Proceedings of Proceedings of the ASME 2018 37th International Conference on Ocean, Offshore and Arctic Engineering
OMAE2018
June 17, 2018, Madrid, Spain

OMAE2018-77079

POWER AND THRUST CAPPING OF TIDAL STREAM TURBINES – A CASE STUDY OF THE PENTLAND FIRTH

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ABSTRACT

It will not be practical for tidal stream turbines to extract all the peaks in energy over the tidal cycle. It is widely assumed that some form of power capping strategy will be required. In this paper we examine this using a 2D shallow water model of tidal hydrodynamics of the Pentland Firth. We argue that in addition to power capping, it is sensible to control the turbines to limit the peak thrust on them. We show that this can lead to a significant reduction in the peak thrust applied to the turbines and smaller changes to the naturally occurring flow rate whilst having a minimal effect of the mean power generated.

INTRODUCTION

Tidal stream turbines are a promising form of clean and renewable energy. They work by extracting energy from fast moving tidal flows. These typically occur when the flow passes around some constraint such as a headland or through a strait. The technology is still at an early stage with the first major installation of an array taking place in the Inner Sound of the Pentland Firth.

It is widely acknowledged that extracting all the possible tidal stream energy over a tidal cycle will not be practical [1–3]. The power will instead be capped when the flow is high and the power available is above a given value, the turbine will be controlled so that power will be limited to the capped value. This limits the required size of the generator and electrical infrastructure whilst having a small effect on the power produced. To the Authors' knowledge, all turbines so far deployed have adopted this strategy. The exact point at which capping should take place is a complex engineering and economic question. However, the best place to cap will be a compromise between the total power produced and the cost per turbine. In this paper we explore this from the view of the large scale hydrodynamics, as well as examining the analogous idea of 'thrust capping' which we believe has not been examined explicitly before.

We choose as our case study for this investigation the Pentland Firth – the strait between mainland Britain and the Orkney Isles. This is probably the most important candidate site for tidal stream power generation in the world. The potential of the strait for tidal energy has been the subject of much academic study [4–13]. The strait has relatively simple hydrodynamics – a strong flow is driven by the difference in water level between its two ends. It is relatively hard for flow to divert around it and the dominant effect of adding turbines (i.e. extra resistance) to the channel is to reduce the flow through the channel. The water depth in the channel is somewhat deeper than would be ideal. However, the high concentration of energy in this region make it an enticing prospect for tidal stream developers.

METHODS

Tidal Hydrodynamic Model

In this paper we model the tidal hydrodynamics using the Shallow Water Equations. The equations used here are:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0, \quad (1)$$

$$\frac{\partial}{\partial t}(uH) + \frac{\partial}{\partial x}\left(Hu^2 + \frac{1}{2}g\left(H^2 - h^2\right)\right) + \frac{\partial}{\partial y}(Huv) = g\zeta\frac{\partial h}{\partial x} - c_f u\sqrt{u^2+v^2} + fv + F_x. \quad (2)$$

Equation 1 is the mass conservation equation, where ζ represents the water elevation above a certain datum, h is the bathymetric depth of the water column below the geoid, H is the total depth of the water column ($H = h + \zeta$), which is equivalent to the sum of the free surface elevation (ζ) and the bathymetric depth, h. The variables u and v represent the depth-averaged velocity components in x and y.

Equation 2 is the horizontal momentum conservation equation in the x direction – a similar equation is used int he y direction. In this g is gravitational acceleration, c_f is an empirical fiction coefficient that depends on the bottom resistance, f is the Coriolis force, and F_x represents additional forces in the system such as tidal potential forces.

We solve these using the discontinuous Galerkin (DG) version of ADCIRC (ADvanced CIRCulation model) [14, 15]. The model domain is based on that used in [7] and used in a variety of subsequent papers. For this study we have increased the resolution of the model around the area of interest relative to these past studies. The mesh resolution was chosen based on experience. Higher resolution is used in areas around the turbines and where we expect from gradients in the flow. The new mesh is shown in Figure 1 – in most of the simulations run we use linear elements. Given the modifications we have made to the mesh for this study a brief validation study is given below which we use to 'tune' the bed friction parameter. Otherwise model details are very similar to those used by the second author in previous work. The boundary is chosen so that it is far enough away that there is negligible change to the hydrodynamics from the tidal turbines at the boundary.

We run the model for 3 days for spin-up before we start using the results. In this study we force the model with just the two dominant tidal constituents (the M_2 and S_2 constituents). This simplification is partly to make the analysis more straightforward to interpret and to allow for shorter runs. Adcock *et al.* [16] found that, for the Pentland Firth, increasing this to eight constituents only increased the power by around $\sim 6\%$. Thus we are capturing most of the important physics by restricting our analysis to just these constituents. A few models have been ran for longer than a spring-neap cycle, the results are sufficiently symmetrical. Therefore the majority of models were ran for only half of a spring-neap cycle to capture the variations between spring and neap tides.

