statement *A* that is composed solely of members of *Terms* is also L_2-determinate. (That is: if $Det_{L1}(A)$ then $Det_{L2}(A)$.) But so long as Adm_1 (and hence Adm_2) is not a singleton class, there will not be any facts of the matter—no existing metasemantic constraints—that require the interpretations of L_1 and L_2 to perfectly or approximately coincide on *Terms*. That is, given such Adm_1 and Adm_2, Ramsey semantics will *not by itself entail* that

$$I_1 = \varepsilon F_1(F_1 \in Adm_1) =/\approx \varepsilon F_2(F_2 \in Adm_2) = I_2.$$

However, one may still consistently *extend* Ramsey semantics for L_1 and L_2 by the perfect/approximate continuity statement '$I_1 =/\approx I_2$' in that case: if one does so, this will not serve the purpose of describing existing metasemantics facts but rather correspond to a "free semantic posit" by which one expresses one's *choice* of talking on the metalevel *as if* the facts had determined the members of *Terms* to precisely or approximately preserve their references/extensions in the transition from L_1 to L_2.[29] *Semantically*, any such interpretational continuity between theoretical terms from consecutive theories would still look like ordinary semantic realism. But not so *metasemantically*: since the continuity would not be grounded in facts, the continuity would remain merely verbal. (One might view '$\varepsilon F_1(F_1 \in Adm_1) =/\approx$

[28]Semantically, the main difference between approximate and perfect continuity is that the former may be accompanied by changes of truth values of atomic statements and, more importantly, of complex law-like statements; not so for perfect continuity (assuming the respective universes of discourse remain invariant).

[29]There would also be the option of enforcing '$I_1 = I_2$' by introducing new metasemantic constraints by which a new class Adm_2' of admissible interpretations of L_2 would be determined so that $Adm_2' = \{I_1\}$, $I_2 \in Adm_2'$, and where $I_1 = \varepsilon F_1(F_1 \in Adm_1)$; which would entail that $I_1 = I_2$. However, that set Adm_2' would be guaranteed to be non-empty in every logically possible case, even if '$\exists F_1 (F_1 \in Adm_1)$' were false, in which case '$\varepsilon F_1(F_1 \in Adm_1)$' would denote an *arbitrary* member of the domain. And thus there would be no guarantee that I_2 would still be a member of the original class Adm_2 of admissible interpretations of L_2, undermining interpretational continuity again.

$\varepsilon F_2(F_2 \in Adm_2)$' as describing a "brute semantic fact" in the sense of Breckenridge and Magidor 2012, Kearns and Magidor 2012, or as a true sentence that is not describing any fact at all.)

I will have to postpone further investigation of that case to a different occasion, but *if* the semantic indeterminacy of scientific theoretical terms happens to be a common phenomenon at all—to which Ramsey semantics is not committed, but for which it is prepared—scientific realists should be regularly forced to seek refuge to such *posited* referential/extensional continuities between scientific terms from successor theories. (Alternatively, they might weaken the notion of referential/extensional continuity in the face of semantic indeterminacy, such that what is meant is only that $Adm_2 \subseteq Adm_1$ or even just $Adm_1 \cap Adm_2 \neq \emptyset$.)[30]

Instead of continuing the previous study of *diachronic* interpretational continuity between theoretical terms in *subsequent* theories, let me turn now to an example of *synchronic* interpretational continuity between theoretical terms from theories held true *at the same time*.

Reconsider the second-order language L' of arithmetic of Example 2 with its class Adm' of admissible interpretations satisfying the second-order Dedekind-Peano axioms, and compare it to the second-order language L^* of *real analysis* with its class Adm^* of admissible interpretations that satisfy Dedekind's second-order axioms of the real numbers. As in the case of number theory, the second-order theory of real numbers is categorical, that is, it pins down the *structure* of the real numbers uniquely. At the same time, infinitely many (pairwise isomorphic) set-theoretic systems instantiate that structure by satisfying the axioms: let us be set-theoretic structuralists again about the real numbers and assume that the class Adm^* consists of all such systems. Ramsey semantics for L^* will thus consists in:

$Adm^* = \{F: F \vDash \mathrm{RA}_2\}$.
$\exists F^* (F^* \in Adm^*)$ (which follows already from the previous equation).
$I^* =_{\mathrm{df}} \varepsilon F^* (F^* \in Adm^*)$.
For all sentences A in L^*: A is true(-in-L^*) iff $I^* \vDash A$.

