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Monetary policy in open economies with production networks[☆]

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ABSTRACT

This paper studies monetary policy design in small open economies with cross-border and input–output linkages. We derive the divine coincidence (DC) Phillips curve linking the output gap to a DC index that weights each sector's inflation by sectoral contents in domestic consumption and exports, and of domestic labor. Output gap targeting can be implemented by stabilizing the DC index, which assigns larger weights to sectors that supply more inputs directly or indirectly to domestic output and face larger expenditure-switching effects. Disregarding openness or treating the economy as one sector *overemphasizes* inflation in sectors that export directly or indirectly, and *underemphasizes* inflation in sectors facing large expenditure-switching effects. We quantify our theoretical results using the World Input–Output Database, showing that the Phillips curves are steeper in open than closed economies, and that output gap targeting is near-optimal as in closed economies and outperforms alternative policies ignoring cross-border or input–output linkages.

1. Introduction

Modern production revolves around intricate input–output (IO) relations within domestic firms and between domestic and foreign firms, and the position and import–export intensity of each domestic firm along the production networks are critical for an economy's response to shocks and the efficacy of stabilizing economic policies. Disruptions to the global supply chain during trade tensions between China and the US since the introduction of the "China Section 301-Tariff Actions" in 2018, the COVID-19 pandemic, and at the outset of the Trump administration's trade actions in 2025 exemplify the primal role of international input–output linkages for the changes in output and prices and the stance of monetary policy.[1]

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[1] See Auray et al. (2024) and Bai et al. (2024, 2025) for discussions on the impacts of trade barriers and COVID-19 on output and monetary policy.

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Yet, there is no systematic research focused on the design of monetary policy in open economies with both cross-border and input–output relations — despite two strands of literature providing distinct insights on the issue. On the one hand, in a one-sector small open economy (SOE) model with nominal price rigidities and without input–output relations as per (Galí and Monacelli, 2005), the optimal monetary policy stabilizes domestic inflation, taking into account the terms of trade. On the other hand, in a multi-sector closed economy with IO linkages as per (Rubbo, 2023), the monetary policy closing the output gap should target a weighted average of sectoral inflation with the weights proportional to Domar weights (i.e., sectoral sales-to-GDP ratio) to account for the propagation of sectoral distortions along input–output linkages.

In light of these separate findings, it remains unknown what the policy prescription should be for a monetary authority that operates in an open, multi-sector economy with both input–output and cross-border relations between firms. Our paper sheds light on this outstanding issue by revealing the separate roles of input–output and cross-border linkages in the design of monetary policy, and studying the pitfalls of monetary policy that disregards either type of linkages.

We study these issues by developing a small open economy model with production networks between domestic and foreign sectors that are subject to nominal rigidities. Our model combines the one-sector open economy framework in Galí and Monacelli (2005) with the multi-sector, production network framework similar to Rubbo (2023), La'O and Tahbaz-Salehi (2022), and Ghassibe (2021b). In our multi-sector economy with nominal rigidities, inflation in the different sectors relates to sectoral markup wedges that prevent attainment of allocations in the flexible-price equilibrium. The cross-border and input–output linkages further propagate these sectoral markup wedges throughout the economy, resulting in the output gap — defined as the difference between the aggregate output in the sticky-price and the flexible-price equilibria.

We show that the output gap is proportional to a weighted average of sectoral markup wedges. The weight assigned to a sector's markup wedge — which we call the sectoral OG weight — crucially depends on the interplay of cross-border and input–output linkages.[2] The size of the sectoral OG weight is determined by three channels that rely on the distinct roles of the sector for the aggregate output in the network economy: (i) the *Consumer Price Index* (*CPI*), (ii) the *expenditure-switching*, and (iii) the *profit* channels. While the *CPI channel* is also present in closed economies, the *expenditure-switching* and *profit* channels are unique to open economies.[3]

In the *CPI channel*, negative sectoral markup wedges are associated with a lower CPI than in the flexible-price equilibrium. The negative CPI gap links to a positive output gap through a higher real wage that increases domestic labor supply and through a lower CPI that increases the net income from trade in units of domestic consumption. In the *expenditure-switching channel*, a negative sectoral markup wedge reduces the prices of domestic relative to foreign products and induces a switching of domestic and foreign expenditures from foreign to domestic goods, thereby increasing domestic labor income and relating to a positive output gap. The *profit channel* comprises two sub-channels of export profit and imported factor, respectively. In the export profit sub-channel, negative sectoral markup wedges lower the domestic prices that are the (opportunity) costs of exported goods, thereby increasing export profits and relating to a positive output gap. In the imported factor sub-channel, negative sectoral markup wedges raise domestic sectors' imported-factor costs relative to sales, thus reducing producers' profits and linking to a negative output gap.

The sizes of the three foregoing channels are determined by the different roles of the sector in the open-economy input–output network as a supplier of inputs to both domestic and foreign demand, as well as a customer for domestic labor inputs — which we measure using different sectoral relevance metrics. Because the CPI is the price of aggregate output, the size of the *CPI channel* is determined by the sector's direct and indirect (via the *downstream* sectors) contribution to domestic aggregate output as a supplier of inputs — measured by the sectoral *total content in domestic consumption*. The size of the *expenditure-switching channel* — measured by the sectoral *generalized expenditure-switching elasticity* — is proportional to three components: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on domestic sectors' prices; (ii) the impacts of domestic sectors' prices on the domestic and foreign expenditures on domestic goods — measured by the *expenditure-switching elasticity*; and (iii) the sector's direct and indirect (via upstream sectors) use of domestic labor factor — measured by the sectoral *total content of domestic labor*.[4] The size of the *export profit sub-channel* is proportional to two components: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on domestic sectors' prices, and (ii) the share of the sector's export value in domestic output. Finally, the size of the *imported factor sub-channel* is also proportional to two components: (i) the share of the sector's sales in domestic output (i.e., the sectoral *Domar weight*), and (ii) the sector's direct and indirect (via upstream sectors) use of imported intermediate inputs — measured by the sectoral *total content of foreign factor*.

Our sectoral relevance metrics and OG weights encompass those in the closed economy framework with production networks à la (Rubbo, 2023; La'O and Tahbaz-Salehi, 2022), showing that the OG weight is equal to the total content in domestic consumption and the Domar weight in closed economies that abstract from cross-border linkages, where the expenditure-switching and profit channels are absent.

Using the sectoral OG weights that link sectoral markup wedges to the output gap, we derive the *divine coincidence Phillips curve* (DC Phillips curve), which links the output gap to an aggregate inflation index — the *divine coincidence index* (DC index) — thereby allowing for the simultaneous stabilization of domestic inflation and the output gap, and achieving the divine coincidence. The DC index is a weighted average of domestic sectoral inflation, with each sector's relative weight given by its *normalized sectoral OG*

[2] We follow Rubbo (2023) to define the aggregate real output as the domestic aggregate consumption. Therefore, our sectoral OG weights essentially relate sectoral markup wedges to the domestic consumption gap.

[3] The *expenditure-switching* channel is standard in the international macroeconomic literature. See Engel (2002) for a review.

[4] The (generalized) expenditure-switching elasticity corresponds to the standard expenditure switching effect in the international macroeconomic literature.

weight. This *normalized OG weight* equals the product of a sector's OG weight and its price rigidity — which links positive sectoral inflation to the negative sectoral markup wedge — divided by a normalizer given by the sum of price-rigidity-adjusted sectoral OG weights. While the *normalized OG weights* determine the relative sectoral weights in the DC index, the *sum of price-rigidity-adjusted sectoral OG weights* is inversely related to the slope of the DC Phillips curve and determines its size through the different channels in the OG weight. At the sector level, the sectoral Phillips curves include both output-gap and cost-push driven inflation, similar to those in closed economies with production networks.

We show that the slopes of the DC and the sectoral Phillips curves in open relative to closed economies depend on the balance between two main countervailing forces: (i) in the CPI channel, the domestic sectors' content in domestic consumption is smaller in open than in closed economies, reducing the elasticity of the output gap to domestic inflation and thereby producing a *steeper* slope in open economies; and (ii) the positive expenditure-switching channel, present only in open economies, increases the elasticity of the output gap to domestic inflation and hence produces a *flatter* slope in open economies.

The DC Phillips curve implies that the monetary policy of *output gap targeting* can be implemented by targeting the DC index to zero. Accordingly, we examine how cross-border and input–output linkages determine output gap targeting through their influence on the *normalized OG weights* in the DC index, focusing on the pitfalls of two alternative output gap targeting policies: (i) one that abstracts from *input–output linkages*, as in the one-sector small open economy literature, and (ii) one that abstracts from *cross-border linkages* and targets the closed-economy DC index based on normalized Domar weights —i.e., the Producer Price Index (PPI) targeting policy in closed economies with production networks à la (Rubbo, 2023).[5]

In the one-sector SOE model — which abstracts from input–output linkages — output gap targeting is the optimal monetary policy, as it simultaneously stabilizes domestic inflation, consistent with the "divine coincidence" result in Galí and Monacelli (2005). However, in the one-sector SOE literature, output gap targeting is implemented by targeting the PPI inflation, which weights sectoral inflation by sectoral sales — proportional to the Domar weights rather than our OG weights. We contribute to this literature by deriving the appropriate sectoral weights in the domestic aggregate inflation index needed to close the output gap, which differ from the PPI weights and account for the interplay of cross-border and input–output linkages.

Output gap targeting that abstracts from cross-border linkages adopts normalized Domar rather than OG weights, and it can overemphasize the relevance of a domestic sector's inflation for the (domestic) output gap. This is because in open economies, a sector's Domar weight is proportional to its *total* sales — which encompasses its direct and indirect (via downstream sectors) contributions to not only *domestic* but also *foreign* demand. Output gap targeting that abstracts from cross-border linkages can also underestimate the relevance of a domestic sector's inflation by ignoring its effect on domestic-to-foreign prices and, in turn, on the demand for domestic products and labor (i.e., by neglecting the expenditure-switching channel).

Overall, we find that output gap targeting policies both used in the one-sector SOE literature and disregarding cross-border linkages are equivalent to the PPI targeting — which uses normalized Domar instead of OG weights. The extent to which the PPI targeting over- or under-emphasizes the relevance of a sector's inflation depends on the quantitative strength of the aforementioned countervailing sectoral forces for a sector relative to the other sectors. This theoretical possibility motivates our quantitative analysis, which calibrates the model to major economies based on data from the World Input–Output Database (WIOD) to assess the different channels across different economies.

We derive the welfare loss function and the resulting optimal monetary policy in our SOEs with production networks. We show that the welfare loss (up to the second-order approximation) comprises the losses from the output gap misallocation and the within- and across-sector misallocation — similar to those in closed economies à la (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023) — as well as the cross-border misallocation that is unique to open economies. The optimal monetary policy — which minimizes the welfare loss subject to sectoral Phillips curves — cannot simultaneously eliminate the output gap, the within- and across-sector, and the cross-border misallocations, and thus needs to trade off among them. In other words, the "divine coincidence" that holds in one-sector SOEs à la (Galí and Monacelli, 2005) breaks down in our multi-sector SOEs.