The standard DG ADCIRC model has been modified to include the presence of tidal stream turbines. Within the shallow water model the energy loss is represented by a change in water level across an element edge where turbines are located. Thus the turbines are a sub-grid scale model. This approach was developed by Scott Draper [17] and implemented in the ADCIRC model as described in [18]. This approach has shown acceptable agreement with experiments [19]. The performance characteristics of the tidal turbines are given by the actuator disc theory of Houlsby *et al.* [20]. This gives the 'blockage', B to be specified – this can be thought of as the proportion of the water column swept by the turbine (and can be modified to an 'effective blockage' if the flow is sheared [21]). In addition the wake velocity coefficient, α_4 of the turbines is required. This is discussed in detail in the subsection below. In this study we only consider the 'available power' – this is the power available to the turbine (i.e. the extracted power minus the power lost in wake mixing behind the turbine). This is given by $P(t) = \frac{1}{2}\rho ABC_p(t)\left|u(t)^3\right|)$ where A is the swept area of turbine, ρ is the density of sea water, B the blockage (which does have minor variations with time as water levels rise and fall), C_p the time-varying power coefficient and u velocity.

In this study the turbines are located as shown in Figure 1. In this paper we have chosen the most favourable location to span as much of the channel as possible. However, we do not put turbines in any region where the water depth is less than 20 m.

CAPPING ALGORITHM

For a real turbine power capping can be achieved by altering the turbine properties – for instance by changing the tip-speed ratio or the pitch of the blades. Because it reduces the loads on the turbine, power capping will usually be achieved by reducing the resistance of the turbine to the flow and thus also reducing both the power and thrust coefficients. In this study we use an actuator disc model to derive the turbine performance characteristics. Thus when power capping we will increase the wake velocity coefficient to reduce the thrust and power of the turbine as required.

Firstly, let us consider a channel with uniform cross section and flow (such as that considered by Garrett & Cummins [22] and numerous subsequent studies. If the turbine properties are held constant over a cycle, Vennell [23] showed that it is not optimal to operate turbines at peak power coefficient but that, to maximise power generation, the wake velocity coefficient should be tuned. However, if turbine properties are allowed to vary over time, then more power can be obtained by optimising the wake velocity coefficient over the cycle [24]. In principle, power

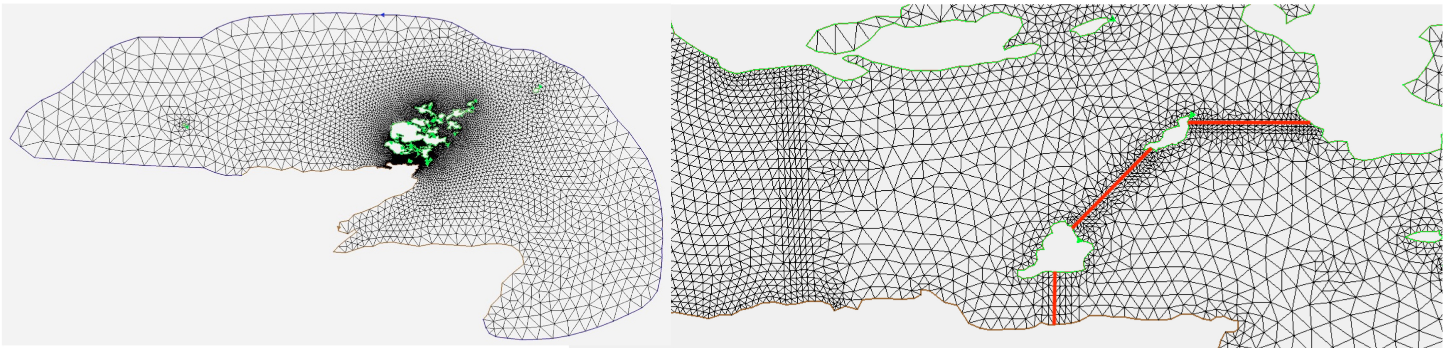

FIGURE 1. COMPUTATIONAL MESH USED IN THIS STUDY LEFT SHOWS FULL DOMAIN AROUND THE NORTH OF SCOTLAND. RIGHT SHOWS ZOOMED IN MESH AROUND THE PENTLAND FIRTH AND SCAPA FLOW. THE LOCATIONS OF TURBINE ROWS IS MARKED IN RED.

and thrust capping could be included in this. Such an approach would be relatively straightforward for a uniform channel but is extremely difficult in a more complex hydrodynamic model.

