On-line Appendix for the paper: State Dependence in Labor Market Fluctuations.*

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Abstract

This document comprises three separate notes that report robustness checks.

- Note 1: Sensitivity analysis for the Threshold Vector Autoregression.
- Note 2: Alternative version of the model with all new matches starting at $\overline{x} = 1$.
- Note 2: Hagedorn and Manovskki (2008)-type calibration.

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1 Note 1: TVAR sensitivity analysis

1.1 Long-Run Restrictions VAR Shock Identification Method

This section illustrates that the results are robust when long-run restrictions, as proposed by Galí (1999), are used to identify the productivity shock.[1] Similar to Michelacci and Lopez-Salido (2007), we identify productivity shocks as being the only driver of the forecast-error variance of productivity at the infinite horizon. There is by now a large number of evidence suggesting that "pure" long-run restrictions have difficulties to recover the "true" shock (see Erceg et al. (2005), Ravenna (2007) and Francis et al. (2014) among others). In our case, this issues becomes more severe as the number of observations per regime is reduced. To bypass these problems, we augment the long-run identification scheme with additional restrictions/information. Namely:

- The productivity shock explains most of the variation of the labour productivity growth between 0 and 40 quarters (as in the benchmark identification strategy);
- A positive productivity shock contemporaneously lowers unemployment and the job separation rate, while increasing the probability of finding a job (sign restrictions).

Table 1: Sign restrictions for the long-run identification scheme

	Labour Productivity	Average Hours	Unemployment Rate	Job Separation Rate	Job Finding Rate
Productivity Shock	+	?	-	-	+

The identification matrix can again be obtained as solution to a similar maximization problem, which augments the standard long-run restriction:

$$\arg\max_{Q} e_1' \left[\sum_{h=0}^{H} \sum_{j=0}^{h-1} \tilde{B}_j \; A \; Q \; Q' A' \tilde{B}_j' \right] e_1$$

subject to:

1. Q is an orthonormal matrix;
2. Q ensures that no other shock can have a permanent effect on the labour productivity;
3. Q satisfies the sign restriction in Table 1.

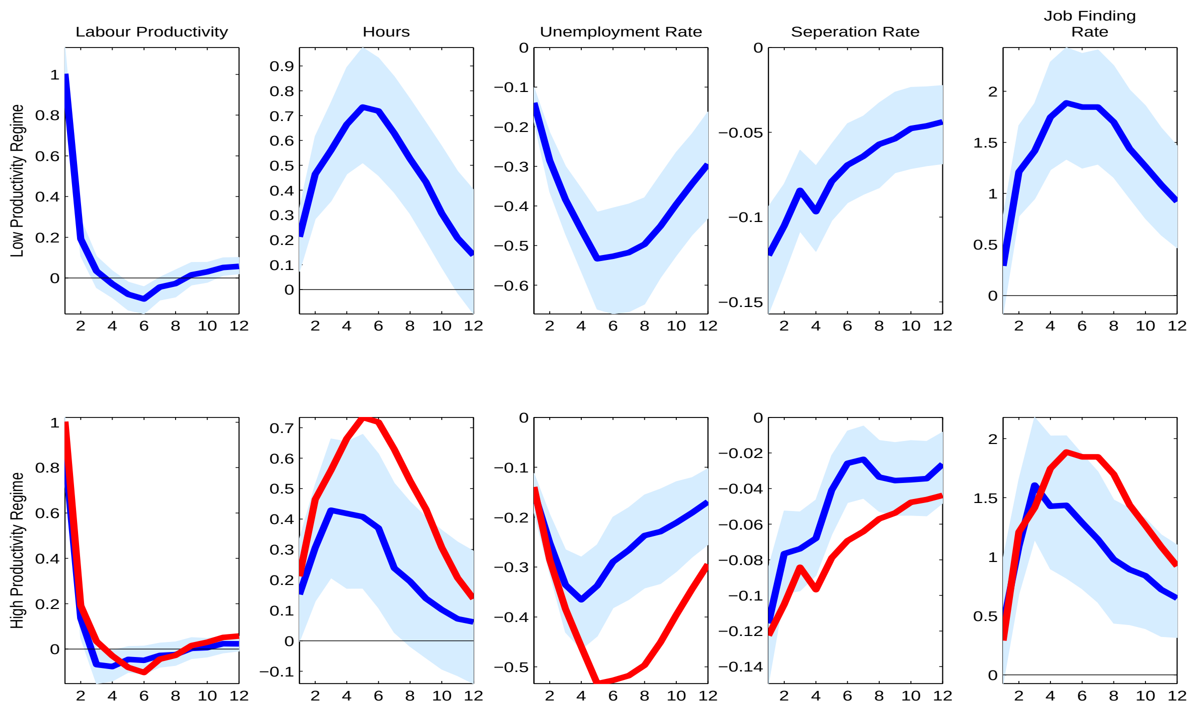
The solution to this highly nonlinear problem is obtained via a minimisation process applied to both regimes and to all posterior draws. Even thought we employ parallel computing techniques to speed up these calculations, the completion of this exercise requires approximately 48 hours.

