Excerpts from Timothy Williamson, *Suppose and Tell: The Semantics and Heuristics of Conditionals*, Oxford University Press, to appear May 2020

* 1. *Supposing and imagining*

What if a strawberry is orange? What if you get lost? What if the government falls? A normal human way to answer hypothetical questions is by using one’s imagination—not arbitrarily, of course, but constrained by one’s background knowledge and expectations of what the world is like. One thinks through various scenarios, imagining and comparing their consequences. Imagining that *A* may prompt one to say something of the form ‘If *A*, *C*’. Conversely, someone else saying something of the form ‘If *A*, *C*’ may prompt one to imagine that *A*.

Imagination is sometimes contrasted with *supposition*. Imagining is envisaged as a much richer and more dynamic activity, typically involving visual imagery, while supposing is a minimal mental or linguistic act, perhaps accomplished simply in the words ‘Suppose that *A*’. But the contrast is overblown. For a start, sensory imagery is not essential to all imagining. Without employing any such imagery, someone might imagine the political effects of a Christian fundamentalist becoming President of the United States, or what it is like to be depressed. Conversely, the point of supposing that *A* is typically to initiate a process of answering a hypothetical question ‘What if *A*?’, by mentally exploring the relevant consequences of the supposition that *A*, often by imagining them.

The method of making suppositions and exploring their consequences can also be applied in more rigorous and systematic ways. The prime example is mathematics, which normally limits itself to strictly deductive consequences. Proofs by reductio ad absurdum establish a proposition by supposing that it does *not* hold and deducing a contradiction. An early example is the Pythagorean proof that √2 is irrational, that is, for any two integers *p* and *q* (with *q* ≠ 0), *p*2 ≠ 2*q*2. The proof starts by supposing that √2 is rational, and climaxes with a contradiction—of course, only *on the supposition* that √2 is rational.

Not all mathematical suppositions are made to be reduced to absurdity. Often they are needed simply to achieve the required level of generality. A typical example is the standard proof of Lagrange’s Theorem, named after the eighteenth century Piedmontese mathematician Joseph-Louis Lagrange, who proved a special case of it. In modern terms, it concerns the algebraic structures known as *groups*; it says that the order (number of members) of any subgroup of a finite group divides the order of the group. To prove it, one starts by supposing that *H* is any subgroup of a finite group *G*, and then establishes, on that supposition, that the order of *H* divides the order of *G*. Here the letters ‘*G*’ and *‘H*’ are variables. Of course, one cannot universally generalize the statement ‘the order of *H* divides the order of *G*’, because for many values of the variables *H* is *not* a subgroup of *G* and the order of *H* does *not* divide the order of *G*. Instead, one must first discharge the assumption and form the conditional conclusion ‘If *H* is a subgroup of a finite group *G*, then the order of *H* divides the order of *G*’. Since that conditional statement depends on no further supposition, it can be universally generalized, which yields the theorem. Thus the conditional form enables one to express the residue of an argument on a supposition from outside that supposition. Without such a form, we could not properly articulate what we have learned by making the supposition and want to generalize—even though, as one might expect from section 1.2, once we have the universal generalization we can paraphrase it without using ‘if’: the order of a subgroup of a finite group always divides the order of the group.

Significantly, mathematicians write and speak their proofs with ordinary logical words, including ‘if’, using them in unreflectively fluent, linguistically standard ways. The standard medium of mathematics is a natural language afforced with mathematical notation and a few diagrams, not a purely formal language. Even when a proof is fully formalized, mathematicians use extended natural language to explain what is going on in it. In particular, there is the normal interplay between conditionals and universal generalizations.

Of course, we often assess and apply conditionals without involving generalizations, at least not consciously. For example, going back to basics, one might ask oneself:

If I start eating that food, will the alpha male notice?

In effect, one is assessing two opposite conditionals:

If I start eating that food, he will notice.

If I start eating that food, he will not notice.

A natural way to do it to imagine starting to eat the food, and imagine how the alpha male will react, taking into account his direction of gaze, his apparent degree of alertness, the sight lines, how noisy the food is to eat, how hungry he is, how well-disposed, and so on. One’s imagining is reality-oriented, but one is typically not conscious of applying any generalizations to the particular case.1 Whether one is applying some unconsciously is another matter. One’s initial supposition is that one starts eating the food. It initiates an imaginative effort. If that exercise robustly leads to the conclusion that he will notice, from outside the initial supposition one accepts the first conditional and rejects the second; one leaves the food well alone. But if the exercise robustly leads to the conclusion that he will not notice, from outside the initial supposition one rejects the first conditional and accepts the second; one starts eating the food. If the exercise is inconclusive—under the constraints one imagines either alternative with similar ease—from outside the supposition one accepts neither conditional; one may or may not start eating the food, depending on one’s hunger and one’s caution. The method of trial and error is too risky when the price of error is high. One must assess the relation between the antecedent and consequent *before* deciding whether to make true the antecedent, ‘I start eating the food’, or to make it false.

Even in contemporary life, one often has no option but to use a similar method. For example, you ask yourself:

If I live in this apartment, will it suit me?

In other cases, one has no control over whether the antecedent holds:

If it rains overnight, will the stream be passable in the morning?

One may have no idea whether it will in fact rain overnight, and no idea whether the stream will in fact be passable in the morning, yet by realistically imagining the effects of overnight rain one may come to know that the stream will *not* be passable in the morning *if* it rains overnight. One need not consciously apply any generalization to the case. One’s imagination enables one to access a connection that verifies the conditional, even when one is in no position to verify or falsify its components, its antecedent and consequent.

* 1. *Ways of assessing conditionals*

There are many contrasting but interrelated ways of assessing conditionals.

Sometimes we can test a conditional ‘If *A*, *C*’ by learning that *A* and whether *C*. When we test the conditional *experimentally*, we bring it about that *A*, then observe whether *C*. For example, the theory that salt dissolves in water predicts that if I drop this quantity of salt in my hand into the bowl of water, it will dissolve. I drop the salt into the water and see whether it dissolves. If not, the conditional fails the test. In such a case, one has grounds to reject the conditional, and with it the theory. But if the salt does dissolve in the water, the conditional passes the test. In such a case, one has grounds to accept the conditional, and to gain confidence in the theory. Schematically, we take ‘*A* and *C*’ to verify ‘If *A*, *C*’ and ‘*A* and not(*C*)’ to falsify it. This way of falsifying a conditional connects closely with the rule of *Modus Ponens*, by which one can deduce ‘*C*’ from ‘If *A*, *C*’ and ‘*A*’. Modus ponens is also known as the *elimination* rule for the conditional, since in formal systems for natural deduction it is the basic way of deriving conclusions in which the conditional symbol does not occur from premises in which it does occur.

However, the experimental testing of conditionals is post hoc. Once we know that *A*, and whether *C*, we typically have no further use for the conditional ‘If *A*, *C*’. Our thinking about the case need no longer be hypothetical. We can use ‘*A* and *C*’ or ‘*A* and not(*C*)’ instead, as a more informative replacement. We need ‘If *A*, *C*’ most when we do *not* know that *A*. Then we need to assess ‘If *A*, *C*’ *prospectively* rather than retrospectively. For instance, you may need to know whether, if you try to climb the cliff, you will fall. You do not test the conditional experimentally, because the cost of falsifying it experimentally is too high. Often, when we most need conditionals, we must test them prospectively.

The examples in section 2.1 suggest a schematic procedure for assessing a conditional ‘If *A*, *C*’ prospectively, the *Suppositional Procedure*. For clarity, the procedure will be characterized as operating on interpreted sentences rather than propositions, though one can regard the former as vehicles for the latter, and the procedure is typically implemented in thought rather than out loud (more detail on this later in the section). Thus supposing, accepting, and rejecting are here relations to interpreted sentences, rather than to propositions, and the variables ‘*A*’ and ‘*C*’ are to be understood accordingly. Informally, however, we sometimes describe examples in terms of attitudes to propositions, when it is more natural to do so. The agent is assumed to engage with the propositions as expressed by corresponding sentences.

The Suppositional Procedure for assessing ‘If *A*, *C*’ works as follows. First, suppose *A*. Then, on that supposition, develop its consequences by whatever appropriate means you have available: constrained imagination, background knowledge, deduction, … . If the development leads to accepting *C* conditionally, on the supposition *A*, then accept the conditional ‘If *A*, *C*’ unconditionally, from outside the supposition. If instead the development leads to *rejecting* *C* conditionally, on the supposition *A*, then *reject* ‘If *A*, *C*’ unconditionally, from outside the supposition. Naturally, the firmness of the unconditional acceptance or rejection of the conditional will correspond to the firmness of the conditional acceptance or rejection of the consequent on the antecedent. Of course, sometimes the exercise is inconclusive: the development leads neither to accepting nor to rejecting *C* conditionally, on the supposition *A*. In that case, neither accept nor reject ‘If *A*, *C*’ unconditionally, from outside the supposition.

An immediate upshot of the Suppositional Procedure is that conditionals of the form ‘If *A*, *A*’ are accepted, for trivially one accepts *A* on the supposition *A*.

Here is a less trivial example. You have to judge whether, if you cancel the meeting, Alex will be disappointed. In your imagination, you suppose that you cancel the meeting and consider Alex’s reaction. If you judge, on that supposition, that Alex will be disappointed, you then non-hypothetically accept that, if you cancel the meeting, Alex will be disappointed. If instead you judge, on the supposition, that Alex will not be disappointed, you then non-hypothetically reject the idea that, if you cancel the meeting, Alex will be disappointed. But if you cannot make up your mind, on the supposition, whether Alex will be disappointed, you also fail to make up your mind non-hypothetically on whether, if you cancel the meeting, Alex will be disappointed.

To take another example, you are considering whether a mushroom is poisonous. You are confident that all mushrooms of type M are poisonous, and that no mushrooms of type N are. On the supposition that this mushroom is of type M, you remain confident that all mushrooms of type M are poisonous, so you are also confident that this mushroom is poisonous. Thus you are confident in unconditionally accepting the conditional ‘If this mushroom is of type M, it is poisonous’, and in unconditionally rejecting the opposite conditional ‘If this mushroom is of type M, it is not poisonous’. Here developing the supposition involves deducing ‘This mushroom is poisonous’ from ‘This mushroom is of type M’ and ‘All mushrooms of type M are poisonous’. But on the supposition that this mushroom is of type N, you remain confident that no mushrooms of type N are poisonous, so you are also confident that this mushroom is not poisonous. Thus you are confident in unconditionally rejecting the conditional ‘If this mushroom is of type N, it is poisonous’, and in unconditionally accepting the opposite conditional ‘If this mushroom is of type N, it is not poisonous’. Here developing the supposition involves deducing ‘This mushroom is not poisonous’ from ‘No mushrooms of type N are poisonous’ and ‘This mushroom is of type N’. In such ways, the procedure upholds the role of conditionals as instances of generalizations, emphasized in section 1.2.

The Suppositional Procedure involves reaching an unconditional attitude to ‘If *A*, *C*’ by means of first reaching a conditional attitude to *C* on the supposition *A*. But it also involves something more specific. Once one has reached an attitude to *C* conditionally on *A*, the question is: *which* attitude to ‘If *A*, *C*’ should one take unconditionally? The natural answer was exemplified above for a variety of attitudes: acceptance and rejection of various degrees of firmness, and agnosticism: take the *same* attitude to ‘If *A*, *C*’ unconditionally that you take to *C* conditionally on *A*. In general, the Suppositional Procedure involves coming to take an attitude to ‘If *A*, *C*’ by first coming to take that very attitude to *C* conditionally on *A*.

The Suppositional Procedure is applicable to a wide range of attitudes, understood as cognitive appraisals. Of course, not everything counts as an attitude for these purposes. On the supposition that it is hot, you may judge the sentence ‘It is not snowing’ not to contain the word ‘if’. From that, the Suppositional Procedure should not license you to conclude unconditionally that the sentence ‘If it is hot, it is not snowing’ does not contain the word ‘if’. That is not a cognitive appraisal in the relevant sense, by contrast with acceptance, rejection, and agnosticism. Although the category of attitudes has not been delimited precisely, it is clear enough to work with.

What has just been described is a single application of the Suppositional Procedure. The Procedure can also be repeated several times, with the same antecedent and consequent. For example, a single application may lead one to *un*confident acceptance or rejection of the consequent on the supposition of the antecedent. In that case one may repeat the procedure, perhaps many times, varying how one imagines the antecedent realized, to see what difference it makes, in the hope of reaching a more robust verdict on the conditional. If each trial of the procedure gives the same result, one can be correspondingly more confident that the result is correct. In that case, the attitude one finally takes to the consequent conditionally on the antecedent becomes the attitude one finally takes to the conditional unconditionally.

One can also run the Suppositional Procedure in reverse. Sometimes, you may first take an attitude to ‘If *A*, *C*’ unconditionally: for example, you may rely on the word of an expert (see below for such cases). You may then come to take the very attitude to *C* conditionally on *A* that you already have to ‘If *A*, *C*’ unconditionally. The governing rule is that the attitudes should be the *same*, in whichever temporal or causal order you come to them:

**Suppositional Rule** Take an attitude unconditionally to ‘If *A*, *C*’ just in case you take it conditionally to *C* on the supposition *A*.

The Suppositional Rule merely requires the attitudes to be the same, whereas the Suppositional Procedure also gives temporal and causal priority to taking the attitude to *A* on *C* over taking it to ‘If *A*, *C*’, in effect to processing the constituents over processing the whole conditional.

The Suppositional Procedure and Rule may also be applied under further background suppositions, held constant between input and output. Many proofs in mathematics do just that. Discussion of this feature is postponed to chapter 3; for now we concentrate on fully unconditional attitudes to conditionals.

So far, we have mainly considered *cognitive* attitudes, such as acceptance and rejection. However, the Suppositional Rule arguably applies to other attitudes too. Some of them are more relevant to conditional commands and questions. For example, to order ‘If *A*, do *X*!’ (‘If the window is open, close it!’) is in effect to order ‘Do *X*!’ (‘Close it!’) on the supposition *A* (‘The window is open’). To query ‘If *A*, *Q*?’ (‘If the window is open, who opened it?’) is in effect to query ‘*Q?*’ (‘Who opened it?’) on the same supposition (see also section 7.2). For now, however, the focus will remain on conditional statements and how we assess them cognitively.

As already suggested, we sometimes assess conditionals prospectively without carrying out the Suppositional Procedure. For example, we sometimes accept conditionals on someone else’s testimony. When an expert on cricket tells you ‘If that batsman stays in for another ten overs, Pakistan will win’, you may simply take his word for it. One may also defer to one’s own past judgments, and accept a conditional because it is so similar to many other conditionals one already accepts, without bothering to apply the Suppositional Procedure to the new conditional itself. In such cases, we pass the buck of assessment: in the first example to the cricket expert’s assessment of the same conditional, in the second example to one’s own previous assessments of other conditionals. Those earlier assessments may themselves have employed the Suppositional Procedure. The final assessments in the examples may be regarded as *non-basic*. All of that is compatible with all prospective assessments of conditionals ultimately depending on applications of the Suppositional Procedure, and in that sense all non-basic prospective assessments of conditionals tracing back to *basic* assessments by the Suppositional Procedure.