The intended domain $Uni(I^*)$ of real numbers includes as a special subset the set $\{I^*(0), I^*(0+1), I^*((0+1)+1), \ldots\}$ of *real "natural" numbers*. However, combining Ramsey semantics for arithmetic with that of analysis still leaves open how these real-valued "natural" numbers relate to the *"actual" natural numbers* in $Uni(I')$.[31]

Suppose that one intends to correct this by *stipulatively identifying the former with the latter*: then Ramsey semantics allows one to express this stipulation by adding

[30] In these ways, Ramsey semantics provides the conceptual resources to describe how "conceptual engineering" (Carus 2007, Cappelen 2018) may be expressed (meta-)semantically: by specifying, diversifying, modifying, or completely revising semantic interpretation. On the metasemantic/pragmatic side, the theory would have to be complemented by an account of speech acts by which the relevant interpretational changes could be brought about. E.g., the most straightforward instances of Carnapian explication (Carnap 1950a) correspond semantically to the specification of the interpretation of a predicate, and one way of effecting such specifications is by *assertion*. For asserting A may have *two* effects on the target subject: the traditional one of belief revision, as in "update your beliefs, by restricting your set of live possibilities to those that satisfy A"; and a novel one of interpretation specification, along the lines of "update the interpretation of your terms, by restricting the class of admissible interpretations to those that satisfy A". The first type of effect may prompt epistemic progress, whereas the second one may lead to semantic progress. Either way, the aims of asserting A would remain the truth of A, where truth is defined by Ramsey semantics as in Section 2.

Similar resources as those of the present section are of course also available to supervaluationists. However, Ramsey semantics is much closer to the classical semantics that scientific realists normally presuppose (see Section 3), and there is no supervaluationist counterpart to epsilon-term formulations of interpretational continuity.

[31] Mathematical structuralists are themselves divided over this question, but e.g. Resnik (1997) defends the view that there is no fact of the matter whether natural number 2 is identical to real number 2.

$$I^*(0) = I'(0), I^*(0{+}1) = I'(s(0)), I^*((0{+}1){+}1) = I'(s(s(0))),\ldots$$

as semantic postulates to one's semantic metatheory, in which, e.g., '$I^*(0) = I'(0)$' is short for

$$(\varepsilon F^*(F^* \in Adm^*))(0) = (\varepsilon F'(F' \in Adm'))(0),$$

and the like. Once again, these are statements of perfect *interpretational continuity* (here between L^* and L'), but this time concerning languages that are used at the same point in time. (*Approximate* continuity would not make much sense in a purely mathematical context.)

Just as in the diachronic case before, continuities established in that manner would not be due to metasemantic facts existing prior to identification but rather result from free semantic choices by which some of the factual gaps are "covered" semantically. While there is no proper *mathematical* reason for identifying real-valued 0, 1, 2,… with natural 0, 1, 2,…, as the existence of an isomorphism between them suffices for all theoretical purposes, it may still be practically convenient for mathematicians to talk *as if* the mathematical facts had engendered the identification—and the continuity statement from above captures that semantically.

We find that Ramsey semantics allows for diachronic and synchronic interpretational continuity between languages even in cases of semantic indeterminacy, for which there is no direct counterpart in classical semantics. According to classical semantics, every instance of interpretational continuity must already have existed "from the start": a theoretical term in the "old" theory must have happened to refer or apply to the same or similar phenomena as the same term does in the successor theory; the extension of 'real natural number' must have coincided with the extension of '(actual) natural number'; and so forth. While one may completely revise the classical interpretation of terms and sentences, and while two classical interpretations may be more similar to each other than to another one, there is no way of literally making a classical interpretation "more specific" or of "identifying" classical interpretations that had not been identical beforehand.[32] Ramsey semantics, in contrast, makes room for linguistic acts by which the (meta-)semantics of scientific and mathematical terms and sentences can be altered without affecting interpretational continuity; and in some cases, continuity may even be established by such acts in the first place.

