Modal Predicates

Volker Halbach
9th February 2020

In this paper I explain why modalities such as necessity should be treated as *predicates* rather than sentential operators, as it is usually done in modal logic. First I explain the distinction. Then various problems of the predicate approach are discussed, in particular to which kind of object these predicates should be applied. In *The Quantification Problem* the expressive weakness of the operator approach is assessed; and in *The Category Problem* I discuss to which kind of object the modal predicates should be attributed. Finally I outline the problems of dealing with *de re* modality if modalities are conceived as predicates.

Predicates & Operators

In contrast to the dominating approach in philosophical logic, we analyze all modal notions including necessity, apriority, analyticity, truth, future truth, provability as predicates in languages of first-order predicate logic. In philosophical logic necessity, knowledge, and future truth, for instance, are commonly studied as a sentential operator of modal logic, while truth is treated as a predicate.

In order to explain the difference between the predicate and the operator conception of a modality, we look at natural language. Without claiming that other notions behave in exactly the same way, we choose necessity as our example. The sentence

> It is necessary that water is H_2O.

can be parsed in at least two different way. According to the first option, 'it is necessary that' is combined with the sentence 'water is H_2O'.

$$\underbrace{\text{It is necessary that}}_{\text{operator, adverb}}\ \underbrace{\text{water is H}_2\text{O.}}_{\text{sentence}}$$

In this sentence the phrase 'it is necessary that' serves the same purpose as the adverb 'necessarily' in the following sentence:

$$\underbrace{\text{Necessarily}}_{\text{operator}}\ \underbrace{\text{water is H}_2\text{O}}_{\text{sentence}}.$$

Adverbs serve various purposes in natural language. Only some behave like 'necessarily' in modifying an entire sentence. Therefore we prefer the term 'sentential operator'. A sentential operator is an expression that yields, combined with a sentence, a new sentence. In the simple case here, the sentential operator 'necessarily' is prefixed to the sentence.

We turn to the second way of parsing the sentence. On this approach, necessity is conceived as a predicate:

$$\underbrace{\text{It is necessary}}_{\text{predicate}}\ \underbrace{\text{that water is H}_2\text{O}}_{\text{singular term}}.$$

In this form the predicate approach looks somewhat awkward; but by reversing the order of singular term and predicate, it can be made more convincing:

$$\underbrace{\text{That water is H}_2\text{O}}_{\text{singular term}}\ \underbrace{\text{is necessary}}_{\text{predicate}}.$$

We can go even further and force the reading of the that-clause as singular term by adding a noun, although reformulations of this kind are somewhat questionable because they add information about the kind of object that is denoted by the that-clause:

$$\underbrace{\text{The proposition that water is H}_2\text{O}}_{\text{singular term}}\ \underbrace{\text{is necessary}}_{\text{predicate}}.$$

Instead of 'proposition' also 'statement', 'belief', or the like could be used, while 'sentence' is at least very awkward.

Both approaches have been implemented in formal systems. Each of the the modal operators $\underline{\Box}$ for necessity and $\underline{\Diamond}$ for possibility yield a sentences when they are written in front of a formula. Thus, if φ is a formula, then $\underline{\Box}\varphi$ and $\underline{\Diamond}\varphi$ are formulæ as well. This means that the operators $\underline{\Box}$ and $\underline{\Diamond}$ syntactically behave like the negation symbol. Modal logic can be based on propositional logic. In this case the language contains sentence parameters and connectives (besides auxiliary symbols such as brackets). It can also be based on more sophisticated languages such as first-order predicate logic.

On the predicate approach, usually first-order languages are used. The predicate $\Box$ for necessity is only another predicate symbol. In English the word 'that' can be used to form a singular term from a sentence: 'that' followed by a sentence such as 'water is H_2O' yields a singular term. In a formal language we may have an analogous device that gives applied to a sentence φ a singular term $\overline{\varphi}$.[1] Then the sentence 'It is necessary that water is H_2O' formalizes as $\Box\overline{\varphi}$, if φ formalizes 'water is H_2O'.

So far we have considered necessity as an example of a modal notion that can be conceived either as an operator or a predicate. Other notions behave differently and have been treated differently. For instance, some intentional attitude predicates cannot so easily be combined with certain singular terms.[2] The problem of substituting 'that *A*' with 'the proposition that *A*' have been discussed in some detail (see, for instance, Moltmann 2003 and Prior 1971). In the case of necessity the substitution seems acceptable, but not in other cases. In the sentence

Nigel fears that there will be more immigrants next year.

the substitution yields the sentence

Nigel fears the proposition that there will be more immigrants next year.

But clearly Nigel is afraid of immigration and not propositions. Even with notions such as knowledge analogous substitutions are at least problematic. Examples of this kind have been used to argue against the conception of that-clauses as singular terms.

Another somewhat tricky kind of modality is future and past truth. Traditionally they have been treated as operators and studied in temporal logic. In English they are commonly expressed neither with an operator nor a predicate, but with the help of tensed predicates. For instance, the sentence

Nigel is in Norway.

can be put in future tense:

Nigel will be in Norway.

It can be argued that changing the tense of a sentence is a modification of the original sentence similar to adding an adverb, just that there is not an adverb added, but rather the verb itself is modified. A corresponding predicate is hard to come by. One could use 'will be the case' or 'will be true', but that is just a tensed truth predicate. There does not seem to be a simple primitive predicate for future truth, unless one resorts to awkward phrases such as

It is future that Nigel is in Norway.

[1] In other languages the rules for generating these singular terms are more complicated, even in closely related languages such as German. The predicate conception can be traced back to medieval logic, long before modern English came into existence. In the Latin texts *accusativus com infinitivo* constructions were used instead of that-clauses.

[2] Usually intentional attitudes are known as *propositional attitude* predicates; but we are going to construe them as relations between subjects and sentences rather than propositions understood in a language independent way; therefore we prefer the less committal term *intentional attitude*.

While necessity, knowledge, belief, future truth, and many other notions have traditionally been formalized as model operators, truth has hardly ever been treated as a sentential operator.