In a real turbine site the turbine size, water depth, blockage, flow velocity, etc. will vary across every turbine. Even in 2D models we usually simplify so that, say, the blockage and wake velocity coefficient are uniform across the site. For a very large tidal turbine farm it is unclear whether turbines will have different sized generators across the farm or use different generators at different points. If we assume the former, then it is clear that because the flow is varying across the site different turbines will be power capping at different times. It is clear that we need to cap turbine performance locally – i.e. have each turbine edge following its own capping algorithm.

For simplicity, the starting point of our model is that all turbines will operate with the same uncapped wake velocity coefficient (Adcock [3] looked at the effect of varying α_4 across an array and found the effect to be small). The optimum time-invariant α_4 is found by running the uncapped model with a range of values of α_4 to find the optimum (following [7]). This is then used as a starting point for the capping algorithm on each turbine.

For a system with a single velocity across all turbines a straightforward way of describing power and thrust capping is by non-dimensionalising the capping value by the the power or thrust in the uncapped case. This is more complex when all turbines have difference properties. After considering a number of options we have chosen to to parameterise the capping by limiting the power and thrust per swept area of turbine. Thus a 'power cap', P_{cap} of 4000 implies that the limiting power is 4000kW/m^2 (i.e. a given power per unit swept area). The thrust capping value, T_{cap}, is similarly defined.

A schematic of the algorithm used on each edge is shown in Figure 2. Essentially, if power or thrust calculated with the default value of α_4 exceeds the cap at any timestep we increase the value of α_4 until the power and thrust fall below the required capped values. The order in which the capping is applied does not change the outcome. The implementation of this has been thoroughly verified on idealised domains [25].

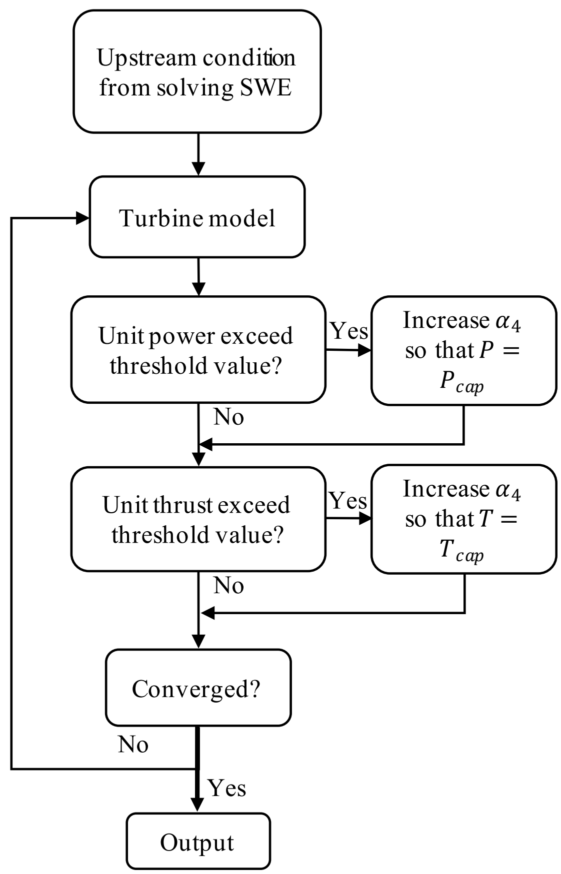

FIGURE 2. CAPPING ALGORITHM USED.

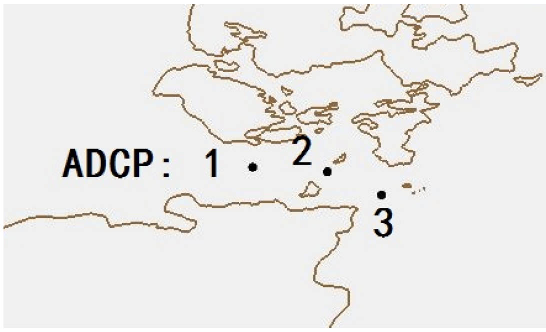

FIGURE 3. SCHEMATIC SHOWING THE LOCATION OF THE THREE ADCPS USED IN THIS STUDY.

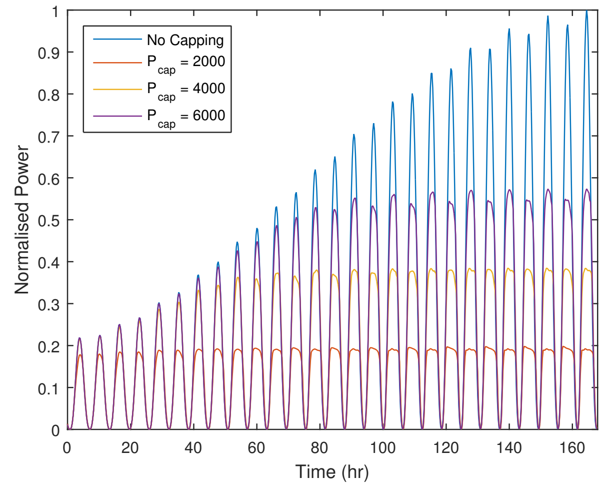

FIGURE 4. TYPICAL NORMALISED TIMESERIES OF POWER WHEN POWER CAPPING IS APPLIED.

