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On Quality and Variety Bias in Aggregate Prices

How do product variety and quality affect the aggregate price bias? We develop a general equilibrium model that accounts for the joint interaction of product quality and variety. Our findings show that the aggregate price bias is procyclical and the contribution of product variety is persistent whereas the contribution of product quality becomes countercyclical in the medium to long run. We show that accounting for product quality and variety has critical implications on the measure of cyclical fluctuations. Measurements of cyclical fluctuations derived using the consumption deflator, which abstracts from changes in product quality and variety, underestimate the variables' true volatility.

CHANGES IN AGGREGATE PRICES ARE biased if they fail to account for variations in prices from shifts to new products or changes in product quality. Errors in the measurement of price changes have far-reaching effects since indices of price changes are key indicators for policymakers, are used by statisticians to deflate nominal data, and serve as reference statistics for updating financial contracts and wages. An accurate measure of aggregate prices, and hence the true cost of living, is critical for the assessment of fiscal and monetary policy stance and the implementation of public policies. Needless to say, it is important to calculate a correct measure of

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economic growth. A number of empirical studies document a significant bias in the measure of aggregate price changes due to inaccurate accounting of product quality and variety. Overall, the consensus is that product quality and variety significantly contribute to movement in aggregate prices.[1]

Despite the extensive empirical research on the topic, no theoretical studies formalize the effect of product quality and variety on the aggregate price bias. This study is the first to develop a simple general equilibrium model that accounts for the joint interaction of product quality and variety and assesses their impact on aggregate price fluctuations.[2] The theoretical framework disentangles the contribution of both components to the aggregate price bias and studies their cyclical properties. Variety creation occurs since sunk entry costs limit the number of newly created products, as in Bilbiie, Ghironi, and Melitz (2012). Likewise, variety destruction occurs since firms have different capability levels, needing to sustain a common operational cost to continue production or otherwise exit the market, as in Hamano and Zanetti (2015). Each firm has a specific productivity and manufactures a distinct product of a specific quality. Hence, both product quality and variety are endogenously determined and interact in the determination of the aggregate price bias.

Numerical simulations of the model enable us to disentangle the contribution of product quality and variety to the aggregate price bias. In the aftermath of a positive productivity shock, the number of producers that use costly technology and manufacture high-quality goods increases, thereby also increasing product quality and variety. Hence, the bias in aggregate prices rises on impact. However, once the positive technology shock vanishes and the economy returns to the long-run equilibrium, firms use efficient technology to produce low-quality goods in order to be profitable in the market. Because of the relatively pronounced fall in product quality, the bias of aggregate prices decreases during the transitory dynamics to the equilibrium.

The analysis shows that improperly accounting for product quality and variety in the measure of aggregate prices has critical implications on the measure of cyclical fluctuations. In particular, we show that if the consumption deflator does not account for changes in the product quality and variety, mismeasurement of aggregate fluctuations occur, implying that the variability of major economic variables is underestimated.

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1. An early comprehensive study by Boskin et al. (1996) finds that U.S. inflation had an upward bias of 1.1% per year. More than half of the total bias was attributed to unmeasured quality improvements. More recently, Bils (2009) shows that price increases due to new product variety and changes in quality are an important source of aggregate price bias, and inflation is overstated by nearly 2 percentage points per year. Similarly, Broda and Weinstein (2010) document a 0.8% annual upward bias in price changes by using a data set that covers around 40% of all expenditures on goods for U.S. households. They also infer that the price bias is procyclical and that official business cycle statistics underestimate the variability of major economic variables.

2. In a recent paper, Aghion et al. (2017) focus on the mismeasurement of GDP growth caused by imperfect accounting of product quality and variety upgrading. In our study, we focus on the cyclical properties of unobservable product quality and variety.

This paper is related to and builds on two distinct strands of literature. First, it is related to the empirical studies that investigate the impact of product variety and quality on aggregate price bias. In addition to the studies previously cited, works by Hausman (2003), Bils and Klenow (2001, 2004), Broda and Weinstein (2004, 2006, 2007), and Nakamura and Steinsson (2008) show that product quality and variety biases are critical for an accurate measurement of aggregate prices. Unlike those studies, our work is primarily theoretical and focuses on disentangling the contribution of product quality and variety as well as studying the cyclical properties of the aggregate price bias. In this respect, our paper also is related to Schmitt-Grohe and Uribe (2012), who investigate the effect of quality bias on the optimal inflation target by using a model with staggered price setting. Our study has a different focus and uses a different model, whose main dynamics are based on endogenous product variety and quality and which abstracts from nominal rigidities. Finally, our work is related to studies that use endogenous growth models enriched with the Schumpeterian idea of creative destruction due to changes in product quality, as described in Aghion and Howitt (1992, 2009). Our model, however, focuses on the effect of changes in variety.

Second, our analysis is related to recent literature on product quality in international trade. Influential studies that use product quality to explain trade patterns across countries are those of Schott (2004), Verhoogen (2008), Baldwin and Harrigan (2011), Hallak and Schott (2011) Johnson (2012), Manova and Zhang (2012), and Feenstra and Romalis (2014). The presence of product quality allows producers with relatively high unit prices to stay in the exporting market due to lower quality-adjusted prices. The key mechanism is a high quality of exporting goods produced with high marginal costs. Different from the trade literature, we focus on business cycle properties and second moments of the aggregate price bias. Our numerical results are also consistent with the recent theoretical and empirical findings in Atkeson and Burstein (2010) Fattal Jaef and Lopez (2014), and Burstein and Cravino (2015). These studies establish the irrelevance of disentangling the empirical-based measure from the welfare-based measure since changes in cross-country variations of product quality and the number of varieties offset each other in the welfare-based measure. We establish a similar result in our closed-economy model, and we focus extensively on the cyclical properties of product quality and variety and their link to the aggregate price bias.

The remainder of the paper is as follows. Section 1 presents the model. Section 2 derives the model-consistent bias in aggregate prices. Section 3 presents the results, focusing on the cyclical properties of the price bias. Section 4 concludes.