Logicians usually decide in favour of one of them and discard the other, presumably because they expect that the chosen approach covers all aspects of the modality and the other approach can be reduced to their preferred conception. Now one could argue that it may be less controversial to keep both, a modal operator $\underline{\Box}$ and the predicate $\Box$. After all, there is an adverb 'necessarily' and a predicate phrase 'is necessary', and both coexist in English. So why should we not be liberal and admit both, a predicate and an operator in a formal language? First, the predicate and the operator would have to be related in some way. The strongest relation would be a reduction of either the operator to the corresponding predicate or *vice versa*.

The reduction of a modal operator $\underline{\Box}$ to a corresponding predicate $\Box$ is technically straightforward under suitable assumptions. The idea is that a sentence $\underline{\Box}\varphi$ is taken as equivalent to $\Box\overline{\varphi}$ where φ does not contain a modal operator, and then the same kind of substitution is defined recursively for sentences with nested occurrences of $\underline{\Box}$ and $\underline{\Diamond}$. We sketch how to define the reduction. We assume that in the language with the predicate we also have a name or some singular term $\overline{\varphi}$ for each formula φ. The choice of the singular term is not without problems. We prefer the quotation of the sentence or some term from which the structure of the named sentence can be read off. The translation O from the language with the operator to the predicate is then defined recursively as follows:[3]

(i) If the formula φ is atomic, then $O(\varphi) = \varphi$;

(ii) If the formula φ is of the form $\neg\psi$, then $O(\neg\psi) = \neg O(\psi)$;

(iii) If the formula φ is of the form $\psi \rightarrow \chi$, then $O(\psi \rightarrow \chi) = O(\psi) \rightarrow O(\chi)$;

(iv) If the formula φ is of the form $\forall x\psi$, then $O(\forall x\, \psi) = \forall x\, O(\psi)$;

(v) If the formula φ is of the form $\underline{\Box}\psi$, then $O(\underline{\Box}\psi) = \Box\overline{O(\psi)}$.

The notation $\Box\overline{O(\psi)}$ may require some explanation. It denotes the application of the predicate symbol $\Box$ to the singular term $\overline{O(\psi)}$, which can be thought of as the quotation of the translation of φ. The symbol O is not a symbol of the object language; it is a symbol for a function defined in the metalanguage. $\overline{O(\psi)}$ is again a singular term of the object language.

Of course, in order to show that this reduction is successful, we have to prove that the translation preserves provability, truth, or even meaning. This task may be far from trivial and require further tweaks and assumptions. However, if this reduction is successful,

[3] The idea for this embedding can be traced back at least to (Carnap 1934, IV.B.e). Carnap's translations is still very different and there are several variations. What caused problems especially in early variants were problems with the iteration of N.

there is no need for a further operator for the modality, if the predicate is available. Conversely, if the predicate is defined in terms of the operator (and perhaps other devices), then the predicate is not required any longer. Such a definition is, as we shall see, more difficult.

Both the operator and the predicate approach have their problems. Deciding in favour of one or the other reading has deep ramifications for many philosophical areas such as metaphysics, epistemology and philosophy of mind. For instance, if we adopt the predicate approach for intentional attitudes, it is plausible to conceive belief, knowledge and various kinds of perceiving as relations between a subject and a belief. This rules out so-called adverbialist theories of these notions. They have been advocated specifically for perceptual notions by Chisholm (1957), Tye (1989), and others; adverbialists deny that there are belief and perceptual contents. Adverbialism can be applied to other modal notions, although some applications may be implausible, for instance in the case of truth.

We do not even try to summarize these discussions about the predicate and operator conceptions of modalities. We are also not able to recommend a comprehensive survey elsewhere, although the reader may start from (Stern 2016). One problem is that many philosophers have focused on a specific kind of modality such as truth, necessity, some intentional attitude and so on. But presumably a general decision is needed as will try to show below.

Problems of the Predicate Approach

Up to the 1960s predicate and operator approaches had been pursued; but then the operator view and modal logic ousted the predicate approach. There were two main factors that led to the triumph of the operator account over the rival predicate approach. The predicate approach is prone to paradox and Montague's (1963) attack on the predicate approach on these grounds was widely seen as very successful (see Theorem **??**, p. **??** below). Even more importantly, Kripke, Kanger, Hintikka, and others developed possible-worlds semantics for modal logics.[4] Possible-world semantics offered a vast playground for philosophical and mathematical theorizing, and the number of papers on modal logic grew rapidly. No comparable semantics was available for the predicate approach. These are the main two factors that lead to the dominance of the operator approach.

Besides the paradoxes and the lack of a neat semantics in the literature, there are more problems with the predicate account on modalities. In the rest of this section we concentrate on some ontological questions. If modalities are conceived as predicates, they apply to objects, according to standard referential semantics. But what kind of object? In my paraphrase of 'It is necessary that water is H_2O' on p. 2 I have used propositions. This is presumably the most popular, but by no means the only choice among philosophers.

[4] See (Copeland 2002) for the history of possible-worlds semantics for modal operators.

If propositions are conceived as abstract objects, nominalists will already see an advantage for the operator approach. We are not worried about an ontological commitment to abstract objects; but there is another problem with propositions. Propositions may have to be individuated in different ways for different modalities. Propositions as objects of beliefs would have to be very fine grained. One can believe that Cicero is a Roman orator without believing that Tully is a Roman orator. Hence the proposition that Tully is a Roman orator and the proposition that Cicero is a Roman orator must be different. Kripke (1979) provided an example of a person who affirms the French sentence 'Londres est jolie' but disagrees with its English translation 'London is pretty.' So both sentences should express different propositions. The puzzle threatens the conception of propositions as entities independent from language: It should not matter in which language a proposition is expressed, especially if only rigid designators such as 'Paris' are used. For necessity these puzzles do not arise in the same way, and coarse grained propositions can be used.