That is still not quite right. Perhaps the cricket expert was hoaxing you: he did not use the Suppositional Procedure to assess that conditional, and your acceptance of it did not ultimately depend on applications of the Procedure. Of course, you may still have relied on the false assumption that he did use the Suppositional Procedure, but in other cases not even that much is assumed.

For example, superstitious people may regard a certain coin as magic, and use it as an oracle on special occasions. They ask ‘*P*?’, toss the coin, and accept ‘*P*’ if it comes up heads, ‘Not(*P*)’ if it comes up tails. They explain away apparent falsifications of the oracle as punishments for tossing the coin in a disrespectful spirit. They may accept ‘If we set sail today, a storm will blow up’ because when they respectfully asked ‘If we set sail today, will a storm blow up?’ and tossed the coin, it came up heads. They do not imagine that the coin or its guiding spirit applied the Suppositional Procedure. Rather, they believe that the coin simply has a direct line to the truth. In that case, their acceptance of the conditional does not depend on real or imagined applications of the Suppositional Procedure. However, if mundane conditionals play anything like the ubiquitous cognitive role which they play in ours, they will presumably also need a more reliable way of assessing them: if not the Suppositional Procedure, what else? With respect to the normal human practice of using conditionals, we may provisionally treat prospective assessments not directly by the Suppositional Procedure as in a broad sense *secondary*.

To classify the testimony-based assessment of conditionals as secondary is not to dismiss it as unimportant. Indeed, taking it seriously will turn out, in chapter 5, to be crucial to understanding our practice of using conditionals, and their meaning. Nevertheless, it obviously depends on some prospective way of assessing conditionals which does not itself depend on conditional testimony, most importantly on something like the Suppositional Procedure.

Assessments of conditionals may also count as secondary when the process is mediated by amateur or professional theorizing about conditionals, in logical, semantic, pragmatic, or other terms. It is not always obvious whether a judgment about a case is theory-laden in such ways. There is no principled limit to what theories, however crazy, may interfere in speakers’ assessments of ordinary sentences in their native language. However, it is too simple to say that the Suppositional Procedure only concerns *unreflective* judgments of conditionals. For any amount of reflective theorizing may legitimately figure in the derivation of *C* from *A* even in straightforward applications of the Procedure. Thus, in the proof of Lagrange’s Theorem, a complex mathematical argument is needed to get from the antecedent ‘*H* is a subgroup of a finite group *G*’ to the consequent ‘The order of *H* divides the order of *G*’. What is typically much less reflective, more automatic, is the mathematician’s subsequent step *from* that derivation *to* the conditional ‘If *H* is a subgroup of a finite group *G*, the order of *H* divides the order of *G*’. That step still constitutes an application of the Suppositional Procedure. By contrast, if a mathematical crank rejects the conditional on the grounds that the word ‘if’ has no place in serious mathematics, theoretical interference is second-guessing the Procedure.

Another kind of violation of the Suppositional Procedure includes utterances like:

If he’s a qualified doctor, I’m the Pope.

(The speaker is not the Pope.) Obviously, the reality-oriented development of the supposition ‘He’s a qualified doctor’ does *not* lead to the conclusion ‘I’m the Pope’. The utterance has a similar effect to this:

If he’s a qualified doctor, I’ll eat my hat.

In both cases, the point of the utterance is to display one’s supreme confidence that the antecedent is false, that he is not a qualified doctor, by gratuitously committing oneself to a humiliating penalty payable if the antecedent is true. In the second example, one is committed to eating one’s hat. In the first example, one is committed to an absurd claim to be the Pope. Normally, it would be unreasonable to make such conditional commitments if the risk of the condition’s being met were serious. The utterances would not sound so extravagant if the consequent were unsurprising on the supposition of the antecedent. The apparent disregard of the Suppositional Procedure signals that more is going on. These cases too can be classified as secondary assessments of the conditionals. On a deeper understanding of our practice of using conditionals, such examples will turn out to be less anomalous than they first look (see section 5.1 and chapter 6).

With these preliminary clarifications of the distinction between primary and secondary prospective assessments of conditionals, everything seen so far is compatible with this conjecture:

**Suppositional Conjecture** The Suppositional Procedure is humans’ primary way of prospectively assessing conditionals.

One main aim of this book is to explore the implications and status of the Suppositional Conjecture.

Admittedly, the Suppositional Conjecture is not super-precise. The remarks above about the distinction between primary and secondary means of assessment are surely not enough to eliminate all vagueness from the term ‘primary’ in the statement of the Conjecture. Think of it this way. The proposal is that we get a good *model* of the practice of using conditionals by treating it as based on the Suppositional Procedure. Like most models in science, that involves various simplifications or idealizations, often quite severe ones—like treating a planet as a point mass, or an environment as containing only two species. Thus we should not expect a perfect or even near-perfect fit with experimental data, since the latter concern complex, messy systems that grossly violate the model’s simplifications and idealizations. Nevertheless, the track record of the natural and social sciences shows that a good simple model of a phenomenon can provide crucial insights into its nature and structure. In the long run, it may also lead to better models of the phenomenon that explain more of its features.2 Indeed, specific intrinsic features of the Suppositional Procedure will turn out to point in such a direction.

The Suppositional Conjecture is a *psychological* hypothesis, which in the end must live or die by psychological evidence. However, this book does not contain much discussion of experimental data. Before we can sensibly test a model against such data, we must properly understand the model itself, and what it does or does not imply. Science needs well-developed theories to make sense of its almost intractably messy data, which often result from dozens of interacting variables. Without such theories to guide it, data-gathering risks becoming a parody of directionless Baconian inductive inquiry. Properly developing a theory is no easy task. It is enough for one book.3

The Suppositional Conjecture is not a *semantic* hypothesis. By itself, it says nothing about conditionals’ truth-conditions and falsity-conditions, or their lack of them, or any other semantic features attributed to conditionals. However, the semantics and the psychology of conditionals should not run completely free of each other, just as the semantics and psychology of natural kind terms should not run completely free of each other. The extension and intension of the noun ‘horse’ are somehow constrained by psychological facts about how speakers of English use the word, and in causal and perceptual relations to which features of the environment they do so, mediated by some principle of charity in interpretation. Other things being equal, speakers of English should come out knowing what they are talking about when they use the word ‘horse’. We should not waste time on the hypothesis that ‘horse’ really refers to waterfalls, while speakers of English use it under the influence of a false dogma that it refers to a kind of animal. Similarly, by the principle of charity, other things being equal, we should come out knowing what we are talking about when we use the word ‘if’. Thus, given the Suppositional Conjecture, we should expect the semantics of conditionals to make the Suppositional Procedure come out reliable, as far as possible. At the very least, the semantics should not gratuitously make the Procedure come out *un*reliable.

One might try to understand the Suppositional Conjecture in terms of a distinction between *competence* and *performance*. On this view, competence with conditionals involves the capacity to assess them by correctly applying the Suppositional Procedure. In actual performance, all sorts of errors and deviations can be expected. They need not all be mere random noise, which can be filtered out over large samples. They may include far more systematic effects, various forms of bias and interference. Since experimental data measure performance rather than competence, they may cast only very indirect light on the Suppositional Conjecture, which concerns competence rather than performance.

There is something to such a view of the Suppositional Conjecture. However, the term ‘competence’ could do with considerable clarification. In particular, we should not assume without argument that a competent attitude to a proposition is *ipso facto* correct, in whatever sense of ‘correct’ is appropriate to that type of attitude.

Compare the case of perceptual judgments. They often rely on various *heuristics*, fast and frugal ways of judging which are normally reliable, especially under ecologically realistic conditions, but are not 100% reliable. For example, we use discontinuities in colour as a heuristic in judging the boundaries of three-dimensional objects, which is mostly reliable but can be exploited by experts in camouflage to make us misjudge. Such ‘glitches’ in our heuristics cause predictable illusions in special circumstances. They are not individual performance errors, failures to execute the heuristic properly, or interferences with its outputs. Rather, they result systematically from its proper execution. We can regard the ability to apply the colour-discontinuity heuristic as a competence. Thus, in unfavourable circumstances, competent judgments can be false.

Similarly, one might understand the Suppositional Procedure as a heuristic, a fast and frugal way of assessing conditionals which is normally reliable, especially under ecologically realistic conditions, but not 100% reliable. It may have glitches, which cause predictable illusions in special circumstances. They too will not be individual performance errors, failures to execute the Procedure properly, or interferences with its outputs. Rather, they will result systematically from its proper execution. We might still regard the ability to apply the Suppositional Procedure as a competence, which, in unfavourable circumstances, can deliver competent but false assessments of conditionals. Chapters 3-8 will confirm this view of the Suppositional Procedure as an imperfectly reliable heuristic.

One might also wonder where to fit such heuristics for assessing sentences into a standard picture of linguistic architecture, with semantics built on syntax and pragmatics built on semantics. The heuristics must come above the semantics, for normally one is in no position to decide between accepting and rejecting a sentence until one knows what it means. But if the semantics takes the syntax of a sentence and a context as inputs and delivers the proposition expressed by the sentence in the context as output, then the semantic output may be too coarsely individuated to provide the heuristic with the information it needs to work on. For a start, the conditional *word*, such as ‘if’, may be needed to activate the heuristic. Moreover, if the proposition expressed by a conditional is a set of possible worlds, or something similarly coarse-grained, not even the propositions expressed by its antecedent and consequent will be recoverable from the proposition expressed by the conditional itself. For example, given the principle of contraposition, the logically equivalent conditionals ‘If Jan is outside, it is sunny’ and ‘If it is not sunny, Jan is not outside’ express the very same proposition (in a given context), even though their antecedents ‘Jan is outside’ and ‘It is not sunny’ express two different propositions, as do their consequents ‘It is sunny’ and ‘Jan is not outside’ (in that context). But the Suppositional Procedure needs to take the antecedent and the consequent as separate inputs.

The short answer is that the heuristic fits in wherever verbal reasoning fits in. Since verbal reasoning fits in *somewhere*, so does the heuristic. Some verbal reasoning may play a role in calculating pragmatic implicatures, but much of it goes far beyond both semantic and pragmatic comprehension. Mathematicians understood the statement of Fermat’s Last Theorem centuries before it was eventually proved. Similarly, conversational participants have considerable discretion in how far they think through the consequences of what is said. Although for gross simplicity we may model them as logically omniscient, that is just a convenient first approximation. When we seek to understand cognitive dynamics in more detail, we must acknowledge that reasoning takes time and is costly in other resources too. For these purposes, a psychologically realistic theory of verbal reasoning will have to treat it as operating over structured mental representations such as interpreted sentences, not over bare coarse-grained propositions. For example, an inference by disjunctive syllogism from ‘*P* or *Q*’ and ‘Not(*P*)’ to ‘*Q*’ depends on the thinker’s capacity for pattern recognition, and in particular for recognizing ‘*P*’ as a constituent common to the two premises. Some participants may notice that the conversation has supplied the premises for a step of disjunction syllogism, and draw the conclusion, while others do not. None of this is to deny that sentences express coarse-grained propositions; it is just to insist that the fine-grained dynamics can only be properly understood at a level where sentential structure has not been left behind.

A framework adequate for understanding the dynamics of verbal reasoning will also help us situate the Suppositional Procedure. The framework will deal with structured mental representations of some sort, just as the Procedure requires. Those representations may still express coarse-grained propositions. The framework will allow thinkers considerable discretion in how far they apply the Procedure: exhaustively applying it is no requirement for ordinary linguistic comprehension. For example, let *A* logically entail *C*. If the entailment is obvious to native speakers, they will normally accept ‘If *A*, *C*’, by a special case of the Suppositional Procedure. But if the entailment is not obvious to them, they are not obliged to continue tracing out the consequences of *A* until (if ever) they eventually come to *C*. Similarly, let *A* be logically inconsistent with *D*. If the inconsistency is obvious to native speakers, they will normally reject ‘If *A*, *D*’, by another special case of the Suppositional Procedure. But if the entailment is not obvious to them, they are not obliged to continue tracing out the consequences of *A* until (if ever) they eventually come to ‘Not(*D*)’.

Further support for this view comes from the need for theories of verbal reasoning to handle *generality*. In arguing for ‘Every *F* is a *G*’, one may argue from the supposition ‘*x* is an *F*’ to the conclusion ‘*x* is a *G*’, where ‘*x*’ is a free variable. In ordinary English, one might put it by arguing from ‘Something is an *F*’ to ‘It is a *G*’, where the pronoun ‘it’ is anaphoric on ‘something’. The reasoning controls the implicit generality of the argument from one open sentence to another by tracking the free variable or pronoun, not by contemplating the relation between two sets of possible worlds. Some alternative theorists regard ‘*x*’ or ‘it’ as a sort of arbitrary name of an object, or even as a name of a sort of arbitrary object, but to control the implicit generality of the argument the reasoning must still operate at the finer-grained level. The same considerations apply to the Suppositional Procedure, when it verifies the open conditional ‘If *x* is an *F*, *x* is a *G*’.4

The comparison between the Procedure and reasoning also helps explain a feature which may initially look troublesome. The Suppositional Procedure and Rule explicitly mention only *unembedded* occurrences of conditionals. What about embedded occurrences, where conditional sentences figure as constituents of more complex sentences? Surely a story needs to be told about cognition of them too. But consider the crucial role of the usual logical connectives, including a conditional one, in mathematical reasoning. At least to a first approximation, it is adequately codified by the introduction and elimination rules for each connective in a standard system of natural deduction. The introduction and elimination rules for a given connective explicitly mention only its unembedded occurrences, where it occurs as the main connective of the given sentence; that applies in particular to the natural deduction rules for the conditional. Nevertheless, when the rules for all the connectives are combined, the result is an adequate background logic for mathematical reasoning, which handles embedded occurrences of conditionals just as well as embedded ones. The Suppositional Rule serves the analogous purpose for ‘if’ in the wider setting of general language use, where non-logical considerations usually dominate. There is no special lacuna for embedded conditionals. Of course, they may occur under other connectives whose associated epistemology remains to be understood, but that problem is not for an account of conditionals to resolve.

The role of the Suppositional Procedure and Rule is epistemological rather than purely semantic. Nevertheless, they play a key role in semantic theorizing—if the Suppositional Conjecture is true. For semanticists continually check what their theories predict about the status of sample sentences against pre-theoretic native speaker judgments. Given the Suppositional Conjecture, those pre-theoretic judgments will normally be products of the Suppositional Procedure. If the Procedure is a less than fully reliable heuristic, the standard data for semantic theorizing will also be less than fully reliable. In that case, even when native speakers strongly agree on the status of a sample sentence in a specified context, they may all be wrong, having applied the heuristic to an unfavourable case. We must be careful not to dismiss good theories on the basis of bad data.

* 1. *Suppositions and updating*

Describing what is in effect the Suppositional Procedure, Frank Ramsey famously wrote in a footnote (1929: 143):5

If two people are arguing ‘If A, then C?’ and are both in doubt as to A, they are adding A hypothetically to their stock of knowledge and arguing on that basis about C

We can regard supposing *A* as a simulated or offline analogue of receiving the new information *A*, and developing the supposition as a simulated or offline analogue of updating one’s knowledge and beliefs on the new information. For example, when I suppose that it will snow tomorrow, what I do is in some ways like what I do when I read a reliable forecast that it will snow tomorrow. Within the scope of the supposition, I accept ‘Warm clothes will be needed tomorrow’. If the news comes as a surprise, I may have to reject some propositions I previously accepted, such as ‘Light clothes will do for tomorrow’.