For the same reason, it is generally difficult for classical semantics to make sense of the project of *rational reconstruction* (Carnap 1928) as applied to language and semantics: the normative project of improving language and interpretation for some purposes by means of clarification, precisification, systematization, simplification, correction, and more.

For example: how should one understand the claim that *number talk* in mathematics can be rationally reconstructed in different ways for different purposes (as suggested by Carnap 1934, 1950b, 1963)? Such as, including contemporary reconstructions: (set-)structuralistically, logicistically, formalistically, category-theoretically, homotopy-type-theoretically, and so on. In classical semantics, the whole project would seem rather pointless: e.g., the numeral '*1*' that is used by number theorists either refers to a Hilbertian sequence of stroke symbols or it does not. If it does, there is no need for rationally reconstructing '*1*'-talk formalistically; if it does not, rationally reconstructing '*1*'-talk formalistically would be a mistake. *Tertium non datur*.

Ramsey semanticists, in contrast, could argue that '*1*' may be semantically indeterminate and hence may be further specified in different ways: by excluding all interpretations except for those that are admissible on (set-)structuralist grounds, or except for those that satisfy all logicist desiderata, or except those that are formalistically admissible; etc. Each such rational

[32] Some of the criticisms of abstract semantics in Wilson (2006) may be understood as criticisms of precisely these feature of classical semantics as applied to terms from applied mathematics and science.

reconstruction may come with its own interpretational continuity statement and remains compatible with classical logic, truth, and mathematics; as Ramsey semantics demonstrates.

7. Conclusions and Extensions

I have argued that Ramsey semantics combines the best of two worlds: Ramsifying classical semantics preserves the syntax, semantic rules, concept of truth, truth values, and logic of classical semantics; but it also avoids the classical presupposition of a unique factually determined intended interpretation. Instead, it follows supervaluationist semantics in merely presupposing a non-empty class of admissible interpretations. The resulting semantics circumvents the Sorites paradox, scales up to higher-order vagueness (if there is such), allows semantically indeterminate theoretical terms in mathematics and science, allows for such terms to retain their interpretation across theories, and makes sense of the rational reconstruction of language and interpretation. When there is no semantic indeterminacy, Ramsey semantics collapses into classical semantics, but when there is, it will be handled more classically by Ramsey semantics than by supervaluationist semantics.

Ramsey semantics can and should be developed into various directions. One was mentioned already at the beginning of Section 1: the Ramsification of classical *intensional* semantics and its application in pragmatics. For instance: say, classical semanticists postulate that what is communicated by the assertion of $B(n)$ is "the" intension of $B(n)$, that is, "the" class of possible worlds in which it is true. Then intensional Ramsey semantics will postulate the existence of an admissible intensional interpretation F, that is, an assignment of references and intensions to the descriptive terms of the object language that fits the metasemantic facts, such that F defines truth, and…, *and asserting* $B(n)$ *conveys the intension of* $B(n)$ *as given by* F. The aim of such an assertion will still be the truth of $B(n)$, but truth will be understood as given existentially, as in Section 2. Metalinguistic intensional epsilon terms will be used to implement this idea formally, and what they "pick" from the space *Adm* of admissible intensional interpretations will also depend on the context of assertion (and perhaps cross-contextual constraints). The resulting Ramsey account of assertion and assertability may be expected to resemble that of supervaluationist semantics, but without changing truth into super-truth.