Input–output and cross-border linkages enter the welfare loss function and, therefore, play an important role in optimal monetary policy. In one-sector SOEs *without input–output linkages*, only within-sector and cross-border misallocations remain, and the output gap is proportional to domestic inflation, making welfare loss proportional to the squares of domestic inflation and hence achieving the divine coincidence as in Galí and Monacelli (2005). In multi-sector closed economies *without cross-border linkages*, the cross-border misallocation in the welfare loss function is absent.

To quantify the relevance of the different channels and countervailing forces in our model for the DC Phillips curve and the relative sectoral weights in the DC index, as well as to determine the welfare differences across alternative monetary policies, we calibrate the model to the data from the WIOD comprising 43 countries with 56 major sectors for the year 2014. We show that both the DC and sectoral Phillips curves are steeper in open economies relative to closed economies. The steeper slopes are mainly driven by the smaller domestic sectors' contents in domestic consumption in open than closed economies, which reduces the CPI channel across all sectors and steepens the Phillips curves.

We show that the CPI and expenditure-switching channels explain the bulk of the variation in the normalized OG weights, with the importance of these two channels decreasing and increasing with the openness of the economy, respectively. The percentage difference between the normalized Domar and OG weight — which captures the pitfalls in the output gap targeting that disregards cross-border linkages — is primarily driven by the sector's *export intensity* component — which captures the sector's direct and indirect (via downstream sectors) contribution to exports — and the expenditure-switching component.

[5] For simplicity throughout the paper, we refer to the monetary policy that targets the DC index using price-rigidity-adjusted PPI (CPI) weights as the *PPI (CPI) targeting*.

We use regression analysis to study the rule-of-thumb combinations of sectoral relevance metrics to approximate the normalized sectoral OG weights. We show that the normalized sectoral OG weights can be closely approximated by a linear combination of *total content in domestic consumption* and *generalized expenditure-switching elasticity*. We also reveal that the normalized Domar-OG differences — capturing the pitfall in the output gap targeting that disregards cross-border linkages, or equivalently, the PPI targeting policy in the one-sector SOE literature — can be closely approximated by a linear combination of *export intensity* and *ratio of generalized expenditure-switching elasticity to the Domar weight*. Therefore, our regression analysis implies that output gap targeting should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption, and that face larger expenditure-switching effects. Disregarding cross-border linkages or treating the economy as a one-sector SOE overstates inflation in sectors that export intensively — directly and indirectly — and understates inflation in sectors facing large expenditure-switching effects.

Finally, we compare the welfare under alternative monetary policies, showing that output gap targeting is close to the optimal monetary policy — as in closed economies with production networks à la (Rubbo, 2023) — and it outperforms three alternatives: (i) the PPI targeting that targets the aggregate inflation index using normalized Domar weights (and therefore abstracts from the cross-border linkages); (ii) the output gap targeting that accounts for cross-border but abstracts from input–output linkages; and (iii) the CPI targeting that targets the aggregate inflation index using normalized CPI weights (and thus abstracts from both cross-border and input–output linkages). For instance, in Mexico, output gap targeting outperforms the PPI targeting and output gap targeting that ignores IO linkages by 67% and 99%, respectively, toward the optimal monetary policy in terms of welfare. In the more open economy of Luxembourg, welfare improvements by output gap targeting are larger at 95% and 99%, respectively. In the relatively closed economy of the US, however, the welfare difference between the output gap and PPI targeting is limited, indicating that international trade plays a minor role in monetary policy design for countries with low openness. Accordingly, our quantitative analysis underscores the importance of accounting for both input–output and cross-border linkages in the design of monetary policy in small open economies with production networks.

Related literature. Our paper is related to four separate strands of literature. First, we relate to literature on the design of monetary policy in closed economies with production networks. Rubbo (2023), La'O and Tahbaz-Salehi (2022), and Xu and Yu (2025) show that in closed economies, output gap targeting is nearly optimal, and that it weights inflation in the different sectors according to the sectoral Domar weights that account for the structure of the domestic production network. La'O and Tahbaz-Salehi (2025) study the optimal fiscal and monetary policies in a closed economy. Compared to the foregoing studies, we show that output gap targeting in open economies is nearly optimal as in closed economies, but it needs to account for the interplay between cross-border and input–output linkages.

Second, our paper relates to literature that investigates the aggregation of sectoral distortions and shocks. Chari et al. (2007) use labor and efficiency wedges to characterize the aggregation of disaggregated shocks and distortions. Acemoglu et al. (2012) show that with input–output linkages, idiosyncratic microeconomic shocks can propagate into aggregate fluctuations. Bigio and La'O (2020) extend that analysis to study a closed economy with production networks; they reveal that the efficiency wedge does not include first-order distortions and that only the labor wedge is critical to first-order economic efficiency. We generalize their results to an open economy with international production networks. Baqaee and Farhi (2024) study distortions in a global economy with interconnected countries and sectors. Elliott and Jackson (2024) examine the propagation of supply chain disruption in international production networks. Compared to their work, we investigate the distortions in SOEs and focus on the design of monetary policy.

Third, our paper relates to literature on the transmission of monetary policy in production networks. Ghassibe (2021a,b) and Afrouzi and Bhattarai (2023) develop an analytical characterization of the transmission mechanism of monetary policy in closed economies with production networks. Nakamura and Steinsson (2010) and Pasten et al. (2020) provide a numerical characterization of the effect of monetary policy on aggregate output and inflation. Silva (2024) explores how the production network alters the propagation of sectoral shocks into the consumer price index in small open economies. Kalemli-Ozcan et al. (2025) develop a New Keynesian open economy model incorporating global production networks and trade distortions to study the interaction between monetary policy and trade. Compared to these works, we focus on the design rather than the transmission of monetary policy in network economies.

Fourth, our paper is linked to the numerous studies on the design of monetary policies in small open economies without production networks. While earlier work focuses on one-sector small open economies (e.g., De Paoli, 2009; Galí and Monacelli, 2005; Soffritti and Zanetti, 2008), more recent work—(Matsumura, 2022; Wei and Xie, 2020)—explore small open economy models with multiple sectors. Compared to these foregoing studies, we derive closed-form solutions for the output gap targeting and optimal monetary policies and provide a comprehensive analysis of the design of monetary policies in small open economies with fully-fledged cross-border and input–output linkages.

Outline. The remainder of the paper is organized as follows. Section 2 describes our model of a small open economy with production networks. Section 3 studies the sectoral OG weights that link sectoral markup wedges to the output gap. Section 4 derives the Phillips curves and analyzes the output gap targeting policy. Section 5 derives the welfare loss function and optimal monetary policy. Section 6 quantifies the theoretical results and compares the welfare under alternative monetary policies. Section 7 concludes the paper.

2. Small open economy with production networks

2.1. Environment

The static, small open economy is populated by a representative household consuming domestic and imported sectoral products and supplying labor in exchange for wage income, a government that levies sector-specific taxes and manages the aggregate demand by controlling the money supply, and producers that operate in $N \in \mathbb{N}_+$ different sectors, indexed by $i \in \{1, 2, \dots, N\}$.

Each sector i comprises two types of producers: (i) a unit mass of monopolistically competitive firms indexed by $f \in [0, 1]$ that transform labor and intermediate inputs into differentiated goods, and (ii) a unit mass of perfectly competitive firms that pack the differentiated goods of each sector into a domestic sectoral product — which is both used domestically and exported to foreign countries. Each domestic sectoral product has a counterpart foreign sectoral product available for import. Consumption and intermediate inputs comprise domestic and foreign sectoral products.

2.2. Producers

Monopolistically competitive firms. Within each sector i, monopolistically competitive firms use a common constant-returns-to-scale production technology to transform labor and intermediate inputs into differentiated goods. The production technology of each firm f in sector i is

$$Y_{if} = A_i \cdot \Big(\frac{L_{if}}{\alpha_i}\Big)^{\alpha_i} \prod_{j=1}^{N} \Big(\frac{X_{if,j}}{\omega_{i,j}}\Big)^{\omega_{i,j}}, \tag{1}$$

where A_i is the sector-specific productivity shock, Y_{if} is the output of firm f in sector i, L_{if} is its labor input, and $X_{if,j}$ is the intermediate input acquired from sector j. Parameter α_i is the share of labor, and $\omega_{i,j}$ is the share of intermediate inputs from sector j. The collection of $\{\omega_{i,j}\}_{i,j}$ characterizes the input–output table. Constant returns-to-scale implies that $\alpha_i + \sum_{j=1}^{N} \omega_{i,j} = 1$.

The openness of the economy is reflected in the composition of $X_{if,j}$, which is aggregated from a domestic sectoral product $X_{Hif,Hj}$ and an imported foreign sectoral product $X_{Hif,Fj}$ according to the following constant-elasticity-of-substitution technology:

$$X_{if,j} = \Big(v_{x,i,j}^{\frac{1}{\theta_j}} X_{Hif,Hj}^{\frac{\theta_j-1}{\theta_j}} + (1 - v_{x,i,j})^{\frac{1}{\theta_j}} X_{Hif,Fj}^{\frac{\theta_j-1}{\theta_j}}\Big)^{\frac{\theta_j}{\theta_j-1}}, \tag{2}$$

where θ_j is the elasticity of substitution between domestic and foreign sectoral products in intermediate input $X_{if,j}$. $v_{x,i,j}$ is the home bias parameter, which in equilibrium is equal to the steady-state expenditure share of $X_{Hif,Hj}$ in the composite intermediate input $X_{if,j}$.

The total cost of inputs used by the firm is

$$TC_{if} = WL_{if} + \sum_{j=1}^{N} (P_j X_{Hif,Hj} + S \cdot P^{*}_{IM,Fj} X_{Hif,Fj}), \tag{3}$$

where W is the nominal wage rate, P_j is the domestic sectoral price, $P^{*}_{IM,Fj}$ is the exogenous sectoral import price denominated in the foreign currency, and S is the nominal exchange rate. Given output Y_{if} and the production technology in Eq. (1), the firm optimally chooses labor and intermediate inputs to minimize the total cost TC_{if}, which yields the marginal cost of production that equals the average cost due to the constant-return-to-scale technology. Moreover, because all firms f in each sector i share the same production technology and face the same input prices, the marginal cost of production is identical across all firms in sector i, and we denote it by Φ_i.

We model nominal rigidity as a static Calvo-pricing friction where only firms indexed by $f \le \delta_i \in [0, 1]$ can choose their desired price $P_i^{\#}$ and the remaining firms maintain the price at the steady-state level. We refer to $(1 - \delta_i)/\delta_i$ as the price rigidity of sector i. In each sector i, firms operate in a monopolistically competitive market and receive a sectoral subsidy rate τ_i on sales. Those firms that can adjust their prices set the desired price to maximize profit.

In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods that the monopolistically competitive firms produce into a sectoral product using a constant-elasticity-of-substitution technology, with the within-sector elasticity of substitution between different firms' products equal to $\varepsilon_i > 1$. The price of the domestic sector i's products — denoted by P_i — is the selling price of its sectoral goods packer. We define the sectoral markup and the desired sectoral markup as $\mu_i \equiv P_i/\Phi_i$ and $\mu_i^{\#} \equiv P_i^{\#}/\Phi_i$, respectively. We further define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz.*, $\ln(\mu_i) - \ln(\mu_i^{\#})$. Shown in Appendix A are the expressions for the nominal profit, demand function, and desired prices of the firms, as well as the sectoral product and price index.