1. THE MODEL

The economy is populated by one unit mass of atomistic households that consume products of different varieties and quality. Firms enter and exit the market and produce

goods of different quality. Upon entry, each firm produces a single product variety, draws a specific capability level, and pays sunk entry costs, as in Ghironi and Melitz (2005). During each period, firms pay fixed operational costs or otherwise terminate production and exit the market, as in Hamano and Zanetti (2015). The firm's capability is associated with a specific quality and productivity level, following findings in Sutton (1998, 2005). The consumption price index accounts for changes in product variety and quality.

1.1 Households

During each period t, the representative household maximizes expected utility,

$$E_t \sum_{i=t}^{\infty} \beta^{i-t} U_t, \tag{1}$$

where $0 < \beta < 1$ is the discount factor. Utility depends on consumption, C_t, and labor supply, L_t, according to $U_t = \ln C_t - \chi L_t^{1+\frac{1}{\varphi}}/(1 + 1/\varphi)$, where $\chi > 0$ is the degree of disutility in supplying labor and φ is the Frisch elasticity of labor supply.

Consumption is defined over a continuum of goods, Ω, and during each period t, only a subset of goods, $\Omega_t \subset \Omega$, is available. Each produced good has a unique variety indexed by $\omega \in \Omega_t$. The consumption aggregator is

$$C_t = V_t \left(\int_{\omega \in \Omega_t} (q(\omega)c_t(\omega))^{1-\frac{1}{\sigma}} d\omega \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{2}$$

where $c_t(\omega)$ is individual demand for variety ω, and $q(\omega)$ is the quality of the variety that is invariant across time. In particular, $V_t = S_t^{\psi - \frac{1}{\sigma-1}}$, where S_t denotes the number of available varieties at time t, and $\sigma > 1$ is the elasticity of substitution among varieties. As in Benassy (1996), ψ represents the marginal utility of an additional increase in the number of varieties in the basket. By imposing $\psi = 1/(\sigma - 1)$, the consumption aggregator (2) nests the standard Dixit–Stiglitz aggregator. The price index that minimizes the consumption expenditure is

$$P_t = \frac{1}{V_t} \left(\int_0^{S_t} \left(\frac{p_t(\omega)}{q_t(\omega)} \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \tag{3}$$

where $p_t(\omega)/q_t(\omega)$ is the quality-adjusted individual price of variety ω. Equation (3) is consistent with a welfare-basis index and shows that for a given variety ω, the price index rises (decreases) when the number of available varieties, S_t, or the quality, $q_t(\omega)$, decreases (rises). The quality-adjusted demand for each variety, ω, is

$$q_t(\omega)c_t(\omega) = V_t^{\sigma-1} \left(\frac{p_t(\omega)/q_t(\omega)}{P_t} \right)^{-\sigma} C_t, \tag{4}$$

where $p_t(\omega)$ denotes the physical unit price of variety ω.

1.2 Quality, Production, Pricing, and Producing Decision

Firms are indexed by their specific "capability," α. As in Sutton (1998, 2005), each capability level is associated with a firm-specific quality, $q(\alpha)$, and productivity level, $z(\alpha)$, as $\alpha \equiv q(\alpha)z(\alpha)$. Capability is thus defined as the amount of quality produced per worker in each production unit.

Here we take the simplest step by assuming $q(\alpha) = \alpha^\phi$ and $z(\alpha) = \alpha^{1-\phi}$, as in Johnson (2012). Thus, the firm's quality and specific productivity are related according to

$$q(\alpha) = z(\alpha)^{\frac{\phi}{1-\phi}}, \tag{5}$$

where the parameter ϕ encapsulates the degree of competition in quality (i.e., "quality ladder"). This relation between firm-specific quality and marginal cost (which is the inverse of firm-specific productivity) is used in the trade literature to obtain data-consistent trade patterns (see Baldwin and Harrigan 2011, among others). In principle, we could generate the same mechanism described in equation (5) with a more complex model for the determination of the optimal firm-specific quality and derive it from the firm's optimization problem. Mandel (2010), Johnson (2012), Kugler and Verhoogen (2012), and Antoniades (2015) develop models where firms set the quality level that maximizes profits and find a functional form that relates firm-specific productivity and quality similar to equation (5). Note that if $\phi = 0$, the quality becomes irrelevant for the dynamics of the model since the quality is the same across firms (i.e., $q(\alpha) = 1$). For a rise in capability, α which is firm-specific, both firm-specific quality and firm-specific productivity rise as long as $0 < \phi < 1$. On one hand, if $\phi > 1$, the correlation between firm-specific quality and productivity is negative since the firm employs a costly technology to produce high-quality goods. On the other hand, the same type of negative correlation between product quality and the firm-specific productivity can be obtained by assuming that $\phi < 0$. In this instance, for the same rise in capability, α, the competition on productivity becomes important to deliver a lower quality-adjusted price since the firm uses the increased capability to produce lower quality goods at lower costs. Ultimately, the value for the parameter of the quality ladder, ϕ, depends on the characteristics of the market. Appendix C shows the effect of ϕ on the results.

Production requires an operational fixed cost of f_t units of labor in every period. During each period t, the labor demand, $l_t(\alpha)$, depends on the scale of effective production, $y_t(\alpha)/Z_t z(\alpha)$. Operational fixed costs are defined in terms of effective labor, f_t/Z_t^θ, so that the total labor demand is

$$l_t(\alpha) = \frac{y_t(\alpha)}{Z_t z(\alpha)} + \frac{f_t}{Z_t^\theta}. \tag{6}$$

According to equation (6), fixed costs fluctuate with aggregate labor productivity level, Z_t, with a degree of spillover, θ. Fixed operational costs, f_t, are exogenous and act as proxy (de)regulation in production.