We investigate whether the benchmark results are robust when a different identification scheme is employed. Here it is studied whether labour market variables display similar asymmetries when the long-run identification scheme discussed in Section 1.1 is used to recover the productivity shock. This exercise allows us to check the sensitivity of the results to the use of

[1] To be clear, in this version the variables enter to the VAR in levels, except for labour productivity, which enters in log differences.

Hamilton filter used in this study to stationarize the data as all series enter in levels (except labour productivity which enters in first differences). Even though the literature expresses serious concerns about the ability of the scheme to correctly identify the true shocks (Erceg et al., 2005; Ravenna, 2007; Francis et al., 2014, among others), it is an intuitive way of identifying a productivity shock. We hope the modifications to scheme discussed in Section 1.1 enhance the power of the scheme to identify the true shocks.

Figure 1: Long Run Identification Scheme: Impulse Responses



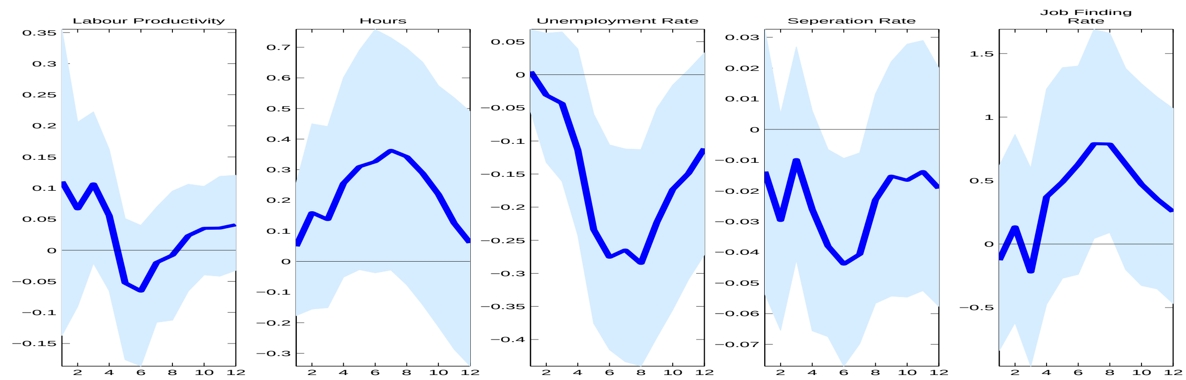
Note. The solid line represents the pointwise median impulse response function, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution. The horizontal axes are in quarters, the vertical axes are in percentage points. The red line in the second row is the pointwise median from the Low Productivity Regime.

Figures 1 and 2 illustrate again that the results are robust to different identification schemes and data detrending approaches.

1.1.1 Gerneralised Impulse Responses: Sensitivity to the Sign of the Shock

Since the model is nonlinear, the responses could be sensitive to the sign of the shock. Figures 3 and 4 show that agents' responses do not display a sensitivity regarding the sign of the shock independently of whether labour productivity is below or above the threshold. The most likely reason for this result is that the TVAR presents a model that is fully linear conditional on being in a given state of the economy. Hence the only source of nonlinearity with respect to the sign of the shock would be coming from the ensuing probability of transitioning across states. Because the average productivity shock is very small relative to the unconditional distribution of productivity, the probability of transitioning across aggregate regimes does not