2.3. Households

The preference of the representative household is described by the utility function defined over domestic aggregate consumption C and labor supply L:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\varphi}}{1+\varphi}, \tag{4}$$

where σ is the coefficient of relative risk aversion, and φ is the inverse of the Frisch elasticity of labor supply. Following Rubbo (2023), in our static model without investment, we define the aggregate (real) output as the domestic aggregate consumption C.

The (domestic) aggregate output C combines sectoral consumption $\{C_i\}_i$ that comprises domestic and imported components, C_{Hi} and C_{Fi}, respectively, for each sector i, represented by:

$$C = \prod_{i=1}^{N}\left(\frac{C_i}{\beta_i}\right)^{\beta_i}, \quad \text{where} \quad C_i = \left(v_i^{\frac{1}{\theta_i}} C_{Hi}^{\frac{\theta_i-1}{\theta_i}} + (1-v_i)^{\frac{1}{\theta_i}} C_{Fi}^{\frac{\theta_i-1}{\theta_i}}\right)^{\frac{\theta_i}{\theta_i-1}}. \tag{5}$$

Vector $\{\beta_i\}_i$ is the set of consumption shares satisfying $\sum_{i=1}^{N}\beta_i = 1$, and v_i is the home bias parameter for the consumption of sectoral products. P_C is denoted as the price index of the aggregate output C — *viz.*, the CPI. The budget constraint of the household is:

$$P_C C = \sum_{i=1}^{N}\left(P_i C_{Hi} + S \cdot P^*_{IM,i} C_{Fi}\right) \le WL + \sum_{i=1}^{N}\int_0^1 \Pi_{if}\, df + T, \tag{6}$$

where Π_{if} is the profit of firm f in sector i, and T is the lump-sum transfer of tax revenues to the household. To purchase the consumption goods, households demand the following amount of money as the medium of exchange: $M_d = P_C C$. Cost minimization by the household yields the CPI:

$$P_C = \prod_{i=1}^{N}\left(v_i P_i^{1-\theta_i} + (1-v_i)(S \cdot P^*_{IM,Fi})^{1-\theta_i}\right)^{\frac{\beta_i}{1-\theta_i}}. \tag{7}$$

2.4. International trade

In addition to the sales subsidy $\{\tau_i\}_i$, the government also imposes sector-specific export tax $\{\tau_{EX,i}\}_i$ on the products exported to foreign countries. The no-arbitrage condition implies that there is no difference between the prices that producers receive from exporting (i.e., $(1-\tau_{EX,i})P_{EX,i}$) or from selling domestically (i.e., P_i): $(1-\tau_{EX,i})P_{EX,i} = P_i, \quad \forall i \in \{1,2,\dots,N\}$.

The export demand for sector i's product is modeled as the reduced-form demand function[6]:

$$Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}, \tag{8}$$

where $D^*_{EX,Fi}$ is the exogenous component of foreign demand, $P_{EX,i}/S$ is the price of the exported domestic sector i goods in units of foreign currency, and the export demand is inversely related to domestic goods' export price, with $\theta_{F,i}$ as the price elasticity of export demand.

Trade is balanced in the static economy, which requires the value of exports to be exactly identical to the value of imports in the whole economy, resulting in the following[7]:

$$\sum_{i=1}^{N} P_{EX,i} Y_{EX,i} = S \sum_{i=1}^{N} P^*_{IM,Fi}\left(\sum_{j=1}^{N}\int_0^1 X_{Hjf,Fi}\, df + C_{Fi}\right). \tag{9}$$

This trade balance condition pins down the endogenous nominal exchange rate S in equilibrium.

2.5. Aggregate states

There are three types of exogenous sector-level states in the economy: productivity $\{A_i\}_i$, foreign demand $\{D^*_{EX,Fi}\}_i$, and import price $\{P^*_{IM,Fi}\}_i$. The aggregate state ξ collects the realized states:

$$\xi \equiv \left\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\right\}_{i\in\{1,2,\dots,N\}} \in \Xi = \mathbb{R}^{3N}_{\ge 0}. \tag{10}$$

2.6. Government: fiscal and monetary policies

The government sets fiscal and monetary policies. Fiscal policy includes a pair of non-contingent sectoral sales and export taxes $\{\tau_i, \tau_{EX,i}\}_i$ that do not respond to changes in exogenous states. The lump-sum transfer T to the households satisfies a fiscal budget balance:

$$T = \sum_{i=1}^{N}\left(\tau_i \int_0^1 P_{if} Y_{if}\, df + \tau_{EX,i} P_{EX,i} Y_{EX,i}\right). \tag{11}$$

The monetary policy is a one-dimensional state-contingent money supply $M(\xi)$ contingent on the aggregate state ξ. We investigate the design of this monetary policy, with a particular focus on the monetary policy of output gap targeting that closes the output gap.

[6] In general, the export demand in Eq. (8) can be written as $Y_{EX,i} = [P_{EX,i}/(S \cdot P^*_{EX,Fi})]^{-\theta_{F,i}} D^*_{EX,Fi}$, where $P^*_{EX,Fi}$ is the exogenous price for foreign-produced sector i's product in foreign markets, and $D^*_{EX,Fi}$ is the exogenous foreign demand given the prices. Therefore, $D^*_{EX,Fi}$ in Eq. (8) captures the effects of both $P^*_{EX,Fi}$ and $D^*_{EX,Fi}$ on export demand.

[7] Engel (2016) advocates using a balanced trade assumption instead of the risk sharing condition in the complete market.

2.7. Equilibrium definition

The market clearing conditions for product, labor, and money markets are:

$$Y_i(\xi) = C_{Hi}(\xi) + \sum_{j=1}^{N} \int_0^1 X_{Hjf,Hi}(\xi) df + Y_{EX,i}(\xi), \tag{12}$$

$$L(\xi) = \sum_{i=1}^{N} \int_0^1 L_{if}(\xi) df, \qquad M(\xi) = M_d(\xi). \tag{13}$$

Definition 1. A *sticky-price equilibrium* is a set of allocations, prices, and policies (i.e., $\{\tau_i, \tau_{EX,i}\}_i$ and $M(\xi)$) such that for any realized state $\xi \in \Xi$,

(i) producers optimally choose inputs to minimize the cost of production;
(ii) monopolistically competitive firms $f \in [0, \delta_i]$ set prices to maximize profits subject to their demand functions, and the remaining firms $f \in (\delta_i, 1]$ do not adjust prices;
(iii) the representative household chooses consumption and labor to maximize utility subject to its budget constraint, and the total expenditure determines the money demand;
(iv) the government budget constraint is satisfied;
(v) all markets clear.

We define the *flexible-price equilibrium* as the special case of the *sticky-price equilibrium* in Definition 1 that involves no Calvo-pricing friction, as stated in the following definition:

Definition 2. A *flexible-price equilibrium* is a set of allocations, prices, and policies satisfying all of the conditions stated in Definition 1, except that for any sector $i \in \{1, 2, \ldots, N\}$, $\delta_i = 1$, *viz.*, all firms can adjust prices flexibly.

While the *sticky-price equilibrium* is our focus, the allocation of the *flexible-price equilibrium* serves as a benchmark to define the distortions and welfare losses that nominal rigidities introduce.

2.8. Flexible-price equilibrium as reference equilibrium

As per Woodford (2003) and Galí (2015), we use non-contingent subsidies and taxes to eliminate domestic-market distortion while allowing domestic producers to exert their market power fully in the international market in the flexible-price equilibrium, as defined by the following assumption[8]:

Assumption 1. The non-contingent tax rates for sales and exports are equal to

$$\tau_i = -1/(\epsilon_i - 1) \text{ and } \tau_{EX,i} = 1/\theta_{F,i}, \text{ respectively, for } \forall i \in \{1, \ldots, N\}. \tag{14}$$

Under Assumption 1, the *flexible-price equilibrium* yields the optimal allocation for the domestic social planner, as stated in the following lemma:

Lemma 1. *Under Assumption 1, the flexible-price equilibrium implements the optimal allocation for the domestic social planner.*

Proof. See Appendix J.2.

Lemma 1 allows use of *flexible-price equilibrium* as the reference equilibrium for our further analyses of the domestic country's aggregate distortion and welfare loss.

2.9. Notations

This section summarizes the notations in the model to facilitate the tracking of variables, vectors, and matrices.

[8] In one-sector closed economies, Woodford (2003) and Galí (2015) show that a sales subsidy eliminates the monopoly distortion and makes the flexible-price equilibrium optimal for the social planner. La'O and Tahbaz-Salehi (2022) and Rubbo (2023) use sector-specific subsidies for the same purpose in a multi-sector closed economy. In small open economies, given that sales subsidies eliminate the monopoly distortion, the monopoly power of domestic producers on the international market needs to be retained for the domestic social planner to restore the optimality of the allocation in the flexible-price equilibrium. Therefore, we use sector-specific subsidies and export taxes to remove the monopoly distortion in the domestic market and exert the monopoly power in the international market, respectively, as in Matsumura (2022).

Table 1
Notations of parameters and steady-state objects.

Name	Expression
Consumption shares and home bias	$\boldsymbol{\beta} \equiv (\beta_1, \beta_2, \dots, \beta_N)^\top$ & $\mathbf{v} \equiv (v_1, v_2, \dots, v_N)^\top$
Labor shares	$\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \dots, \alpha_N)^\top$
Intermediate input shares and home bias	$\boldsymbol{\Omega} \equiv \{\omega_{i,j}\}_{i,j\in\{1,2,\dots,N\}}$ & $\mathbf{V}_x \equiv \{v_{x,i,j}\}_{i,j\in\{1,2,\dots,N\}}$
Elasticity of home-foreign substitution	$\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \dots, \theta_N)^\top$ & $\boldsymbol{\theta}_F \equiv (\theta_{F,1}, \theta_{F,2}, \dots, \theta_{F,N})^\top$
Frequency of price adjustment	$\boldsymbol{\Delta} = diag(\delta_1, \delta_2, \dots, \delta_N)$
Steady-state sectoral Domar weight	$\lambda \equiv (\lambda_1, \lambda_2, \dots, \lambda_N)^\top \equiv \left(\frac{P_1^{ss}Y_1^{ss}}{P_C^{ss}C^{ss}}, \frac{P_2^{ss}Y_2^{ss}}{P_C^{ss}C^{ss}}, \dots, \frac{P_N^{ss}Y_N^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state sectoral export-to-GDP ratio	$\lambda_{EX} \equiv (\lambda_{EX,1}, \dots, \lambda_{EX,N})^\top \equiv \left(\frac{P_1^{ss}Y_{EX,1}^{ss}}{P_C^{ss}C^{ss}}, \dots, \frac{P_N^{ss}Y_{EX,N}^{ss}}{P_C^{ss}C^{ss}}\right)^\top$
Steady-state economy-wise labor share	$\Lambda_L \equiv W^{ss}L^{ss}/P_C^{ss}C^{ss}$
Total content in domestic consumption & exports	$\widetilde{\lambda}_D \equiv \left(\widetilde{\lambda}_{D,1}, \widetilde{\lambda}_{D,2}, \dots, \widetilde{\lambda}_{D,N}\right)^\top$ & $\widetilde{\lambda}_F \equiv \left(\widetilde{\lambda}_{F,1}, \widetilde{\lambda}_{F,2}, \dots, \widetilde{\lambda}_{F,N}\right)^\top$
Total content of domestic labor	$\widetilde{\boldsymbol{\alpha}} \equiv (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \dots, \widetilde{\alpha}_N)^\top$
Expenditure-switching (ES) & generalized ES elasticities	$\rho_{ES} \equiv (\rho_{ES,1}, \rho_{ES,2}, \dots, \rho_{ES,N})^\top$ & $\widetilde{\rho}_{ES} \equiv (\widetilde{\rho}_{ES,1}, \widetilde{\rho}_{ES,2}, \dots, \widetilde{\rho}_{ES,N})^\top$