In every period, a number of new entrants, H_t, enters the market. Prior to entry, these new entrants are identical and face a sunk entry cost of $f_{E,t}$ effective labor units. Entry cost is therefore equal to $w_t f_{E,t}/Z_t^\vartheta$ units of consumption goods, where w_t is the real wage and ϑ denotes the degree of spillover from aggregate productivity shock. Upon entry, each firm draws a capability level, α, from a distribution, $G(\alpha)$, with support on $[\alpha_{\min}, \infty)$.

Because of fixed operational cost, only a subset of firms with a capability level, α, superior to the cutoff level, $\alpha_{s,t}$, charges sufficiently lower quality-adjusted prices and earns positive profits, despite the existence of fixed operational cost f_t. Destruction of the production unit is thus endogenous and depends on the cutoff capability level. In addition to the endogenous destruction, an exogenous depreciation shock, which takes place with probability $\delta \in (0, 1)$, hits producers in every period. This exit-inducing shock is independent of the firm-specific capability level and is assumed to take place at the very end of the period. Therefore, $G(\alpha)$ also represents the productivity distribution of all firms that have production potential.

Each firm faces a residual demand curve with constant elasticity, σ, described by equation (4), which affects the production scale. The firm's real profits are the difference between the revenue and the cost of total production: $d_t(\alpha) = \rho_t(\alpha)y_t(\alpha) - w_t l_t(\alpha)$, where $\rho_t(\alpha) = p_t(\alpha)/P_t$ is the real price of goods produced by the firm with capability level α. The firm chooses the real price to maximize profits subject to the demand curve described by equation (4), which yields the following optimal real prices:

$$\rho_t(\alpha) = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t z(\alpha)}. \tag{7}$$

Equation (7) shows that changes in unit real price and unit marginal cost depend on the quality ladder, ϕ, since $z(\alpha) = \alpha^{1-\phi}$, for any given capability level. Depending on α, the firm may or may not produce. Thus, using equations (6) and (7), if production materializes, the firm's real profits are

$$d_t(\alpha) = \frac{1}{\sigma} S_t^{\psi(\sigma-1)-1} \left(\frac{\rho_t(\alpha)}{q(\alpha)} \right)^{1-\sigma} C_t - \frac{w_t f_t}{Z_t^\theta}. \tag{8}$$

1.3 Firm Average

Given the distribution of capability level, $G(\alpha)$, the mass of producers, N_t, is defined over the productivity levels $[\alpha_{\min}, \infty)$. Among these firms, $S_t = [1 - G(\alpha_{s,t})]N_t$ engage in production after surviving establishment destruction, as described below.

Following Melitz (2003) and Ghironi and Melitz (2005), we define the capability of average surviving firms, $\widetilde{\alpha}_{s,t}$, as follows:[3]

$$\widetilde{\alpha}_{s,t} \equiv \left[\frac{1}{1 - G(\alpha_{s,t})} \int_{\alpha_{s,t}}^{\infty} \alpha^{\sigma-1} dG(\alpha) \right]^{\frac{1}{\sigma-1}}, \tag{9}$$

which contains all of the information about the distribution of capabilities. Provided this average, we define the real price of the average surviving firm as $\widetilde{\rho}_{s,t} \equiv \rho_{s,t}(\widetilde{\alpha}_{s,t})$. We define real profits for the average surviving firm as $\widetilde{d}_{s,t} \equiv d_{s,t}(\widetilde{\alpha}_{s,t})$, where

$$\widetilde{d}_{s,t} = \frac{1}{\sigma} \frac{C_t}{S_t} - \frac{w_t f_t}{Z_t^{\theta}}. \tag{10}$$

Finally, we define average operational profits among total producers as $\widetilde{d}_t = (S_t/N_t)\widetilde{d}_{s,t}$.

1.4 Firm Entry and Exit

We assume that entrants at time t only start producing at time $t + 1$. These entrants discount the stream of their expected profits $\{\widetilde{d}_i^s\}_{i=t+1}^{\infty}$ by using the stochastic discount factor of households adjusted by exogenous exit inducing shock δ. Thus, their expected post entry value is

$$v_t = E_t \sum_{i=t+1}^{\infty} [\beta(1 - \delta)]^{i-t} \left(\frac{C_i}{C_t} \right)^{-1} \widetilde{d}_i, \tag{11}$$

which represents the share price of equities and mutual funds across different firms. Entry occurs until the expected firm value (11) is equal to the entry cost, leading to the free entry condition,

$$v_t = \frac{w_t f_{E,t}}{Z_t^{\vartheta}}. \tag{12}$$

The timing of entry and production implies that the number of domestically producing firms evolves according to

$$N_t = (1 - \delta)(N_{t-1} + H_{t-1}), \tag{13}$$

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3. We define the average capability level as a harmonic mean weighted by quality-adjusted output. From the goods market clearing condition, we have $[q_t(\alpha_{s,t})y_t(\alpha_{s,t})]/[q_t(\widetilde{\alpha}_{s,t})y_t(\widetilde{\alpha}_{s,t})] = (\alpha_{s,t}/\widetilde{\alpha}_{s,t})^{\sigma}$. Thus, $\widetilde{\alpha}_{s,t}$ can be defined as

$$\widetilde{\alpha}_{s,t}^{-1} \equiv \frac{1}{1 - G(\alpha_{s,t})} \int_{\alpha_{s,t}}^{\infty} \alpha^{-1} \frac{q_t(\alpha_{s,t})y_t(\alpha_{s,t})}{q_t(\widetilde{\alpha}_{s,t})y_t(\widetilde{\alpha}_{s,t})} dG(\alpha).$$

where N_{t-1} and H_{t-1} refers to the number of total producers and new entrants, respectively, in period $t-1$.

Establishments that engage in production, S_t, are a subset of the number of total producers, N_t. Therefore, in any given period t, the number of destroyed establishments is the sum of those that are endogenously destroyed, D_t^S, and exogenously destroyed, D_t^{δ}:

$$D_t \equiv D_t^S + D_t^{\delta},$$

where $D_t^S \equiv N_t - S_t$ and $D_t^{\delta} \equiv \delta(N_t + H_t)$.