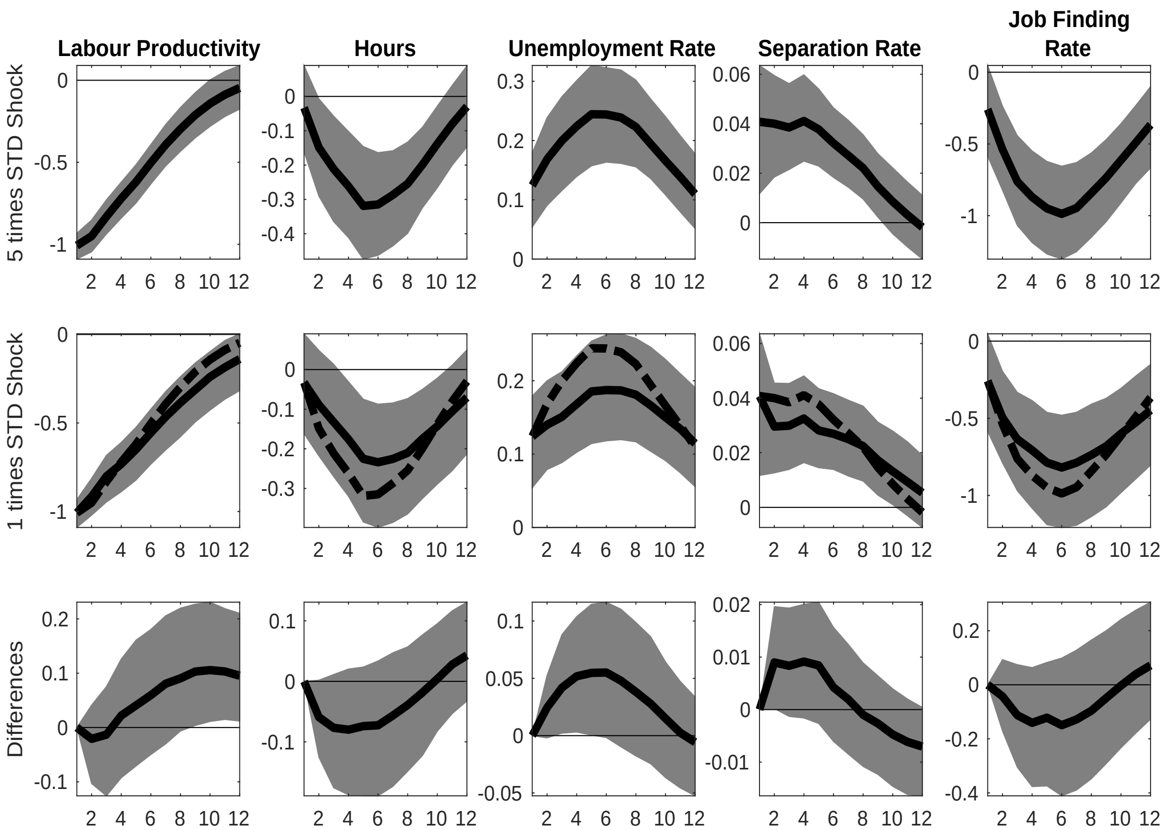
Figure 2: Long Run Identification Scheme: Differences



Note. The solid line represents the pointwise median impulse response function, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution.