These problems have been discussed in some detail in the literature; and we do not enter the this discussion here. There is a much bigger problem that is not discussed so frequently. There are modalities that do not apply to propositions, but other objects. Usually a sentence is said to be analytic if and only if it is true in virtue of the meanings of the expressions in the sentence. It does not make much sense to ascribe analyticity to propositions conceived as language independent objects, because analyticity is standardly seen as defined in terms of the meaning of linguistic items in a sentence. Similarly, provability in a formal system is a property of sentences, according to the standard view. If we go along with the usual assumptions, it becomes hard to compare analyticity and necessity. If we ask whether everything that is necessary is also analytic, the answer is negative and trivial, because some propositions, but no sentences are necessary. So the proposition that water is water is necessary, but not analytic, because only the *sentence* 'water is water' is analytic, but not the proposition. Similarly, if we ask whether provability in a certain formal system such as Peano arithmetic implies truth and assume that only propositions can be true, we have to conclude that no provable sentence is true, simply because sentences are never true; only propositions are.

These examples show that it is difficult to come up with one single kind of object of which all the modalities can be predicated. Among the candidates are sentences, conceived as types or tokens and as mental or natural, propositions with various granularities, statements, beliefs, utterances, judgements, and so on. It is not necessarily clear how they are related to propositions and sentences. We call the problem of finding a single class of objects to which all the modalities can sensibly be ascribed the category problem.

Various solutions to the category problem have been tested in the literature. The truth predicate has been used in various solutions. Very often claims about the relation between analyticity, necessity, apriority, knowledge, and so on and couched in terms of truth. So instead of saying 'There are synthetic a priori propositions' or 'All analytic sentences are necessary', philosophers write 'There are synthetic truths that are a priori' or 'All analytic truths are necessary'. The use of the term 'truth' elegantly hides the problem, but does not solve it. Presumably the most natural way to understand these reformulations is the following:

> There is something that is synthetic, true and a priori.

and

> Everything that is analytic and true is necessary.

But the category problem remains. If only sentences are analytic and only propositions – conceived as language independent entities – are necessary, then the last sentence is false.

One way to dodge the category problem could lie in a partial departure from the predicate approach. Halbach and Welch (2009) considered the option of retaining a predicate for truth and treating other modalities as sentential operators. The two sentences can then be reformulated in the following ways:

> There is something that is synthetically true and a priori true.

and

> Everything that is analytically true is necessarily true.

Thus only adverbs or sentential operators for syntheticity, apriority, and analyticity are needed. Objects that can be true are still required, but we do not have to worry whether they can also be analytic or necessary, because these other modal notions are not conceived as predicates any longer. This mixed approach means that the predicate approach is abandoned with the exception of the truth predicate.

The Quantification Problem

By adopting the operator view of modalities we can avoid the drawbacks of modal predicates: There are no analogues to Montague's paradox in operator modal logic (unless diagonalization is added artificially); modal operators are adverbial modifiers of sentences or perhaps formulæ, but they do not apply to anything, and thus the category problem vanishes; finally we have mathematically and philosophically successful semantics for modal operators.

However, there is a steep price to pay for dodging the problems of modal predicates and settling for modal operators. The price is a lack of expressive power. If modalities are treated as sentential operators, many philosophical claims cannot be expressed. Above we considered the following example: 'There are synthetic a priori propositions.' This sentence cannot easily be reformulated if only modal operators are available. Assume we have a modal operator Syn for syntheticity and an operator Ap for apriority, then we can write Syn $\varphi \wedge$ Ap φ to express that φ is synthetic and a priori. With the sentence 'There are synthetic a priori propositions' we do not intend to claim this for a specific φ, but rather only make an existential claim. However, in a first-order language the expression $\exists x(\mathrm{Syn}\, x \wedge \mathrm{Ap}\, x)$ is only a well-formed formula if Syn and Ap are *predicates* and not sentential operators. On the operator view, Syn x fails to be well-formed in the same way $\neg x$ fails to be well-formed; the operators Syn and Ap need to be combined with formulæ, not singular terms such as the individual variable x. When we form quantified claims of this kind in English, we also resort to predicate phrases and not to adverbs.

There are many more philosophically important quantified statements. In particular, there are also many universally quantified claims, for instance, the claim that necessity implies truth (in the sense that whatever is necessary is also true), that what is known is true, that analyticity implies necessity, and that whatever is true can be known. Hence, as we mentioned above, the quantification problem is especially pertinent to philosophical discourse. For the purposes of a linguistic analysis of everyday language, it may be less pressing; but, as has been mentioned before, philosophers are interested in general statements about modalities and their relations. If modal logic does not allow us to state that there are synthetic a priori judgements (or propositions), we are unable to discuss claims that are at the very centre of philosophy.

Defenders of the operator account have tried to show in various ways that the quantified statements can be expressed with operators. A first defence is obtained by claiming that universal statements can be expressed by schemata. According to this suggestion, the empiricist claims that apriority implies non-syntheticity would be expressed by all sentences $\mathrm{Ap}\,\varphi \rightarrow \neg\mathrm{Syn}\,\varphi$. This strategy is not very promising. First, empiricism cannot be stated as a single sentence, but only by employing infinitely many sentences. The opponent of empiricism would be in an even worse position, as they have to negate a schema. It is unclear how to do this. We know how to negate formulæ, but not schemata. Quantifiers and connective can be iterated and combined. If a claim involves alternating quantifier and quantifiers embedded in sentences, then schemata are useless.

Propositional quantifiers have been used to boost the expressive power of a language with operators. The claim that apriority implies non-syntheticity would be expressed as $\forall P(\mathrm{Ap}\, P \rightarrow \neg\mathrm{Syn}\, P)$ on this strategy. The letter P is here a propositional variable. It can stand in the place of formula as in Ap P. The negation of this claim can be expressed as $\neg\forall P(\mathrm{Ap}\, P \rightarrow \neg\mathrm{Syn}\, P)$ or simply as $\exists P(\mathrm{Ap}\, P \wedge \mathrm{Syn}\, P)$. Usually modal operators and propositional quantifiers can be freely combined.[5]

[5] Quantification into sentence position is as old as modal logic and was considered already by Lewis and Langford (1932). There are numerous accounts of propositional quantification with subtle differences. They can be understood as quantifiers ranging over sets of possible worlds, as suggested already by Kripke (1959). Bull (1969), Fine (1970), and Kaplan (1970) elaborated on Kripke's suggestion. These accounts are commonly based on propositional logic, which is sufficient for expressing quantified claims such as $\exists P(\mathrm{Ap}\, P \wedge \mathrm{Syn}\, P)$, if suitable modal operator are available. Extending this to predicate

At this point a thorough discussion of higher-order modal logic would be needed for a fair comparison with the predicate approach. We mention only a few general reasons why we prefer the predicate approach, which may not apply to all variants of propositional quantification or higher-order modal logic more generally.