1.5 Parameterization of Productivity Draw

To solve the model, we assume a distribution of capability levels, α. We assume the following Pareto distribution for $G(\alpha)$:

$$G(\alpha) = 1 - \left(\frac{\alpha_{\min}}{\alpha}\right)^k,$$

where $\alpha_{\min}$ is the minimum productivity level and k ($>\sigma - 1$) determines the shape of the distribution.[4] With this parameterization, we can express the capability of average surviving firms, $\widetilde{\alpha}_{s,t}$, in equation (9) as

$$\widetilde{\alpha}_{s,t} = \alpha_{s,t} \left[\frac{k}{k - (\sigma - 1)}\right]^{\frac{1}{\sigma - 1}}, \tag{14}$$

and the fraction of surviving producers is

$$\frac{S_t}{N_t} = \alpha_{\min}^k \left[\frac{k}{k - (\sigma - 1)}\right]^{\frac{k}{\sigma - 1}} \widetilde{\alpha}_{s,t}^{-k}.$$

As mentioned above, one firm that has a cutoff capability level that earns zero profits from production, such that $d_t(\alpha_{s,t}) = 0$, under which production becomes unprofitable. Substituting equation (14) in the firm's real profits (8) yields the equation that determines the cutoff capability level:

$$\frac{1}{\sigma} \frac{C_t}{S_t} = \frac{k}{k - (\sigma - 1)} \frac{w_t f_t}{Z_t^{\theta}}. \tag{15}$$

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4. The parameter k indexes the dispersion of capability across firms. The dispersion decreases as k increases, and the capability levels are concentrated toward the lower bound $\alpha_{\min}$. To ensure that variance of the capability distribution is finite, we assume that $k > \sigma - 1$.

1.6 Household Budget Constraint and Intertemporal Problems

We choose the consumption-based price index, P_t, as numéraire. The household receives income by supplying labor, L_t, at the real wage rate, w_t, by acquiring dividends income among producers, $\tilde{d}_t$, and by selling its initial share position, v_t, of each mutual fund shareholdings, x_t, of producers, N_t. The household spends its income on consumption, C_t, buying x_{t+1} shares of the mutual funds of producers, N_t, and new entrants, H_t, at the share price, v_t. The household budget constraint is thus

$$L_t w_t + x_t N_t (v_t + \tilde{d}_t) = C_t + x_{t+1} v_t (N_t + H_t). \tag{16}$$

During each period t, the representative household chooses consumption, C_t, shareholding, x_{t+1}, and the labor supply, L_t, to maximize the expected utility function (1), subject to the budget constraint (16). The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation of

$$\chi(L_t)^{\frac{1}{\varphi}} = w_t C_t^{-1}.$$

The first-order condition with respect to share-holdings once it is combined with the firm's law of motion (13) and with the first-order condition for consumption yields

$$v_t = \beta(1 - \delta)E_t \left(\frac{C_{t+1}}{C_t}\right)^{-1}(v_{t+1} + \tilde{d}_{t+1}), \tag{17}$$

which, once iterated forward, shows that share prices are an expected discounted sum of future dividends.

1.7. Model Equilibrium and Solution

To derive the aggregate equilibrium, we impose labor market clearing. Aggregate labor supply, L_t, is employed in either the production of consumption goods (intensive margins, i.e., production scale) or the creation of new firms (extensive margins):

$$L_t = S_t l_t(\tilde{\alpha}_{s,t}) + H_t \frac{v_t}{w_t},$$

which can be expressed as[5]

$$L_t = S_t \left[(\sigma - 1)\frac{\tilde{d}_{s,t}}{w_t} + \sigma \frac{f_t}{Z_t^\theta}\right] + H_t \frac{v_t}{w_t}. \tag{18}$$

Equation (18) is equivalent to the aggregated accounting identity of GDP obtained by aggregating budget constraints among households, $Y_t \equiv C_t + v_t H_t =$

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5. Note that $\tilde{d}_{s,t} = \frac{\tilde{\rho}_{s,t}}{\sigma}\tilde{y}_{s,t} - \frac{w_t f_t}{Z_t^\theta}$, where $\tilde{y}_{s,t}$ represents average intensive margins.

TABLE 1

SUMMARY OF THE BENCHMARK MODEL

Average pricing	$\widetilde{\rho}_{s,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \widetilde{z}_{s,t}}$
Quality-adjusted real price	$\widetilde{\rho}_{s,t}/\widetilde{q}_{s,t} = S_t^{\psi}$
Average survivors' profits	$\widetilde{d}_{s,t} = \frac{1}{\sigma} \frac{C_t}{S_t} - \frac{w_t f_t}{Z_t^{\theta}}$
Average profits	$\widetilde{d}_t = \frac{S_t}{N_t} \widetilde{d}_{s,t}$
Free entry condition	$v_t = \frac{w_t f_{E,t}}{Z_t^{\theta}}$
Motion of firms	$N_{t+1} = (1-\delta)(N_t + H_t)$
Euler equation	$v_t = \beta(1-\delta)E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(v_{t+1} + \widetilde{d}_{t+1}\right)$
Optimal labor supply	$\chi(L_t)^{\frac{1}{\phi}} = w_t C_t^{-1}$
ZCP	$\frac{1}{\sigma} \frac{C_t}{S_t} = \frac{k}{k-(\sigma-1)} \frac{w_t f_t}{Z_t^{\theta}}$
Schumpeterian surviving rate	$\frac{S_t}{N_t} = \alpha_{\min}^k \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}} \widetilde{\alpha}_{s,t}^{-k}$
Labor market clearing	$L_t = S_t\left[(\sigma-1)\frac{\widetilde{d}_{s,t}}{w_t} + \sigma \frac{f_t}{Z_t^{\theta}}\right] + H_t \frac{v_t}{w_t}$
Average quality	$\widetilde{q}_{s,t} = \widetilde{\alpha}_{s,t}^{\phi}$
Average productivity	$\widetilde{z}_{s,t} = \widetilde{\alpha}_{s,t}^{1-\phi}$

$L_t w_t + S_t \widetilde{d}_{s,t}$, where Y_t is real GDP measured in the welfare basis of expenditures and income. The model consists of 13 equations and 13 endogenous variables, among which the number of producers, N_t, is the state variable. Finally, we assume that aggregate productivity follows the law of motion, $\ln(Z_t) = \rho \ln(Z_{t-1}) + \varepsilon_t$, where ε_t is a normally distributed innovation with zero mean and variance equal to σ_v^2. Table 1 summarizes the benchmark model.