The analogy between developing a supposition and updating on new information helps make our ability to develop suppositions less mysterious. For it is not mysterious *that* we are able to update on new information, even though much remains to be explained about *how* we do it. Intelligent life is impossible without the ability to update on new information. The natural hypothesis is that we re-employ some of the very same abilities offline when we develop a supposition, although again much remains to be explained about how we do that.

Notes

1. For more on this function of the imagination see Williamson 2016d.
2. On models in natural science see Weisberg 2013. On models in philosophy and semantics see Williamson 2017b.
3. In psychology, there are several contrasting traditions of work on conditionals, including the mental models approach (Johnson-Laird and Byrne 2002, Byrne and Johnson-Laird 2009, Khemlani, Byrne, and Johnson-Laird 2019), the suppositional approach (Evans and Over 2004), the Bayesian approach (Oaksford and Chater 2007), and more recently the erotetic approach (Koralus 202X, Koralus and Mascarenhas 2013). Given the present emphasis on the role of supposition in the primary heuristic for conditionals, the obvious comparison is with Evans and Over’s work; however, the view to be defended in this book differs sharply from them on the semantics of conditionals, and makes a wider separation than they do between the semantics and the psychology. Most work on the semantics of conditionals has engaged very little with the psychology of conditionals, and indeed the two traditions are often hard to relate to each other. One might wonder whether a subsidiary reason has been that, within psychology, conditionals have mainly been studied within the psychology of reasoning, with the conditionals figuring as premises or conclusions (or components thereof), and subjects having to consider questions about what does or does not follow from what, which in turn raises questions about how they understand relations of logical or non-logical consequence or validity. One can generate a conditional such as ‘If she doesn’t come, I’ll be disappointed’ without addressing issues of validity. However, the shortage of interaction between semantics and psychology is a more general phenomenon.

1. See section 6.5 for more on the relation between the semantic and cognitive levels, in light of the specific semantics to be proposed for ‘if’.
2. Schematic letters adjusted to present notation.

* 1. *From conditional probabilities to conditional proof*

The first task of this chapter is to elicit more precise and general consequences of the Suppositional Rule.

One main application of the Rule is to probabilistic attitudes, *credences*, often regarded as *degrees of belief* or *subjective probabilities*. The credences of a rational agent are supposed to satisfy standard axioms of the probability calculus, and so are measured by real numbers between 0 (the probability of a contradiction) and 1 (the probability of a tautology). Such an agent’s credence in a disjunction of mutually inconsistent disjuncts is the sum of the agent’s credences in the disjuncts, and logical equivalents share the same credence for the same agent at the time.

Credences have been understood in various radically different ways: as states operationally defined in terms of betting behavior, as the fine-grained natural psychological reality underlying the coarse-grained folk term ‘belief’, and as outright beliefs about probabilities on one’s evidence.1 For present purposes, we need not decide between these views of credence. We can just work on the usual basis that, to a first approximation, one’s credence in ‘*P*’ can be estimated from one’s response to the question ‘How probable is it that *P*?’

A mathematical probability space is based on a set Ω, whose members are conceived as mutually exclusive, jointly exhaustive possible *outcomes*; subsets of Ω are *events*. Probabilities are ascribed to such events. Since outcomes function like possible worlds, events function like coarse-grained propositions, sets of possible worlds. Thus probabilities are in effect ascribed to propositions. However, the assumption that conditional sentences express propositions is controversial (Edgington 1986, 1995; Bennett 2003). For now, we can finesse that issue by ascribing probabilities to declarative sentences implicitly relative to a background context of utterance, rather than to propositions. One may, but need not, equate the probability of a sentence with the probability of the proposition it expresses in the given context. Thus logically equivalent sentences will have the same probability, whether or not they express the same proposition. We will return to the question whether conditional sentences express propositions in section 3.3.

The mathematical apparatus of probabilities may seem far too precise to be psychologically realistic for human agents. Proposals have been made for adjusting the framework to achieve greater realism, although they tend to involve a massive loss of mathematical power and tractability, without coming much closer to the psychological reality of actual human uncertainty. Anyway, we will stick to classical probabilities here. They provide a perspicuous, well-understood, working model of uncertainty in simple cases. That is the best place to start in working through the consequences of the Suppositional Rule.

The Rule is easily applied to credences, for we have a natural, standard account of probabilities on a supposition, as noted in section 2.3.

Let Prob(*X*) be the probability of a sentence *X*, and Prob(*X*|*Y*) the probability of *X* conditional on a sentence *Y*. Then Prob(*X*|*Y*) is standardly equated with the ratio Prob(*X* and *Y*)/Prob(*Y*), in effect the proportion of *Y*-cases which are also *X*-cases (as measured by Prob). In conditioning on *Y*, we in effect eliminate all possibilities where *Y* fails and keep the proportions between the remaining possibilities constant, while recalibrating to make the new probabilities sum to 1. The probability of *X* on the supposition *Y* is naturally equated with the probability of *X* conditional on *Y*, Prob(*X*|*Y*).

Of course, the ratio definition crashes when Prob(*Y*) = 0. One can extend conditional probabilities to at least some such cases, by treating conditional probability as primitive rather than defining it in terms of the ratio, while still requiring the ratio equation to hold whenever Prob(*Y*) > 0.2 For simplicity, however, we will usually treat Prob(*X*|*Y*) as undefined when Prob(*Y*) = 0.

The Suppositional Rule for credences (SRC) tells us to equate our unconditional credence in ‘If *A*, *C*’ with our credence in *C* conditional on *A*. We can therefore state its requirement thus:3

SRC Cred(if *A*, *C*) = Cred(*C*|*A*) (when Cred(*A*) > 0)

For example, how probable is it that if a fair die comes up more than 1, it comes up more than 3? SRC predicts the answer 3/5, which is indeed the natural answer.

SRC has a significant history. It was propounded by Ernest Adams, Brian Ellis, and Richard Jeffrey in the 1960s, and published by Robert Stalnaker in 1970.4 It is sometimes called ‘Stalnaker’s Hypothesis’, sometimes simply ‘the Equation’. There is much experimental evidence that it fits speakers’ assessments of probability well.5 Theorists of conditionals tend to agree that it is a good fit. However, some cases are also known where the fit is less good. Moreover, as a general principle, SRC has some alarming theoretical consequences, explained in section 3.3.

Given the Suppositional Conjecture, SRC results from the proper application of our primary way of assessing conditionals. It does not follow that SRC holds for ideally rational beings, since they may have better ways of assessing conditionals than we have. Nor does it follow that SRC always holds for ordinary speakers of English, since we may sometimes rely on secondary ways of assessing conditionals. Rather, SRC is the default for ordinary speakers.

One consequence of SRC is that ‘If *A*, *C*’ and ‘If *A*, not(*C*)’ are treated similarly to contradictories, for one’s credences in them should sum to 1 (at least, when Cred(*A*) > 0):

Cred(if *A*, *C*) + Cred(if *A*, not(*C*)) = Cred(*C*|*A*) + Cred(not(*C*)|*A*)

= Cred(*C*|*A*) + 1 − Cred(*C*|*A*) = 1

As the probability of one conditional goes up, the probability of the other goes down, and *vice versa*. Thus one should neither confidently accept both (since then one’s credences in them would sum to more than 1) nor confidently reject both (since then one’s credences in them would sum to less than 1). Much of our practice seems to fit that prediction. The more confident I am that if she went out, she took an umbrella, the less confident I am that if she went out, she did not take an umbrella, and *vice versa*. However, as we shall see later, that default is defeasible.

SRC can be generalized. We can assess conditionals on background suppositions. For example, suppose that we have a fair 20-sided die. How probable is it that if it comes up more than 1, it comes up more than 3? The natural answer is now 17/19. Such examples can be multiplied without limit. We are at ease with assigning conditional probabilities to conditionals. The Suppositional Rule already covers such cases, since credences on suppositions are themselves attitudes. When we replace credences in SRC by credence conditional on a background supposition *B*, the result is SRCC:

SRCC Cred(if *A*, *C*|*B*) = Cred(*C*|*A* and *B*) (when Cred(*A* and *B*) > 0)

One can recover SRC from SRCC by substituting a tautology for *B*. SRCC is also called ‘Generalized Stalnaker’s Hypothesis’.

David Lewis (1976) derived SRCC from SRC by arguing that the rationality of a pattern of credences should be preserved under the standard updating procedure of conditionalization. By contrast, SRCC is motivated here directly by the Suppositional Rule, generalizing the motivation for SRCC rather than treating it as derivative. Thus we can regard ‘Cred’ in SRC and SRCC as meaning exactly the same distribution of credences, not separated by any process of updating. SRCC does not do as well as SRC on current experimental evidence (Douven 2016: 74-5), but that is not very surprising: the extra complexity of SRCC, with three propositions rather than two to keep track of, may well cause more confusion. After all, we need *some* way of cognitively assessing conditionals on suppositions, since in complex decision-making we often have to make suppositions within suppositions: the idea that we do so by quite different means from those we employ in assessing them without suppositions is hardly plausible. Thus SRCC is the natural form of probabilistic assessment for conditionals on suppositions.

A further issue is that some theorists have used SRC to argue that conditionals lack truth-conditions and so do not express propositions. They hold that credence in a conditional is a genuine doxastic state even though it does not correspond to credence in a proposition (Edgington 1986, 1995). One consequence is that they deny that truth-functional operators such as negation, conjunction, and disjunction are well-defined on conditionals. But on the ratio definition of conditional credence, the left-hand side of SRCC is defined in terms of credence in a conjunction: the numerator is Cred((if *A*, *C*) and *B*). Of course, the claim that conjunctions such as ‘The dog is large and, if hungry, aggressive’ are ill-defined is somewhat implausible.

In any case, no-proposition theorists may accept SRCC by treating conditional credence as primitive, rather than defining it in terms of the ratio formula. On an austere version of the view, they may allow conditionals to occur as inputs to the unconditional credence function only as in SRC, and as inputs to the corresponding conditional credence function only as in SRCC. They can still require Cred to satisfy all standard principles of probability whenever the terms are well-defined.

Treating conditional probability as primitive rather than defined by the ratio formula is arguably more realistic psychologically, since one key function of conditionals, which the Suppositional Procedure captures, is to enable us to access connections between antecedent and consequent *without* assessing the antecedent and consequent. For example, how probable is it that if you bet on the outcome of a fair coin toss, you will win? You can answer that question ’1/2’ without having any idea how probable it is that you *will* bet: psychologically, you do not go via the ratio definition.

A no-proposition theorist might deny that conditionals really have well-defined conditional probabilities, but such a view is highly revisionary of ordinary practice. More specifically, it is quite consistent with the status of SRCC as a consequence of the Suppositional Rule; it is merely an error theory about that aspect of the Suppositional Rule.

Given SRCC, opposite conditionals behave even more like contradictories, even if they do not express propositions, for one’s credences in them conditional on any supposition should sum to 1, whenever the probabilities are well-defined:

Cred(if *A*, *C*|*B*) + Cred(if *A*, not(*C*)|*B*) = Cred(*C*|*A* and *B*) + Cred(not(*C*)|*A* and *B*)

= Cred(*C*|*A* and *B*) + 1 − Cred(*C*|*A* and *B*) = 1

By similar reasoning, if *D* is a logical consequence of *C*, then one should have at least as much credence in ‘If *A*, *D*’ as in ‘If *A*, *C*’, on any supposition for which the probabilities are well-defined:

Cred(if *A*, *C*|*B*) = Cred(*C*|*A* and *B*) ≤ Cred(*D*|*A* and *B*) = Cred(if *A*, *D*|*B*)

Hence, if *C* and *E* are logically inconsistent, ‘If *A*, *C*’ and ‘If *A*, *E*’ behave like contraries, in the sense that one should not be confident of both, for ‘Not(*E*)’ is a logical consequence of *C*, so on any supposition for which the probabilities are well-defined:

Cred(if *A*, *C*|*B*) + Cred(if *A*, *E*|*B*) ≤ Cred(if *A*, not(*E*)|*B*) + Cred(if *A*, *E*|*B*) = 1

So far, SRCC seems to be imposing a reasonable structure on credences.

The special case of SRCC for *maximal* credence merits separate attention:

SRCC1 Cred(if *A*, *C*|*B*) = 1 Cred(*C*|*A* and *B*) = 1 (when Cred(*A* and *B*) > 0)

By interpreting the epistemic modal ‘must’ in terms of credence 1, we can rewrite SRCC1 as an application of the Suppositional Rule to ‘must’:

SRmust Must(if *A*, *C*|*B*) Must(*C*|*A* and *B*)

In other words, ‘If *A*, *C*’ must hold on the supposition *B* just in case *C* must hold on the supposition ‘*A* and *B*’. We can restate this in terms of *conditional epistemic necessity*, understood as analogous to conditional probability: ‘If *A*, *C*’ is epistemically necessary conditional on *B* just in case *C* is epistemically necessary conditional on ‘*A* and *B*’.

Similarly, SRCC also entails:

SRCC>0 Cred(if *A*, *C*|*B*) > 0 Cred(*C*|*A* and *B*) > 0 (when Cred(*A* and *B*) > 0)

By interpreting the epistemic modal ‘may’ in terms of credence greater than 0, we can rewrite SRCC>0 as an application of the Suppositional Rule to ‘may’:

SRmay May(if *A*, *C*|*B*) May(*C*|*A* and *B*)

In other words, ‘If *A*, *C*’ may hold on the supposition *B* just in case *C* may hold on the supposition ‘*A* and *B*’. We can restate this in terms of *conditional epistemic possibility*, understood as analogous to conditional probability: ‘If *A*, *C*’ is epistemically possible conditional on *B* just in case *C* is epistemically possible conditional on ‘*A* and *B*’.

Those interpretations of epistemic modals in terms of credence need refining. First, epistemic modality concerns what follows from or is compatible with what is *known*, rather than with what is *believed*. What *must* hold *does* hold, and what *does* hold *may* hold. ‘Must’ and ‘may’ are *epistemic* not *doxastic* modals. Second, in infinite probability spaces, some possibilities must have probability 0, for combinatorial reasons, so not even epistemic probability 1 entails truth, and truth does not even entail probability greater than 0 (Williamson 2007b). Thus SRmust and SRmay are better understood as analogous to SRCC than as strict entailments of it. However, epistemic modals can still be interpreted over the same space of possibilities over which epistemic probabilities are defined: what must hold is what holds in every epistemic possibility; what may hold is what holds in some epistemic possibility. By thus eliminating the dependence on SRCC, we can free SRmust and SRmay of the irksome constraint that ‘*A* and *B*’ must have positive probability, which was needed only to guarantee that the probabilities were well-defined. Instead, we can understand the conditional epistemic modalities in SRmust and SRmay simply by a restriction on the quantifiers over possibilities, to those in which the given condition holds. With these glosses, both SRmust and SRmay are legitimate applications of the Suppositional Rule in their own right.