Without explicitly specified semantics, it is not straightforward to distinguish between a multi-sorted first-order (in the sense of section **??**) and a higher-order language. Propositional variables could be understood as a sort of first-order variables, and Ap and Syn as predicates that are combined with them. Formulæ of the form $\Box\varphi$ could be understood as a predicate combined with a singular term for φ. The same effect could be achieved, so one could argue, by using a single sort of variable, introducing two new predicate symbols, and relativizing all quantifiers with one of the two predicates. So $\forall P$ would become $\forall x\,(\mathrm{Pro}\,x \to \ldots)$, where Pro is a predicate symbol for restricting the quantifier to propositions. Here we will not make this reduction precise, because it depends on many further decisions and assumptions; but the general strategy should be clear.

There is one point where the reduction of modal operators with propositional quantifiers to modal predicates runs into problems. Presumably expressions of the kind $\forall P(\mathrm{Ap}\,P \to P)$ count as well-formed. Replacing the last occurrence of the variable P with a first-order variable will give an ill-formed expression. The reduction could be saved by using truth. The sentence $\forall P(\mathrm{Ap}\,P \to P)$ could be expressed with predicates Ap for apriority and $\Box$ for truth as $\forall x\,(\mathrm{Pro}\,x \to (Ap\,x \to \Box x))$. Actually, the first-order version with a truth predicate is closer to English. For propositional quantification there is no obvious equivalent in English: The sentence *All a priori beliefs are true* is well formed.

To make reduction more precise, we would have to spell out the assumptions in much more detail. In particular, we would have to make precise the syntax or semantics of the propositional quantifiers. We expect, however, that the problems of this kind of quantifier is very similar to that of adding a truth predicate. Kripke (1976) provided an account of such quantifiers that is very much based on his theory of truth in (Kripke 1975).

There are reasons to reject both, the propositional quantification and the multi-sorted approach. On both, the propositions and other objects are separated and cannot be related in a straightforward way. Sentences of the following kind mix propositions and other objects:

(0.1) Only propositions are necessary.

(0.2) The things that are necessary are not located in space.

(0.3) All sentences provable in PA are a priori and necessary.

We assume that *is a proposition*, *is located in space*, *is provable in* PA, and *is a logical truth* should be treated as normal first-order predicates. With a modal operator $\Box$ for necessity and a predicate Prop the following is not well formed: $\forall P\,(\Box P \to \mathrm{Prop}\,P)$, because Prop

logic is not trivial, and there is a vast literature on higher-order modal logic based on predicate logic.

requires a first-order, not a propositional variable. Equally, $\forall x\,(\Box x \rightarrow \mathrm{Prop}\,x)$ is not well formed, because $\Box$ requires a propositional variable.

The point is general. If we treat some notions as predicates and others as operators, then it will be difficult to relate them. The treatment of metaphysical necessity, knowledge, and belief as operators has become standard in philosophical logic. Truth, in contrast, is almost always conceived as a predicate. Similarly, provability in a fixed formal system is conceived as a predicate in the tradition of (Gödel 1931). In provability logic the properties of these predicates are analyzed using modal operator languages; but the primary analysis of provability is always as a predicate. Surprisingly, absolute general mathematical provability is then often treated as an operator. In limited contexts where we focus only on one modality it does not matter how the different theories fit together. But philosophical logicians should be able to provide a formal framework in which the various notions can be related and compared. The most obvious way to do so is a uniform treatment of all modalities and all notions to which they are related. As we have seen in the previous paragraph, even 'normal' notions such as being located in space may be related to a modality. So unless we treat them as operators as well, we are pushed towards the predicate conception. The predicate approach to modalities offers the most general uniform framework for studying modal notions, their interaction among each other and with other notions.

The proponent of the operator approach with propositional quantifiers has still options to deal with examples such as (0.3): They can still try to defend the account using additional devices to boost the expressive strength of the language. It could be argued that being provable in PA is a first-order predicate of sentences, while being a priori and necessary are operators or predicates of propositions (understood as objects different from sentences). The claim (0.3) would then the understood as an abbreviation of *All sentences provable in* PA *express a priori and necessary propositions*. That is, a bridging particle 'express' taking sentences as first argument and a proposition as second would be invoked. Also truth would become an operator or predicate of propositions. We suspect that a language with propositional quantification in conjunction with operators can be intertranslatable with a language that has only the corresponding predicates. But then the problems of the predicate approach will also affect such an elaborate account.[6]

The Category Problem

The predicate conception of modalities offers the most powerful, flexible, and straightforward way to theorize about modalities. If we opt for modal predicates, we have to pay a price for its expressive strength and the uniform treatment of quantified statements: We need to address the paradoxes of modal predicates such as Montague's paradox; they

[6] For an attempt to formalize 'express' in such contexts see (Mount 2019). His approach is a formalization of Ramsey's informal theorizing.

will be the topic of subsequent chapters. Moreover, we need to equip our object theory with resources to reason about the things to which modalities can be ascribed. At least we need to build some assumptions about the ontology of these objects into our object theory. These assumptions will depend on whether we think of them as beliefs, sentences of a natural or mental language, structured or unstructured propositions, fine or coarse grained propositions, or still something else.