The equilibrium conditions do not have an analytical solution. Consequently, we approximate the system by log-linearizing its equations around the stationary steady state. In this way, a linear dynamic system describes the path of the endogenous variables' relative deviations from their steady-state value, accounting for the exogenous shocks.

2. EQUILIBRIUM PRICE BIAS

Feenstra (1994) and Broda and Weinstein (2010) derive a measure of welfare-consistent price inflation that nests constant elasticity of substitution (CES) preference. Consequently, our corresponding measure of welfare-consistent (gross) price inflation is P_{t+1}/P_t. We express the welfare-based price index (3) as a function of average individual price and quality as well as the number of product varieties, $P_t = \widetilde{p}_{s,t}/(S_t^{\psi} \widetilde{q}_{s,t})$. We then use it to determine the total bias in price changes as the difference between average inflation, $\widetilde{\pi}_{s,t+1}$, and welfare-consistent inflation, π_{t+1}:

$$\widetilde{\pi}_{s,t+1} - \pi_{t+1} = \underbrace{\psi\left(\widehat{S}_{t+1} - \widehat{S}_t\right)}_{\text{Variety bias}} + \underbrace{\left(\widehat{\widetilde{q}}_{s,t+1} - \widehat{\widetilde{q}}_{s,t}\right)}_{\text{Quality bias}}, \tag{19}$$

where $\widetilde{\pi}_{s,t+1} = \widehat{\widetilde{p}}_{s,t+1} - \widehat{\widetilde{p}}_{s,t}$ is the change in average prices, $\pi_{t+1} = \widehat{P}_{t+1} - \widehat{P}_t$ is the change in welfare-consistent prices, and the symbol "$\widehat{}$" denotes the log-deviation of a variable from the steady state. Equation (19) shows that fluctuations in the price bias depend on movements in product variety and quality and that the extent to which changes in product variety affect the price bias depends on the preference for variety, controlled by parameter ψ. By using equation (19) in the context of the fully defined general equilibrium model, we study how exogenous productivity shocks and changes to the underlying structure of the economy affect fluctuations in the price bias. Importantly, equation (19) disentangles how the contribution of product quality and variety in the measure of prices bias change over time. To the best of our knowledge, this study is the first to quantify the two sources of price bias and investigate their cyclical properties in the context of a general equilibrium model.

3. RESULTS

In this section, we investigate the effect of product quality and variety on the price bias using the general equilibrium model. After calibrating the model, we study the cyclical properties of the price bias and disentangle the contribution of product quality and variety. Finally, we focus on the effect of price bias on the measure of cyclical fluctuations.

3.1 Calibration

To produce a quantitative assessment of the theoretical framework, we assign numerical values to the model's parameters, summarized in Table 2.

We calibrate the model on quarterly frequencies. We set the value of discount factor, β, and the Frisch elasticity of labor supply, φ, to 0.99 and 2, respectively.

TABLE 2

CALIBRATION OF THE MODEL

β	Discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	Elasticity of substitution among varieties	3.8
k	Distribution parameter	3.4
ψ	Marginal utility of an increase in varieties	0.36
ϕ	Quality ladder	-0.942
χ	Disutility of supplying labor	0.8549
$\alpha_{\min}$	Minimum idiosyncratic capability level	1
δ	Exogenous destruction rate	0.0308
A	Steady-state level of aggregate productivity	1
ρ	Persistence of aggregate productivity	0.979
σ_υ	Standard deviation of productivity shocks	0.0072
f_E	Steady-state fixed entry costs	1
f	Steady-state value of subsidies	0.0093
θ	Propagation on fixed operational cost	0.291
ϑ	Propagation on entry cost	0.709

These values are within the range of those used in the literature. We set the elasticity of substitution among varieties, σ, to 3.8, as in Ghironi and Melitz (2005), based on empirical findings on U.S. manufacturing in Bernard et al. (2003). We calibrate the parameter k that determines the shape of the distribution of firm-specific capability, as in Ghironi and Melitz (2005). We set the parameter that establishes the marginal utility of an increase in the number of varieties, ψ, to 0.36 (since $\psi = 1/(\sigma - 1)$), consistent with the standard Dixit–Stiglitz preferences. We set the parameter that determines the quality ladder, ϕ, to -0.942 to generate an equilibrium annual CPI bias of 0.8%, as estimated in Broda and Weinstein (2010). For the version of the homogenous quality model, we set the degree of competition in quality equal to zero ($\phi = 0$). We set the value of the disutility of supplying labor, χ, to 0.8549, consistent with Mumtaz and Zanetti (2015), to deliver a steady-state labor supply equal to one. We normalize A, f_E, and $\alpha_{\min}$ to one.

We calibrate the steady-state value of subsidies, f, and the exogenous establishment destruction rate, δ, to match the average annual exit and entry rate equal to 11% and 12.7%, respectively, for all U.S. establishments from the Business Dynamics Statistics (BDS) for the period 1977–2011. By setting the entry rate equal to 12.7% in equation (13) that tracks the law of motion of the firms, we derive the value for the exogenous establishment destruction rate, δ, of 0.0308. The values of the exogenous destruction rate and the exit rate pin down the steady-state value for subsidies, f, of 0.0093.[6] Broda and Weinstein (2010) report annual rates for product destruction and creation equal to 24% and 25%, respectively. Since product turnover is more frequent than establishment turnover, the annual exit and entry rate of 11% and 12.7% at the establishment level observed in the BDS data seem reasonable figures.