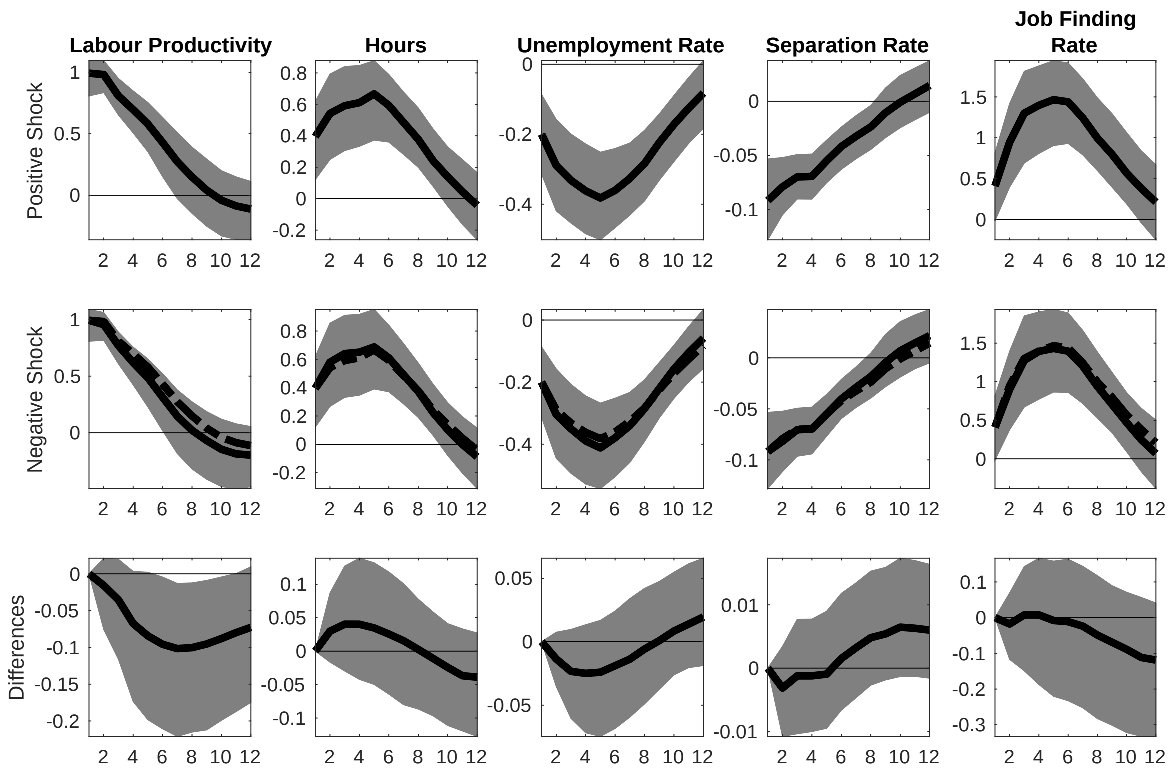
change much after a positive or negative shock if the initial level of productivity is not close to the threshold. A similar intuition explains why there is almost no difference in the response to one and two-standard deviation shocks.

Figure 3: Large versus normal-size shock in a low-productivity regime



Note. The solid line represents the pointwise median impulse response function, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution. The horizontal axes are in quarters, the vertical axes are in percentage deviations from the trend. The third row displays the posterior distribution of the difference between a large (first row) and a normal (second row) productivity shock.

Figure 4: Negative versus Positive Shock in a High Productivity Regime



Note. The solid line represents the pointwise median impulse response function, and the shaded area is the corresponding 16th and 84th percentiles of the posterior distribution. The horizontal axes are in quarters, the vertical axes are in percentage deviations from the trend. The third row displays the posterior distribution of the difference between a negative (first row) and positive (second row) productivity shock.

2 Note 2: Alternative model with new matches starting at $\bar{x}$

This version of the model assumes that newly established matches from unemployed workers and workers searching while on the job have an individual productivity with value $\bar{x}$ that is equal to the mean of the distribution $F(x)$. In subsequent periods, workers have a productivity x different from mean productivity $\bar{x}$ upon receiving of a new productivity draw. This assumption has two important implications. First, it avoids the possibility for a worker searching on the job to accept an offer for a job with lower productivity. Second, and important for our analysis, this version of the model disconnects the job creation from the reservation productivity threshold. As long as the surplus of a match at $(a_t, \bar{x})$ is positive, all new matches from unemployment will turn into employment. Hence, changes in x^r cannot drive the state dependence in the job finding rate.

The surplus function is defined by the following equations

$$S(a_t, x) = \max\{S^{n,c}(a_t, x), S^{s,c}(a_t, x), 0\}, \tag{1}$$

$$\begin{aligned} S^{n,c}(a_t, x) = a_t x - b + \beta \mathbb{E}_t \Big\{ (1-s) \Big[(1-\lambda) S(a_{t+1}, x) + \lambda \int S(a_{t+1}, x') dF(x') \Big] \\ - p(\theta_t) \phi S(a_{t+1}, \bar{x}) \Big\}, \end{aligned} \tag{2}$$

$$\begin{aligned} S^{s,c}(a_t, x) = a_t x - k^s - b + \beta \mathbb{E}_t \Big\{ \Big[1 - p(\theta_t) \overline{F(x^r_{t+1})} \Big] (1-s) \Big[(1-\lambda) S(a_{t+1}, x) \\ + \lambda \int S(a_{t+1}, x') dF(x') \Big] \Big\}. \end{aligned} \tag{3}$$

Assuming that $S(a_{t+1}, \bar{x}) > 0$, the free-entry condition is

$$\frac{k}{q(\theta_t)} = (1-\phi) \beta \mathbb{E}_t \Big[S(a_{t+1}, \bar{x}) \Big]. \tag{4}$$

Finally, the job finding rate is $JFR_t = UE_{t+1}/u_t = p(\theta_t)$.