We do not think that there is a definite solution to the category problem that can be unearthed by looking at linguistic data in English or other languages. By paying selective attention to certain examples one can force an answer in the one or other direction.[7] We do not assume that there is a coherent underlying ontology to be found in the linguistic data. We take the protracted discussions in the literature about the objects of necessity, truth, belief, knowledge, and other modalities as an indication that there is not enough evidence from linguistic data to prefer one category of objects over the other. In the end some regimentation in Quine's (1960) sense may be required.

For the purposes here, no full solution to the category problem is required. We consider highly regimented formal languages; they lack many elements, such as indexicals, that cause trouble in natural languages for the category problem. In this section we give only a few reasons why we think an approach with sentences as bearer of modal properties are a promising answer to the category problem.

Whatever the objects may be that can be true, analytic, necessary, be known, be a priori, and so on, our main access to them are sentences and, in some cases also formulæ. Therefore we focus on predicates applying to sentences and formulæ. We look at the truth predicate as an example.

If the predicate symbol $\Box$ expresses truth, a sentence such as $\Box\overline{\varphi}$, where $\overline{\varphi}$ is a name for φ, can be read as *'φ' is true* or as *The proposition expressed by the sentence 'φ' is true* (as Quine 1970, p. 10 suggested). In the latter case the analysandum would be 'expresses a true proposition'. This is compatible with viewing propositions as the primary objects of truth.

The predicate *expresses a true proposition* can be taken to primitive. Which elements are taken to be primitive is a methodological question and one might have to explore different options. Even if propositions are employed as primary bearers of truth, one does not have to start with a unary predicate *is true* applying to propositions, a unary predicate for *is a proposition*, and a binary relation *expresses*. Treating *expresses a true proposition* as primitive does not rule out a later analysis from other notions.

Whether we read $\Box\overline{\varphi}$ as *'φ' is true* or *the proposition expressed by the sentence 'φ' is true*, we seem to have lost a very important device: Neither of the two readings gives us an obvious formal counterpart of *it is true that φ*, and it is not clear how we can express that-sentences in our framework. Often these that-clauses are taken to be singular terms denoting propositions. It is here where our approach of viewing modalities as predicates

[7] Moltmann (2018) provides many examples.

of sentences seems to fall short.

An obvious reply would be that these that-clauses are coreferential with quotations, and that therefore nothing is lost by conceiving modalities as syntactic predicates. There are various reasons why philosophers have thought that neither of the two readings mentioned above are equivalent to *it is true that* φ. In *it is true that* φ the sentence φ is used, while it is only mentioned in *'*φ*' is true* and *the proposition expressed by the sentence '*φ*' is true.* Sometimes the following observation is given as evidence for the claim that they are not equivalent: *'snow is white' is true* is about the sentence *snow is white*, while *it is true that snow is white* is about snow. We do not think that this is sufficient to establish non-equivalence of the two sentences. What is needed are examples that show that they are not substitutable *salva veritate*. And indeed, in certain modal contexts they seem to come apart. Whether *it is true that snow is white* obtains depends on the colour of snow. Whether *'snow is white' is true* depends also on what *snow is white* means. If *snow is white* had meant *coal is white*, then *'snow is white' is true* would be false. The meanings of sentences understood as strings of symbols or sounds, or so the argument goes, are contingent. Therefore the truth conditions of *'snow is white' is true* and *it is true that snow is white* are different. The origins of these arguments can be traced back at least to Plato's *Cratylus*. They have been used to show that the modal status of T-equivalences depends on whether they are formulated with sentences or propositions (suitably understood). The equivalence

(T) The proposition that snow is white is true iff snow is white

is necessary, according to this view, while

The sentence 'snow is white' is true iff snow is white

is only contingently true: If *snow is white* had meant *coal is white* the second equivalence would fail, while the first would still hold. Similarly, the equivalence

The proposition expresssed by the sentence 'snow is white' is true iff snow is white

is only contingently true, because the sentence *snow is white* could have expressed some other proposition.

To capture (T), it may be argued, a predicate applying directly to propositions is required, probably in addition to some term forming device (corresponding to *that*); predicates of sentences are insufficient to analyze (T), it could be claimed.[8]

[8] One of the first to make an argument along these lines on the status of these equivalences was Lewy (1947). G. E. Moore (1966, p. 142) also mentioned a similar argument already in lectures 1925–26 (edited by Lewy). I thank Graham Solomon for making me aware of this passage in Moore's lectures. The argument, including a variant with translations by Lewy (1947), is a precursor to the Church–Langford argument by Church (1950), which concerns intentional attitudes rather than truth.

The example of the T-sentences above involves iterated modalities: (T) contains the truth predicate and then we ask whether (T) is necessary. There are variants of this argument. The sentence

(Log) " 'All logicians are logicians' is necessary" is necessary.

is false under the following assumption: The expression *All logicians are logicians* could have been used to express that snow is black; hence the sentence *All logicians are logicians* is only contingently necessary, because its necessity depends on our contingent linguistic conventions.

We do not think that these arguments, purported to show that truth and necessity should primarily be understood as predicates of propositions, are convincing. The truth predicate can be read in different ways. We could focus on a reading of the truth predicate as *is a true sentence now in my actual idiolect from my perspective*. Some deflationist about truth, including Field (1994), understand the truth predicate in this way. On this reading, *snow is white* is still true even if *snow is white* had meant *snow is black*; it would still be true in my idiolect. In the same way (Log) has a true reading if 'is necessary' is understood as *is necessary now in my actual idiolect from my perspective*.

These readings are not completely unnatural. When ascribing truth to particular utterances and sentences we usually mean *true in our language*. *Is a true sentence now in my actual idiolect from my perspective* is taken as primitive not as a truth predicate that takes additional arguments such as a parameter for the language, context etc. One could then hope to define truth predicates applied to sentences in other languages from this predicate via translations or interpretations. Starting with such a reading of *is true* and *is necessary* would be our preferred methodological starting point. Of course there are many problems. In particular, we do not claim that all other philosophically relevant uses of truth and necessity can be reduced to such primitive predicates. Even if one is sceptical about the possibility of such a reduction, we can still theorize about those predicates.They still go a long way in providing a formal framework for the use of the truth, necessity and other predicates in philosophical discourse.