We set the persistence parameter, ρ, and the standard deviation of innovations, σ_υ, to 0.979 and 0.0072, respectively, as in King and Rebelo (1999). The coefficients that govern the propagation of productivity on fixed operational costs, θ, and entry costs, ϑ, are set to minimize the distance between some key moments in the observed data and those implied by the theoretical model. In particular, we numerically solve for $J = \min_{\theta, \vartheta} [\widehat{\Psi} - \Psi(\theta, \vartheta)]' V^{-1} [\widehat{\Psi} - \Psi(\theta, \vartheta)]$, where $\widehat{\Psi}$ is the vector containing the standard deviation of establishment entry and exit in the data, $\Psi(\theta, \vartheta)$ is the vector containing the corresponding standard deviation implied by the theoretical model and V^{-1} is the inverse of the variance–covariance matrix of empirical data for establishment entry and exit. This procedure gives the value of $\theta = 0.291$ and $\vartheta = 0.709$.

3.2 Cyclical Properties of Price Bias

We use the general equilibrium model to investigate the cyclical properties of the price bias and determine the contribution of product quality and variety. In the model, exogenous shocks to technology generate cyclical fluctuations. We isolate the effect of changes in quality by comparing the benchmark version of the model that embeds
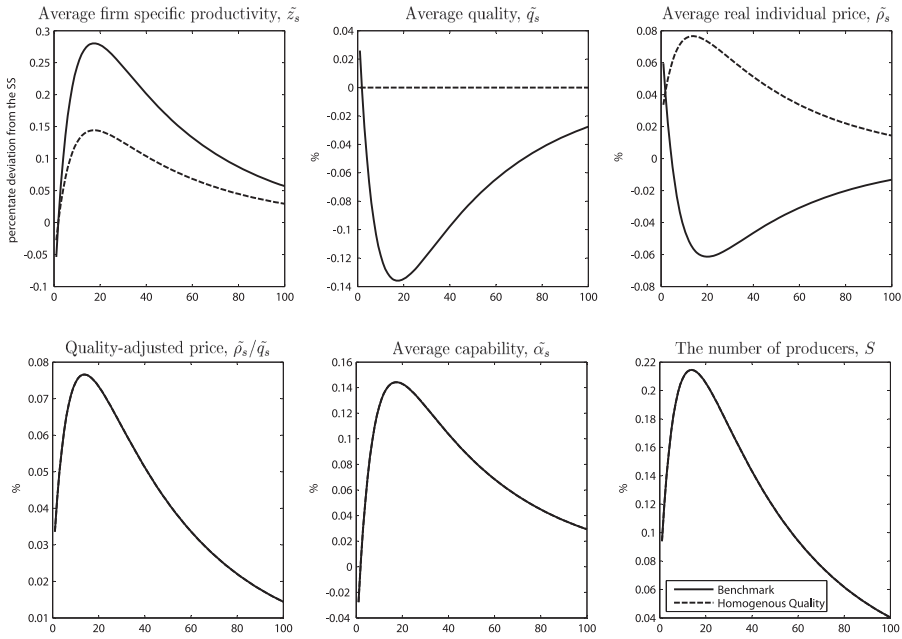
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6. See equation (A3) in Appendix A.

Fig. 1. Responses of Selected Variables to a Productivity Shock.

NOTES: Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the shock for the benchmark model (solid line) and the model with homogeneous quality (dashed line).

both product quality and variety against the alternative model with homogenous quality (i.e., $\phi = 0$).

Since movements in the price bias depend on fluctuations in product quality and variety, it is instructive to outline how the model reacts to the technology shock before focusing on the price bias. Figure 1 shows the responses of key variables to a positive productivity shock for the benchmark model (solid line) and the model with homogeneous quality (dashed line).[7] On impact, the positive technology shock lowers the producer's average capability, $\widetilde{\alpha}_{s,t}$, which, in turn, increases the producer's average quality, $\widetilde{q}_{s,t}$, and reduces the productivity of the average surviving firms, $\widetilde{z}_{s,t}$, in accordance with equation (5). The increase in average marginal cost raises current-period real prices, $\widetilde{\rho}_{s,t}$ to a greater extent in the version of the model that accounts for product quality. In the aftermath of the shock, the average capability and the firm-specific productivity increase and return to the steady state with a hump-shaped trajectory. The producer's average quality decreases and reaches its steady-state level with a hump-shaped pattern. In response to the shock, entry occurs, and the producer's profit increases more than the cost. Therefore, the number of producers, S_t, in the economy also increases. Note that the increase in the number of producers is the

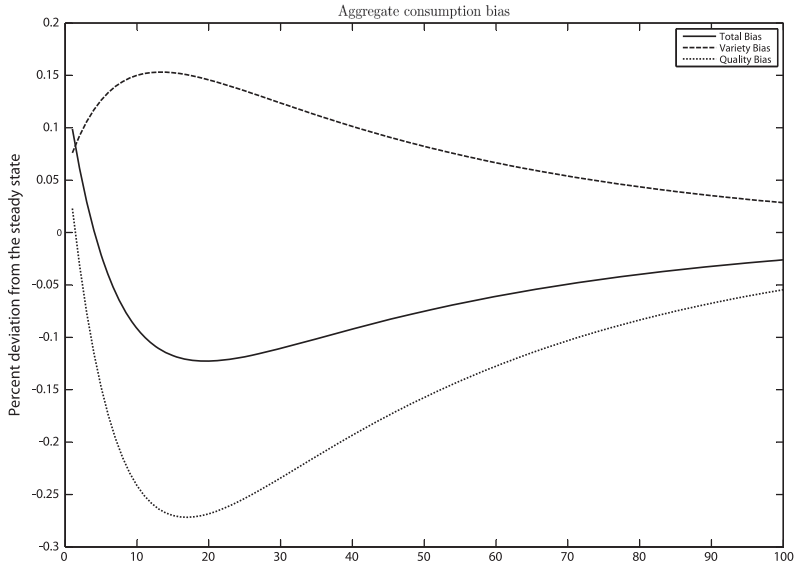7. The Matlab code for the simulated model is available on the website of the journal.