The conditional nature of the epistemic modalities in SRmust and SRmay is crucial. SRmust would be obviously hopeless if ‘must(*C*|*A*)’ were understood as saying in effect that *C* is unconditionally or independently epistemically necessary if *A* is true. For when *C* = *A* and *B* is a tautology, SRmust would then tell us, absurdly, that since ‘If *A*, *A*’ is always trivially epistemically necessary, any truth is unconditionally or independently epistemically necessary—a fallacy of just the sort noted at the end of section 2.3. Consequently, ‘may(*C*|*A*)’ should not be understood as saying in effect that *C* is unconditionally or independently epistemically possible if *A* is true, for that would undermine the natural duality between ‘may’ and ‘might’, on which ‘must(*C*|*A*)’ is equivalent to ‘not(may(not(*C*)|*A*))’ and ‘may(*C*|*A*)’ to ‘not(must(not(*C*)|*A*))’. Instead, given the quantificational truth-conditions, the conditional epistemic necessity of *C* on *A* is equivalent to the unconditional epistemic necessity of the material conditional *A* *C*, and the conditional epistemic possibility of *C* on *A* is equivalent to the unconditional epistemic possibility of the conjunction ‘*A* and *C*’.

Now consider a context in which unconditional epistemic necessity reduces to logical truth, and conditional epistemic necessity to logical consequence. Thus we can replace ‘Must(*Y*|*X*) by ‘*Y* *X*’. Hence SRmust becomes SR:

SR *B* if *A*, *C* *A* and *B* *C*

Let *BB* be a finite set (possibly empty) of ordinary declarative sentences. Then *BB* is equivalent to a single sentence *B*, in effect its conjunction. Thus SR yields an equivalent form with multiple premises:

SR *BB* if *A*, *C* *BB*; *A* *C*

Now divide SR into its right-to-left and left-to-right directions:

SR *BB*; *A* *C* *BB* if *A*, *C*

SR *BB* if *A*, *C* *BB*; *A* *C*

In other words, the Suppositional Rule tells us to treat ‘if’ as subject to these joint constraints.

Those last two rules may well look familiar. They are the standard introduction and elimination rules for a conditional operator in many formal proof systems of the kind known as *natural deduction*. They are also exactly the rules needed to support standard mathematical reasoning with ‘if’.

SR is also known as *conditional proof*, the deductive form of prospective reasoning for conditionals. When one has deduced *C* from the supposition *A*, given background assumptions *BB*, it lets one discharge the assumption *A* and conclude ‘If *A*, *C*’ on just the background assumptions. It is the introduction rule for ‘if’ because it lets one argue from a deductive relation involving only unconditional sentences to a deductive relation involving a conditional sentence. In brief, the introduction role takes one *to* conditionals.

Similarly, SR is the elimination rule for ‘if’ because it takes one *from* conditionals: it lets one argue from a deductive relation involving a conditional sentence to a deductive relation involving only unconditional sentences. In fact, SR is equivalent to the standard rule of *modus ponens*:

MP *A*; if *A*, *C* *C*

To establish the equivalence rigorously, we need some *structural rules* for , rules not specific to any particular vocabulary in the sentences of the object language. Here are the three standard structural rules, where double letters stand for (possibly empty) sets of sentences, single letters for single sentences, and set-theoretic notation is avoided for the sake of readability:

Assumptions *A* *A*

Monotonicity *AA* *C* *AA*; *BB* *C*

Cut [*AA* *B* and *B*; *CC* *D*] *AA*; *CC* *D*

The rule of Assumptions says that deductive consequence is reflexive. The Monotonicity rule says that adding extra premises never breaks deductive connections, by contrast with inductive and default reasoning. The Cut rule is a kind of generalized transitivity for deductive consequence; it is needed to chain together many short arguments into a long one, and is unreflectively assumed in most mathematical proofs.

We can now establish the equivalence of SR with MP. To derive the latter from the former, let *BB* in SR comprise just ‘If *A*, *C*’; then we have the left-hand side by Assumptions and the right-hand side is MP. Conversely, just feeding the left-hand side of SR and MP into Cut yields the right-hand side of SR (the sentence cut is ‘If *A*, *C*’).

The derivability of modus ponens from the Suppositional Rule shows that the Rule supports *retrospective* as well as prospective assessments of conditionals, for modus ponens supports the predictions from a conditional which may be retrospectively verified or falsified, when one verifies the minor premise *A*, turning the supposition into known fact (see section 2.2).

Of course, the plausibility of conditional proof and modus ponens is by no means confined to mathematical contexts. Indeed, their plausibility in mathematical contexts is just a special case of their much more general plausibility. Even those who allege counterexamples against one or both of them usually acknowledge the plausibility of the rules, which they seek to explain away. Their plausibility is just what the Suppositional Conjecture would lead one to expect.

The strong analogy between SRCC and the natural deduction rules for the conditional confirms the naturalness of allowing auxiliary assumptions in SRC (Stalnaker’s Hypothesis) and generalizing it to SRCC (Generalized Stalnaker’s Hypothesis). For although one can consider an artificially restricted version of conditional proof, with auxiliary premises forbidden, it lacks much of the unrestricted version’s power. It fails to support much normal reasoning both inside and outside mathematics, for we often have to make suppositions within suppositions, when working through a branching tree of apparently open possibilities.

The derivation of SR from SRmust is not really needed, for we can obtain SR directly from the Suppositional Rule itself, simply by applying it to the attitude of treating something as a logical consequence, with side premises *BB*. One advantage of this more direct route is that it frees us from the constraint that *BB* must be a *finite* set of premises. There is no such restriction on SR. Thus the Suppositional Rule yields the very rules we standardly need for deductive reasoning with a conditional.

* 1. *Deductive paradoxes for the Suppositional Rule*

Unrestricted conditional proof and modus ponens together impose tight constraints on a conditional: they make it equivalent to the material conditional , where ‘*A*  *C*’ is true just in case either *A* is false or *C* true, and is false otherwise. For obeys Modus Ponens: *A*; *A* *C* *C*. Thus by conditional proof for ‘if’, *A* *C* if *A*, *C*. Conversely, also obeys conditional proof; applying it to modus ponens for ‘if’: if *A*, *C* *A* *C*.

This is not good news for the Suppositional Rule. The present worry is not that the material reading of the natural language ‘if’ is highly controversial. Rather, the more urgent concern is that the material reading does not fit other cases of the Suppositional Rule, in particular SRC. For when Cred obeys the probability axioms, Cred(*A* *C*) = Cred(*C*|*A*) only in two very special cases: when Cred(*A* *C*) = 1 and when Cred(*A*) = 1. Otherwise Cred(*A*) > 0 andCred(*C*|*A*) < Cred(*A* *C*), for:

Cred(not(*A* *C*)) = Cred(*A* and not(*C*)) < Cred(*A* and not(*C*))/Cred(*A*) = Cred(not(*C*)|*A*)

So:

Cred(*C*|*A*) = 1 – Cred(not(*C*)|*A*) < 1 − Cred(not(*A* *C*)) = Cred(*A* *C*)

This is in effect an internal tension in the Suppositional Rule, between what it says about attitudes to logical consequences and what it says about intermediate credences.

We can see the tension from a different angle by considering the special case of SSRC for *minimal* credence:

SSRC0 Cred(if *A*, *C*|*B*) = 0 Cred(*C*|*A* and *B*) = 0 (when Cred(*A* and *B*) > 0)

SSRC0 is the analogue of SSRC1 for probability 0 in place of probability 1. Just as SSRC1 turned out to be strongly analogous to the principle SR about the logic of ‘if’, via an analogy between logical consequence () and probability 1, so SSRC0 is strongly analogous to the principle SR− about the logic of ‘if’, via an analogy between logical inconsistency (−) and probability 0:

SR− *B* − if *A*, *C* *A* and *B* − *C*

Just as SR is equivalent to SR, so SR− is equivalent to SR−, where *BB* is a finite set of sentences:

SR− *BB* − if *A*, *C* *BB*; *A* − *C*

As with SR, we can lift the restriction to finite *BB* by deriving SR− directly from the Suppositional Rule. Either way, we can also separate the right-to-left and left-to-right directions of SR−:

SR− *BB*; *A* − *C* *BB* − if *A*, *C*

SR− *BB* − if *A*, *C* *BB*; *A* − *C*

The introduction rule S− says that when we can deductively rule out *C* on the basis of *A* and other assumptions, we can deductively rule out ‘If *A*, *C*’ on the basis of those other assumptions alone. The elimination rule SR− says that when we can deductively rule out ‘If *A*, *C*’ on the basis of assumptions, we can deductively rule out *C* on the basis of *A* and those other assumptions.

Like SR, we can obtain SR− directly from the Suppositional Rule itself, simply by applying it to the attitude of treating something as deductively ruled out, with side premises *BB*.

Of course, if − were simply interpreted as the negation of , SR− and SR− would simply be equivalent to SR and SR respectively, so nothing would have been added. But that is analogous to interpreting − in terms of probability less than 1. That is not the intended interpretation of −, which is analogous to interpreting it in terms of probability 0. For *BB* − *C* is to mean that *BB* is inconsistent with *C*, not that *BB* fails to entail *C*. But even on that intended interpretation of − in terms of inconsistency, the rules have some pull.

Nevertheless, the ‘if’ rules for − are in tension with those for . Here is one case. Let *A* and *C* be any declarative sentences. Since the ‘if’ rules for make material conditionals entail the corresponding ‘if’ conditionals, as already seen, we have:

(1) not(*A*) *C* if not(*A*), *C*

But the three sentences ‘not(*A*)’, ‘not(*A*) not(*C*)’, and *C* form an inconsistent triad by modus ponens for , so:

(2) not(*A*); not(*A*) not(*C*) − *C*

But SR− on (2) yields (3):

(3) not(*A*) not(*C*) − if not(*A*), *C*

The semantics of yields (4a) and (4b):

(4a) *A* not(*A*) *C*

(4b) *A* not(*A*) not(*C*)

By Cut, (1) and (4a) yield (5):

(5) *A* if not(*A*), *C*

Similarly, (3) and (4b) yield (6) by a Cut-like rule linking and −:

(6) *A* − if not(*A*), *C*

The required Cut-like structural rule is this:6

Cut− [*AA* *B* and *B*; *CC* − *D*] *AA*; *CC* − *D*

For if *B* entails *AA*, and *B* and *CC* are jointly inconsistent with *D*, then *AA* and *CC* are jointly inconsistent with *D* (where all the connections are deductive). This is a disastrous result, for (5) and (6) together make the same sentence (‘If not(*A*), *C*’) both a deductive consequence of *A* and deductively inconsistent with *A*, which is to make *A* itself logically inconsistent. But nothing very special was assumed about *A*! Of course, no-proposition theorists who deny that conditionals embed coherently under logical operators will not allow *A* or *C* to be a conditional sentence, since the argument involves embedding both of them under ‘not’ and ‘if’. Still, the rules make any simple declarative sentence inconsistent.

We can eliminate the premise in (5) and (6) by substituting any logical truth T for *A*, so:

(7) T if not(T), *C*

(8) T − if not(T), *C*

(9) T

Then Cut on (5) and (7) yields (10), and Cut− on (6) and (8) yields (11):

(10) if not(T), *C*

(11) − if not(T), *C*

Thus the same sentence is both provable and refutable: the rules themselves are inconsistent. But it is worth remembering (5) and (6), since we can instantiate them to have an everyday premise, with its everyday negation as the antecedent of the conditional.

We can simplify the derivation of an inconsistency by using a Monotonicity rule for instead of Cut−:

Monotonicity− *AA* − *C* *AA*; *BB* − *C*

Adding extra premises does not remove a deductive inconsistency. Again, let *A* be any declarative sentence. By Assumptions:

(12) *A* *A*

Since *A* is deductively inconsistent with not(*A*):

(13) not(*A*) − *A*

By Monotonicity, (12) yields (14):

(14) *A*, not(*A*) *A*

Similarly, by Monotonicity−, (13) yields (15):

(15) *A*, not(*A*) − *A*

By SR, (14) yields (16):

(16) *A* if not(*A*), *A*

By SR−, (15) yields (17):

(17) *A* − if not(*A*), *A*

This is another disastrous result, for (16) and (17) together make the same sentence (‘If not(*A*), *A*’) both a deductive consequence of *A* and deductively inconsistent with *A*, which is to make *A* itself logically inconsistent. Again, nothing very special was assumed about *A*. Of course, no-proposition theorists who deny that conditionals embed coherently under logical operators will again not allow *A* to be a conditional sentence, since the argument involves embedding it under ‘not’ and ‘if’. Still, in this way too, the rules make any simple declarative sentence inconsistent.

The inconsistency essentially depends on the interaction of the rules for ‘if’ with those for and −. Neither SR and SR nor SR− and SR− produce any such formal inconsistency by themselves. That is clear for SR and SR, since both are valid when ‘if’ is interpreted as a material conditional, as already observed. To see that SR− and SR− are equally harmless by themselves, we can interpret ‘*XX* − *Y*’ as equivalent to ‘*XX* not(*Y*)’, which fits its intended interpretation, and ‘If *X*, *Y*’ as the conjunction ‘*X* and *Y*’ which does not. Thus SR− and S− are interpreted as these two rules:

R1 *BB*; *A* not(*C*) *BB* not(*A* and *C*)

R2 *BB* not(*A* and *C*) *BB*; *A* not(*C*)

Both R1 and R2 are classically valid. That the interpretation of ‘if’ is so far from its intended one does not matter for the purpose of showing the formal consistency of SR− and SR−, even when combined with the structural rules.

The pairing of conjunction with the material conditional as readings of ‘if’ is anyway less odd, structurally, than it may seem. Both readings agree that when *A* is true, ‘If *A*, *C*’ has the same truth-value as *C*. They disagree over the cases where *A* is false, but both treat them uniformly. On the material conditional reading, whenever *A* is false, ‘If *A*, *C*’ is true. On the conjunctive reading, whenever *A* is false, ‘If *A*, *C*’ is false.

The inconsistency proofs do *not* show that we have not really been implicitly using the Suppositional Rule. Nor do they show that the Rule does not really have the consequences we took it to have. We may have been using an inconsistent rule for ‘if’ all along. After all, both derivations involve applications of the rules which do not readily come to mind, because they go via mutually inconsistent suppositions (*A* and ‘Not(*A*)’), even though the overall result is to derive an inconsistency from the most harmless of premises. The problem is severe, but from a practical perspective easy to overlook, like semantic paradoxes such as the Liar. There may be a glitch in our primary heuristic for assessing conditionals, just as the semantic paradoxes reveal a glitch in our way of assessing ascriptions of truth and falsity.

The tensions just identified in the Suppositional Rule do *not* depend on any assumption that ‘if’ conditionals express propositions, and are not resolved by denying any such assumption. Although the rules for force ‘if’ to be equivalent to , that is an *outcome* of applications of the Rule, not an imposition on it from outside. Of course, ‘if’ conditionals were treated as entering into deductive relations ( and −), but those relations were not assumed to be characterized in terms of truth-preservation or the like. For all the argument required, those relations might be purely proof-theoretic. Indeed, following Ernest Adams (1965, 1975), no-proposition theorists have gone to great lengths to explain how there can nevertheless be a logic of conditionals. Nor did the derivations embed ‘if’ conditionals in more complex sentences of the object-language, as would be problematic on the no-proposition view. Denying that ‘if’ conditionals express propositions in no way reduces the urgency of the problem posed by the derivations for the Suppositional Rule.

One might guess that the glitch in the Rule is quite local, confined to the handling of inconsistent suppositions. But that is over-optimistic. If the problem arose only in cases of inconsistent suppositions, the probabilistic application SRCC of the rule would avoid it, since the conditional probabilities could be left undefined in those cases. That is not so: SRCC faces a closely related problem, discussed in the next section.