Even though we call our formal systems *theories of syntax*, one may try to reinterpret them as theories of propositions, properties, and relations in some way. We like to call this kind of reinterpretation of our theories the *Henry Ford theory of universals*. Ford allegedly pronounced that his customers could purchase his model T in any colour as long as it is black. Readers may take our formal theories as theories of universals as long as they are content to believe that there are operations on universals and their constituents that correspond to our syntactic operations. Universals would then truly be mere shadows of syntactic objects, to use Quine's phrase. We do not think that the Henry Ford theory will appeal to many readers; but they may try to come up with a theory of universals that is intertranslatable with our syntax theory to make the theories more appealing.

Ultimately we prefer to think of our predicates as syntactic predicates, that is, as predicates applying to sentences. Analyticity, provability (in some formal system), logical validity, and so on will have to be understood as predicates of sentences; and in order to make all predicates comparable we rather see truth and necessity as predicates of sentences than provability and analyticity as predicates of propositions.

We have said little about intentional attitudes such as believing or knowing. As mentioned above, Lewy's argument in its form with translations has a counterpart for intentional attitudes in the form of the Church–Langford argument, which is also known as *argument from translation*. There is an extensive literature on this argument and its refinements. As intentional attitudes are not at the centre of our account, we only refer the reader to the relevant literature by Leeds (1979), Salmon (2001), Felappi (2014), Sackris (2016), and many others.

De re modality

Our conception of modalities as predicates of objects – whether they are sentences or propositions – faces another challenge. So far we have considered only *de dicto* modalities, but no *de re* modalities. If we cannot incorporate the latter into our account, large parts of modern philosophy of language and metaphysics would be incompatible with our predicate conception of modalities.

We begin with explaining the distinction and the problems of 'quantifying in' from the perspective of modal logic and the operator approach. We start with the following situation:

> The people at the table in a Gothenburg restaurant are Mary's three best friends. Mary believes of each of them that she is in England.

We can conclude from this that Mary believes of every person at the table in Gothenburg that she is in England. Of course, this does not mean that she believes that every person at the table in Gothenburg is in England. The distinction can me made in modal logic with an operator $\Box$ for 'Mary believes that'. The claim that Mary believes of every person at the table in Gothenburg that she is in England can be formalized as the following formula:

$$\forall x\,(Px \rightarrow \Box Qx) \tag{0.4}$$

Here Px expresses that x is a person at the table in Gothenburg and Qx expresses that x is in England. In the formalization of the claim that Mary believes that every person at the table in Gothenburg is in England, the modal operator takes a wider scope: $\Box\forall x\,(Px \rightarrow Qx)$.

In the sentence (0.4) the quantifier $\forall x$ binds all occurrences of x, including the last one that is in the scope of the belief operator $\Box$. Thus in (0.4) we quantify into a modal context, more precisely a belief context.

Analogous examples can be given for other modalities. Kripke (1980) provided many examples for what he called metaphysical necessity. Assume we have a glass with pure water. It is necessary for each of the molecules in the glass that it contains a hydrogen atom. If it did not contain a hydrogen atom, it would not be a water molecule and thus a different molecule. The claim that it is necessary of every molecule in the glass that it contains a hydrogen atom can be formalized as $\forall x\,(Px \rightarrow \Box Qx)$ again. Now $\Box$ stands for metaphysical necessity, and the predicate symbols have to be understood in the obvious way. Clearly, we cannot move the modal operator out and write $\Box\forall x\,(Px \rightarrow Qx)$, as this would be the incorrect claim that it is necessary that every molecule in the glass contains a hydrogen atom; the glass could be empty.

Beyond necessity and intentional attitudes such as belief and knowledge, truth provides further examples of this kind. We can use the above example and replace necessity with truth: It is true for each of the molecules in the glass that it contains a hydrogen atom. However, as we have seen above, truth has hardly ever been treated as an operator. But, at least on the surface, it seems to behave syntactically in a similar way. If we were to formalize truth as an operator, we would quantify into the scope of the truth operator.

In many places, among them (1943, 1976), Quine argued that quantifying into certain modal contexts is incomprehensible. How problematic quantifying-in is depends on the modality. Quine focused on a much smaller class of modalities than we do. For many modalities quantifying-in does not pose any problem. In the case of truth, for instance, there seems little reason to worry about the comprehensibility of quantifying in; in other cases *de re* conceptions make little sense (at least in a non-trivial sense), as in the case of analyticity.

Important objections against quantifying-in arise from puzzles about *de re* modality in general. In a sentence such as

> Jana believes that the greatest sane logician was born in Poland.

does not imply that Jana believes that the author of *The Concept of Truth* was born in Poland, even if it happens that he and the greatest sane logician are the same person. Modal contexts are intensional and identicals may not be substitutable *salva veritate*. The displayed sentence and the analogous sentence with the author of *The Concept of Truth* ascribe *de dicto* beliefs to Jana. Whether the sentence is true depends on the specific singular term that is used. Using a different singular term, even if it refers to the same person or object, can affect the truth value of the entire sentence. In the case of a *de re* belief, roughly, the belief is about the person or the thing, independently of the any specific singular term. For instance, we could talk about a person and say even things about that person Jana does not know and then say that she believes of them that they were born in Poland. Quantifying-in seems to presuppose that *de re* modality is comprehensible. If we have a variable in the scope of a modal operator, then we do not have any specific singular term and just an unspecific pronoun or a variable. Thus the modality is *de re*.

There are many puzzles that arise from *de re* modalities. *De re* modalities have also been at the heart of the modern revival of metaphysics, as *de re* necessity is closely tied to essential properties. Here we do not intend to go any deeper into the analysis to *de re* necessity or take a specific stance. We are only interested in the question whether we can deal with such modalities and quantifying-in, if modalities are conceived as predicates rather than sentential operators. Whatever our specific stance on *de re* modality and quantifying-in may be, our adoption of the predicate approach should not bar us from developing a theory of *de re* modality as it has been done for the operator approach. Here possible worlds semantics has been used to sharpen and refine informal accounts of *de re* modality, and metaphysicians and philosophers of language have availed themselves to the methods of modal logic.