Deviations from the steady state and flexible-price equilibrium. We define the steady state of the static economy as the equilibrium in which all exogenous states A_i, $P^*_{IM,Fi}$, and $P^*_{EX,Fi}$ are at the steady state. We denote with x^{ss} and x^{flex} the values for the variable x in the steady state and in the flexible-price equilibrium, respectively. We express the log deviation of the variable x from the steady state x^{ss} and the flexible-price equilibrium x^{flex} as:

$$\hat{x} \equiv \ln(x) - \ln(x^{ss}) \quad \text{and} \quad \hat{x}^{gap} \equiv \ln(x) - \ln(x^{flex}), \tag{15}$$

respectively.[9] We denote the output gap by $\hat{C}^{gap}$.[10] The sectoral markup wedge is $\ln(\mu_i) - \ln(\mu_i^\#) = \ln(\mu_i) - \ln(\mu_i^{ss}) \equiv \hat{\mu}_i$ as the steady-state markup is equal to the desired markup.

Parameters and steady-state objects. Summarized in Table 1 are the key parameters and steady-state variables. Throughout the paper, for any variable x, we use bold fonts to denote the corresponding vector or matrix — i.e., $\mathbf{x} \equiv \{x_i\}_i$ or $\mathbf{x} \equiv \{x_{i,j}\}_{i,j}$. For expositional simplicity, the superscript "ss" to denote the steady state is omitted when there is no obvious confusion. In particular, we introduce the open-economy version of the Leontief-inverse matrix: $\mathbf{L}_{vx} \equiv \left(\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x\right)^{-1} = \left\{l_{vx,i,j}\right\}_{i,j}$, where $l_{vx,i,j} = \mathbf{I}_{i,j} + \left(\boldsymbol{\Omega} \odot \mathbf{V}_x\right)_{i,j} + \left(\boldsymbol{\Omega} \odot \mathbf{V}_x\right)^2_{i,j} + \cdots$ captures the total elasticity of sector i's cost to a change in domestic sector j's price, directly and indirectly through the use of domestic products as intermediate inputs.

2.10. Sectoral relevance metrics in an open economy with networks

To facilitate the study of the link between sectoral inflation (or markup wedges) and the output gap, we define the sectoral metrics below that depend on the cross-border and input–output linkages of the economy and represent the relevance of a sector in the economy across different dimensions.

Definition 3 (*Sectoral Relevance Metrics*)**.** For each domestic sector i, the **total content in domestic consumption** $\widetilde{\lambda}_{D,i}$ and the **total content in exports** $\widetilde{\lambda}_{F,i}$ are defined as[11]:

$$\widetilde{\lambda}_{D,i} \equiv \sum_r \beta_r v_r l_{vx,r,i} \quad \text{and} \quad \widetilde{\lambda}_{F,i} \equiv \sum_r \lambda_{EX,r} l_{vx,r,i}, \quad \text{respectively.} \tag{16}$$

The **total content of domestic labor** $\widetilde{\alpha}_i$ and the **total content of foreign factor** are:

$$\widetilde{\alpha}_i \equiv \sum_r l_{vx,i,r}\alpha_r \quad \text{and} \quad 1 - \widetilde{\alpha}_i, \quad \text{respectively.} \tag{17}$$

The **expenditure-switching elasticity** $\rho_{ES,r}$ is:

$$\rho_{ES,r} \equiv \underbrace{\left(\theta_{F,r} - 1\right)\lambda_{EX,r}}_{\text{foreign expenditure}} + \underbrace{\left(\theta_r - 1\right)\left[\beta_r v_r\left(1 - v_r\right) + \sum_s \lambda_s \omega_{s,r} v_{x,s,r}\left(1 - v_{x,s,r}\right)\right]}_{\text{domestic expenditure}}, \tag{18}$$

based on which the **generalized expenditure-switching elasticity** $\widetilde{\rho}_{ES,i}$ is defined as:

$$\widetilde{\rho}_{ES,i} \equiv \sum_r \rho_{ES,r}\widetilde{\alpha}_r l_{vx,r,i}. \tag{19}$$

[9] In our static model, the sectoral inflation is identical to the log deviation of the sectoral price from its steady-state level.

[10] As we stated above, we follow Rubbo (2023) to define the aggregate (real) output as the domestic aggregate consumption C, and with a slight abuse of notation, refer to $\hat{C}^{gap}$ as the output gap throughout the paper.

[11] In matrix forms, the total contents in domestic consumption and exports, the total content of domestic labor, and the generalized elasticity of expenditure switching are equal to $\widetilde{\lambda}_D^\top \equiv (\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx}$, $\widetilde{\lambda}_F^\top \equiv \lambda_{EX}^\top \mathbf{L}_{vx}$, $\widetilde{\boldsymbol{\alpha}} \equiv \mathbf{L}_{vx}\boldsymbol{\alpha}$, and $\widetilde{\rho}_{ES}^\top \equiv (\rho_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top \mathbf{L}_{vx}$, respectively.

The *total content in domestic consumption* $\widetilde{\lambda}_{D,i}$ (vs. *total content in exports* $\widetilde{\lambda}_{F,i}$) of a domestic sector i in Eq. (16) encapsulates the importance of a domestic sector in the network economy as both a direct and an indirect supplier (via downstream sectors) — captured by the Leontief inverse $l_{vx,r,i}$ — for the domestic aggregate consumption or output (vs. exports).[12] As a result, a sector's total content in domestic consumption decreases in the import shares of the sector and its downstream sectors, as shown in Proposition G.1 of Appendix G. The *total content of domestic labor* $\widetilde{\alpha}_i$ (vs. *total content of foreign factor* $1-\widetilde{\alpha}_i$) of a domestic sector i in Eq. (17) summarizes the sector's role in the network economy as both a direct and an indirect customer (via upstream sectors) — captured by the Leontief inverse $l_{vx,i,r}$ — of domestic labor factor (vs. imported foreign factor).

The *expenditure-switching elasticity* $\rho_{ES,i}$ of a domestic sector i in Eq. (18) — corresponding to the standard expenditure switching effect in international macroeconomic literature — captures the elasticity of the domestic and foreign expenditures on domestic products to the relative prices of foreign versus domestic sector i's products. To further capture the impact of the domestic-to-foreign price on domestic labor income through the expenditure switching effect, we define the *generalized expenditure-switching elasticity* $\widetilde{\rho}_{ES,i}$. Specifically, it is equal to the elasticity of domestic labor income — evinced by the total content of domestic labor $\widetilde{\alpha}_r$ in Eq. (19) — to the relative price of foreign versus domestic sector i, *through* the direct and indirect (via downstream sectors) impact of the foreign-to-domestic price on the expenditures on domestic products — captured by the expenditure-switching elasticity $\rho_{ES,i}$ multiplied by the Leontief inverse $l_{vx,r,i}$. The vector formats for these defined sectoral metrics are summarized in Table 1.

3. Sectoral markup wedges and the output gap

In this section, we derive the output gap as a weighted average of the sectoral markup wedges, and the sectoral weights are functions of the sectoral relevance metrics, as introduced in Section 2.10, thereby depending on the cross-border and input–output linkages.[13]

Under nominal rigidities, as a fraction $(1-\delta_i)$ of sector i's firms cannot adjust prices in response to changes in marginal costs, sectoral inflation is linked to sectoral markup wedges — encapsulating sectoral distortions — through sectoral price rigidities as follows[14]:

$$\hat{\mu}_i(\xi) = -(1-\delta_i)/\delta_i \cdot \hat{P}_i(\xi). \tag{20}$$

These negative sectoral markup wedges relate to a positive output gap through three distinct channels, as outlined in the following theorem:

Theorem 1 (*Output Gap and Sectoral Markup Wedges*). *In a sticky-price equilibrium, the output gap $\hat{C}^{gap}(\xi)$ is proportional to a weighted average of sectoral markup wedges $\{\hat{\mu}_i(\xi)\}_i$*[15]*:*

$$\kappa_C \cdot \hat{C}^{gap}(\xi) = -\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot \hat{\mu}_i(\xi), \tag{21}$$

where the vector of sectoral OG weights ($\boldsymbol{\mathcal{M}}_{OG} \equiv (\mathcal{M}_{OG,1}, \mathcal{M}_{OG,2}, \dots, \mathcal{M}_{OG,N})$) is equal to:

$$\boldsymbol{\mathcal{M}}_{OG} \equiv \underbrace{\widetilde{\boldsymbol{\lambda}}_D}_{\text{CPI channel}} + \underbrace{\kappa_{CPI}^{-1} \cdot \widetilde{\boldsymbol{\rho}}_{ES}}_{\text{expenditure-switching channel}} + \underbrace{\kappa_{CPI}^{-1} \cdot \left(\widetilde{\boldsymbol{\lambda}}_F - \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right)}_{\text{profit channel}}, \tag{22}$$

$$\kappa_{CPI} \equiv \frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} + 1, \tag{23}$$

$$\kappa_C \equiv \Big[\frac{(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} \left(\sigma + \varphi/\Lambda_L\right) + \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} \left(\sigma + \varphi/\Lambda_L\right) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\Big] / \kappa_{CPI}. \tag{24}$$

Proof. See Appendix K.6.

Eq. (21) shows that negative sectoral markup wedges relate to a positive output gap. The sectoral OG weight ($\mathcal{M}_{OG,i}$) in Eq. (22) measures the relevance of the sector's markup wedge for the output gap, with its size determined by three distinct channels: (i) the *CPI*, (ii) the *expenditure-switching*, and (iii) the *profit* channels. To illustrate the three channels, we first present and interpret the two equilibrium conditions that we combine to derive Eq. (21), contrasting the cases of closed and open economies.

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12 For a pair of domestic sectors $r \neq i$, r is defined as a downstream (vs. upstream) sector of i if $l_{vx,r,i} > 0$ (vs. $l_{vx,i,r} > 0$).

13 In Appendix B.1, we show that up to the first-order approximation, the aggregate distortion is proportional to the output gap.