* 1. *Probabilistic paradox for the Suppositional Rule*

David Lewis initiated a large literature on combinatorial problems facing Stalnaker’s Hypothesis (SRC), Generalized Stalnaker’s Hypothesis (SRCC), and other variations on the same theme. The proofs are usually interpreted as showing, or attempting to show, that no assignment of propositions to conditional sentences satisfies the stipulated constraints. However, that is to underestimate what the proofs achieve. They also make trouble for the idea that conditionals do not express propositions but can still be assigned probabilities in accordance with the proposed equation, even though they are not probabilities of truth.7

Lewis’s original argument (1976) can be applied to SRCC. We will rework it to allow for the option of assigning probabilities to conditionals according to SRCC without taking them to express propositions.

Consider a restricted language with two types of sentence. Type 1 sentences are assumed to express propositions; they include three mutually inconsistent sentences *P*, *Q*, and *R*, and their truth-functional combinations. Type 2 sentences are not assumed to express propositions; they are ‘if’ conditionals whose antecedent and consequent are type 1 sentences. Type 2 sentences are not subject to further embedding. Let Cred be a probability distribution over sentences of both types such that Cred(*P*) > 0, Cred(*Q*) > 0, Cred(*R*) > 0. Conditional probabilities are taken as primitive, but as obeying standard principles of conditional probability whenever they are defined. Type 2 sentences are permitted in the *X* position of Cred(*X*|*Y*) but not in the *Y* position. Then Cred does not satisfy SRCC.

Here is a proof. Assume that SRCC holds. Then:

Cred(if (*P* or *Q*), *P*|(*P* or *Q*) *P*) = Cred(*P*|(*P* or *Q*) and ((*P* or *Q*) *P*)) = Cred(*P*|*P*) = 1

Therefore, by SRCC again and the principle that Cred(*X*) ≤ Cred(*Y*) whenever Cred(*Y*|*X*) = 1:

Cred((*P* or *Q*) *P*) ≤ Cred(if (*P* or *Q*), *P*) = Cred(*P*|*P* or *Q*)

But we saw in section 3.2 that Cred(*A* *C*) ≤ Cred(*C*|*A*) only when either Cred(*A*) = 1 or Cred(*A*  *C*) = 1. Thus either Cred(*P* or *Q*) = 1 or Cred((*P* or *Q*) *P*) = 1. But, *P*, *Q*, and *R* are mutually incompatible by hypothesis, so ‘*P* or *Q*’ entails ‘not(*R*)’ and ‘((*P* or *Q*) *P*’ entails ‘not(*Q*)’, hence:

Cred(*P* or *Q*) ≤ Cred(not(*R*)) < 1 and Cred((*P* or *Q*) *P*) ≤ Cred(not(*Q*)) < 1.

This contradicts the disjunction just established. Thus SRCC does not hold.

Nothing in that argument requires ‘if’ conditionals to be treated as expressing propositions, yet it still shows that SRCC is unacceptably restrictive. We usually face more than two live, mutually exclusive possibilities.

Following Lewis, there is a large literature on ways of restricting SRCC to avoid the triviality results, and proofs of further triviality results from weaker assumptions. We need not go into details, for SRCC is the natural heuristic with which to calibrate credences in conditionals by conditional probabilities. It combines the required generality, by covering conditionals under suppositions, with the required simplicity, by avoiding the clutter of gerrymandered restrictions.

The triviality results for credences cannot quite be strengthened to proofs of outright inconsistency like those in section 3.2 for the deductive rules. The restriction to cases where the conditional probabilities are well-defined is just enough to rescue SRCC from that even worse fate. In fact, we can show that SRCC is satisfiable when only two mutually exclusive possibilities are relevant.

Here is the proof. Let W be a set of just two worlds, and treat propositions as subsets of W. Let [*X*] be the proposition expressed by the sentence *X*. We assign propositions to conditional sentences thus:

When [*A*][*C*] ≠ {}, let [if *A*, *C*] = [*A*  *C*].

When [*A*][*C*] = {}, let [if *A*, *C*] = {}.

Propositions are assigned to unconditional sentences, and to truth-functional combinations of sentences (including conditional sentences) in a standard compositional way. Let Prob be any probability distribution over W; set Cred(*X*) = Prob([*X*]) for every sentence *X*. Then SRCC is satisfied, as we show thus (considering only cases where Cred is well-defined):

If [*A*] = W, then Cred(if *A*, *C*|*B*) = Cred(*C*|*B*) = Cred(*C*|*A* and *B*).

If [*A*] ≠ W and [*A*][*C*] ≠ {}, then [*A*] [*C*], so Cred(if *A*, *C*|*B*) = 1 = Cred(*C*|*A* and *B*).

If [*A*][*C*] = {}, then Cred(if *A*, *C*|*B*) = 0 = Cred(*C*|*A* and *B*).

This shows that the use of *three* mutually exclusive cases with nonzero probability (*P*, *Q*, *R*) in the triviality proof was the best possible: one cannot get the number down to two or one, let alone to zero.

Still, even though the trivialization implied by SRCC does not amount to outright inconsistency, it is bad enough to show that SRCC is unsatisfiable under reasonable conditions. Since denying that conditionals express propositions does not block the trivialization, the no-proposition view loses its main theoretical motivation.

The no-proposition view also fails to give good explanations of the linguistic data, as has often been noted. Many sentences with embedded ‘if’ seem to make perfectly good sense and to express propositions of a perfectly ordinary kind. Many universally quantified indicative sentences with conditionals are cases in point. Here is just one example, where ‘if’ is embedded under both conjunction and a universal quantifier:

(18) Everything was put in the suitcase if it belonged to Mary and on the table if it

belonged to John.

That seems to make perfectly good sense, and to be equivalent to (19), whose truth-conditions are not in doubt:

(19) Everything that belonged to Mary was put in the suitcase and everything that belonged to John was put on the table.

Sometimes no-proposition theorists simply claim that we ‘interpret’ a sentence with an embedded conditional as another sentence with no embedded conditional, for example (18) as (19), even though on their view the reinterpretation is unwarranted by the compositional semantics of English. In the absence of any explanation as to *why* we reinterpret one sentence as another, such a strategy inherits the *ad hoc* character of the repairs it postulates. Moreover, the phenomenology of smoothly reading (18) as equivalent to (19) is quite unlike that of guessing at the intended interpretation of a string of words that makes no literal sense.

We can parse (18) thus, in overall form:

(20) Everything*x* ((if *x* belonged to Mary, *x* was put in the suitcase) and (if *x* belonged to John, *x* was put on the table))

To evaluate (18)/(20), as we surely can, we must be able to evaluate its matrix (the open formula following ‘Everything*x*’), and so the conditionals (21) and (22), for each value of the variable ‘*x*’ in the domain:

(21) If *x* belonged to Mary, *x* was put in the suitcase.

(22) If *x* belonged to John, *x* was put on the table.

For (18)/(20) to be true, as it may well be with a suitably restricted domain of contextually relevant things, (21) and (22) must also be true for each value of ‘*x*’ in the domain, not least for those on which the antecedent is false. The component conditionals must also be true for values of ‘*x*’ of which the speaker has no idea, and which play no role in structuring the speaker’s credences or epistemic and doxastic state more generally.8 Such values of variables are often contextually relevant, and falsify our rash universal generalizations, or verify our existential ones: I can truly say ‘There are more things in heaven and earth than are dreamt of in my philosophy’.

To deny that conditionals such as (21) and (22) have truth-conditions obstructs the natural understanding of generalizations such as (18). As we already saw, it also fails to solve the paradoxes created by the Suppositional Rule. In what follows, indicative conditionals will therefore be assumed to be meaningfully embeddable in more complex sentences in all the ways standard in natural languages. For example, the quantified sentence (18) can be freely embedded, just like any other ordinary declarative sentence.

* 1. *The Suppositional Rule for complex attitudes*

Sections 3.2 and 3.3 considered paradoxical consequences of the Suppositional Rule when applied to credences and deductive attitudes respectively. It also has paradoxical consequences when applied to other attitudes. However, before we consider them, we must enlarge our understanding of what the Suppositional Rule implies.

We can use logical operators to define new attitudes from old ones. For example, from negation and a non-probabilistic attitude of *acceptance* we can define a corresponding non-probabilistic attitude of *rejection*, by stipulating that to reject something is just to accept its negation (under the same suppositions, if any). Some authors reserve the word ‘reject’ for a negative attitude unmediated by an added negation in the content, but that terminological issue should not obscure the legitimacy of the attitude just defined, whatever we call it. Such newly defined attitudes allow us to draw further consequences from the Suppositional Rule. For it involves taking *any* cognitive attitude to ‘If *A*, *C*’ just in case one takes that attitude to *C* on the supposition *A*.

We use the symbol || for an arbitrary cognitive attitude; *AA* || *C* means that the attitude || is taken to *C* on the set of suppositions *AA*. We can therefore rewrite the Suppositional Rule as a generalization of the natural deduction rules for the conditional, for any attitude ||, sentences *A*, *C*, and set of sentences *BB*:

SR *BB*; *A* || *C* just in case *BB* || if *A*, *C*

For any attitude || (such as acceptance), we can now define a corresponding negative attitude ||not of ‘rejection’, in the way just explained:

||not *AA* ||not *C* just in case *AA* || not(*C*)

In other words, to take the negative attitude to something is just to take the positive attitude to its negation (under the same suppositions).

The combination of SR and ||not has significant consequences. It makes negating the consequent of an indicative conditional equivalent to negating the whole conditional, in the sense that exactly the same attitudes to them are mandated on any given set of suppositions. For SR and ||not together yield this chain of mandated equivalences, for any attitude ||, sentences *A*, *C*, and set of sentences *BB*:

*BB* || (if *A*, not(*C*)) just in case *BB*; *A* || not(*C*) (by SR)

just in case *BB*, *A* ||not *C* (by ||not)

just in case *BB* ||not (if *A*, *C*) (by SR)

just in case *BB* || not(if *A*, *C*) (by ||not)

In this sense, given the Suppositional Rule and the way of defining negative attitudes ||not, the conditional commutes in the consequent with negation:

CCCN *BB* || (if *A*, not(*C*)) just in case *BB* || not(if *A*, *C*)

In particular, we can apply CCCN to the attitude of acceptance, written ||a. Since anything is accepted on a set of suppositions to which it belongs, we have:

(23) *BB*; (if *A*, not(*C*)) ||a (if *A*,not(*C*))

(24) *BB*; not(if *A*, *C*) ||a not(if *A*, *C*)

But then, by applying opposite directions of CCCN to (23) and (24) respectively, we have:

(25) *BB*; (if *A*, not(*C*)) ||a not(if *A*, *C*)

(26) *BB*; not(if *A*, *C*) ||a (if *A*, not(*C*))

Thus, still given SR and ||not, negating a conditional and negating its consequent are also equivalent in the additional sense that each is accepted on the supposition of the other and any given set of further suppositions. This equivalence, unlike CCCN, is specific to the attitude of acceptance, for not every cognitive attitude is to be taken to something on a set of suppositions to which it belongs; *rejection* is an obvious counterexample.

For most purposes, then, negating a conditional is tantamount to negating its consequent. This idea has attracted theorists of conditionals in natural language. For example, Dorothy Edgington tentatively proposes that ‘*A* is to ¬*A* as “If *A*, *B*” is to “If *A*, ¬*B*”’, and she asserts that ‘”It’s not the case that if *A*, *B*” has no clear established sense distinguishable from this’ (1995: 283).9 As noted in section 3.1, there is considerable evidence that we tend to conform to SRC (the Suppositional Rule for credences), which requires credences for ‘If *A*, *C*’ and ‘If *A*, not(*C*)’ to sum to 1, so in that respect we treat ‘If *A*, not(*C*)’ like the contradictory of ‘If *A*, *C*’.

A striking feature of the derivations of both CCCN and (25)-(26) is that they are independent of the logic of ‘not’, of which they exploit no characteristic principle. That suggests generalizing them to other operators. To reach other non-trivial truth-functions, we must generalize from one-place operators like negation to many-place operators.

A salient candidate is *conjunction*. The analogue of CCCN is the conditional commuting in the consequent with conjunction, for any attitude ||, sentences *A*, *C*, *D*, and set of sentences *BB*:

CCCC *BB* || (if *A*, (*C* and *D*)) just in case *BB* || ((if *A*, *C*) and (if *A*, *D*))

In other words, conjoining conditionals with the same antecedent is equivalent to conjoining their consequents. That sounds plausible. Just as (25) and (26) were derived from CCCN, so we can derive (27) and (28) from CCCC:

(27) *BB*, (if *A*, (*C* and *D*)) ||a ((if *A*, *C*) and (if *A*, *D*))

(28) *BB*, ((if *A*, *C*) and (if *A*, *D*)) ||a (if *A*, (*C* and *D*))

These are natural outcomes of the Suppositional Rule. If you accept a conjunction on some suppositions, you can apply the elimination rule for conjunction to accept each of its conjuncts, on the same suppositions. Conversely, if you accept each of the conjuncts on some suppositions, you can apply the introduction rule for conjunction to accept the conjunction, again on the same suppositions. The Suppositional Rule then takes one the rest of the way to (27) and (28). Of course, the introduction rule for conjunction does not work for a probabilistic standard of acceptance, with a threshold for acceptance less than 1, since a conjunction may be less probable than each of its conjuncts, but it still works for *full acceptance*, which gives us what we want in (27) and (28). When scientists develop the consequences of a hypothesis, they seem quite willing to conjoin different consequences which they have separately extracted from it.

However, those considerations are specific to conjunction. For example, disjunction does not satisfy the elimination rule for conjunction: you can accept a disjunction without accepting all its disjuncts, even without accepting any of them. You saw the coin land, but you were too far away to see which way it came up. You accept ‘It came up heads or tails’, but you do not accept ‘It came up heads’, nor do you accept ‘It came up tails’ (nor do you accept their negations). But (27) and (28) were derived from CCCC, the analogue of CCCN. If CCCC is properly analogous to CCCN, its rationale should not depend on the logic of conjunction, any more than the rationale for CCCN depends on the logic of negation. Indeed, such a rationale for CCCC should smoothly generalize to a rationale for the corresponding principle about disjunction, the conditional commuting in the consequent with disjunction:

CCCD *BB* || (if *A*, (*C* or *D*)) just in case *BB* || ((if *A*, *C*) or (if *A*, *D*))

In other words, *dis*joining conditionals with the same antecedent is equivalent to *dis*joining their consequents. Just as (27) and (28) were derived from CCCC, so we can derive (29) and (30) from CCCD:

(29) *BB*, (if *A*, (*C* or *D*)) ||a ((if *A*, *C*) or (if *A*, *D*))

(30) *BB*, ((if *A*, *C*) or (if *A*, *D*)) ||a (if *A*, (*C* or *D*))

Perhaps surprisingly, these generalized expectations can be implemented. Just as deriving CCCN required defining negative attitudes, so deriving CCCC and CCCD requires defining conjunctive and disjunctive attitudes respectively. These new forms of complex attitude will be attitudes to *sequences* of sentences (or, through them, to sequences of propositions). For simplicity, we just consider attitudes to ordered pairs, but the generalization to longer sequences is straightforward. Here are the definitions:

||and *AA* ||and *C*, *D* just in case *AA* || *C* and *D*

||or *AA* ||or *C*, *D* just in case *AA* || *C* or *D*

In other words, to take the conjunctive attitude to some things is just to take the original attitude to their conjunction (under the same suppositions); to take the disjunctive attitude to the things is just to take the original attitude to their disjunction (again, under the same suppositions). For instance, to accept a plurality conjunctively is to accept its conjunction; to accept the plurality disjunctively is to accept its disjunction. These definitions have singular attitudes on the right-hand side, the definiens, so any initial mystery about the plural attitudes on the left-hand side, the definiendum is cleared up by the definition itself.

