Hannes Leitgeb: *Ramsification and Semantic Indeterminacy*

Simon Nagler

New College Oxford
Munich Center for Mathematical Philosophy

9th March 2020

Outline

The main idea

Ramsifying classical semantics

Leitgeb wants to

- ramsify[1] classical semantics and
- thereby solve problems of semantic indeterminacy from
  - vagueness from natural language ('bald'), including
    - the Sorties paradox and
    - higher-order vagueness,
  - technical terms in mathematics ('natural number')
  - and science ('mass').

[1]The outcome of which is that a set of new terms can be defined by reference to their relations to each other and to other old terms already understood.

Classical semantics
(1) satisfaction

For all F, s:

- $F, s \vDash P(a)$ iff $F(a) \in F(P)$;
- $F, s \vDash \neg A$ iff $F, s \nvDash A$;
- $F, s \vDash C \vee D$ iff $F, s \vDash C$ or $F, s \vDash D$;
- $F, s \vDash \exists x A$ iff $\exists d \in Uni(F)$, such that $F, s\frac{d}{x} \vDash A$.

Classical semantics
(2) logical consequence

$$A_1 \dots A_n \vDash C \text{ iff } \forall F, s: \text{if } F, s \vDash A_1 \dots A_n, \text{ then } F, s \vDash C.$$

Classical semantics

(3) intended interpretation

for all sentences A in L: A is true iff $I \vDash A$.

Classical semantics

the meta-semantic facts determining *I*

I is supposed to be determined by

i all linguistic facts concerning the competent usage of predicated and singular terms (individual constants, individual variables, function terms) in *L*,[2] and

ii all non-linguistic facts that are relevant as to whether the atomic formulas in *L* are satisfied.[3]

[2] This is supposed to determine the truth conditions of the atomic formulae as well as the universe (the intensions of predicates/singular terms).

[3] This is supposed to be determine whether the truth conditions by (i) is met by the universe from (i) (mathematical and scientific investigation).

Classical semantics

the meta-semantic laws determining *I*

(i) and (ii) is supposed to be governed by

iii all meta-semantic laws taken together that concern the atomic formulas, and hence the predicates and singular terms, of L.

Classical semantics

Adm

(i)-(iii), the pre-supposed meta-semantic constraints constitute a class of interpretations of L: Adm.

Classical semantics

(4) uniqueness of the intended interpretation

$\exists! F (F \in Adm)$ and $I \in Adm$.

Classical semantics

summary

Classical semantics are composed of:

1. satisfaction
2. logical consequence
3. intended interpretation
4. uniqueness of the intended interpretation

Classical semantics

and semantic indeterminacy

- Vague terms not semantically indeterminate, another explication is available (cf. e.g. epistemicism).
- Structuralism about arithmetic is wrong (no more arithmetical terms than their structural content).
- Newtonian mechanics gives ‘mass’ a unique interpretation.

Ramsey semantics (1)

the ramsification

1. Treat 'I' and 'true' as theoretical terms.
2. Delete the classical definitions of an intended interpretation and its uniqueness.
3. Substitute 'F' for 'I and 'T' for 'true' and thereby arrive at:

Ramsey semantics (1)

(5) existence of an admissible interpretation

$$\exists F \exists ! T(F \in Adm \text{ and for all sentences } A : A \in T \text{ iff } F \vDash A).$$

Ramsey semantics (1)

summary

Ramsey semantics (1) are composed of:

1. satisfaction
2. logical consequence
3. ~~intended interpretation~~
4. ~~uniqueness of the intended interpretation~~
5. existence of an admissible interpretation

Ramsey semantics (2)

ramsifying with the ε-operator

The ramsification

1. Add the ε-operator to the meta-language.
2. Extend the classical meta-logical semantics by the axioms of the ε-calculus:
   2.1 $\exists F C[F] \leftrightarrow C[\varepsilon F\ C[F]]$
   2.2 $\forall F(C[F] \leftrightarrow C[F]) \rightarrow \varepsilon F\ C[F] = \varepsilon F\ C[F]$ (extensionality)
3. derive from 5.:

Ramsey semantics (2)

(8) *an* admissible interpretation

$$\exists F(F \in Adm)$$

Ramsey semantics (2)

(9) indefinite 'I'

$$I = \varepsilon F(F \in Adm)$$

Ramsey semantics (2)

(10) ‘true’ relative to ‘I’

for all sentences A in $L : A$ is true iff $I \models A$.

Ramsey semantics (2)

summary

Ramsey semantics (2) are composed of:

1. satisfaction
2. logical consequence
8. *an* admissible interpretation
9. indefinite 'I'
10. 'true' relative to 'I'

and the ε-calculus.

Ramsey semantics at work

Classical truth

Ramsey semantics employs a classical concept of truth, i.e.:

- it derives T-biconditionals for all L-sentences,
- truth is compositional,
- it proves LEM.

Ramsey semantics at work

logical consequence

Ramsey semantic defines logical consequence as *truth preservation* and, hence, validates all

- theorems,
- rules, and
- meta-rules

of classical logic.

Ramsey semantics at work

Det

The classical meta-rules remain valid if we add the *Det* operator:

1. Expand the logical vocabulary L by *Det* (by changing every '$F, s \vDash \dots$ to $F, s, X \vDash \dots$', where X is a class of interpretations and $X \neq \varnothing$).
2. Augment the semantic rules by $F, s, X \vDash Det(A)$ iff $\forall F', s, X \vDash A$.
3. Change (10) to for all sentences A in $L : A$ is true iff $I, Adm \vDash A$.

Ramsey semantic at work

Det

This will give us:

- For all $A \in L : Det(A)$ is true $\Rightarrow A$ is true.
- In a borderline case: $\neg Det(B(n)) \wedge \neg Det(\neg B(n))$ and $(B(n) \vee \neg B(n))$.

Ramsey vs classical vs supervaluationist semantics

	classical truth	classical consequence	semantic determinacy
Ramsey semantics			
Classical semantics			
Supervaluationist semantics			

A solution to the Sorties

13. $B(0)$
14. $\forall x(B(x) \rightarrow B(x+1))$
15. Therefore, B(100000)

A solution to the Sorties

the solution

Ramsey semantics renders (14) false, since it derives:

$$\exists n \in U, \text{ such that: } n \in \varepsilon F(F \in Adm)(B) \text{ and } n+1 \notin \varepsilon F(F \in Adm)(B).$$

It views the meta-semantic facts in the borderline case as incomplete,

$$\text{`}\exists x Det(B(x) \wedge \neg B(x+1))\text{' is false,}$$

but retains classical truth simultaneously:

$$\text{`}\exists x(B(x) \wedge \neg B(x+1))\text{' is true.}$$

Higher-order vagueness

the problem: indeterminacy of the indeterminacy of vague L

Using a meta-semantic predicate formalisation of Det:

$$\text{for all sentences } A \in L\text{: } Det(A) \text{ iff for all } F \in Adm, F \vDash A,$$

'Det' will only be vague if 'Adm' is also vague.

If 'Adm' has a factually uniquely determinate interpretation, no higher-order vagueness occurs; if it does not, there is indeed higher-order vagueness.

Higher-order vagueness

the solution

In the latter case, the ramsification schema can be extended to the meta-languages. This means, that one can account for higher-order vagueness by ramsifying up the Tarskian hierarchy.