MODEL VALIDATION

The model used here is based on the model which was validated against field measurements in Adcock *et al.* [7]. However, the resolution within the Pentland Firth strait has been significantly improved in the present model and, as the bed friction used in such models is expected to be grid dependant, we have re-calibrated this for the present model. In this paper we do not present a comparison of measured and modelled water levels outside the strait as these are similar to those in the previous model and are in good agreement with field data (see Wang [25]).

The data available for calibration is harmonic decomposition from three ADCPs (Acoustic Doppler Current Profilers) placed in the strait [26] (see also [7, 27]). The locations of these are given in the tables below and shown in Figure 3. This data has been used to confirm that the model is capturing the leading order physics and to tune the bed friction coefficient C_d where the force from the sea bed is given by $\frac{1}{2}C_d\rho Au|u|$. Data on both the magnitude and the phase of the current is measured. Of these we place more weight on the phase as this is determined by the dynamic balance over the length of the channel (see for instance [22]) rather than the velocity which is strongly dependent on local water depth, velocity profile and measurement error. Tables 1 and 2 present the comparison for the M_2 tidal constituent. The data for the S_2 constituent is presented in [25]. From these comparisons a drag coefficient of $C_d = 0.005$ was used for this modelling. We also concluded that, based on very limited available field data, our model was capturing the leading order physics of the flow through the Pentland Firth.

RESULTS

In this paper we consider two general scenarios – a high blockage case with three rows of turbines with $B = 0.4$ and a low blockage case with a single row with $B = 0.1$ (following [16]).

Power and thrust

In Figures 4, 5 and 6 we present typical timeseries of normalised available power for different values of power capping, thrust capping, and combined power and thrust capping over half a spring/neap tidal cycle. Further, in Figure 7 we present the normalised thrust for the case with power capping only where the thrust is taken as the sum of the instantaneous force on all the turbines.

The graphs show clearly the spring/neap tidal cycle as the tidal constituents come in and out of phase. It can be seen that, by doing the capping in this way, we do not cap all turbines at once, which would instead produce timeseries where during power capping the power remained constant. The other key take away message is that where just power capping, or thrust capping, are applied then the one not being capped is rather spikey. For instance, the variation in thrust when power capping (Figure 7) shows sharp spikes in thrust for short time duration. However, without the dual capping strategy proposed in this paper, the turbine structure would have to be designed for the peaks in the force.

We now explore the impact of different combinations of power and thrust capping. We present a series of Figures where the capped value of power is on the x-axis and the capped thrust is on the y-axis and we present contour plots for key outputs over the spring-neap cycle to show how these vary with different strategies. Of these outputs, the mean power and maximum thrust are straightforward. We also present the capacity factor (following say [2]) which we take as the mean power divided by the maximum power over a certain period of time (in this case half a spring-neap cycle).

Figure 8 presents how the mean available power varies for

ADCP no	Lat	Long	ADCP	$C_d = 0.0025$	$C_d = 0.004$	$C_d = 0.005$	$C_d = 0.0075$
1	58.670	2.976W	1.78	2.25	2.01	1.87	1.61
2	58.726	3.236W	2.64	2.72	2.45	2.28	1.99
3	58.717	3.086W	1.85	1.75	1.44	1.34	1.17

TABLE 1. M_2 VELOCITY MAGNITUDE (M/S)

ADCP no	Lat	Long	ADCP	$C_d = 0.0025$	$C_d = 0.004$	$C_d = 0.005$	$C_d = 0.0075$
1	58.726	3.236W	248	255	251	248	240
2	58.717	3.086W	242	249	244	243	238
3	58.670	2.976W	248	259	247	246	240

TABLE 2. M_2 PHASE

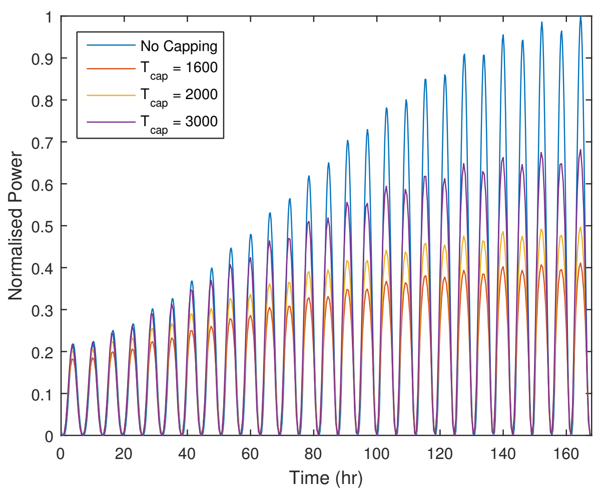

FIGURE 5. TYPICAL NORMALISED TIMESERIES OF POWER WHEN THRUST CAPPING IS APPLIED.