We calibrate the model using the same approach for the benchmark model, described in the main paper. However, we were unable to match all the same empirical targets because even with a calibrated value for κ_s of 0, we could not match a mean job-to-job rate of 3.2 percent. We thus use λ to match the average job-to-job rate rather than the autocorrelation of the separation rate. As a result, we do not match the latter moment in this version of the model.

The third column in Table 3 the values for the calibrated parameters, while Table 2 shows the simulated target moments. Figure 5 plots the generalized IRFs and shows that the response of the job finding rate to the shock is substantially similar between states of low and high productivity and therefore the variable fails to exhibit state dependence. The key implication of the analysis is that changes in reservation productivity are important for the state dependence in the job finding rate. Once this mechanism is excluded from the model, the other discussed potential channel does not generate large state dependence under this calibration. Despite the fact that the flow value of unemployment (b) is larger than in the benchmark model and therefore it increases the sensitivity of vacancies to changes in productivity, as outlined in Hagedorn and Manovskii (2008), the value is not sufficiently large to generate sizeable state dependence in job creation.

Table 2: Empirical targets and simulated moments from the baseline and alternative model.

		Empirical Targets	Baseline	Mean-x matches
SR	Mean	3.3	3.3	3.2
SR	Standard Deviation	1.7	1.7	1.6
SR	Autocorrelation	0.79	0.81	0.86
JFR	Mean	45	44	46
JFR	Standard Deviation	3.3	3.3	3.1
Job-to-Job	Mean	3.2	3.2	3.1

Note. The first column reports the targets for the moments of the separation rate (SR), the job finding rate (JFR), and the job-to-job rate. The targeted standard deviations of the separation rate and the job finding rate are derived by taking the share of the stationary variance explained by the TVAR (weighted across regimes) multiplied by the unconditional variance. Means and standard deviations are multiplied by 100 for presentational purposes.

Table 3: Parameter values to match empirical evidence for baseline and alternative models

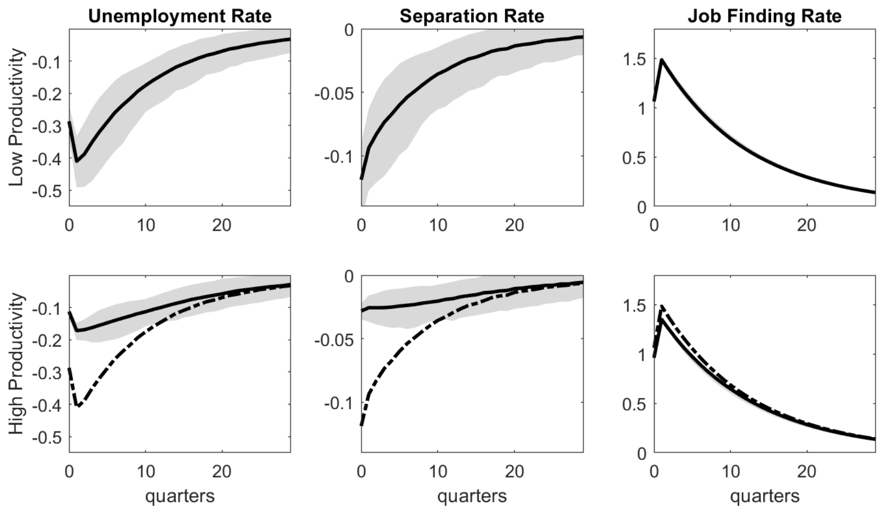
	Description	Baseline	$\overline{x}$ for new matches
b	Flow value of unemployment	0.72	0.84
γ	Matching function efficiency	0.47	0.51
s	Exogenous separation probability	0.03	0.03
σ_x	Standard deviation of x draw	0.115	0.115
λ	Probability of new x draw	0.03	0.155
c_s	Cost of OJS	0.13	0

Table 4: State dependence for the with new matches starting at $\overline{x} = 1$ calibration relative to the TVAR.