We need the Suppositional Rule to apply to plural attitudes as well as singular ones. That is entirely within the spirit of the rule. It mandates taking whatever attitude one takes to some sentences conditionally on the supposition *A* unconditionally to the corresponding conditionals with those sentences as consequents and *A* as the antecedent (with the same background suppositions). For instance, one is to treat *C* and *D* as *mutually incompatible* conditionally on the supposition *A* just in case one treats ‘If *A*, *C*’ and ‘If *A*, *D*’ as unconditionally mutually incompatible. Here is the Rule stated explicitly in plural form, where *A*, *C*1, . . ., *Cn* are any sentences, *BB* any set of sentences, and || any attitude:

SR+ *BB*, *A* || *C*1, . . ., *Cn* just in case *BB* || (if *A*, *C*1), . . ., (if *A*, *Cn*)

Here is the derivation of CCCC from SR+ and ||and, analogous to the derivation of CCCN from SR and ||not, for any attitude ||, sentences *A*, *C*, *D*, and set of sentences *BB*:

*BB* || if *A*, (*C* and *D*) just in case *BB*, *A* || *C* and *D* (by SR+)

just in case *BB*, *A* ||and *C*, *D* (by ||and)

just in case *BB* ||and (if *A*, *C*), (if *A*, *D*) (by SR+)

just in case *BB* || (if *A*, *C*) and (if *A*, *D*) (by ||and)

By substituting ‘or’ for ‘and’ throughout, one obtains a parallel derivation of CCCD from SR+ and ||or. These derivations make no use of any characteristic principles of the logic of conjunction or disjunction. We then obtain the equivalences (27)-(28) and (29)-(30) from CCCC and CCCD respectively, as already observed.

The main point of these derivations of CCCC and CCCD is to check that both principles follow simply from the Suppositional Rule as applied to a suitable variety of attitudes, without appeal to any special principles of the logic of conjunction or disjunction. For, as we also noted, the principle that the conditional commutes with conjunction in the consequent is pre-theoretically very plausible, without explicit appeal to the Suppositional Rule, and all truth-functions, including disjunction, are definable in terms of conjunction and negation. Thus, by repeatedly apply the commutativity principles for conjunction and negation, we can derive the corresponding commutativity principle for *any* truth-function, at least in its defined form. In the case of disjunction, we could anyway argue for the equivalence of the De Morgan equivalents of ‘if *A*, (*C* or *D*)’ and ‘(if *A*, *C*) or (if *A*, *D*)’, in other words, ‘if *A*, not(not(*C*) and not(*D*))’ and ‘not(not(if *A*, *C*) and not(if *A*, *D*))’. From that, the equivalence of the explicitly disjunctive versions is a comparatively small step. One can also derive the new commutativity principle, along the same lines as CCCC and CCCD.

The commutativity principles enable us to derive some much-discussed theses in the logic of conditionals. One is *Conditional Excluded Middle*, this schema:

CEM (if *A*, *C*) or (if *A*, not(*C*))

For, given that ‘if’ commutes with ‘not’ in the consequent, CEM is equivalent to (31):

(31) (if *A*, *C*) or not(if *A*, *C*)

But (31) is just a special case of the classical law of excluded middle (‘*B* or not(*B*)’), and classical logic is not here in question. Equally, we can argue for CEM from the principle that ‘if’ commutes with ‘or’ in the consequent, for it makes CEM equivalent to (32):

(32) if *A*, (*C* or not(*C*))

Again given classical logic, (32) has a tautologous consequent, and so should be accepted.

The position is similar for *Conditional Non-Contradiction*, this schema:

CNC not((if *A*, *C*) and (if *A*, not(*C*))

For, given that ‘if’ commutes with ‘not’ in the consequent, CNC is equivalent to (33):

(33) not((if *A*, *C*) and not(if *A*, *C*))

But (33) is just a special case of the classical law of non-contradiction (‘not(*B* and not(*B*))’), and classical logic is not here in question. Equally, we can argue for CNC from the principle that ‘if’ commutes with ‘and’ in the consequent, for it makes CNC equivalent to (34):

(34) not(if *A*, (*C* and not(*C*))

Again given classical logic, the conditional in (34) has a contradictory consequent, which one might take to merit rejection on any supposition.

Together, CEM and CNC guarantee that ‘not(if *A*, *C*)’ is materially equivalent to ‘if *A*, not(*C*)’. For CEM guarantees that ‘not(if *A*, *C*)’ materially implies ‘if *A*, not(*C*)’, and CNC guarantees the converse material implication. Thus they jointly yield a version of the principle that the conditional commutes in the consequent with negation.

How do the commutativity principles fare on various interpretations of ‘if’? There is a trivial interpretation on which all such principles hold: it treats the antecedent as idle, and ‘if *A*, *C*’ as having the same semantics as *C* itself. Thus for any *n*-place sentential operator O and sentences *A*, *C*1, . . ., *Cn*, ‘if *A*, O(*C*1, . . ., *Cn*)’ has the same semantics as ‘O(*C*1, . . ., *Cn*)’, which in turn has the same semantics as ‘O((if *A*, *C*1), . . ., (if *A*, *Cn*))’, provided that O itself has a compositional semantics, in the sense that the semantics of the output sentence ‘O(*B*1, . . ., *Bn*)’ is always determined just by the semantics of the input sentences *B*1, . . ., *Bn*. Negation, conjunction, and disjunction all have a compositional semantics in that sense. The trivial reading also validates both CEM and CNC.

That unintended interpretation of ‘if’ suffices to show that the commutativity principles *by themselves* generate no inconsistency. However, it is obviously inadequate, since it implies that ‘If *A*, *A*’ is acceptable only if *A* is acceptable, which is false: one can rationally accept ‘If there is life in other galaxies, there is life in other galaxies’ without accepting ‘There is life in other galaxies’. In such cases, the acceptability of ‘If *A*, *A*’ is a trivial upshot of the Suppositional Rule, since *A* is acceptable on the supposition *A*, so this unintended interpretation does not even validate the full consequences of the Suppositional Rule, which therefore exceed the commutativity principles. Thus we restrict attention to interpretations of ‘if’ which vindicate the acceptability of ‘If *A*, *A*’.

The material interpretation of ‘if’ validates the commutativity principles for both the binary conjunction and disjunction operators, as well as ‘If *A*, *A*’. For when *A* is true, it makes the truth-value of any conditional with antecedent *A* the truth-value of the consequent, so both ‘if *A*, (*C* and *D*)’ and ‘(if *A*, *C*) and (if *A*, *D*)’ have the same truth-value as ‘*C* and *D*’, while ‘if *A*, (*C* or *D*)’ and ‘(if *A*, *C*) or (if *A*, *D*)’ have the same truth-value as ‘*C* or *D*’ ; when *A* is false, the material interpretation makes any conditional with antecedent *A* true, so ‘if *A*, (*C* and *D*)’, ‘(if *A*, *C*) and (if *A*, *D*)’, ‘if *A*, (*C* or *D*)’, and ‘(if *A*, *C*) or (if *A*, *D*)’ are all true; either way, commuting ‘if’ with conjunction or disjunction in the consequent is logically guaranteed to preserve truth-value. Thus the material interpretation validates the principles that the conditional commutes with binary conjunction and disjunction in the consequent. It also validates CEM. However, it does not fully validate the commutativity principle for negation. When *A* is true, the material interpretation also gives both ‘if *A*, not(*C*)’ and ‘not(if *A*, *C*)’ the truth-value of ‘not(*C*)’. But when *A* is false, it makes ‘if *A*, not(*C*)’ true and ‘not(if *A*, *C*)’ false, so it fails hopelessly to validate the principle that the conditional commutes with negation in the consequent. It also invalidates CNC, because it makes both ‘If *A*, *C*’ and ‘If *A*, not(*C*)’ true when *A* is false. Thus the material interpretation fails validate fully the applications of the Suppositional Rule at issue in this section.

A truth-functional interpretation of ‘if’ which validates the commutativity principle for negation, as well as ‘If *A*, *A*’, is the material *biconditional* reading, on which ‘If *A*, *C*’ is true or false according to whether the truth-values of *A* and *C* are the same or different. This reading makes both ‘if *A*, not(*C*)’ and ‘not(if *A*, *C*)’ true or false according to whether the truth-values of *A* and *C* are different or the same. Consequently, it also validates both CEM and CNC. However, the material biconditional principle invalidates the commutativity principles for both conjunction and disjunction. For let *C* and *D* have opposite truth-values, so ‘if *A*, *C*’ and ‘if *A*, *D*’ also have opposite truth-values. Hence ‘*C* and *D*’ and ‘(if *A*, *C*) and (if *A*, *D*)’ are false, while ‘*C* or *D*’ and ‘(if *A*, *C*) or (if *A*, *D*)’ are true, irrespective of the truth-value of *A*. Hence, when *A* is false, ‘if *A*, (*C* and *D*)’ is true while ‘(if *A*, *C*) and (if *A*, *D*)’ is false, and ‘if *A*, (*C* or *D*)’ is false while ‘(if *A*, *C*) or (if *A*, *D*)’ is true. Thus the material biconditional interpretation also fails to validate fully the applications of the Suppositional Rule at issue in this section.

The approach that arguably comes closest to validating the applications of the Suppositional Rule in this section is Stalnaker’s (1968); Williams (2010) and Dorr and Hawthorne (2018) hold relevantly similar views. Such a semantics is modal; formulas are assigned truth-values at worlds. Each model is equipped with a selection function *f*, which maps each nonempty set X of worlds and world *w* to a world *f*(X, *w*) in X (informally, one might regard *f*(X, *w*) as the closest world in X to *w*).10 Let [*A*] be the set of worlds in the model at which *A* is true. Stalnaker ensures that [*A*] is nonempty for every sentence *A* by stipulating that each model includes an ‘absurd world’ λ where all sentences whatsoever are true. By contrast, the evaluation of truth-functional combinations at worlds other than λ is standard. His rule for the conditional is that the truth-value of ‘if *A*, *C*’ at *w* is the truth-value of *C* at *f*([*A*], *w*). Informally, we can regard *f*([*A*], *w*) as the world we imagine where *A* is true when we suppose *A* from the perspective of *w*. As a result, this framework has many consequences friendly to the applications of the Suppositional Rule at issue in this section.

Obviously, Stalnaker’s semantics makes ‘if *A*, *A*’ true at every world. Moreover, it makes a conjunction true at a world (possible or impossible) just in case every conjunct is true at the world, which ensures that ‘if *A*, (*C* and *D*)’ and ‘(if *A*, *C*) and (if *A*, *D*)’ coincide in truth-value at every world. Thus the conditional commutes in the consequent with conjunction. Similarly, a disjunction is true at a world (possible or impossible) just in case some disjunct is true at the world, which ensures that ‘if *A*, (*C* or *D*)’ and ‘(if *A*, *C*) or (if *A*, *D*)’ coincide in truth-value at every world. Thus Stalnaker’s conditional also commutes in the consequent with disjunction.

Stalnaker’s semantics *almost* validates commutativity in the consequent with negation too. For whenever *w* is a world other than λ, and X contains at least one world other than λ, *f*(X, *w*) is required to be a world other than λ too. Thus when the sentence *A* expresses a possibility, so [*A*] contains at least one possible world, *f*([*A*], *w*) is required to be possible too. Informally: when we make a possible supposition from a possible perspective, we should not imagine it in an impossible way. Now ‘not(*C*)’ is true at a possible world just in case *C* is not true at that world. Thus, when *A* expresses a possibility and *w* is possible, the truth-values of *C* and ‘not(*C*)’ at the possible world *f*([*A*], *w*) are opposite, so the truth-values of ‘if *A*, *C*’ and ‘if *A*, not(*C*)’ back at the world *w* are also opposite, so the truth-values of ‘not(if *A*, *C*)’ and ‘if *A*, not(*C*)’ at *w* are the same.

Unfortunately, this nice pattern breaks down when *A* expresses an impossibility. For then [*A*] is simply {λ}, so *f*([*A*], *w*) must be λ, so both *C* and ‘not(*C*)’ are true at *f*([*A*], *w*), so both ‘if *A*, *C*’ and ‘if *A*, not(*C*)’ are true at any possible world *w*, for any sentence *C*, while ‘not(if *A*, *C*)’ is not true. Thus CNC fails in this special case, though CEM holds unrestrictedly, since at least one of *C* and ‘not(*C*)’ is true at any possible world, possible or impossible. Nevertheless, the upshot is that Stalnaker’s conditional does not fully commute in the consequent with negation.

Stalnaker’s semantics can be tweaked to validate CNC as well as CEM. For one can drop λ, treat the truth-functors classically at all worlds, and make the brute stipulation that *f*({}, *w*) = *w* for every world *w*. Thus whenever *A* expresses an impossibility, ‘if *A*, *C*’ has the same truth-value as *C*. That is in effect to treat counterpossible conditionals as don’t-cares. But it makes ‘if *A*, *A*’ impossible whenever *A* is impossible. Since it invalidates ‘if *A*, *A*’, this tweaked semantics also violates the blatant requirement of the Suppositional Rule.

Tweaked or untweaked, Stalnaker’s approach faces obvious worries about how the selection function *f* is determined. There is the problem of equally good candidates. For example, let *A* be ‘the coin is tossed’. If *A* is false at *w*, what determines whether *f*([*A*], *w*) is a world where the coin come up heads or one where it comes up tails? Worse, there is also the problem of sequences not containing their own limit. For example, let the rod R be exactly *m* long in *w*, and *B* be ‘R is longer than *m*’. Consider any world *x* in [*B*]. R is exactly *m* + δ long in *x*; but in another world *y* in [*B*], R is only *m* + δ/2 long, so *y* seems to beat *x* as a candidate to be *f*([*B*], *w*), since *y* is closer than *x* to *w* in the relevant respect. Thus every candidate is bettered by another candidate. Stalnaker handles these problems by suggesting that it can be indeterminate which selection function is operative, even though it is determinate that some selection function or other is operative. Thus CEM is still determinately true, because each selection function verifies it, even though it may be indeterminate which disjunct of CEM is true, because some selection functions verify one disjunct while other selection functions verify the other disjunct. The theory of *supervaluationism* supplies the apparatus to implement this approach.11

Theorists who like a modal treatment of indicative conditionals but dislike the arbitrariness of selection functions may prefer to generalize over a plurality of relevant worlds at which the antecedent is true, perhaps regarding the worlds as epistemically rather than objectively possible. Thus they regard the indicative conditional as an epistemically strict conditional, true just in case *all* relevant antecedent-worlds are also consequent-worlds. The strict reading does at least validate ‘If *A*, *A*’. However, it invalidates CEM, since some but not all antecedent-worlds may be consequent-worlds. Moreover, CNC remains invalid, since counterpossibles with mutually contradictory consequents are both vacuously true.