A different kind of extension concerns Ramsey semantics for a *type-free* truth predicate that avoids semantic paradoxes, such as the Liar paradox, while maintaining classical logic and truth. In that extended semantics, 'true' and 'determinate' would be type-free predicates of sentences, the extension of 'true' would be given by *some* admissible interpretation, while the extension of 'determinate' would coincide with the set of sentences satisfied in all admissible interpretations. And each admissible interpretation would itself interpret the same predicates 'true' and 'determinate' again (which is the type-free aspect). The relevant literature that comes closest to this are McGee (1991) and Cantini (1996), though neither of them treats type-free truth as given by Ramsification. Ramsey semantics might provide suitable variants of their theories with new philosophical support. At the same time, a type-free version of Ramsey semantics might improve the hierarchical account of higher-order vagueness that was sketched in Section 5 and the Appendix in similar ways in which Kripke's (1975) type-free theory of truth improved Tarski's classical typed theory. But all that is left for future work.

The new semantics may also have philosophical applications beyond the philosophy of language proper. Let me conclude this paper by mentioning one of them: it might partially reconcile (versions of) semantic *anti-realism about mathematics* with classical logic and truth.

Consider intuitionism about mathematics: clearly, Ramsey semantics will *not* be suitable to orthodox intuitionists who insist that 'existence' means constructibility, 'truth' means provability, and a disjunction is provable only if one of its disjuncts is. However, there might also be more liberal intuitionists who are driven by very different considerations: they might worry about the existence of mathematical statements A, such that all existing metasemantic constraints on the interpretation of A would be insufficient to determine A to be true, while also being insufficient to determine A to be false—no facts fill the gap. If one assumes the facts in question to be matters of mental construction, as intuitionists normally do, the worry should be quite obvious. But even someone who merely thinks that mathematical facts supervene on mental, linguistic, physical,… facts, might well question whether these facts will be able to settle the status of, say, '$x \in y$', for all sets x and y at whatever ordinal height in the cumulative hierarchy of sets. And that worry also becomes relevant to complex mathematical statements, if one accepts the plausible semantic thought that truth conditions are compositional.

Indeed, if compositionality is implemented by the semantic rules of intuitionistic logic, this clearly casts doubt on the truth of $A \vee \neg A$ and hence on classical logic and mathematics. The rationale for these semantic rules is to *track determinacy by truth*, much as supervaluationist semantics does, but *compositionally*, unlike supervaluationism: for these rules are compositional but may be understood as employing (in present terminology) a determinacy predicate or operator in the clauses for atomic formulas, negation formulas, conditionals, and universal quantifications (as can be seen from Kripke semantics for intuitionistic logic or from Gödel's translation of intuitionistic logic into the modal system S4[33]).

It is to intuitionists of such more liberal stripe that Ramsey semantics may provide a profitable offer, since it accepts both the possibility of metasemantic gaps and the compositionality of truth, and yet delivers the logical truth of the excluded middle. The way it manages to do so is by excluding determinacy from the semantic rules for truth (see Section 2), and by postulating instead, at the outset, one non-constructive existence statement: a metalinguistic Ramsey sentence. If liberal intuitionists could make their peace with that Ramsey sentence by interpreting it as expressing some kind of "regulative ideal" (cf. Shapiro 1997, p. 209) that is meant to systematize semantics in a practically beneficial manner, they might be able to embrace the logical truth of classical logic and the truth of classical mathematics without alarming factual commitments.[34]

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[33] Though one should stress that the axiom scheme 4 of S4 is actually rather questionable for *determinacy*.

[34] It is well known that extending intuitionistic logic by the logical laws of the epsilon calculus with extensionality (as used in Section 2) yields full classical logic (see Bell 1993). If intuitionists were to accept the epsilon calculus on similarly pragmatic grounds as a kind of "regulative ideal", they should be able to embrace classical logic.

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Appendix: Iterating Ramsey Semantics Throughout the Tarskian Hierarchy

The metalinguistic Ramsey semantics of the first-order extensional object language L from Example 1 of Section 1 has been described in Section 2. I am now going to introduce Ramsey semantics for the *meta*language L_2 of L in which Ramsey semantics for L has been formulated. In an analogous manner, Ramsey semantics may be introduced at yet higher levels of the Tarskian hierarchy. The main technical challenge, on which I did not comment in Section 5, consists in the interpretation of the metalinguistic epsilon terms that figure prominently in L_2 and in the formal interaction between determinacy and truth predicates.