The transition from a modal operator that allows for quantifying-in to a predicate, is not straightforward. If we replace the operator in $\forall x\,(Px \rightarrow \underline{\Box} Qx)$ with a predicate, we obtain the sentence $\forall x\,(Px \rightarrow \Box\overline{Qx})$, where $\overline{Qx}$ is a name for the formula Qx or an object corresponding to Qx such as the property expressed by Q. This substitution does not achieve the desired effect. It cannot express what it is supposed to express, because the variable x is not free in $\Box\overline{Qx}$; it is only mentioned, not used. Of course, we have not provided an axiomatic system or a semantics for the language with the predicate $\Box$, but in any reasonable axiomatization and under every reasonable semantics, $\forall x\,(Px \rightarrow \Box\overline{Qx})$ will never be an adequate rendering of the above English sentences with quantifying-in. Only variables that are used can be bound; variables that are merely mentioned cannot be bound. In $\overline{Qx}$ the x is merely part of a name for the formula Qx. It is a feature of our notation that the variable x shows up within the name $\overline{Qx}$ for the formula Qx. For instance, in the English singular term 'The letter *Q* followed by the third-from-last letter of the alphabet' the variable *x* does not occur in any way.

There have been various attempts to address the problem (see Bealer 1982 for a detailed treatment). Our preferred method is a technique that is well-known from the theory of truth. As mentioned above, the grammar of the truth predicate is very similar to that of the necessity and other modal predicates. In particular, we can form sentences with the truth predicate that involve quantifying-in. The sentence

> It is true for each of the molecules in the glass that it contains a hydrogen atom.

displays the same grammatical structure as

> It is necessary for each of the molecules in the glass that it contains a hydrogen atom.

In the case of the first sentence a common way to deal with quantifying-in would rely on a predicate for satisfaction. The sentence could be formalized as

$$\forall x\,(Px \rightarrow \mathsf{Sat}\,\overline{Qx}\,x) \tag{0.5}$$

Here Sat is a binary predicate symbol that is applied to the two terms $\overline{Qx}$ and x. As before, $\overline{Qx}$ is a name for the formula Qx. It is important that we use the specific variable x rather than the metavariable, because now it really matters which variable is used. In predicate logic renaming a bound variable cannot transform a provable into an unprovable sentence, as long as the new variable does not occur in the original sentence; also the truth or falsity of a sentence does not depend on such a renaming. In the present case this means that, concerning provability or truth in any given model, there is no difference between $\forall \mathsf{x}\,(P\mathsf{x} \rightarrow \mathsf{Sat}\,\overline{Q\mathsf{x}}\,\mathsf{x})$ and, say, $\forall \mathsf{v}_{27}\,(P\mathsf{v}_{27} \rightarrow \mathsf{Sat}\,\overline{Q\mathsf{x}}\,\mathsf{v}_{27})$. Of course, when we rename bound variables, the name $\overline{Q\mathsf{x}}$ for the formula $Q\mathsf{x}$ is not replaced with the name of another formula; in particular, it is not replaced with a name of $Q\mathsf{v}_{27}$. Thus, in (0.5), there is no connection between the variable x that occurs as a symbol in (0.5) and the letter x, which is used in a way to communicate the name of $Q\mathsf{x}$.

There are different ways to understand $\mathsf{Sat}\,x\,y$, where x and y are some variables. It could be read as 'y satisfies x if the free variable v_0 (that is, x) of x is interpreted by y'. In this case Sat would be tied to the first variable v_0. Alternatively, we could read $\mathsf{Sat}\,x\,y$ as 'y satisfies x if the free variable of x (whatever that free variable may be) is interpreted by y'. At any rate such as reading of Sat always confines us to a predicate expressing a relation between a formula with a single free variable and a single object. However, there are examples where we quantify into modal contexts with two variables:

> If *A* and *B* are distinct, then they are necessarily distinct.

We could employ a ternary predicate Sat to formalize claims of this kind with the additional intricacies of determining which variables are interpreted by which object. Moreover, the binary and the ternary predicate would have to be related in some way.

The obvious solution is to follow Tarski's lead for truth and to conceive the necessity predicate as a binary predicate applying to formulæ and variable assignments. Variable assignments are lists of objects associating an object with some or all variables. If variable assignments are finite, then we have to make sure that the variable assignment matches the free variables in the formula. Here we do not go deeper into the details. Whatever the details are, the operator and the predicate accounts of quantifying-in will diverge in some important aspects. On the predicate approach we will need a theory of variable assignments, usually a theory of sequences of objects. These sequences may be finite and their theory can be developed in weak systems of arithmetic already, but they still require more than pure logic. On the operator account nothing comparable is needed.

The predicate conception of *de re* modalities has also advantages over the operator conception. First, as with unary predicates, we can express quantification and generalizations as explained in section . With a binary necessity predicate applying to formulæ and variable assignments we gain even more expressive power, which is not easily matched by a language with an operator. With such a predicate we can express that a formulæ is necessary of some objects, for instance. We can also use such a predicate to formulæ the *ab necesse ad esse* or factivity of *de re* necessity as a universally quantified principle. In English the principle may be expressed as follows:

> If a formula is necessary of some objects, then the formula is true of these objects.

This principle can be expressed using the binary necessity predicate and quantification over variable assignments. It is stronger that the well-known *de dicto* version stating that whatever is necessary is also true.

Of course, to provide a deeper discussion, we would have to specify a semantics or a deductive system for the *de re* modalities. In chapter **??** we develop possible worlds semantics for languages with modal predicates, but only for unary modal predicates. This semantics can be adapted to binary modal predicates, that is, *de dicto* modalities. There are some tricky and metaphysically interesting problems, some of which are discussed in (Halbach 2020). We may consider a semantics where worlds have different domains. Presumably then also different variable assignments exist in different worlds, because a variable assignment cannot contain an only possibly existing object. Here we break off the discussion and refer the reader to work has been done in this direction, for instance, by Bealer (1993), Halbach and Sturm (2004), and Halbach, Leitgeb, and Welch (2003); but many questions in this area are still open and modal metaphysics could benefit from considering languages with great expressive power and predicates for *de re* modalities.