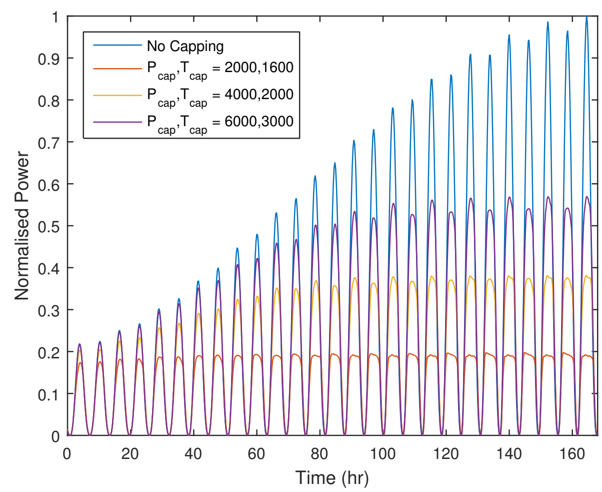

FIGURE 6. TYPICAL NORMALISED TIMESERIES OF POWER WHEN COMBINED POWER AND THRUST CAPPING ARE APPLIED.

different values of capping. Figure 9 presents the variations in capacity factor for different values of capping. Figure 10 presents how the maximum thrust varies for different values of capping. For the Pentland Firth there is relatively little difference between the two turbine deployments considered in this study – the differences would be expected to be larger for an inertia dominated channel [25].

With minimal capping (top right of figures) the power production is maximised but the capacity factor is very poor. If power capping is introduced, to say a value of 4kW/m^2 the mean power falls to around 70% of the uncapped value but the capacity factor is considerably improved making this a more realistic design. However, if we consider the maximum thrust we have to design for (Figure 10), then it is clearly advantageous to thrust cap as well as power capping. For the same value of power capping, we get virtually the same mean power if we cap the thrust at 2.5kN/m^2 as if this were uncapped. This greatly reduces the load that has to be carried by the turbines. If turbines are operated in this region, combined capping is the optimal strategy.

In general, sensible operating strategies would be around the leading diagonal on these Figures. Exactly where the optimal trade off between mean power, power capping and thrust are will depend on economic and practical factors outside the scope of this high level study.

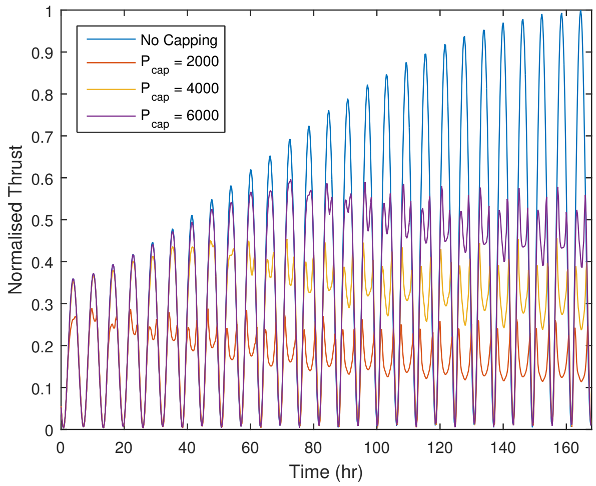

FIGURE 7. TYPICAL NORMALISED TIMESERIES OF TOTAL THRUST ON THE TURBINES WHEN POWER CAPPING ONLY IS APPLIED.

Flow rate

Tidal stream turbines will have a significant impact on the marine environment [28]. It is obviously desirable to minimize this impact. Changes to the environment could come from changes to sediment transport patterns, changes in the dispersion of pollutants, changes to the biological environment etc. In this study we briefly consider these by simplistically considering the changes to the peak flow rate and the impact of capping on this. To examine this we define the metric

$$\Delta Q = \frac{\max(Q_{capping})}{\max(Q_{natural})} - 1, \tag{3}$$

where Q values are the modelled flow rates.

Figure 11 presents the change in flow rate for the higher blockage case. We considered the flow rate through the whole channel as well as through individual subchannels. The subchannels considered are the areas of higher resolution between the two major islands in the strait shown in Figure 1. Thus the Inner Sound between mainland Britain and Stroma is the 'bottom channel'; the channel between Stroma and Swona the 'middle channel'; and the channel between Swona and South Ronald say the 'top channel'.

The overall flowrate gives the clearest picture of the benefits of the different capping strategies. Here the picture is very similar to that above – a combined power and thrust capping can help mitigate the change in the tidal hydrodynamics.