For simplicity, let us assume that L_2 is a first-order extensional language again; e.g. the metalinguistic expression ‘$F \in Adm$’ in L_2 is really a first-order formula of the form ‘$Adm(F)$’ in which ‘F’ is a first-order variable and ‘Adm’ a predicate. Classical interpretations F_2 of L_2 are defined as described in Section 1, except that now also the epsilon terms of L_2 require interpretation: here one may simply follow the standard choice semantics of epsilon terms (see Leisenring 1969) by defining each F_2 to come equipped with some choice function $Ch(F_2)$ by which epsilon terms in L_2 are interpreted. E.g., $Ch(F_2)$ applied to a non-empty subclass X of $Dom(F_2)$ yields some member of X, and F_2(‘$\varepsilon F(F \in Adm)$’) is defined equal to $Ch(F_2)(F_2$(‘Adm’)). Two interpretations F_2 and F'_2 may well differ only in their respective choice functions. ‘$\vDash$’ for satisfaction is defined as in (1) from Section 1 but now relative to interpretations F_2 of L_2, and logical consequence is still defined by (2) from Section 1.

Next, one assumes all existing metametasemantic constraints on the interpretation of L_2 to be summed up by a class Adm_2 of classical (L_2-)admissible interpretations F_2 of L_2. The universe of each F_2 is going to be a set that includes all classical interpretations F of L for which $Uni(F) = U$ (which was the intended set universe of L). For simplicity again, let us assume all interpretations in Adm_2 to have one and the same set domain U_2. Each (L_2-)admissible F_2 assigns over U_2, amongst others, an (L_2-)admissible interpretation to the metalinguistic expression ‘conforms to the existing metasemantic constraints on the interpretation of L’, hence also to ‘Adm’, and thereby, as in (22), to ‘Det’. Amongst the metametasemantic constraints on the interpretation of ‘Adm’, there will be conceptual constraints, such as: for each $F_2 \in Adm_2$, F_2(‘Adm’) is a subset of U and indeed a non-empty set of classical interpretations F of L. And all $F_2 \in Adm_2$ interpret the metalinguistic terms ‘I’ and ‘true(-in-L_1)’ so that the postulates of Ramsey semantics for L from Section 2 are made true: e.g., F_2(‘I’) = F_2(‘$\varepsilon F(F \in Adm)$’).

Once again, Ramsey semantics for the metalanguage L_2 of L commits itself to Adm_2 being non-empty, “the” intended interpretation I_2 of L_2 being given now by a *metameta*linguistic epsilon term, and truth for L_2 being defined by classical satisfaction by I_2; that is,

$$\exists F_2\,(F_2 \in Adm_2),$$

$I_2 =_{df} \varepsilon F_2 \, (F_2 \in Adm_2)$,
for all sentences A in L_2: A is true(-in-L_2) iff $I_2 \vDash A$.

The epsilon term '$\varepsilon F_2 \, (F_2 \in Adm_2)$' in L_3 that defines 'I_2' "picks" an interpretation in Adm_2. In that way, the same epsilon term also "picks" the set I_2('*Adm*') of admissible interpretation of L that serves as "the" intended interpretation of the term '*Adm*' that was used in the metalanguage of L when Ramsey semantics was stated in previous sections (and from which I got "picked" by a metalinguistic epsilon term). "The" intended interpretation of the metalinguistic predicate '*Det*' is thereby defined, too, and similarly for 'I' and 'true(-in-L_1)'.

For instance, by the definition of 'I_2' and the previous assumptions, it holds that

$$I_2(\text{'}I\text{'}) = I_2(\text{'}\varepsilon F(F \in Adm)\text{'}) = Ch(I_2)(I_2(\text{'}Adm\text{'})) = Ch(\varepsilon F_2(F_2 \in Adm_2))((\varepsilon F_2(F_2 \in Adm_2))(\text{'}Adm\text{'})),$$

where the second and third occurrence of 'ε' denotes the epsilon operator of the metalanguage L_3 of L_2, that is, the *metameta*language of L. L_3 is the language in which Ramsey semantics for L_2 is formulated; but I will the details of *its* syntax to one side here.