FIG. 2. Responses of the Aggregate Price Bias to a Productivity Shock.

NOTES: Each entry shows the percentage-point response of the aggregate price bias (solid line) and the share of the bias due to product quality (dotted line) and variety (dashed line) to a one-percentage deviation of the technology shock.

same in both models since firms have the same marginal cost of production once costs are adjusted for quality in the benchmark model. Since the quality-adjusted price, $\widetilde{\rho}_{s,t}/\widetilde{q}_{s,t}$, which depends on firm-specific capability alone, increases in exactly the same way in the two models, average profits and therefore the number of surviving producers are the same in both models. Overall, the impulse responses show that firms with costly technology enter the market and produce high-quality goods on impact, but they subsequently employ more efficient technology and lower the average quality in the production process.

We can now use the insights from the model's dynamics to focus on the cyclical properties of the aggregate price bias. Figure 2 shows the bias in aggregate prices (solid line) and the share of the bias due to product quality (dotted line) and variety (dashed line). A few interesting findings emerge. First, the price bias in quality is procyclical on impact, but it becomes countercyclical at longer horizons. Movements in the contribution of product variety to the price bias are procyclical and display high persistence. In response to a positive productivity shock, new firms find it profitable to enter the market, and as a result, the number of varieties increases. However, it takes time to create a new variety due to the assumption of time to build embedded in equation (13), which translates to a persistent dynamic in the number of available varieties. The figure shows that the initial response of the total bias is predominantly driven by the quality bias since the variety bias remains broadly constant. However, in subsequent quarters, the quality bias decreases whereas the variety bias remains

TABLE 3

SECOND MOMENTS

		Y_R	C_R	I_R	L	H	D	Bias
St. dev. (%)	U.S. data	1.23	0.93	5.30	1.83	4.19	4.68	n/a
	Benchmark	1.17	0.80	4.03	0.36	3.82	5.06	0.12
	Homogenous quality	1.17	0.78	4.05	0.36	3.82	5.06	0.10
	Welfare-consistent	(1.21)	(0.83)	(4.08)	(0.36)	(3.82)	(5.06)	(0)
Relative to Y_R	U.S. data	1.00	0.75	4.30	1.49	3.40	3.80	n/a
	Benchmark	1.00	0.69	3.45	0.31	3.28	4.33	0.10
	Homogenous quality	1.00	0.67	3.47	0.31	3.27	4.33	0.08
	Welfare-consistent	(1.00)	(0.69)	(3.38)	(0.30)	(3.17)	(4.19)	(0)
Corr(Y_R. X)	U.S. data	1.00	0.88	0.93	0.92	0.68	−0.15	n/a
	Benchmark	1.00	0.97	0.95	0.99	0.93	0.05	0.51
	Homogenous quality	1.00	0.97	0.96	0.92	0.95	−0.02	0.85
	Welfare-consistent	(1.00)	(0.98)	(0.96)	(0.91)	(0.95)	(0.00)	(0)

stable. Quality and variety bias cancel each other out over medium- and long-run horizons, reducing the total bias. The cancelation effect that materializes in the long run is consistent with the theoretical and empirical findings in Atkeson and Burstein (2010), Fattal Jaef and Lopez (2014), and Burstein and Cravino (2015). These studies establish the irrelevance of disentangling the empirical-based measure from the welfare-based measure since cross-country variations in product quality and the number of varieties cancel each other out in the aftermath of a trade cost shock. Our analysis shows that a similar mechanism holds in a closed economy characterized by productivity shocks.

What is the effect of price bias on the measure of cyclical fluctuations? To answer this question, we compare the second moments of real variables deflated by the welfare-consistent prices, P_t, against those of empirically based prices, $P_{e,t}$. To derive such a measure, for any variable X_t, we deflate the corresponding real variable by the average price index as $X_{R,t} \equiv P_t X_t / P_{e,t}$. Using this method, we define the empirically based aggregate output, $Y_{R,t}$, consumption, $C_{R,t}$, and investment, $I_{R,t}$.[8]

Table 3 provides second moments of selected variables for the U.S. data for benchmark and homogenous-quality economies.[9] The models replicate second moments in the data fairly well. In particular, since the determination of product variety is endogenous, the model is able to replicate movements in product creation and destruction. In the benchmark model, product creation is strongly procyclical whereas product destruction is mildly procyclical, in line with Broda and Weinstein (2010) and Lee and Mukoyama (2015).

Broda and Weinstein (2010) establish that official statistics understate cyclical fluctuations due to their biased measure of aggregate price. The second moments

8. Investment is defined as: $I_{R,t} \equiv v_{R,t} H_t$.

9. All series are detrended by the HP filter, using a smoothing parameter equal to 1,600. Second moments of the theoretical models are computed by the frequency domain technique in Uhlig (1998). Appendix B reports data sources.

of the theoretical model in the table corroborate this fact. For the benchmark model, the second moments omit fluctuations in both product quality and variety. On the other hand, the homogenous quality model does account for variety bias, but the second moments omit fluctuations in product variety. For comparison, we report the welfare-consistent second moments of output, consumption, and investment in parentheses. The entries show that fluctuations in welfare-consistent variables are larger due to movements in product quality and variety. Hence, measurements of cyclical fluctuations, derived using the conventional consumption deflator that abstracts from changes in both product quality and variety, underestimate the variables' true volatility in welfare-consistent measure.

The last column in the table shows that the standard deviation of the bias is lower in the model with homogenous quality compared to the benchmark model, suggesting that product quality predominantly drives movements in the bias. Finally, both models deliver a procyclical price bias, in line with the empirical findings in Broda and Weinstein (2010). Since these result depend on the marginal utility of an increase in variety, ψ, and the quality latter parameter, ϕ, Appendix C shows the sensitivity of the results to alternative calibrations of these parameters.