The complicated picture in the other channels was somewhat unexpected. Due to the complexities of the hydrodyanmics there are some scenarios where the flow rate in these increases even whilst the overall flow rate through the strait decreased. This is somewhat inconsistent with the previous studies of Draper *et al.* [29, 30] particularly for the top channel. However, in this previous work the model used was cruder and the Authors identified that their modelled data was not fully consistent with the very limited data available for this channel. The present study improved the resolution significantly and until better measurements are available all we can do is note this discrepancy.

CONCLUSIONS

It is generally assumed that some form of power capping will be employed in tidal stream turbine developments. In this paper we argue that it is sensible to use both power capping and thrust capping. Adding thrust capping limits the loads on the structure and reduces the environmental impact with minimal impact of the mean power production.

In this study we have endeavoured to model power capping in slightly more detail than previous work. Capping algorithms have been applied to individual parts of the array rather than being applied to very simplistic models (such as those based on Garrett & Cummins [22]) where the flow through all turbines is assumed to be the same. In this paper we have not carried out a direct comparison between simpler models but our qualitative findings are very similar to those drawn from simpler channel models [25]. However, our representation of the turbines in the present study is based on a highly idealised representation of tidal stream turbines. The real method for implementing capping will be a strong function of the real turbine characteristics.

The tidal resource of the Pentland Firth, probably the most important site for tidal stream energy generation in the world, is still an open question. The study of Adcock *et al.* [7] used an approach which sought to place an upper bound on the resource. As noted in Neill *et al.* [31], various factors such as shear [21], drag from support structure [32] and most importantly the difference between real turbines and actuator discs means this is a considerable overestimate of the practical resource. To these factors must be added power (and thrust) capping – it will not be feasible to extract all the power.

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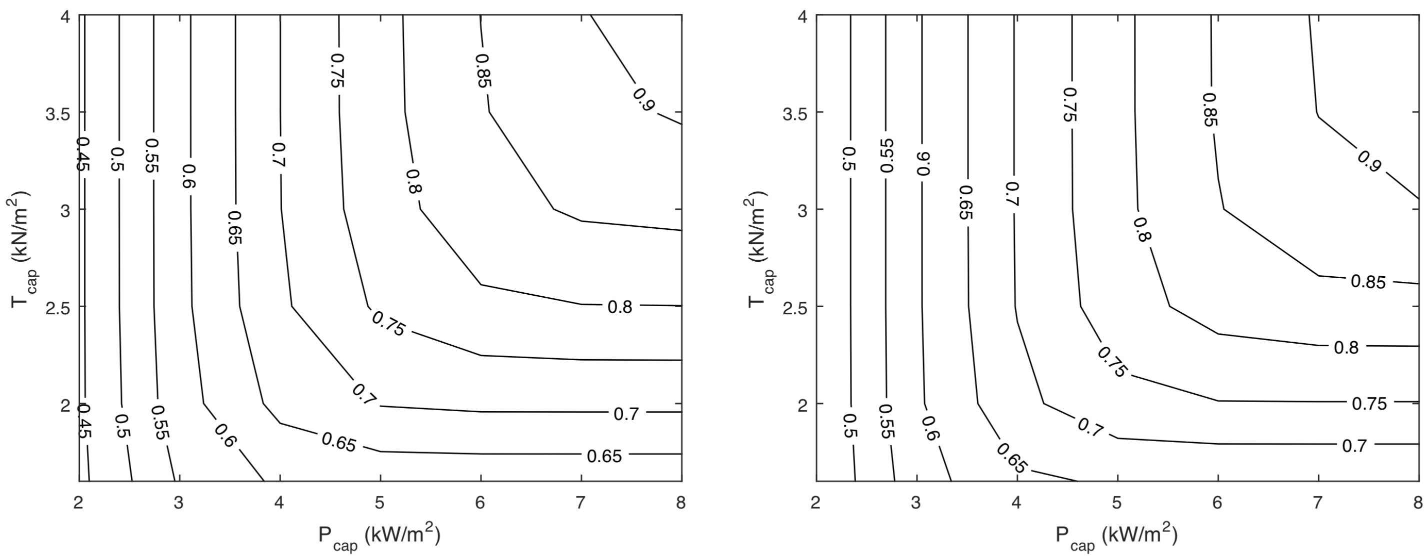

FIGURE 8. VARIATION IN MEAN POWER NORMALISED BY UNCAPPED POWER OVER THE SPRING-NEAP CYCLE FOR DIFFERENT CAPPING VALUES. LEFT – THREE ROWS WITH $B = 0.4$; RIGHT – ONE ROW WITH $B = 0.1$