	σ|TVAR			Model		
	$\sigma_{p<58pct.}$ (1)	$\sigma_{p>58pct.}$ (2)	Ratio (3)	$\sigma_{p<58pct.}$ (4)	$\sigma_{p>58pct.}$ (5)	Ratio (6)
Prod	1.61	1.25	1.29	1.34	1.17	1.17
U	1.22	0.6	2.04	0.72	0.39	1.90
JFR	5.01	2.49	2.01	1.29	1.06	1.24
SR	0.4	0.17	2.28	0.00	0.00	1.43

Note. Entries are averages of 1,000 simulations over 768 monthly periods aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV. $\sigma_{p<(>)58pct.}$ represents the standard deviation of the variable for the productivity state below (above) the 58th percentile of the productivity distribution, which corresponds with the estimated treshold in the TVAR. All standard deviations are multiplied by 100 for presentation purposes.

Figure 5: Generalized IRFs: model with OJS and new matches at $\bar{x}$.



Note. The solid line represents the mean IRF value in each period. The shaded area represents the 5^{th} and 95^{th} percentiles of the IRF values. Responses of the variables in periods with high (low) aggregate productivity are in left (right) panels. Units on the y-axis are percentage points. In the second row, the dashed line reports the mean IRF from the first row for comparison.

Note 3: Hagedorn and Manovskii Calibration

Baseline model

We assess whether the results hold using a calibration *à la* Hagedorn and Manovskii (2008), henceforth HM08. The essence of this strategy is to internally calibrate the flow value of unemployment b and the worker's bargaining power ϕ (hence moving away from the Hosios condition) to match the mean ratio of wages to productivity and the elasticity of wages with respect to productivity. Based on Hornstein et al. (2011), these values are 0.97 and 0.5, respectively. This strategy is meant to solve the so called "Shimer puzzle" of low volatility in the job finding rate. A high value of b, combined with a low bargaining power for workers, implies that firms derive very small but highly volatile profits from the employment relationship and are therefore highly sensitive to fluctuations in aggregate productivity.

Two observations are in order. First, the HM08 strategy was initially proposed for the DMP model with exogenous separations and no OJS, which was the subject of the original analysis in Shimer (2005). The addition of endogenous separation and OJS already provide improvements over the results of this model even with a more standard calibration. The reason why it may be worth considering the results of our model under the HM08 approach is that state-dependent volatility hinges on the interaction between individual productivity and the flow value of unemployment. Second, the original calibration in HM08, and its extension to the OJS case in Fujita and Ramey (2012), henceforth FR12, are in weekly frequency. We attempted this calibration at both monthly and weekly frequencies with similar results and present only the former for the baseline model.

Similar to the experience of FR12, in our attempts we could not successfully match all the empirical moments targeted in the baseline calibration. As shown also in FR12, it is unfeasible to obtain an average job-to-job rate of 0.032. In fact, FR12 set $k^s = 0$ and still obtain an average job-to-job rate that is effectively 0 (see Section V in FR12 and their replication codes). The almost entire absence of OJS is due to the very low value of the worker's bargaining power needed to produce the empirical elasticity of wages with respect to productivity. A value of ϕ close to 0 effectively eliminates the value of OJS for the worker relative to current employment. Similarly, it is not feasible to use λ to target the autocorrelation of the separation rate, as no value of λ yields an autocorrelation as high as 0.80. In fact, FR12 do not pin down this parameter in their HM08 calibration but simply use the value from their baseline calibration. We follow a similar approach, leaving $\lambda = 0.05$ as in the baseline calibration.

Keeping these shortcomings in mind, the HM08 approach is still illustrative of the quantitative importance of b for determining the asymmetries. Table 5 lists the parameters of this calibration. Table 6 reports the calibration targets. As shown in Table 7, the HM08 calibration generates some differences in fluctuations across states of the business cycle that are much too large compared to the data. In particular, the separation rate exhibits extreme differences across regimes of low and high labor productivity. Fluctuations in job destruction are almost absent in good times(in some simulations the standard deviation is equal to 0) but are very large in bad times.

Overall, this assessment suggests that the HM08 calibration for the model with OJS and endogenous separation overstates the main mechanisms in the model well beyond the empirical magnitudes.