Claudio Pizzi has shown how to validate CNC within the modal approach by strengthening strict implication to *consequential implication* (Pizzi 1977; Pizzi and Williamson 1997, 2005). The idea is that ‘if *A*, *C*’ is true just in case *A* strictly implies *C* and *A* and *C* have the same *modal status*, in the sense that the propositions they express are either both necessary, or both contingent, or both impossible. In other words, the truth-condition is this: every antecedent-world is a consequent-world, and if there are consequent-worlds, there are antecedent worlds, and if there are only consequent-worlds, there are only antecedent-worlds. To see that this truth-condition validates CNC, suppose that ‘If *A*, *C*’ and ‘If *A*, not(*C*)’ are both true. Then every A-world is both a C-world and a ‘not(*C*)’-world, so there are no *A*-worlds. Since both conditionals must have consequents with the same modal status as their antecedent, there are no *C*-worlds and no ‘not(*C*)’-worlds, which is a contradiction, since the actual world is either a *C*-world or a ‘not(*C*)’-world. The consequential reading also validates ‘If *A*, *A*’, for *A* always both strictly implies itself and has the same modal status as itself.

However, consequential implication is even worse than strict implication for CEM. When *A* strictly implies neither of *C* and ‘not(*C*)’, it also consequentially implies neither of them. Moreover, consequential implication creates additional problems for CEM. When one of *A* and *C* expresses an impossibility or necessity, while the other expresses a contingency, *A* differs in modal status from both *C* and ‘not(*C*)’, so all these cases are counterexamples to CEM.

Moreover, despite the feeling that CEM is metaphysically ungrounded, the natural language data suggest that much of our ordinary thought and talk respects CEM. Imagine yourself contemplating two hypotheses about a fair coin, not knowing whether it will be tossed:

(35) If it is tossed, it will come up heads.

(36) If it is tossed, it will come up tails.

Consider (35) and (36) as uttered in your context. If an oracle tells you that (35) is false, it seems natural for you to conclude that (36) is true. But the strict and consequential readings make that inference fallacious. For it ignores the possibility that the coin is never actually tossed, in which case both heads and tails are symmetric open possibilities. Thus the relevant counterfactual worlds where the coin is tossed should include some where it comes up heads and some where it comes up tails. Consequently, on those readings, both (35) and (36) are false. By contrast, CEM licenses the inference, via disjunctive syllogism: eliminating the first disjunct leaves you with the second (here we can treat ‘not heads’ as tantamount to ‘tails’, given that the coin was tossed).

Similarly, if the oracle instead tells you that (35) is *true*, it seems natural for you to conclude that (36) is false. That is further confirmation that much of our ordinary thought and talk respects CNC too.

The strict and consequential readings also invalidate the principle CCCD that the conditional commutes in the consequent with disjunction, for they disallow the deduction from (37) to (38):

(37) If it is tossed, (it will come up heads or it will come up tails).

(38) (If it is tossed, it will come up heads) or (if it is tossed, it will come up tails).

The strict reading does at least validate the principle CCCC that the conditional commutes in the consequent with conjunction, for every *A*-world is a ‘*C* and *D*’-world just in case every *A*-world is a *C*-world and every *A*-world is a *D*-world. By contrast, the consequential reading invalidates even that principle. For if *C* expresses a contingency and *D* a necessity, ‘If (*C* and *D*), (*C* and *D*)’ is true on the consequential reading, while ‘If (*C* and *D*), *D*’ is false, since its antecedent and consequent differ in modal status.

Notably, *none* of the interpretations of ‘if’ just surveyed validates both the unrestricted commutativity principles and the unrestricted schema ‘If *A*, *A*’. There is a more general reason for that, as we shall soon see.

* 1. *The inconsistency of the Suppositional Rule for complex attitudes*

Anything is to be accepted on the supposition of itself. Therefore, by the Suppositional Rule, ‘If *A*, *A*’ is always to be accepted, for any meaningful sentence *A.* As a special case of that schema, we have for any meaningful sentence *A*:

[I] If (*A* and not(*A*)), (*A* and not(*A*))

For meaningful sentences are closed under conjunction and negation. Therefore, by applying commutativity in the consequent with conjunction (CCCC), we can derive [II] from [I]:

[II] (If (*A* and not(*A*)), *A*)) and (if (*A* and not(*A*)), not(*A*))

By applying commutativity in the consequent with negation (CCCN) to the second main conjunct of (II), we can derive [III] from [II]:

[III] (If (*A* and not(*A*)), *A*) and not(if (*A* and not(*A*)), *A*)

But [III] is an explicit contradiction; it conjoins ‘if (*A* and not(*A*)), *A*’ with its own negation.

For example, less formally: if it is raining and not raining, it is raining and not raining; so if it is raining and not raining, it is raining; but also, if it is raining and not raining, it is not raining, so it is not the case that if it is raining and not raining, it is raining; that is an outright contradiction.

The resources from conditional logic needed for the derivation can be pared down to three schematic principles: ‘If *A*, *A*’ to get [I], the inference rule from ‘If *A*, (*B* and *C*)’ to ‘(If *A*, *B*) and (if *A*, *C*)’ to get from [I] to [II], and the inference rule from ‘If *A*, not(*C*)’ to ‘Not(if *A*, *C*)’ (in effect, CNC) to get from [II] to [III]. Since the Suppositional Rule delivers all the required resources, this is another direct proof of its inconsistency.

One initial reaction to the paradox [I]-[III] may be that it does not matter, because we have independent evidence that conditionals with self-contradictory antecedents are pathological. After all, probabilities conditional on a contradiction are normally undefined. Moreover, indicative conditionals normally presuppose that their antecedents are epistemically possible.

However, that initial reaction is too quick. Of course, to utter a conditional with a self-contradictory antecedent out of the blue is likely to cause puzzlement: what point can uttering it have? But some pointless utterances are meaningful, and even true; Queen Anne *is* dead. Moreover, uttering an indicative conditional with a self-contradictory antecedent is not always pointless. There was a theoretical point to propounding the paradox [I]-[III]. To take a different example, people who know the logician Graham Priest are not taken aback when he asserts something like (39):

(39) If the Liar sentence is true and not true, dialetheism is vindicated.

Priest, a sane, rational native speaker of English, regards many contradictions, including the antecedent of (39), as epistemically possible (indeed true); in asserting (39), he may easily make a relevant contribution to a philosophical conversation. Even classical logicians and mathematicians may assert an indicative conditional in English with an inconsistent antecedent, in the course of proving the negation of the antecedent by reductio ad absurdum. In some cases, both they and their audience already regard the antecedent as epistemically impossible, when an old or new proof of an already known result is being explained; they are merely treating the antecedent as open for purposes of the proof.

In less technical conversation, imagine an utterance of (40), with the indefinite articles used to express a universal generalization:

(40) If Mary hates a man and a woman doesn’t hate John, the man is nastier than the woman.

In context, Mary may be clearly in the domain for ‘a woman’, and John clearly in the domain for ‘a man’. Thus (41) is an instance of (40):

(41) If Mary hates John and Mary doesn’t hate John, John is nastier than Mary.

Since (40) is intended as a universal generalization, it is true only if every instance of it is true. Hence when (40) is true, (41) is also true. But when a sentence crashes, it is not true. Thus when (40) is true, (41) does not crash. Since (40) is true in some situations, (41) need not crash.

To take a simpler case, (42) understood as a universal generalization is clearly true:

(42) If one tower is tall and one tower is not tall, the former is taller than the latter.

Here is an instance of (42):

(43) If the Eiffel Tower is tall and the Eiffel Tower is not tall, the Eiffel Tower is taller than the Eiffel Tower.

The truth of (42) requires the truth of (43). Similarly, in natural mathematical English, one can truly say about the natural numbers:

(44) If *m* belongs to an initial segment S and *n* does not belong to S, *m* is less than *n*.

But (44) entails (45) by universal instantiation:

(45) If 7 belongs to S and 7 does not belong to S, 7 is less than 7.

Thus the paradox [I]-[III] cannot be so easily dismissed. Normal intelligent use of English commits us to treating some indicative conditionals with self-contradictory antecedents as not only intelligible but true.

Although most proponents of CEM restrict CNC in such cases, they sometimes argue for CEM in ways which also commit them to CNC quite generally. Moreover, since they normally take for granted that conditionals commute in the consequent with conjunction, they are vulnerable to the paradox [I]-[III]. In particular, there is an argument for CEM just like one given by Robert Williams (2010); he gives it for subjunctive rather than indicative conditionals, but the difference plays no role in his argument. He uses considerations from James Higginbotham (1986) and Sabine Iatridou (2002).

Here is the argument. Consider pairs such as (46) and (47):

(46) No student passed if they goofed off.

(47) Every student failed if they goofed off.

In this context, failing is equivalent to not passing. Then (46) and (47) seem to be equivalent. Moreover, they can be regimented as (46a) and (47a) respectively:

(46a) [no *x*: student *x*] (if *x* goofed off, *x* passed)

(47a) [every *x*: student *x*] (if *x* goofed off, not(*x* passed))

Generalizing this pattern, we have the equivalence of (48) and (49):

(48) [no *x*: *Fx*] (if *Gx*, *Hx*)

(49) [every *x*: *Fx*] (if *Gx*, not(*Hx*))

But ‘[no *x*: *Fx*] *A*’ is quite generally equivalent to ‘[every *x*: *Fx*] not(*A*)’. Thus, in particular, (48) is quite generally equivalent to (50):

(50) [every *x*: *Fx*] not(if *Gx*, *Hx*)

Thus (49) and (50) are quite generally equivalent. But that requires ‘if *Gx*, not(*Hx*)’ and ‘not(if *Gx*, *Hx*)’ to be quite generally equivalent. For given any value *o* of the variable ‘*x*’, we can always interpret *F* to be true of *o* and nothing else, in which case ‘if *Gx*, not(*Hx*)’ and ‘not(if *Gx*, *Hx*)’ for that value of ‘*x*’ are equivalent to (49) and (50) respectively. Although the conclusion usually drawn in the literature is only CEM, the argument delivers something stronger: that the conditional commutes in the consequent with negation (CCCN), which requires CNC as well as CEM.

Very similar arguments conclude that the conditional commutes in the consequent with conjunction (CCCC). For pairs such as (51) and (52) seem to be equivalent:

(51) Every student sang and danced if they passed.

(52) Every student sang if they passed and every student danced if they passed.

They can be regimented as (51a) and (52a) respectively:

(51a) [every *x*: student *x*] (if *x* passed, (*x* sang and *x* danced))

(52a) [every *x*: student *x*] (if *x* passed, *x* sang) and [every *x*: student *x*] (if *x* passed, *x* danced)

Generalizing this pattern, we have the equivalence of (53) and (54):

(53) [every *x*: *Fx*] (if *Gx*, (*Hx* and *Ix*))

(54) [every *x*: *Fx*] ((if *Gx*, *Hx*) and (if *Gx*, *Ix*))

By reasoning parallel to that above, the general equivalence of (53) and (54) requires the equivalence of ‘if *Gx*, (*Hx* and *Ix*)’ and ‘(if *Gx*, *Hx*) and (if *Gx*, *Ix*)’ for any value of the variable ‘*x*’. Thus the conditional commutes in the consequent with conjunction.

Once we recognize that the commutativity principles generate contradictions, we cannot simply pick and choose which conclusions they support, restricted to avoid inconsistency—for example, by taking them to support unrestricted CEM but only restricted CNC. The source is tainted; that fact must qualify our view of all its products. Of course, the taint of paradox does not make it evidentially worthless, any more than perceptual error motivates abandoning perception as a source of evidence. Rather, we need to take a more critical attitude towards our sources of evidence, which is a normal part of a scientific approach. That includes understanding their principles of operation, and the implications for the scope and limits of their reliability. For apparent equivalences like those of (46) to (47) and (51) to (52), the relevant source is the Suppositional Rule.

Of course, arguments like those just given are not the only support for the commutativity principles. CEM and CNC also gain support from probability judgments, as discussed in sections 3.1 and 3.4. For example, we tend to treat the probabilities of ‘if *A*, *C*’ and ‘if *A*, not(*C*)’ as summing to 1. But the relevant probability judgments too are explained by the Suppositional Rule, and more specifically by our tendency to equate the probability of ‘if *A*, *C*’ with the conditional probability of *C* on *A*, which was also seen to generate untenable results, by the triviality proofs. Indeed, the paradoxical consequences of the Suppositional Rule for credences in section 3.3 and for complex attitudes in this section are in a way complementary, for the latter concern only conditionals with logically inconsistent antecedents, while the former concern only conditionals with logically *consistent* antecedents, since probabilities conditional on logically inconsistent suppositions are normally undefined. The paradoxical consequences of the Suppositional Rule for deductive attitudes in section 3.2 concern both kinds of conditional. Thus merely making an exception of conditionals with logically inconsistent antecedents fails to resolve the underlying problem.

We can draw a further moral from the way in which the Suppositional Rule supports implicitly inconsistent commutativity principles. We should be suspicious of the tempting idea that epistemic modals obey such principles: that ‘It must be that if *A*, *C*’ is somehow equivalent to ‘If *A*, it must be that *C*’, and that ‘It may be that if *A*, *C*’ is somehow equivalent to ‘If *A*, it may be that *C*’. Even if such principles result from legitimate applications of the Suppositional Rule, they may well be in cases of just the sort where the Rule is most unreliable.

* 1. *Plain counterlogicals*

A *counterlogical* is a conditional with a logically inconsistent antecedent, like those in the [I]-[III] paradox (section 3.5). Logical considerations pressure semantic theories of conditionals to make all counterlogicals true. For the schema ‘if *A*, *A*’ seems trivially correct, whether *A* is consistent or not; the Suppositional Rule endorses that pre-theoretic impression. Moreover, this principle of single-premise deductive closure in the consequent seems plausible:

SPCC *C* *D* if *A*, *C* if *A*, *D*

The Suppositional Rule arguably endorses SPCC too, since iterated deductive reasoning is as reliable a way of developing a supposition as any. But, at least in classical logic, an inconsistent premise entails any conclusion. Thus, whenever *A* is inconsistent, *A* *C*, for any *C*, so by SPCC (if *A*, *A*) (if *A*, *C*). But (if *A*, *A*) as above, so (if *A*, *C*) by the Cut Rule from section 3.1. In other words, all counterlogicals are logical truths.

As observed in section 3.1, the Suppositional Rule yields standard natural deduction rules for the conditional, of which SPCC is an easy consequence. For they yield modus ponens, and so *A*; (if *A*, *C*) *C*; therefore, when *C* *D*, Cut gives *A*; (if *A*, *C*) *D*, so by conditional proof (if *A*, *C*) (if *A*, *D*). These results are robust with respect to the background standard of logical consequence and logical truth represented by . The rules just invoked hold on both very narrow conceptions of logicality and much broader conceptions. The rest of this section will employ a rather loose standard of logicality, but similar arguments apply to much tighter standards.

Despite such strong reasons for regarding the Suppositional Rule as forcing the logical truth of counterlogicals, other aspects of the Rule seem to pull in the opposite direction. Consider examples like these:

(55) If nothing is self-identical, everything is self-identical.

(56) If parthood is non-transitive, parthood is transitive.

(57) If the Russell set both is and is not a member of itself, dialetheism explodes.