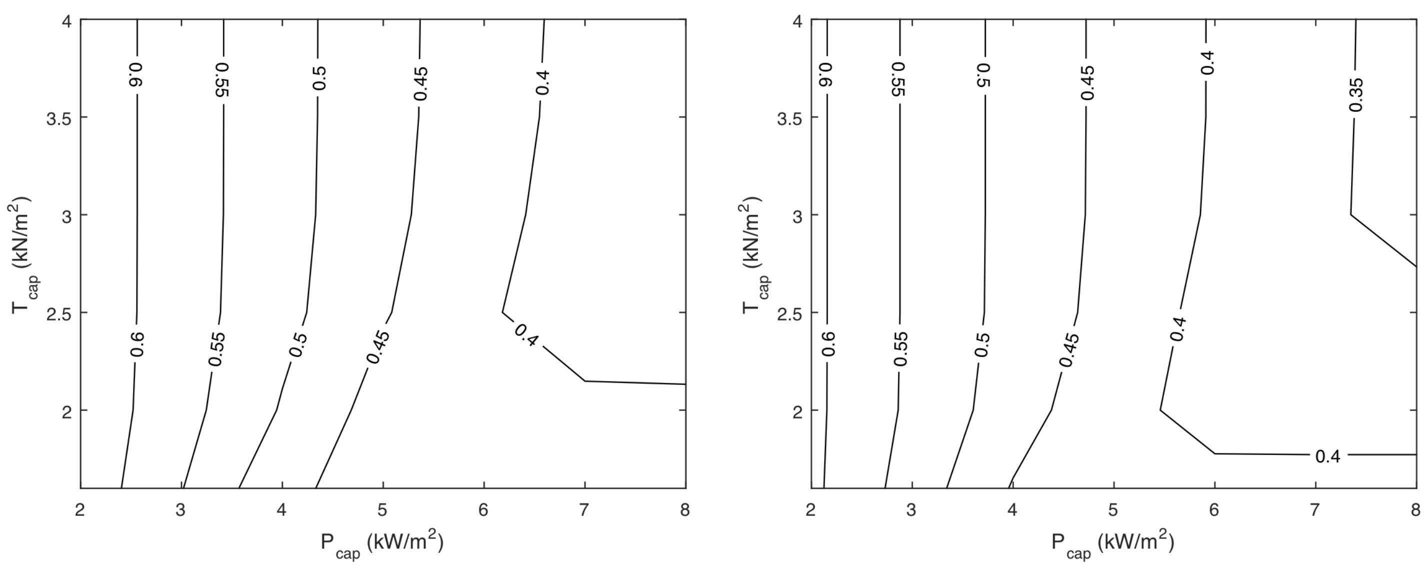

FIGURE 9. VARIATION IN CAPACITY FACTOR FOR DIFFERENT CAPPING VALUES. LEFT – THREE ROWS WITH $B = 0.4$; RIGHT – ONE ROW WITH $B = 0.1$

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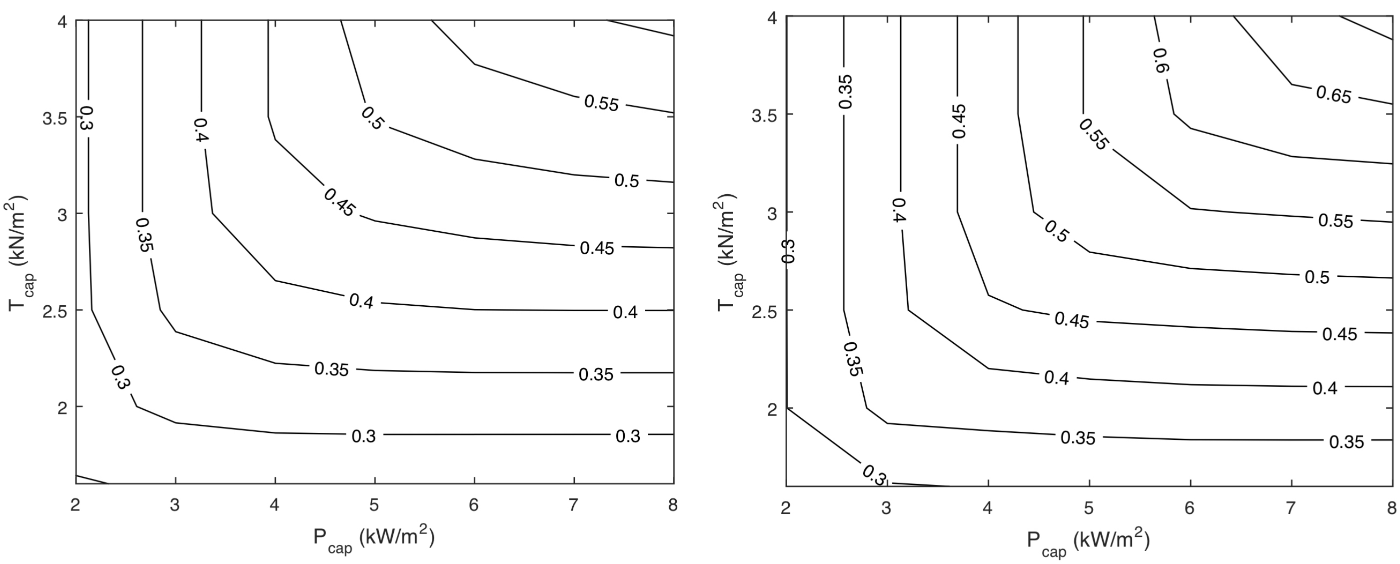