4. CONCLUSION

This paper develops a general equilibrium model that embeds endogenous product quality and variety and assesses the impact on the aggregate price bias. The analysis shows that the aggregate price bias is procyclical, mainly driven by procyclical movements in product variety. The contribution of product quality becomes countercyclical in the medium to long run whereas the contribution of product variety is procyclical and highly persistent throughout the business cycle. The analysis shows that measurements of fluctuations derived using the conventional consumption deflator underestimate the variables' true volatility.

To simplify the analysis, our model assumes that each firm produces a distinct product variety. However, Broda and Weinstein (2010) document that turnover of product variety may take place within firms. Extending the model to include multiproduct variety and an endogenous determination of product quality, as in Bernard, Redding, and Schott (2010), Eckel and Neary (2010), and Minniti and Turino (2013), would certainly be a useful extension for future research. Finally, the analysis is based on a closed economy model that abstracts from the effect of foreign producers. The presence of foreign firms, however, is potentially important for adjustments in product quality and variety in the aggregate price index, and thus also for changes in the aggregate price bias. Extending the analysis to consider this additional channel remains a task for future research.

APPENDIX A: STEADY STATE

We start by deriving the steady state of the benchmark model. The Euler equation (17) provides

$$\frac{1}{\beta} = (1 - \delta)\left(1 + \frac{\widetilde{d}}{v}\right). \tag{A1}$$

Using the equation of profit of average surviving firms (10) and the equation that determines the cutoff capability level (15), we write equation (A1) as

$$\frac{\widetilde{d}_s}{w} = \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{A2}$$

From the definition of operational profits among producers, we have $\widetilde{d} = S\widetilde{d}_s/N$, and the free entry condition (12) implies $v = w$. Using these relations, we can express equation (A1) as

$$\frac{1}{\beta} = (1 - \delta)\left(1 + \frac{S}{N}\frac{\sigma - 1}{k - (\sigma - 1)}f\right), \tag{A3}$$

which provides the steady-state exit rate, $1 - S/N$, given operational fixed costs, f.

We set the value of χ so that the steady-state labor supply is one. From the law of motion of producers (13), we derive the number of new entrants, $H = \delta N/(1 - \delta)$. Using these relations in the labor market clearing condition (18), it yields

$$\frac{1}{N} = (\sigma - 1)\frac{S}{N}\frac{\sigma - 1}{k - (\sigma - 1)}f + \sigma\frac{S}{N}f + \frac{\delta}{1 - \delta}, \tag{A4}$$

which provides a unique solution for the number of producers, provided the steady-state exit rate, $1 - S/N$. Once the value S is obtained, the steady-state values of other variables is straightforward to derive.

APPENDIX B: DATA

The quarterly data on establishment entry and exit are taken from private sector establishment births and deaths, reported by the Bureau of Labor Statistics (BLS). The annual data on establishment births and deaths are taken from the Business Dynamics Statistics (BDS).[10]

For each variable, the mnemonics and data source are:

Domestic Product, GDP, BEA.

Fixed Private Investment, FPI, BEA.

Personal Consumption Expenditures: Services, PCESV, BEA.

Personal Consumption Expenditures: Nondurable Goods, PCND, BEA.

Gross Domestic Product: Implicit Price Deflator, GDPDEF, BEA.

All Employees: Total Nonfarm, PAYEMS, BLS.

10. These data sets are available at http://www.bls.gov/web/cewbd/table9_1.txt and http://www..census.gov/ces/dataproducts/bds/, respectively.
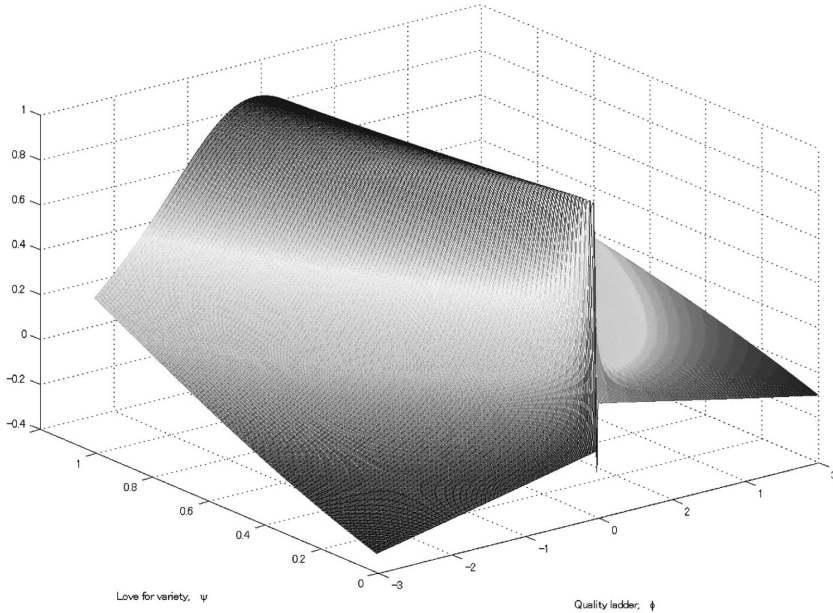
FIG. C1. Sensitivity Analysis.

NOTES: The figure shows how different values for the quality ladder, ϕ, and preference for variety, ψ, affect the correlation between output and the aggregate price bias.

Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing, AWHMAN, BLS.

APPENDIX C: SENSITIVITY ANALYSES

In this appendix, we perform sensitivity analyses on how values for the parameters controlling the quality ladder parameter (ϕ) and the preference for variety parameter (ψ) affect the correlation of the aggregate price bias with the empirically consistent measure of output, $Y_{R,t}$. Figure C1 shows how different values for the degree of competition in quality (i.e., the quality ladder), ϕ, and preference for variety, ψ, affect the correlation between output and the aggregate price bias. Total bias is procyclical for a broad range of values for parameters ϕ and ψ. The contemporaneous correlation between output and total bias is primarily driven by the variety bias. In response to a positive productivity shock, the number of producers increases, which leads to a rise in the variety bias that is proportional to the preference for variety parameter, ψ. Figure C1 shows that for any given value of ϕ the total bias increases proportionally with the value of parameter ψ.

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