14 Exogenous shocks to sectoral productivity, import prices, and export demand drive sectoral inflation in the sticky-price equilibrium. Shown in Appendix J.6 is the derivation of Eq. (20).

15 As we stated in Section 2.3, we followed Rubbo (2023) to define the aggregate (real) output as the domestic aggregate consumption. It follows that our sectoral OG weights essentially relate sectoral markup wedges to the domestic consumption gap. If, instead, we define the output gap as a weighted average of sectoral value-added gap, following the spirit of Galí and Monacelli (2005), it should deviate from the domestic aggregate consumption gap by a weighted average of the differences between sectoral price gaps and the exchange rate gap. Another policy-relevant concept, the employment gap, is proportional to the domestic consumption gap (i.e., our output gap) up to the first-order approximation, as we show in Proposition B.1.

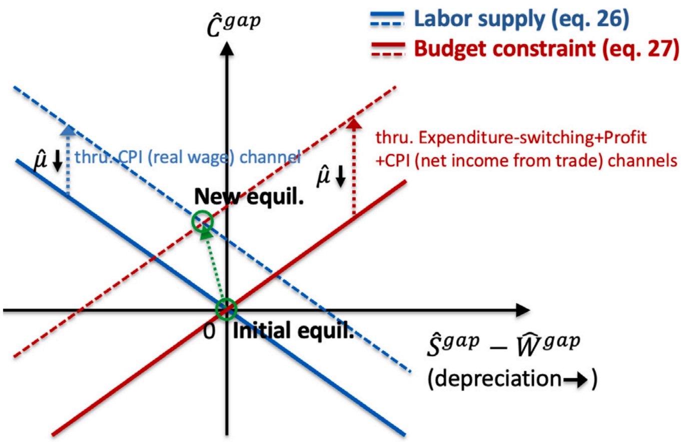


Fig. 1. Determination of equilibrium output and real exchange rate gaps.
Notes: The figure shows the labor supply curve (blue) in Eq. (26) and the open-economy budget constraint curve (red) in Eq. (27), respectively. The two curves determine the equilibrium output gap (i.e., $\hat{C}^{gap}$ on the vertical axis) and real exchange rate gap (i.e., $\hat{S}^{gap} - \widehat{W}^{gap}$ on the horizontal axis). The solid and the dashed curves correspond to the curves in the initial steady state without shocks and in the new equilibrium with shocks and the resulting negative markup wedges, respectively. As we show later, under standard assumptions on the household's preferences, the new equilibrium real exchange rate gap under negative markup wedges is always negative due to a larger shift up in the budget constraint curve (red) than the labor supply curve (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The first condition is the log-linearized labor supply equation around the flexible-price equilibrium (Eq. (25)) that is combined with the equation of real wage gap (Eq. (26)) as follows:

$$\left(\sigma + \varphi/\Lambda_L\right)\hat{C}^{gap}(\xi) = \widehat{W}^{gap}(\xi) - \hat{P}_C^{gap}(\xi) \tag{25}$$

$$= -(1 - \tilde{\lambda}_D^{\top}\boldsymbol{\alpha})\left(\hat{S}^{gap}(\xi) - \widehat{W}^{gap}(\xi)\right) - \tilde{\lambda}_D^{\top}\hat{\boldsymbol{\mu}}(\xi) + o(\|\hat{\xi}\|). \tag{26}$$

The second condition is the log-linearized open-economy budget constraint around the flexible-price equilibrium, given by[16]:

$$(1 - \tilde{\lambda}_D^{\top}\boldsymbol{\alpha})\hat{C}^{gap}(\xi) = \underbrace{-(\boldsymbol{\rho}_{ES} \odot \tilde{\boldsymbol{\alpha}})^{\top}\left(\hat{\mathbf{P}}^{gap}(\xi) - \mathbf{1}\hat{S}^{gap}(\xi)\right) + \lambda_F^{\top}\boldsymbol{\alpha}\left(\hat{S}^{gap}(\xi) - \widehat{W}^{gap}(\xi)\right)}_{\text{expenditure-switching channel}}$$
$$+ \underbrace{[\lambda \odot (\mathbf{1} - \tilde{\boldsymbol{\alpha}})]^{\top}\hat{\boldsymbol{\mu}}(\xi)}_{\text{imported factor channel}} \underbrace{-\lambda_{EX}^{\top}\left(\hat{\mathbf{P}}^{gap}(\xi) - \mathbf{1}\hat{S}^{gap}(\xi)\right) + (1 - \lambda^{\top}\boldsymbol{\alpha})\left(\hat{S}^{gap}(\xi) - \widehat{W}^{gap}(\xi)\right)}_{\text{export profit channel}}$$
$$\underbrace{-(1 - \tilde{\lambda}_D^{\top}\boldsymbol{\alpha})\left(\hat{P}_C^{gap}(\xi) - \widehat{W}^{gap}(\xi)\right)}_{\text{CPI channel through net income from trade}} + o(\|\hat{\xi}\|), \tag{27}$$

where the gap in the CPI relative to domestic wage is given by Eq. (26), and the gaps in domestic sectoral prices relative to foreign prices are functions of the *real exchange rate gap* — which we *define* as $\hat{S}^{gap} - \widehat{W}^{gap}$ — and sectoral markup wedges as in the following Eq. (28):[17]

$$\hat{\mathbf{P}}^{gap}(\xi) - \mathbf{1}\hat{S}^{gap}(\xi) = -\tilde{\boldsymbol{\alpha}}\left(\hat{S}^{gap}(\xi) - \widehat{W}^{gap}(\xi)\right) + \mathbf{L}_{vx}\hat{\boldsymbol{\mu}}(\xi) + o(\|\hat{\xi}\|), \tag{28}$$

Given sectoral markup wedges, Eqs. (26) and (27) jointly determine the equilibrium output gap ($\hat{C}^{gap}$) and real exchange rate gap ($\hat{S}^{gap} - \widehat{W}^{gap}$), which is illustrated in Fig. 1 by the interaction of the blue labor supply curve indicating Eq. (26) and the red budget constraint curve indicating Eq. (27).

The labor supply Eq. (26) is similar to that in closed economies, by relating the output gap to sectoral markup wedges through the real wage gap and labor supply. In particular, while the real wage gap is a function of only sectoral markup wedges in closed economies — where the total share of domestic labor in domestic output $\tilde{\lambda}_D^{\top}\boldsymbol{\alpha} = 1$— it also relates to the real exchange rate gap in open economies. The blue labor supply curve in Fig. 1 is downward sloping, because a depreciation of domestic currency (i.e., an

[16] The nominal exchange rate (S) is the endogenous component in foreign goods prices (in units of the domestic currency) that relates to sectoral markup wedges and, therefore, monetary policy. As such, the exchange rate gap ($\hat{S}^{gap}$) reflects the prices of foreign products or the values of foreign demand in the log-linearization of equilibrium conditions around the flexible-price equilibrium.

[17] We normalize both the gaps in the nominal exchange rate and the CPI by domestic wage — i.e., $\hat{S}^{gap} - \widehat{W}^{gap}$ and $\hat{P}_C^{gap} - \widehat{W}^{gap}$ — so that the real exchange rate and CPI gaps are measured in units of domestic labor. This normalization is necessary because gaps in nominal prices are indeterminate in equilibrium.

increase in $\widehat{S}^{gap} - \widehat{W}^{gap}$) increases the CPI through the prices of imported goods, thereby lowering the real wage gap and reducing labor supply and the output gap.

The open-economy budget constraint Eq. (27) is specific to open economies.[18] It relates the output gap to sectoral markup wedges and the real exchange rate gap, by equating changes in domestic consumption of imported goods (LHS) to changes in net income from trade (RHS). In particular, changes in net income from trade are deflated by the CPI — as evinced by $-(1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha})(\widehat{P}_C^{gap} - \widehat{W}^{gap})$ on the RHS — to align with the domestic consumption gap on the LHS. The red budget constraint curve in Fig. 1 that represents Eq. (27) is upward sloping due to two forces: first, a depreciation of domestic currency (i.e., an increase in $\widehat{S}^{gap} - \widehat{W}^{gap}$) increases the domestic sectoral prices relative to foreign prices — as Eq. (28) shows. The higher domestic-to-foreign prices increase net income from trade and, in turn, the domestic consumption gap, through the switch of expenditures from foreign to domestic products and increased profits from exports — as evinced by the terms $-(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})$ and $-\lambda_{EX}^\top(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})$ on the RHS of Eq. (27), respectively; second, a depreciation of domestic currency increases the output gap by directly increasing the net income from trade in units of domestic currency — as evinced by the real exchange rate terms $\lambda_F^\top \boldsymbol{\alpha}(\widehat{S}^{gap} - \widehat{W}^{gap})$ and $(1-\lambda^\top \boldsymbol{\alpha})(\widehat{S}^{gap} - \widehat{W}^{gap})$ on the RHS of Eq. (27).[19] Appendix B.2 derives the slopes of the labor supply and budget constraint curves.

Sectoral markup wedges relate to the equilibrium output gap by shifting both the labor supply curve in Eq. (26) and the open-economy budget constraint curve in Eq. (27) through the three channels in the OG weight in Theorem 1. We now interpret each of the three channels.

(i) CPI channel. The *CPI channel* describes the relationship between sectoral markup wedges and the output gap through distortion in the price of the aggregate output — i.e., the CPI. Specifically, negative sectoral markup wedges are associated with a lower CPI in the sticky-price relative to the flexible-price equilibrium. This negative CPI gap relates to a positive output gap through two complementary mechanisms that shift *upward* the labor supply and the budget constraint curves in Eqs. (26) and (27), respectively. One operates through the real wage — a standard mechanism in closed economies with nominal rigidities — where a lower CPI increases the real wage (W/P_C) and stimulates a higher supply of domestic labor, thereby being linked to a positive output gap, as evinced by the negative coefficient $-\widetilde{\lambda}_D^\top$ of sectoral markup wedges in Eq. (26). The second mechanism operates through the domestic consumption of imported goods financed by net income from trade and is specific to open economies, where a lower CPI increases the net income from trade in units of domestic consumption and fosters a higher domestic output gap — as evinced by the negative coefficient of the CPI gap $-(1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha})$ on the RHS of Eq. (27).

For a domestic sector in an open economy with production networks, the size of the CPI channel is proportional to the domestic sector's total content in domestic consumption $\widetilde{\lambda}_{D,i}$ — as shown in the CPI channel in the OG weight of Eq. (22). In particular, both the CPI channel and the OG weight in Eq. (22) reduce to the corresponding Domar weight in closed economies à la (Rubbo, 2023).[20]

(ii) Expenditure-switching channel. The *expenditure-switching channel* — specific to the open economy and standard in the international macroeconomic literature — describes how markup wedges relate to the output gap through the switch of expenditure from foreign toward domestic products. Specifically, Eq. (28) shows that negative sectoral markup wedges — directly and indirectly via the Leontief inverse $\mathbf{L}_{vx}$ — reduce the prices of domestic products relative to foreign products, which are captured by the difference between the sectoral inflation gap $\widehat{\mathbf{P}}^{gap}$ and the exchange rate gap $\mathbf{1}\widehat{S}^{gap}$. The reduced domestic-to-foreign goods prices generate a switch of both domestic and foreign expenditures from foreign to domestic products — as evinced by the negative coefficient $-\boldsymbol{\rho}_{ES}$ in Eq. (27). As a result, domestic labor income from international trade increases — as captured by the total content of domestic labor $\widetilde{\boldsymbol{\alpha}}$. Therefore, the expenditure-switching channel connects negative sectoral markup wedges with a positive output gap, as is shown by the first term of expenditure-switching on the RHS of Eq. (27) — through which negative markup wedges shift *upward* the budget constraint curve and increases the output gap.