(58) If intuitionistic logic is the right logic, every mathematical hypothesis is either true or false.

By orthodox logic, broadly conceived, (55)-(58) are all counterlogicals, so by the preceding considerations they should be logical truths. But if they occurred in a student paper, they might well attract the grader’s red pen. Indeed, it is far from obvious that the Suppositional Procedure would endorse them. However orthodox the grader’s own logical outlook, (55)-(58) are about what holds *if that outlook fails* in a specified way. To assess them properly, using the Suppositional Procedure, one must take the antecedent seriously, and assess the consequent from an appropriately unorthodox perspective. In the case of (55)-(58), doing so requires one to reject the consequent, in that hypothetical spirit. Thus each conditional will be assessed as false, even if its consequent is *in fact* a logical consequence of its antecedent—by the correct (orthodox) logic, but not by the unorthodox sort of logic its antecedent demands. A formal semantic framework appropriate to the evaluation of such conditionals may well need logically impossible worlds, or something like that: in which case, the more the merrier. Or so the usual arguments go.

In effect, such arguments describe the application of the Suppositional Procedure to indicative counterlogicals. They also rely in effect on its correctness. In doing so, they are of course in good company. Indeed, despite their counterlogicality, examples like (55)-(58) are in a way peculiarly well-suited to applying the Procedure. For the deviation stipulated by the antecedent from orthodoxy is of such an abstract theoretical character that one is forced hypothetically to adopt some sort of unorthodox logic in order to suppose it seriously; one can then assess the consequent in the light of that logic. In some cases, the unorthodox logic is explicitly specified, as in (58). On the intuitionistic approach to logic, the universal generalization ‘Every mathematical hypothesis is either true or false’ is false, so the Suppositional Procedure supports assessing (58) itself as false. In other cases, the unorthodox approach to logic lies very close to hand, as with (57). Dialetheism, the view that there are true contradictions, typically rejects the claim that it explodes (in the sense of entailing everything) as just plain false, and not also true, so the Suppositional Procedure supports assessing (57) itself as just plain false, and not also true. In still other cases, the unorthodox logic is characterized only negatively and unspecifically, as with (55) and (56). Nevertheless, given that in those cases the unorthodox logic is not meant to endorse contradictions, the antecedents of (55) and (56) hint at unorthodox logics on which their respective consequents are false, so the Suppositional Procedure supports rejecting (55) and (56) themselves as false.

However, that the Suppositional Rule supports the rejection of some counterlogicals does *not* mean that the Rule does not, after all, support the derivation of the paradox [I]-[III]. In particular, the relevant commutativity principles CCCN and CCCC for negation and conjunction were derived in section 3.4 from the Suppositional Rule on very general structural grounds, without appeal to principles specific to the classical logic of negation and conjunction as contrasted with other operators of the same grammatical type. Thus rejecting the classical logic of negation and conjunction would not block the derivation of the paradox [I]-[III].

Of course, since the Suppositional Rule is implicitly inconsistent, it indirectly supports incompatible attitudes to the same conditional or conditional-involving argument, though typically one direction of support will be more salient psychologically than the others.

Those who reject some counterlogicals like (55)-(58) as false are also in no position to reject the paradox [I]-[III] on the grounds that counterlogicals are meaningless. For what is meaningless is neither true nor false. Indeed, our ability to apply the Suppositional Procedure non-trivially to assess examples like (55)-(58) provides good evidence of their meaningfulness. Even when one is quite certain that a theory is wrong, one may still understand the theory well enough to develop it as a hypothesis appropriately, by drawing out its consequences in the way its proponents intend. Thus one may be able to apply the Suppositional Procedure non-trivially to conditionals with the theory as antecedent.

Philosophers who envisage judgment on a supposition exclusively in probabilistic terms may underestimate the applicability of the Suppositional Procedure to counterlogicals and similar examples. It can make sense even when probabilities conditional on the antecedent are undefined. For instance, Dorothy Edgington claims ‘there is no thought which begins “if I don’t exist now”: this is a non-starter’ (1995: 265). But, although I am quite sure that I do exist now, I can still intelligibly and reasonably use the Suppositional Procedure to reach conclusions such as this:

(59) If I don’t exist now, thinking occurs without a thinker.

Even if I lack well-defined credences conditional on *A*, I can still make well-defined assessments of other kinds on the supposition *A*. Not all our attitudes are probabilistic.

Of course, our ability to apply the Suppositional Procedure to a conditional does not guarantee that its verdicts on that conditional are *correct*. We have already seen ample reason to treat the Procedure as fallible. In particular, the paradox [I]-[III] shows that it can generate inconsistent results when applied to counterlogicals. Philosophers who uncritically appeal to examples like (55)-(58) to show that there are false counterlogicals presumably fail to realize that their verdicts on them are likely to result from applying a heuristic which is provably inconsistent, not least in its application to counterlogicals. Without a more sophisticated methodology, one cannot expect to reach adequately supported verdicts on such cases.

There is further reason to doubt the reliability of the Suppositional Procedure for conditionals with philosophically wayward antecedents. For example, most semantic (and non-semantic) theories of conditionals validate conditionals of the form ‘If *A*, *A*’: even if *A* is true only at an impossible world *w*, *A* is still true at *w*. As already noted, the Suppositional Procedure seems to agree: surely one is warranted in asserting *A* on the supposition of *A* itself, so the Procedure supports one in asserting ‘If *A*, *A*’ on no supposition. But now consider the hyper-sceptical theory that nothing is assertible (on a supposition or otherwise). Try applying the Suppositional Procedure to this:

(60) If nothing is assertible, nothing is assertible.

Suppose that nothing is assertible. According to that hypothesis, that nothing is assertible is itself not assertible. Hence, by the Suppositional Procedure, (60) is itself not assertible. Indeed, asserting (60) can feel like a refusal to take its antecedent seriously, in something like the way in which asserting one of (55)-(58) can feel like a refusal to take *its* antecedent seriously. But must we assume that the Suppositional Procedure is correct in such a recherché case? That natural semantic theories of conditionals validate (60) suggests a negative answer.

At this stage of the argument, the appropriate attitude to counterlogicals is to suspend judgment on their truth-value. Theory-driven reasoning suggests that they are all true, but it is not yet strong enough to be conclusive. At first sight, some of them look true, while others look false. However, counterlogicals are one main case where our primary pre-theoretic way of assessing conditionals has turned out to be inconsistent. Given those problems, it would be naive to take appearances uncritically at face value in a special case so marginal to normal use of the language, for example by offering them as clear counterexamples to a proposed semantics of conditionals.

What is far from plausible is that the ordinary meaning of the word ‘if’ has been crafted to give special treatment to counterlogicals: that would be the tail wagging the dog, a dysfunctional complication favouring a tiny minority of theoretical uses over the robust simplicity needed for most practical applications. A far more plausible assumption is that the semantic behaviour of counterlogicals is just a byproduct of the general patterns governing the semantic behaviour of other conditionals, not something for which special provision has been made. Thus it is good methodological practice to concentrate on conditionals with less bizarre antecedents in determining our best semantic theory of conditionals, and then follow its verdicts on the semantics of counterlogicals.

* 1. *Inconsistent linguistic practices*

This chapter has explained various ways in which the Suppositional Rule is inconsistent. Such inconsistency does not imply inconsistency in the Suppositional *Conjecture*. It is consistent to hold that people implicitly rely on inconsistent rules in speaking and understanding their native language.12 In that sense, linguistic competence may be inconsistent.

Such a view is not at all unprecedented. For an extended example, we may consider the ordinary use of the words ‘true’ and ‘false’, and their analogues in other languages. It is governed by schematic principles like these:

TRUE ‘*P*’ is true if and only if *P*.

FALSE ‘*P*’ is false if and only if not(*P*).

Here declarative sentences are to be substituted for ‘*P*’. For example, the statement ‘There was a sea battle at Salamis’ is true if and only if there was a sea battle at Salamis; it is false if and only if there was no sea battle at Salamis. To avoid problems with context-dependent statements, we must interpret the statement ‘*P*’ as if made in the same context as the corresponding instances of TRUE and FALSE, for *your* statement in the words ‘I am hungry’ is true if and only if *you* are hungry, rather than if and only if *I* am hungry.

Plato and Aristotle were already aware of the importance of principles like TRUE and FALSE. Nevertheless, as the ancient Greeks also knew, they have paradoxical consequences. By standard reasoning, they have inconsistent instances when tricky Liar-like sentences are substituted for ‘*P*’. For example, consider:

The underlined statement is not true

When we substitute the meaningful sentence ‘The underlined statement is not true’ for ‘*P*’ in TRUE, this is the result:

(61) ‘The underlined statement is not true’ is true if and only if the underlined statement is not true.

But the underlined statement *is* ‘The underlined statement is not true’, so (61) is equivalent to (62):

(62) The underlined statement is true if and only if the underlined statement is not true.

But (62) is of the form ‘*Q* if and only if not(*Q*)’, which is inconsistent in classical logic. One can derive a similar paradox from FALSE, by putting ‘false’ in place of ‘not true’ in the underlined statement.

Centuries of research on the semantic paradoxes, in both the medieval period and the past hundred years, have shown how desperately hard it is to devise a satisfactory treatment of them, one that avoids inconsistency and less formal absurdities without imposing over-draconian restrictions on ordinary uses of ‘true’ and ‘false’. In light of that track record of recurrent disappointment, it is not remotely plausible that any solution to the semantic paradoxes, an escape clause, is somehow written into our ordinary understanding of ‘true’ and ‘false’. To overcome the paradoxes, we must build new theories, not uncover old ones already hidden in our heads.

Some theorists blame the semantic paradoxes on classical logic, not on TRUE and FALSE, but for present purposes the upshot is the same: our ordinary practice has relied on reasoning with jointly inconsistent rules, whether they are rules for reasoning with ‘true’ and ‘false’, or rules for reasoning with ‘if’ and ‘not’, or some other rules for reasoning. We need not know *which* of our rules are inconsistent to know that some of them are, perhaps jointly.

Despite the semantic paradoxes, in practice our ordinary use of ‘true’ and ‘false’ is robustly stable. Such examples demonstrate that a practice can combine several striking features. First, a stable practice can rely on inconsistent rules. Second, participants can be immersed in the practice with no inkling that its rules are inconsistent. Third, through theoretical inquiry participants can gain good evidence that their stable practice relies on inconsistent rules (the appendix, 3.8, discusses another example with similar features). Our stable practice of using ‘if’ is turning out to have all three features. First, it relies on the Suppositional Rule, which is inconsistent. Second, we ordinarily use ‘if’ with no inkling that in doing so we rely on an inconsistent rule. Third, through theoretical inquiry we can gain good evidence for the Suppositional Conjecture, good evidence that our practice of using conditionals relies on the Suppositional Rule and so is inconsistent.

According to some philosophers, principles like TRUE and FALSE, despite their inconsistency, are built into competence with the words ‘true’ and ‘false’. That would be a precedent for regarding the Suppositional Rule, despite its inconsistency, as built into competence with ‘if’. However, the possibility of inconsistent competence puts more pressure on the very idea of competence.

Consider a native speaker of English who uses the word ‘if’ in the normal way, respecting the Suppositional Rule, and relying primarily on the Suppositional Procedure in her prospective assessments of conditionals. If anyone is competent with ‘if’, she is. She becomes an expert on the logic, psychology, and linguistics of conditionals, and discovers by experimentation and theoretical reasoning both that she is relying on the Suppositional Rule and that it is inconsistent. She consciously modifies her practice to achieve consistency. She no longer relies on the Suppositional Rule itself. What she relies on instead is some consistent, complex modification of it, which she devised through a long process of trial and error. At first, she still has a *disposition* to use the original Suppositional Rule, which she consciously inhibits. Gradually, however, she becomes habituated to the revised rule, which she uses increasingly automatically. She loses even the disposition to use the original Rule. But she still remembers vividly what it was like to rely on the original Rule. She can use the Suppositional Procedure in imagination whenever she wants to, although she does not automatically accept the results. She knows that most other speakers of English rely on the original Rule. Does such enlightenment constitute loss of competence with ‘if’?

Our imagined expert on conditionals still *knows how* to use the original Rule for ‘if’, but that is a very weak condition for competence. Someone who uses ‘red’ as if it meant *green* and ‘green’ as if it meant *red* will normally be counted as incompetent with those words, even though he also knows how to use ‘red’ as if it meant *red* and ‘green’ as if it meant *green*, just as you know how to use ‘red’ as if it meant *green* and ‘green’ as if it meant *red*—you know how to pretend to be someone who is making just that mistake. The distinction between ‘competence’ and ‘performance’ is too blunt an instrument to clarify the position of our expert on conditionals.

Irrespective of the competence/performance distinction, the inconsistency of the Suppositional Rule limits its capacity to constrain, by a principle of charity, the semantics of conditionals. *No* semantics can fully validate an inconsistent rule. Nevertheless, we can still hope for the semantics to validate the Rule as far as possible. The trouble is that the inconsistences in sections 3.2-6 do not reveal exactly where the Rule goes wrong, *which* of the constraints it mandates the semantics violates. However, as we shall see later in the book, that does not make it impossible to find out.

Inconsistencies in the Suppositional Rule also limit its capacity to guide our assessments of conditionals. In problematic cases, we find ourselves conflicted, pulled in opposite directions, and we resort to more or less *ad hoc* repair strategies. That also happens with our assessments of applications of ‘true’ and ‘false’ in semantic paradoxes, but the problematic cases for conditionals are more widespread and often arise in more natural ways. That is further reason to be cautious of reading too much into experimental studies of the use of conditionals, despite their obvious relevance in principle. When we have theoretical reasons to expect a variety of alternative processes to be at work in assessments of conditionals, aggregated data from heterogeneous samples may well present a confusing picture.

Notes

1. See Weisberg 2019 and Williamson 2019 for recent discussion.
2. The approach of treating conditional probability as undefined but constrained by suitable axioms goes back to Popper 1936.
3. Quotation marks will often be omitted for readability.
4. See Bennett 2003: 58 and Stalnaker 1970.
5. Evans and Over 2004 was a pioneering work in this respect; see Douven 2016: 66-71 for a more recent (positive) assessment.
6. Less naturally but more concisely, one can also derive (5) by SR from the classically vacuously valid principle that a contradiction entails everything: *A*, not(*A*) *C*. Similarly, one can also derive (6) by SR− from the corresponding classically vacuously valid principle for −: *A*, not(*A*) − *C*. The longer derivations in the text show that there is no need to go via the vacuous case.
7. See for example Lewis 1976 and 1986, Hájek 1989, 2011, 2012, Hájek and Hall 1994, Bennett 2003: 60-77, Rumfitt 2013, Kaufman 2015.
8. This presents a challenge for expressivist frameworks for compositional semantics on which sentences express constraints on the speaker’s mental state, rather than propositions about the world, within which attempts have been made to make sense of indicative conditionals as entering more freely into complex embeddings without expressing propositions (Yalcin 2007, Moss 2018). Even over a finite domain, one can coherently give high credence to an existential generalization but credence 0 to every instance of it one can entertain. See also Williamson 2007a: 16-17.
9. See also Dummett 1982: 82 for the view that a subjunctive conditional is to be negated by negating its consequent.
10. The text slightly simplifies Stalnaker’s original presentation, for instance by ignoring his accessibility relation between worlds, but not in ways that affect the present issues.