Last but not least, one may introduce a new *metameta*linguistic determinacy(-in-L_2) predicate 'Det_2' into L_3, such that 'Det_2' applies to sentences of L_2 in the expected manner:

for all sentences A in L_2: $Det_2(A)$ iff for all $F_2 \in Adm_2$, $F_2 \vDash A$.

Similarly, one may introduce a *binary* metametalinguistic determinacy(-in-L_2) predicate into L_3 that applies to all open formulas $A[x]$ of L_2 with precisely one free variable x, and to all objects d in U_2. I will use the predicate 'Det_2' again for that purpose:

for all $A[x]$ in L_2, for all $d \in U_2$: $Det_2(A[x], d)$ iff for all $F_2 \in Adm_2$: F_2, $s\frac{d}{x} \vDash A[x]$,

where s is a variable assignment over U_2. '$Det_2(A[x], d)$' is read as: $A[x]$ is determinate *of* d.

For instance, one may now express the semantic indeterminacy of the metalinguistic term '$\varepsilon F(F \in Adm)$' in L_2 by means of

it is not the case that there is a $d \in U_2$, such that Det_2('$\varepsilon F(F \in Adm) = x$', d).

And that indeterminacy of '$\varepsilon F(F \in Adm)$' can indeed be derived in the Ramsey semantics for L_2 whenever one can derive that Adm_2 includes at least two interpretations F_2 and F'_2 of L_2 that assign distinct interpretations to '$\varepsilon F(F \in Adm)$', that is, when

$$F_2(\text{'}\varepsilon F(F \in Adm)\text{'}) = Ch(F_2)(F_2(\text{'}Adm\text{'})) \neq Ch(F'_2)(F'_2(\text{'}Adm\text{'})) = F'_2(\text{'}\varepsilon F(F \in Adm)\text{'}).$$ [35]

As another example, consider the sentence $B(0)$ of L: '$Det(B(0))$' is a sentence of L_2 that is short for

for all $F \in Adm$, $F \vDash B(0)$.

Similarly, 'Det_2('$Det(B(0))$')' is a sentence of L_3 that is short for

[35] One can show that no Evans (1978)-style argument against statements of the form 'it is indeterminate that $a = b$' with a sentential (in-)determinacy operator can be run against statements of the form '$\neg\exists d Det_2$('$\varepsilon F(F \in Adm) = x$', d)' in which the predicate Det_2 is applied to an identity statement with an indefinite description under quotation marks. This said, the semantics of identity statements leads to lots of important questions both in Ramsey semantics and in supervaluationist semantics: for the latter, see Varzi (forthcoming).

for all $F_2 \in Adm_2$, $F_2 \vDash Det$('$B(0)$').

Taken together, and using the semantic rules by which '$\vDash$' is defined, 'Det_2('Det('$B(0)$')')' expresses: for each interpretation $F_2 \in Adm_2$, such that F_2 assigns an interpretation F_2('*Adm*') to '*Adm*', and for each interpretation $F \in F_2$('*Adm*'), the sentence $B(0)$ is true relative to F.

For instance, so long as for all $F_2 \in Adm_2$, and for all $F \in F_2$('*Adm*'), it holds that $0 \in F(B)$, it is also going to be the case that Det_2('Det('$B(0)$')')', which in turn implies (by the definitions sketched before) that Det_2('*True-in-L*('$B(0)$')') and *True*-in-L_2('Det('$B(0)$')'), and hence also that *True-in-*L_2('*True-in-L*('$B(0)$')'). In plain words, and using operator-talk: it follows that having no hair at all on one's head is not just a case of being bald (*True-in-*L_2('*True-in-L*('$B(0)$')')) but also one of being determinately bald (Det_2('*True-in-L*('$B(0)$')') and *True*-in-L_2('Det('$B(0)$')')) and of being determinately determinately bald (Det_2('Det('$B(0)$')')).