For a domestic sector of an open economy with production networks, the size of the expenditure-switching channel is determined jointly by the magnitudes of (i) the Leontief inverse $\mathbf{L}_{vx}$ in Eq. (28) that links negative sectoral markup wedges to the gaps in domestic-to-foreign goods prices and (ii) the elasticity of domestic labor income to the domestic-to-foreign goods prices ($-(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}})^\top$ in Eq. (27)). These two sub-components are combined to yield the generalized expenditure-switching elasticity in the expenditure-switching channel in the OG weight of Eq. (22).

(iii) Profit channel. The *profit* channel — also specific to the open economy — describes how domestic sectoral markup wedges relate to the output gap through both the profits from exports and the costs of imported foreign inputs. In the export profit sub-channel (the fourth term $-\lambda_{EX}^\top(\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})$ on the RHS of Eq. (27)), negative sectoral markup wedges directly and indirectly reduce domestic prices or, equivalently, the (opportunity) costs of exported goods. The lower costs of exports increase the profits

[18] In closed economies, the open-economy budget constraint equation does not affect the output gap, and the labor supply Eq. (26) alone determines the relationship of the output gap to sectoral markup wedges.

[19] A depreciation in domestic currency can also reduce the output gap by increasing the CPI and reducing the net income from trade in units of domestic consumption — as evinced by $-(1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha})(\widehat{P}_C^{gap} - \widehat{W}^{gap})$ on the RHS of Eq. (27). However, this negative effect of currency depreciation on the output gap is strictly dominated by the positive effect via the increased net income from trade in units of domestic currency — as evinced by the strictly smaller coefficient of $\widehat{S}^{gap} - \widehat{W}^{gap}$ in the former effect (i.e., $(1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha})(1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha})$) than that in the latter effect (i.e., $\lambda_F^\top \boldsymbol{\alpha} + (1-\lambda^\top \boldsymbol{\alpha}) = 1-\widetilde{\lambda}_D^\top \boldsymbol{\alpha}$). Therefore, the budget balance curve is strictly upward sloping even with the negative effect of currency depreciation on the output gap through the CPI.

[20] The OG weight reduces to the Domar weight in closed economies under our normalization by the scalar κ_{CPI} in Eq. (23). κ_{CPI} captures the size of the CPI channel in units of the CPI gap, and is equal to the sum of the sizes of two CPI channels through the real wage and through the net income from trade. Appendix B.2 provides the formula and the intuition of κ_{CPI}.

from exports and the net income from trade, thereby relating to a positive output gap. In the imported factor sub-channel (the third term $[\lambda \odot (\mathbf{1} - \tilde{\boldsymbol{\alpha}})]^\top \hat{\boldsymbol{\mu}}$ on the RHS of Eq. (27)), negative sectoral markup wedges are associated with higher sectoral costs of imported foreign factors in production (relative to sectoral sales) and lower domestic producers' profits, thus linking to a negative output gap.[21] In Fig. 1, through the export profit (vs. imported factor) sub-channel, negative markup wedges shift *upward* (vs. *downward*) the budget constraint curve (27), generating a positive (vs. negative) output gap in equilibrium.

For a domestic sector in an open economy with production networks, the size of the export profit sub-channel is determined by both the Leontief inverse $\mathbf{L}_{vx}$ in Eq. (28) —which links negative sectoral markup wedges to the gaps in domestic prices — and the share of the sector's exports in domestic output — i.e., $\lambda_{EX}^\top$ in Eq. (27). The size of the imported factor channel is determined by both the sector's size and its direct and indirect (via upstream sectors) use of imported inputs — captured by the product of the sectoral Domar weight (i.e., λ_i) and the sectoral content of foreign factor (i.e., $1 - \tilde{\alpha}_i$) in Eq. (27). These two sub-channels are combined to yield the profit channel in the OG weight of Eq. (22).

Responses of output gap and real exchange rate gap to negative sectoral markup wedges in equilibrium. Summarizing the three channels above and illustrated in Fig. 1, negative sectoral markup wedges shift *upward* both the labor supply curve in Eq. (26) — through the CPI channel — and the budget constraint curve in Eq. (27) — through the expenditure-switching, profit, and the CPI channels.[22] The upward shifts of both curves lead to an increase in the equilibrium output gap, consistent with Eq. (21) in Theorem 1 that shows negative sectoral markup wedges relate to a positive output gap.

The response of the equilibrium real exchange rate gap (i.e., $\widehat{S}^{gap} - \widehat{W}^{gap}$) to negative markup wedges depend on whether the labor supply or the budget constraint curve shifts up more. Specifically, under our *key assumption of balanced trade*, the domestic currency responds to preserve the trade balance. When negative markup wedges shift up the labor supply curve in Eq. (26), the labor supply and domestic output gap increase, generating a trade surplus. In response to it, the real exchange rate depreciates (i.e., $\widehat{S}^{gap} - \widehat{W}^{gap}$ increases) to increase the CPI and reduce the real wage, thereby reducing the labor supply and the output gap to preserve the trade balance. In contrast, when negative markup wedges shift up the budget constraint curve in Eq. (27) and generate a trade surplus, the real exchange rate appreciates (i.e., $\widehat{S}^{gap} - \widehat{W}^{gap}$ decreases) to reduce export profit and domestic labor income through expenditure switching, thereby reducing the net income from trade to preserve the trade balance.

Under standard assumptions on the household's preference of $\sigma \geq 1$ and $\varphi > 0$, the labor supply is less elastic to the real wage gap. Therefore, negative markup wedges shift up the open-economy budget constraint curve strictly more than the labor supply curve, thereby making the equilibrium real exchange rate appreciate (i.e., $\widehat{S}^{gap} - \widehat{W}^{gap}$ decreases) in response to negative markup wedges.[23] As we show in equation (B.6) of Appendix B.2, the equilibrium real exchange rate gap is also proportional to a weighted average of sectoral markup wedges. The sectoral weights of markup wedges in the real exchange rate gap are composed of the same three channels in the OG weight, with a smaller share of the CPI and larger shares of the expenditure-switching and profit channels than in the OG weight, consistent with the intuition that the real exchange rate gap is more exposed to international trade than the overall output gap.

In Appendix B.3, we show that in a dynamic environment with the standard assumption of a *complete (asset) market* à la (Galí and Monacelli, 2005) — such that the Backus-Smith and uncovered interest rate conditions hold — the consumption gap (i.e., $\widehat{C}^{gap}$) is proportional to the real exchange rate gap according to the international risk-sharing condition à la (Galí and Monacelli, 2005; Corsetti et al., 2010), rather than determined by the open-economy budget constraint in Eq. (27) that is based on the trade balance condition. Using the risk-sharing condition, we derive the sectoral OG weight in complete markets and show that the share of the CPI channel in the OG weight of complete markets is smaller than that in our baseline case of balanced trade.[24]

Sectoral OG weights without input–output linkages. In multi-sector horizontal small open economies without input–output linkages — i.e., $\boldsymbol{\Omega} = \mathbf{0}_{N\times N}$ — the Leontief-inverse matrix reduces to $\mathbf{L}_{vx} = \mathbf{I}$, and the sectoral total content of domestic labor and labor share reduce to $\tilde{\alpha}_i = \alpha_i = 1 - \sum_r \omega_{i,r} = 1$. Therefore, the imported factor sub-channel in Eq. (27) reduce to zero, and the other channels in Eqs. (26) and (27) remain — but without the Leontief-inverse matrix capturing IO linkages — which leads to the following OG weights $\mathcal{M}_{OG,i}^{NoIO}$ without IO linkages[25]:

$$\mathcal{M}_{OG,i}^{NoIO} \equiv \underbrace{\beta_i v_i}_{\text{CPI channel}} + \underbrace{(\kappa_{CPI}^{NoIO})^{-1}\left[(\theta_{F,i}-1)\lambda_{EX,i} + \left(\theta_i - 1\right)\beta_i v_i \left(1 - v_i\right)\right]}_{\text{expenditure-switching channel}} + \underbrace{(\kappa_{CPI}^{NoIO})^{-1}\lambda_{EX,i}}_{\text{profit channel}}. \tag{29}$$

[21] In general, the two sub-channels of export profits and imported factors do not perfectly offset one another. For example, in the special case of a multi-sector small open economy that imports foreign goods only for consumption but not as intermediate inputs, the export profit sub-channel is nonzero while the imported factor sub-channel reduces to zero, thereby allowing sectoral markup wedges to link positively to the output gap through the profit channel. Discussed in Appendix C is why the optimization by the domestic planner does not imply a zero profit channel in general.

[22] As we show quantitatively in panel (a) of Figure I.1 in Appendix I.2, the negative imported factor sub-channel is mostly offset by the positive export profit sub-channel, generating an almost zero profit channel in total. Therefore, driven by the positive expenditure-switching and CPI channels, negative markup wedges shift the budget constraint curve upward in general.

[23] Appendix B.2 shows why negative markup wedges shift up the budget constraint curve more than the labor supply curve. Appendix B.2 also discusses why the terms of trade has a limited relation to the output gap in our multi-sector SOEs.

[24] According to the analysis in Section 4.1, the smaller share of the CPI channel in the OG weight in complete markets than under balanced trade implies a smaller (vs. larger) relative sectoral weights in the DC index in complete markets for sectors with a larger (vs. smaller) CPI channel. In Appendix B.3, we also show that the slope of the DC Phillips curve is flatter in complete markets than under balanced trade. Provided in Appendix B.3 are the detailed DC Phillips curve and DC index, with the economic intuition behind the flatter slope in complete markets.

[25] The scalar $\kappa_{CPI}^{NoIO} \equiv \left[\sum_{i=1}^N (\theta_{F,i}\lambda_{EX,i} + (\theta_i - 1)\beta_i v_i (1 - v_i))\right] / (1 - \sum_{i=1}^N \beta_i v_i) + 1$. Without IO linkages, negative markup wedges still shift both labor supply and budget balance curves up and, particularly, shift up budget balance curve more according to equation (B.6) in Appendix B.2. Therefore, as in the case with IO linkages, negative markup wedges relate to a positive output gap and an appreciation of domestic currency.