Bibliography

George Bealer. *Quality and Concept*. Clarendon Press, Oxford, 1982. (document)

George Bealer. Universals. *Journal of Philosophy*, 90:5–32, 1993. (document)

R.A. Bull. On modal logic with propositional quantifiers. *Journal of Symbolic Logic*, 34: 257–263, 1969. 5

Rudolf Carnap. *Logische Syntax der Sprache*. Springer, Wien, 1934. 3

Roderick Chisholm. *Perceiving*. Routledge & Kegan Paul, London, 1957. (document)

Alonzo Church. On Carnap's analysis of statements of assertion and belief. *Analysis*, 10: 97–99, 1950. 8

B. Jack Copeland. The genesis of possible worlds semantics. *Journal of Philosophical Logic*, 31:99–137, 2002. 4

Giulia Felappi. In defence of sententialism. *dialectica*, 68:581–603, 2014. (document)

Hartry Field. Disquotational Truth and Factually Defective Discourse. *The Philosophical Review*, 103:405–452, 1994. (document)

Kit Fine. Propositional quantifiers in modal logic. *Theoria*, 36:336–346, 1970. 5

Kurt Gödel. Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. *Monatshefte für Mathematik*, 38:173–198, 1931. (document)

Volker Halbach. The fourth grade of modal involvement. In Carlo Nicolai and Johannes Stern, editors, *Modes of Truth: The Unified Approach to Modality, Truth, and Paradox*, pages 331–356. Routledge, New York, 2020. (document)

Volker Halbach and Holger Sturm. Bealers Masterargument: Ein Lehrstück zum Verhältnis von Metaphysik und Semantik. *Facta Philosophica*, 6:97–110, 2004. (document)

Volker Halbach and Philip Welch. Necessities and Necessary Truths: A Prolegomenon to the Metaphysics of Modality. *Mind*, 118:71–100, 2009. (document)

Volker Halbach, Hannes Leitgeb, and Philip Welch. Possible Worlds Semantics For Modal Notions Conceived As Predicates. *Journal of Philosophical Logic*, 32:179–223, 2003. (document)

David Kaplan. S5 with quantifiable propositional variables. *Journal of Symbolic Logic*, 35: 355, 1970. abstract. 5

Saul Kripke. Outline of a Theory of Truth. *Journal of Philosophy*, 72:690–716, 1975. reprinted in Martin (1984). (document)

Saul Kripke. Is There a Problem about Substitutional Quantification? In Gareth Evans and John McDowell, editors, *Truth and Meaning: Essays in Semantics*, pages 325–419. Clarendon Press, Oxford, 1976. (document)

Saul Kripke. A Puzzle About Belief. In A. Margalit, editor, *Meaning and Use*, pages 239–83. Reidel, Dordrecht, 1979. (document)

Saul A. Kripke. A completeness theorem in modal logic. *Journal of Symbolic Logic*, 24: 1–14, 1959. 5

Saul A. Kripke. *Naming and Necessity*. Harvard University Press, 1980. (document)

Stephen Leeds. Church's translation argument. *Canadian Journal of Philosophy*, 9:43–51, 1979. (document)

Clarence I. Lewis and Cooper H. Langford. *Symbolic Logic*. Century, London, 1932. 2nd edition 1959, New York: Dover. 5

Casimir Lewy. Truth and Significance. *Analysis*, 8:24–27, 1947. 8

Robert L. Martin, editor. *Recent Essays on Truth and the Liar Paradox*. Clarendon Press and Oxford University Press, Oxford and New York, 1984. (document)

Friederike Moltmann. Propositional Attitudes without Propositions. *Synthese*, 135:77–118, 2003. (document)

Friederike Moltmann. Truth predicates, truth bearers, and their variants. *Synthese*, 2018. doi: doi.org/10.1007/s11229-018-1814-8. 7

Richard Montague. Syntactical Treatments of Modality, with Corollaries on Reflexion Principles and Finite Axiomatizability. *Acta Philosophica Fennica*, 16:153–67, 1963. Reprinted in (Montague 1974, 286–302). (document)

Richard Montague. *Formal Philosophy: Selected Papers of Richard Montague*. Yale University Press, New Haven and London, 1974. Edited and with an introduction by Richmond H. Thomason. (document)

George Edward Moore. Selections from a Course of Lectures Given in 1925-26. In Casimir Lewy, editor, *Lectures on Philosophy*, pages 107–149. George Allen & Unwin, London, 1966. 8

Beau M. Mount. Type-free Ramseyan truth theories: A preliminary report. 2019. draft. 6

Arthur Prior. *Objects of Thought*. Clarendon Press, Oxford, 1971. (document)

Willard V. Quine. Notes on existence and necessity. *Journal of Philosophy*, 40:113–127, 1943. (document)

Willard Van Orman Quine. *Word and Object*. MIT Press, Cambridge (Mass.), 1960. (document)

Willard Van Orman Quine. *Philosopy of Logic*. Harvard University Press, Cambridge, Massachusetts, 1970. (document)

Willard Van Orman Quine. Quantifiers and Propositional Attitudes. In *The Ways of Paradox*, pages 185–196. Harvard University Press, Cambridge, Mass., revised and enlarged edition, 1976. (document)

David Sackris. Salmon's translation argument. *Southwest Philosophy Review*, 32:163–182, 2016. (document)

Nathan Salmon. The very possibility of language: A sermon on the consequences of missing church. In C. Anthony Anderson and Michael Zelëny, editors, *Logic, Meaning, and Computation: Essays in Memory of Alonzo Church*, pages 573–595. Kluwer, 2001. (document)

Johannes Stern. *Toward Predicate Approaches to Modality*, volume 44 of *Trend in Logic*. Springer, Cham, 2016. (document)

Michael Tye. *The Metaphysics of Mind*. Cambridge University Press, Cambridge, 1989. (document)