Table 5: Parameters for the model with OJS and endogenous separations, with HM08 calibration.

Parameter	**Description**	**Value**
κ^s	OJS cost	0
b	Flow value of unemployment	0.905
γ	Matching function efficiency parameter	0.22
ϕ	Worker's bargaining power	0.05
s	Exogenous job separation rate	0.028
σ_x	Standard deviation of individual productivity shock	0.055

Table 6: Targets for the model with OJS and endogenous separations, with HM08 calibration.

	Target	Model
Job Finding Rate - mean	0.45	0.44
Separation Rate - mean	0.03	0.028
Separation Rate - standard deviation	0.017	0.016
$\epsilon_{w,p}$	0.5	0.49
Mean wage/productivity	0.97	0.96

Table 7: State dependence for the baseline model calibrated with the Hagedorn and Manovskii (2008)-type calibration relative to the TVAR.

	σ|TVAR			Model		
	$\sigma_{p<58pct.}$ (1)	$\sigma_{p>58pct.}$ (2)	Ratio (3)	$\sigma_{p<58pct.}$ (4)	$\sigma_{p>58pct.}$ (5)	Ratio (6)
Prod	1.61	1.25	1.29	1.40	1.31	1.17
U	1.22	0.6	2.04	1.12	0.46	1.90
JFR	5.01	2.49	2.01	4.41	3.54	1.24
SR	0.4	0.17	2.28	0.20	0.03	10.19

Note. Entries are averages of 1,000 simulations over 768 monthly periods aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV. $\sigma_{p<(>)58pct.}$ represents the standard deviation of the variable for the productivity state below (above) the 58th percentile of the productivity distribution, which corresponds with the estimated treshold in the TVAR. All standard deviations are multiplied by 100 for presentation purposes.

Table 8: Parameters for the model with exogenous separation, with the Hagedorn and Manovskii (2008) calibration.

Parameter	**Description**	**Value**
β	Discount factor	0.9991
κ	Vacancy cost	0.17
b	Flow value of unemployment	0.925
η	Elasticity of matching with respect to vacancies	0.5
γ	Matching function efficiency paramete	0.08
ϕ	Worker's bargaining power	0.06
s	Exogenous job separation rate	0.0094
ρ	Persistence parameter of aggregate productivity	0.9895
σ	Standard deviation of aggregate productivity shock	0.0034

2.1 No OJS and exogenous separations

The simplest version of the DMP model, as reported in Shimer (2005), abstracts from both OJS and endogenous separations. HM08 proposed their calibration approach as a solution to the "Shimer puzzle" related to this version of the model. Like the original paper, we use here a weekly calibration. The weekly exogenous separation rate is set to 0.0094 and the average job finding rate targeted in the internal calibration is 0.129. When turned into monthly rates (see HM08), these values correspond to a separation rate of 0.033 and a job finding rate of 0.45. The discount rate β is also set to the same value as in HM08.

Tables 8, 9, and 10 report the results.

Table 9: Targets for the model with OJS and endogenous separations, with HM08 calibration.

	Target	Model
Job Finding Rate - mean	0.139	0.139
$\epsilon_{w,p}$	0.5	0.49
Mean wage/productivity	0.97	0.96

Table 10: State dependence for the model with exogenous separation calibrated and no OJS with the Hagedorn and Manovskii (2008)-type calibration relative to the TVAR.

	σ|TVAR			Model		
	$\sigma_{p<58pct.}$ (1)	$\sigma_{p>58pct.}$ (2)	Ratio (3)	$\sigma_{p<58pct.}$ (4)	$\sigma_{p>58pct.}$ (5)	Ratio (6)
Prod	1.61	1.25	1.29	1.34	1.1743	1.1696
U	1.22	0.6	2.04	0.72	0.39	1.8995
JFR	5.01	2.49	2.01	1.29	1.06	1.2432
SR	0.4	0.17	2.28			

Note. Entries are averages of 1,000 simulations over 3,328 weekly periods aggregated at quarterly frequency and have the same length as the period 1950:I-2014:IV. $\sigma_{p<(>)58pct.}$ represents the standard deviation of the variable for the productivity state below (above) the 58th percentile of the productivity distribution, which corresponds with the estimated treshold in the TVAR. All standard deviations are multiplied by 100 for presentation purposes.

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