Eq. (29) shows that, omitting IO linkages under-estimates the relevance of a sector's inflation for the output gap, as it fails to account for its *indirect* impacts as an input supplier in the network—through both the CPI (i.e., ignoring $\sum_{r\neq i}\beta_r \upsilon_r l_{\upsilon x,r,i}$), the expenditure-switching (i.e., ignoring $\sum_{r\neq i}\rho_{ES,r}\widetilde{\alpha}_r l_{\upsilon x,r,i}$ and $\sum_s \lambda_s \omega_{s,i}\upsilon_{x,s,i}\left(1-\upsilon_{x,s,r}\right)$), and the export profit channels (i.e., ignoring $\sum_{r\neq i}\lambda_{EX,r} l_{\upsilon x,r,i}$). On the other hand, omitting IO linkages fails to account for domestic sectors' direct and indirect use of imported foreign inputs, thereby omitting the imported factor sub-channel in Eq. (29). Appendix B.4 uses a simple example to show that omitting IO linkages under-estimate the upstream sectors' CPI, expenditure-switching, and export profit channels, while ignoring the downstream sectors' imported factor channel.

4. The Phillips curves and the output gap targeting policy

In this section, we use the sectoral OG weight introduced in the previous section to derive an aggregate-level Phillips curve that links an aggregate inflation index to the output gap, which allows for the divine coincidence.[26] We follow Rubbo (2023) to refer to this aggregate Phillips curve as the divine coincidence Phillips curve and to the associated aggregate inflation index as the divine coincidence index.

In Section 4.1, we derive the DC and sectoral Phillips curves. We show that the slope of the DC Phillips curve is inversely related to the *sum of the sectoral OG weights*, while the *relative OG weights* determine the *relative sectoral weights* in the DC index. Section 4.2 compares the slopes of the DC and sectoral Phillips curves to the counterfactual slopes in closed economies and without IO linkages. Section 4.3 studies how the *relative sectoral weights* in the DC index — which is targeted to zero to implement the policy of output gap targeting — depend on cross-border and input–output linkages.

4.1. The divine coincidence and sectoral Phillips curves

The divine coincidence Phillips curves. Based on the sectoral OG weights in Eq. (22) that relate sectoral markup wedges to the output gap, we construct the following divine coincidence index as a weighted average of sectoral inflation.

Definition 4 (*Divine Coincidence Index*)**.** Assume that no sector has perfectly rigid prices (i.e., $\delta_i \neq 0\ \forall i$). The *divine coincidence index (DC index for short)* weights sectors according to their sectoral OG weights — adjusted by the sectoral price rigidity — as in the following equation:

$$\pi_{DC} \equiv \sum_{i=1}^{N} \widetilde{\mathcal{M}}_{OG,i}\widehat{P}_i, \tag{30}$$

where the *normalized OG weight* ($\widetilde{\mathcal{M}}_{OG,i}$) is equal to:

$$\widetilde{\mathcal{M}}_{OG,i} \equiv \frac{\mathcal{M}_{OG,i}\cdot(1-\delta_i)/\delta_i}{\sum_{i'=1}^{N}\mathcal{M}_{OG,i'}\cdot(1-\delta_{i'})/\delta_{i'}}, \tag{31}$$

and we denote the normalization factor by κ_{OG} as follows:

$$\kappa_{OG} \equiv \sum_{i'=1}^{N}\mathcal{M}_{OG,i'}\cdot(1-\delta_{i'})/\delta_{i'}. \tag{32}$$

Notably, the normalized sectoral OG weight ($\widetilde{\mathcal{M}}_{OG,i}$) in Eq. (31) is the price-rigidity-adjusted sectoral OG weight normalized by its sum across all sectors κ_{OG}. Thus, the normalized sectoral OG weight sums to one (i.e., $\sum_{i=1}^{N}\widetilde{\mathcal{M}}_{OG,i}=1$) and reflects the sectoral weight relative to the sum of weights, in contrast to the original sectoral OG weight that determines both the sum of sectoral weights and the relative sectoral weights.[27] Based on the DC index defined in Eq. (30), we derive the divine coincidence Phillips curve that is consistent with the simultaneous stabilization of domestic aggregate inflation and the output gap, as stated in the next proposition.

Proposition 1 (*Divine Coincidence Phillips Curve*)**.** *For any realized state $\xi \in \Xi$, the divine coincidence Phillips curve (DC Phillips curve for short) is given by:*

$$\pi_{DC}(\xi) = \frac{\kappa_C}{\kappa_{OG}}\widehat{C}^{gap}(\xi). \tag{33}$$

Proof. Straightforward substitution of Eq. (20) in Eq. (21) from Theorem 1.

[26] In multi-sector open economies, sector-level Phillips curves include a residual of exogenous shocks (see Section 4.1), preventing simultaneous stabilization of inflation and output gap — i.e., the "divine coincidence" fails to hold — both at the sector level and under arbitrary aggregation, similar to the case of closed economies à la (Rubbo, 2023).

[27] We distinguish between normalized and original sectoral OG weights because the original OG weight reduces to the Domar weight in closed economies, making our analysis of the DC Phillips curve directly comparable to the case of the closed economy.

The divine coincidence Phillips curve in Proposition 1 links domestic inflation in the form of the DC index in Eq. (30) to the output gap, allowing for the simultaneous stabilization of domestic aggregate inflation and the output gap that achieves the divine coincidence, as in closed economies à la (Rubbo, 2023). In particular, Eq. (31) reveals that the DC index assigns higher weights to sectors with high nominal rigidities — as captured by the sectoral price rigidity $(1-\delta_i)/\delta_i$ — which is consistent with the results in closed economies (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). However, in an open economy, Eq. (31) indicates that the weight assigned to sector i is proportional to the OG weight ($\mathcal{M}_{OG,i}$) defined in Eq. (22), which internalizes the structure of both input–output and cross-border linkages, as stated in Theorem 1. Our DC index in Eq. (30) nests the DC index of Rubbo (2023) in the case of the closed economy with production networks.

Eq. (33) shows that, in our small open economy with production networks, the slope of the DC Phillips curve is equal to κ_C/κ_{OG}, which is inversely related to the *sum of the price-rigidity-adjusted OG weights* (i.e., κ_{OG}). In contrast, the *relative sectoral OG weights* determine the shares of sectoral inflation in the DC index, as evinced by the normalized OG weights in Eq. (31).

In Appendix E, we extend our baseline model of producer-currency pricing (PCP) to the setting of foreign-currency pricing — which encompasses the alternatives of local-currency pricing (LCP) and dominant-currency pricing (DCP). We show that the divine coincidence index under foreign-currency pricing includes sectoral inflation of both domestic-market prices and export prices in the foreign market (Corollary E.1). Particularly, while the CPI and profit channels depend on domestic inflation, the expenditure-switching channel depends on both domestic and export price inflation.

The sectoral Phillips curves. In addition to the DC Phillips curve that simultaneously stabilizes domestic aggregate inflation and the output gap, we also derive the sectoral Phillips curves linking sectoral inflation to both the output gap and the exogenous sectoral shocks, as stated in the next proposition.

Proposition 2 (*Sectoral Phillips Curves*). *In the sticky-price equilibrium, the following sectoral-level Phillips curves hold:*

$$\widehat{\mathbf{P}}(\xi) = \underbrace{\mathcal{B}\widehat{C}^{gap}(\xi)}_{\text{output-gap-driven inflation}} + \underbrace{\mathcal{V}\widehat{\xi}}_{\text{cost-push inflation}} + o(\|\widehat{\xi}\|), \tag{34}$$

where $\widehat{\mathbf{P}}(\xi)$ is an N-by-1 vector of sectoral inflation, and parameters $\mathcal{B}$ (an N-by-1 vector) and $\mathcal{V}$ (an N-by-3N matrix) are the slopes of Phillips curves and the coefficients of exogenous shocks, respectively.

Proof. See Appendix L.1.

In Proposition 2, the slopes of the sectoral Phillips curves are equal to:

$$\mathcal{B} \equiv \boldsymbol{\Delta}_{\Phi}\Big\{\underbrace{\boldsymbol{\alpha}\left(\sigma + \varphi/\Lambda_L + \Gamma_{CPI,C}\right)}_{\text{nominal wage component}} + \underbrace{\left(\boldsymbol{\Omega}\odot\mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1}\Gamma_{S,C}}_{\text{nominal exchange rate component}}\Big\}, \tag{35}$$

$$\boldsymbol{\Delta}_{\Phi} \equiv \left[\boldsymbol{\Delta}^{-1} - \boldsymbol{\Omega}\odot\mathbf{V}_x - \boldsymbol{\alpha}\boldsymbol{\Gamma}_{CPI,P}^{\top} - (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}_{S,P}^{\top}\right]^{-1} \tag{36}$$

where the scalar $\Gamma_{CPI,C} \equiv \left(\boldsymbol{\beta}^{\top}\mathbf{v} + \mathcal{M}_p^{\top}\mathbf{1}\right)^{-1}(1-\boldsymbol{\beta}^{\top}\mathbf{v})$ and vector $\boldsymbol{\Gamma}_{CPI,P}$ are the elasticities of the CPI to the output gap and domestic sectoral inflation, respectively. The scalar $\Gamma_{S,C} \equiv \left(1 + \mathcal{M}_p^{\top}\mathbf{1}\right)^{-1}\left(1 + \Gamma_{CPI,C}\right)$ and the vector $\boldsymbol{\Gamma}_{S,P}$ are the elasticities of the nominal exchange rate to the output gap and domestic sectoral inflation, respectively.[28]

Consistent with the sectoral OG weight in Eq. (22), the slopes of sectoral Phillips curves — which also link domestic inflation to the output gap — include the same three channels. The terms representing the domestic contents in domestic consumption (i.e., $\boldsymbol{\beta}^{\top}\mathbf{v}$ and $\boldsymbol{\beta}\odot\mathbf{v}$ in $\Gamma_{CPI,C}$ and $\boldsymbol{\Gamma}_{CPI,P}$, and $\mathbf{V}_x$ in $\boldsymbol{\Delta}_{\Phi}$) capture the *CPI channel* in the slopes of sectoral Phillips curves. The terms $\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}$ and $\boldsymbol{\lambda}_{EX}$ in the vector $\mathcal{M}_p \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})$ and the vector $\mathcal{M}_{\mu} \equiv (1-\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha})^{-1}(\boldsymbol{\Delta}^{-1}-\mathbf{I})\left[\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})\right]$ capture the relations of domestic sectoral inflation to the output gap in the open-economy budget constraint, through the *expenditure-switching*, *export profit*, and *imported factor channels*, respectively.