FIGURE 10. VARIATION IN MAXIMUM THRUST OVER SPRING-NEAP CYCLE FOR DIFFERENT CAPPING VALUES. LEFT – THREE ROWS WITH $B = 0.4$; RIGHT – ONE ROW WITH $B = 0.1$

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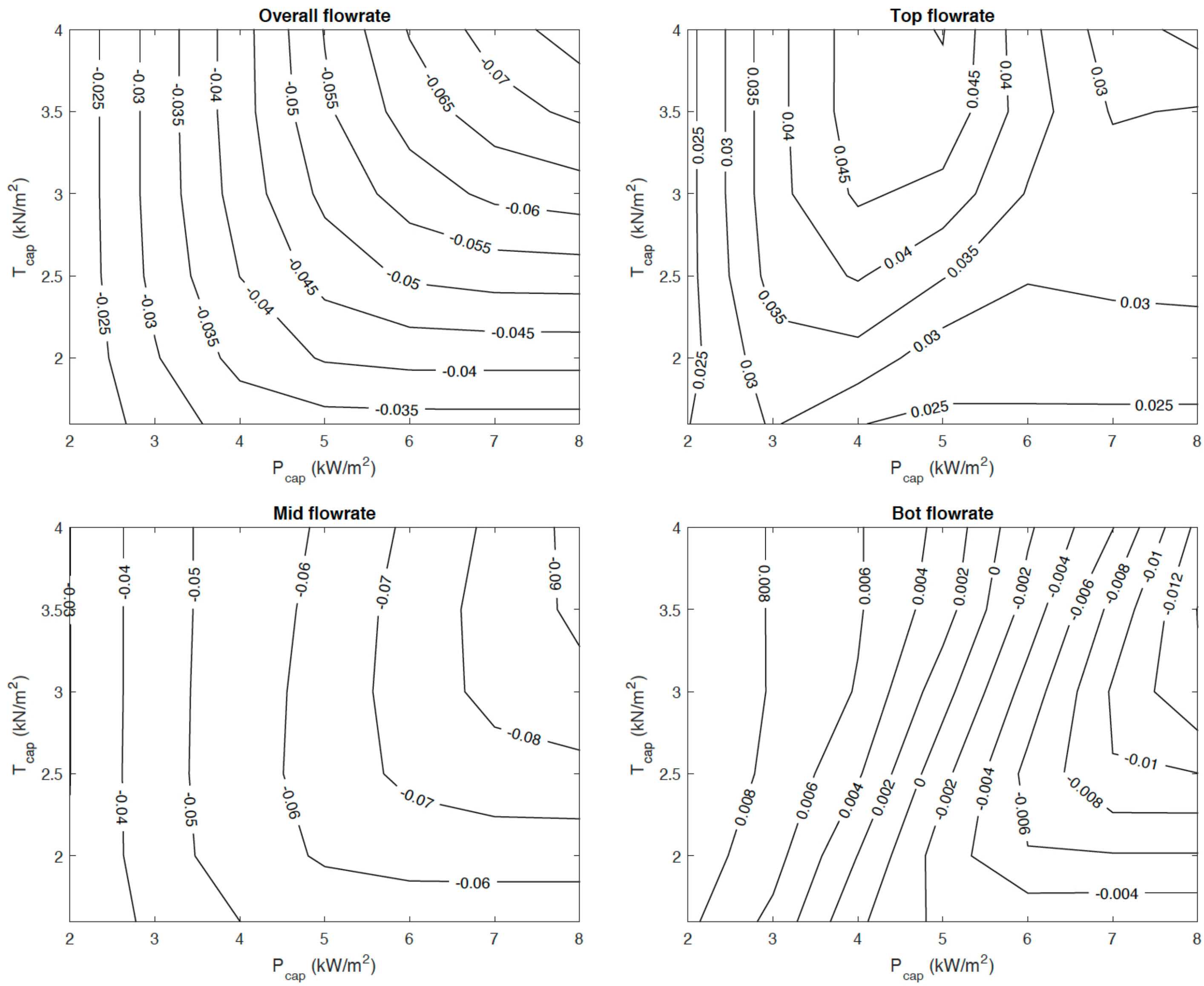

FIGURE 11. CHANGE IN FLOW RATE THROUGH THE PENTLAND FIRTH (TOP LEFT) AS WELL AS THE INDIVIDUAL CHANNELS IN THE STRAIT FOR DIFFERENT CAPPING STRATEGIES.

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