More generally, from the axioms and assumptions above one can derive every instance of the four material conditional claims

$$\begin{array}{ccc} & \rightarrow Det_2(\text{'}True\text{-}in\text{-}L(\text{'}A\text{'})\text{'}) \rightarrow & \\ Det_2(\text{'}Det(\text{'}A\text{'})\text{'}) & & True\text{-}in\text{-}L_2(\text{'}True\text{-}in\text{-}L(\text{'}A\text{'})\text{'}) \\ & \rightarrow True\text{-in-}L_2(\text{'}Det(\text{'}A\text{'})\text{'}) \rightarrow & \end{array}$$

in which 'A' may be replaced by an arbitrary sentence of L. The converses of these claims are not necessarily derivable, nor can one derive all instances of 'Det_2('*True-in-L*('A')') $\rightarrow$ *True*-in-L_2('Det('A')')' or of its converse. (But if one assumes 'Det_2('Det('A')')', then both conditionals can be derived from that assumption.)

By unpacking the two truth predicates and the determinacy predicate '*Det*' in the four conditionals above, one may reformulate the conditionals as statements involving epsilon terms. In the case of semantic indeterminacy, these epsilon terms cannot be replaced *salva veritate* by terms with a unique factually determined intended interpretation. And the same holds for 'Det_2', if 'conforms to the existing metametasemantic constraints on the interpretation of L_2' and hence 'Adm_2' turn out to be indeterminate, too (which may or may not be the case).

In order to be prepared for *that* possibility, too, Ramsification may be repeated on the next level (on which L_3 would be interpreted), and so on and so forth, throughout the Tarskian hierarchy—if necessary including level of transfinite ordinal α. In short: for whatever ordinal level of metameta…metalanguage of L, classical semantics ought to be Ramsified if one wants to stay in the safe side.[36]

The following image shows what the resulting structure of intended interpretations $I, I_2, I_3,\ldots$ (of, respectively, $L, L_2, L_3,\ldots$) and corresponding non-empty classes $Adm, Adm_2, Adm_3,\ldots$ looks like, where each intended interpretation $I, I_2, I_3,\ldots$ gets "picked" by an epsilon term from its corresponding set Adm, Adm_2, $Adm_3,\ldots$ of admissible interpretations (the "picking" being visualized by arrows), and where in turn each such set of admissible interpretations coincides with I_2('*Adm*'), I_3('Adm_2'), I_4('Adm_3'),…, respectively:

[36] With sufficient syntactic resources, and the height of the ordinal α sufficiently constrained, it will also be possible to express on level α that a sentence A is true at every level $\gamma<\alpha$, as well as that a sentence B is determinate at every level $\gamma<\alpha$. It will not be possible to express at any level α that A is true at every level whatsoever, including α, for the usual Tarskian reasons. Such restrictions could be avoided by turning Ramsey semantics into a type-free theory of truth and determinacy, on which I have commented briefly in the Conclusions section.

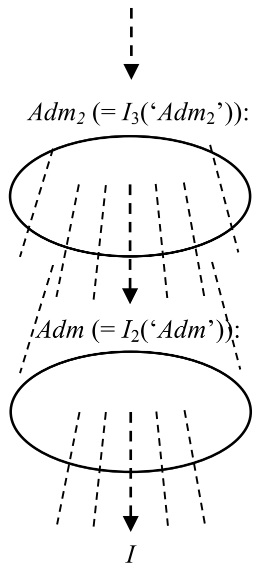

Fig. 2: Ramsey semantics throughout the Tarskian hierarchy

On each level, hypotheses concerning the corresponding class of admissible interpretation may be added to the respective metameta…-theory. E.g., as far as the 'B' in the baldness case is concerned, the following kind of axiom would be plausible for each $n \geq 2$ (where $F_1 = F$):

for all $F_n \in Adm_n$, for all $F_{n-1} \in F_n$('Adm_{n-1}'),…, for all $F \in F_2$('Adm'): $0 \in F(B)$.

But generally one should not expect all plausible hypotheses concerning (L_n-)admissible interpretations in Adm_n to be added as *axioms* to one's semantic metameta…-theory at level n+1: after all, assumptions concerning (L_n-)admissible interpretations do not belong to semantics proper (recall Section 1), and one might simply not be confident enough of them to "dignify" them with the status of *provability* in one's metameta…-theory.