Eq. (35) shows that sectoral inflation is linked to the output gap *via* the positive nominal wage and nominal exchange rate components, thereby making the slopes of sectoral Phillips curves positive for all sectors. The nominal wage component demonstrates that a positive output gap is linked to a higher wage via the labor supply — as captured by the term $(\sigma + \varphi/\Lambda_L)$ — thus relating to higher sectoral inflation. The nominal exchange rate component demonstrates that a positive output gap relates to an increased nominal expenditure and a worsened current account, hence occurring with a depreciation of the domestic currency and an increase in the real exchange rate, captured by the term $\left(1 + \mathcal{M}_p^{\top}\mathbf{1}\right)^{-1}$ in $\Gamma_{S,C}$. The increase in the nominal exchange rate propagates into the cost of imported inputs and accordingly into sectoral inflation, captured by the term $\left(\boldsymbol{\Omega}\odot\mathbf{V}_{\mathbf{1}-\boldsymbol{x}}\right)\mathbf{1}$. The nominal wage and exchange rate components are in nominal terms and, therefore, affected by the CPI — as captured by the term $\Gamma_{CPI,C}$ in both components.[29]

[28] The vectors $\boldsymbol{\Gamma}_{CPI,P}$ and $\boldsymbol{\Gamma}_{S,P}$ are equal to: $\boldsymbol{\Gamma}_{CPI,P} \equiv \left(\boldsymbol{\beta}^{\top}\mathbf{v} + \mathcal{M}_p^{\top}\mathbf{1}\right)^{-1}\left[\left(1 + \mathcal{M}_p^{\top}\mathbf{1}\right)(\boldsymbol{\beta}\odot\mathbf{v}) + (1-\boldsymbol{\beta}^{\top}\mathbf{v})\left(\mathcal{M}_p + \mathcal{M}_{\mu}\right)\right]$ and $\boldsymbol{\Gamma}_{S,P} \equiv \left(1 + \mathcal{M}_p^{\top}\mathbf{1}\right)^{-1}\left(\mathcal{M}_p + \mathcal{M}_{\mu} + \boldsymbol{\Gamma}_{CPI,P}\right)$, respectively. Appendix L.1 reports the definition of the matrix $\mathcal{V}$.

[29] Sectoral inflation also directly and positively links to the nominal CPI and exchange rate — as evinced by the positive vectors $\boldsymbol{\Gamma}_{CPI,P}$ and $\boldsymbol{\Gamma}_{S,P}$ in the denominator of $\boldsymbol{\Delta}_{\Phi}$ in Eq. (36) — which generates a positive feedback effect that increases the elasticity of domestic inflation to the output gap (i.e., flattens the slope of the sectoral Phillips curve).

4.2. The slopes of Phillips curves

In this section, we study the slopes of the DC and sectoral Phillips curves, comparing them to their counterparts in the counterfactual cases of closed economies and no input–output linkages.

Eq. (33) in Proposition 1 shows that the slope of the DC Phillips curve is inversely related to the *sum of the price-rigidity-adjusted OG weights*, thereby depending on the three channels that comprise the sectoral OG weight in Eq. (22). Similarly, the slopes of the sectoral Phillips curves in Eq. (36) also depend on these three channels, as we discussed in the previous subsection. In closed economies, the slopes of the DC and sectoral Phillips curves reduce to $(\sigma+\varphi)\,/\left(\sum_i \lambda_i(1-\delta_i)/\delta_i\right)$ and $(\boldsymbol{\Delta}^{-1}-\boldsymbol{\Omega}-\boldsymbol{\alpha}\boldsymbol{\beta}^{\top})^{-1}\boldsymbol{\alpha}(\sigma+\varphi)$, respectively, consistent with the results of Rubbo (2023).

Compared to the closed-economy case, the slope of the DC Phillip curves in open economies can be flatter or steeper, depending on the quantitative strength of two main countervailing forces. First, in the CPI channel, the content of domestic sectoral goods in domestic consumption is smaller in open than in closed economies — as shown by $\widetilde{\lambda}_D^{\top} < \lambda^{\top}$ in the *sum of the CPI channels of the OG weights* in equation (D.2) for the DC Phillips curve. Therefore, the elasticity of the (domestic) output gap to domestic sectoral inflation through the CPI channel is smaller in open economies, thereby *steepening* the slope of the DC Phillips curve relative to the closed-economy case.[30] Second, the positive, open-economy-specific expenditure-switching channel increases the elasticity of the output gap to domestic sectoral markups and inflation — as shown by the *sum of the expenditure-switching channels* in equation (D.1) for the DC Phillips curve — thereby *flattening* the slope in open relative to closed economies.

Compared to the closed-economy case, the slopes of sectoral Phillip curves in open economies can be flatter or steeper, depending on the quantitative strength of two main countervailing forces similar to those for the DC Phillips curve slope. First, the content of domestic sectoral goods in domestic consumption is smaller in open than in closed economies, as evinced by $\boldsymbol{\beta}^{\top}\mathbf{v}$ smaller than its closed-economy counterpart $\boldsymbol{\beta}^{\top}\mathbf{1} = 1$ in the denominators of $\Gamma_{CPI,C}$ (equation L.10) and $\boldsymbol{\Gamma}_{CPI,P}$ (equation L.11). As a result, $\Gamma_{CPI,C}$ and $\boldsymbol{\Gamma}_{CPI,P}$ — which capture the CPI channel in the slopes of sectoral Phillips curves — are larger in open than in closed economies, thereby increasing the slopes $\boldsymbol{\mathcal{B}}$ and *steepening* sectoral Phillips curves. Second, the positive, open-economy-specific expenditure-switching channel — captured by $\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}}$ in $\boldsymbol{\mathcal{M}}_p$ (equation L.5) — tend to reduce $\Gamma_{CPI,C}$, $\boldsymbol{\Gamma}_{CPI,P}$, $\Gamma_{S,C}$, and $\boldsymbol{\Gamma}_{S,P}$ in open relative to closed economies by increasing their denominators, thereby reducing $\boldsymbol{\mathcal{B}}$ and *flattening* sectoral Phillips curves in open economies.

In Section 6.1, we calibrate our model using the WIOD data and show that the first force — namely, the smaller content of domestic goods in domestic consumption in open relative to closed economies — dominates in most cases, generally making the slopes of both the DC and the sectoral Phillips curves steeper in open economies. In Figure D.2 of Appendix D.1, we also show that the DC Phillips curve slope in the baseline economy with IO linkages is flatter than that in one-sector SOEs and in multi-sector SOEs without IO linkages, consistent with the results in closed economies à la (Rubbo, 2023). Intuitively, with the introduction of IO linkages, domestic sectoral goods are not only *directly* but also *indirectly* used by domestic output, which increases the elasticity of the domestic output gap to domestic sectoral inflation, thereby *flattening* the slope of the DC Phillips curve. In particular, the figure shows that the introduction of IO linkages flattens the slope more in closed (relative to open) economies for those economies that are relatively open.

Takeaways for the DC Phillips curve slope. Overall, our analysis shows that the *sum of the sectoral OG weight* — through the sum of each of the three channels across sectors — determines the slope of the DC Phillips curve —which, therefore, depends on cross-border and input–output linkages.

4.3. Relative sectoral weights in the divine coincidence index for output gap targeting

In this section, we show that the monetary policy of output gap targeting that closes the output gap is implemented by targeting the divine coincidence index — which depends on the *relative sectoral OG weights* — to zero. Therefore, we study the role of input–output and cross-border linkages for output gap targeting through their impacts on the *relative sectoral weights in the DC index*. We do so by focusing on the pitfalls of the normalized sectoral OG weights in alternative output gap targeting that: (i) disregards the role of input–output linkages as in the one-sector small open economy literature, and (ii) disregards the role of cross-border linkages and targets the output gap in closed economies with production networks.

Output gap targeting. Proposition 1 implies that the DC index is a sufficient statistic for the output gap in multi-sector open economies with production networks, as in closed economies à la (Rubbo, 2023). Therefore, the monetary policy of *output gap targeting* that fully closes the output gap can be implemented by targeting the DC index to zero. As we show later in the paper, although output gap targeting is not optimal in multi-sector open economies because the divine coincidence fails to hold (Section 5), it remains a useful policy that closely approximates the optimal monetary policy in minimizing welfare loss, similar to the case of multi-sector closed economies à la (Rubbo, 2023) (Section 6.3).

[30] As we show in Appendix D.3, due to the smaller content of domestic goods in domestic consumption, the domestic consumer price Phillips curve — an important case in policy practice — is flatter (conditional on shocks and foreign prices) in open than in closed economies in general.

Pitfall in output gap targeting in one-sector small open economy literature. A well-established result in one-sector SOE models without input–output linkages is that optimal monetary policy should stabilize domestic inflation (Galí and Monacelli, 2005), which simultaneously closes the output gap, as well as the terms-of-trade gap — i.e., the divine coincidence holds. This result is consistent with our theoretical finding in the special case of the one-sector version of our model (Section 5).

However, in the one-sector SOE literature, the optimal policy of output gap targeting is usually implemented by targeting the PPI inflation, where domestic sectoral inflation rates are weighted by the sectoral sales that are proportional to the sectoral Domar weights. Below, we analyze the pitfalls of using Domar weights in place of OG weights for output gap targeting. Thus, our results are consistent with the findings in one-sector SOE models that domestic inflation should be stabilized. We contribute to this line of research by deriving the appropriate sectoral weights for output gap targeting in open economies with production networks.

Pitfall in output gap targeting that disregards cross-border linkages. To assess the relevance of openness for output gap targeting, we study the pitfalls of using OG weights that ignore cross-border linkages in the DC index.

As a first step, we determine the OG weight in closed economies. In a closed economy, only domestic demand exists; consequently, the total content of domestic goods in exports is zero (i.e., $\widetilde{\lambda}_{F,i} = 0,\ \forall i$). Moreover, the *expenditure-switching* and *profit channels* are equal to zero. Thus, *total content in domestic consumption* uniquely determines the closed-economy OG weight, which is equal to the Domar weight and consistent with the results of Rubbo (2023), as summarized in the next lemma.[31]

Lemma 2. *In a closed economy, the OG weight of each sector $i \in \{1, 2, \dots, N\}$ reduces to the Domar weight, i.e., $\mathcal{M}_{OG,i} = \lambda_i$. In the open economy, the Domar weight of each sector i equals the sum of the sectoral total contents in domestic consumption and in exports, viz.:*

$$\lambda_i = \widetilde{\lambda}_{D,i} + \widetilde{\lambda}_{F,i}. \tag{37}$$

Proof. See Appendix L.2.

Lemma 2 implies that output gap targeting ignoring cross-border linkages will adopt the Domar weight in place of the OG weight. Thus, we construct the normalized sectoral Domar weights for closed economies as $\widetilde{\lambda}_i \equiv (\lambda_i(1-\delta_i)/\delta_i)/\kappa_\lambda$, where $\kappa_\lambda \equiv \sum_{i'} \lambda_{i'}(1-\delta_i')/\delta_i'$ — similar to the normalized sectoral OG weights for open economies in Eq. (31). The corresponding monetary policy that targets the aggregate inflation index using the normalized Domar weights — or, equivalently, the *(price-rigidity-adjusted) PPI targeting* policy — coincides with output gap targeting in closed economies à la (Rubbo, 2023). For simplicity, we refer to the price-rigidity-adjusted PPI targeting policy as "*PPI targeting*" throughout the paper.[32]

Eq. (37) in Lemma 2 further shows that, unlike in closed economies, the Domar weight in open economies includes not only *total content in domestic consumption* ($\widetilde{\lambda}_{D,i}$), but also *total content in exports* ($\widetilde{\lambda}_{F,i}$) as domestic output in open economies is supplied to both domestic and foreign customers. Combining Lemma 2 and Theorem 1 gives the percentage deviation of the normalized closed-economy OG weight (i.e., the Domar weight) from the normalized open-economy OG weight, as stated in the following proposition:

Proposition 3. *The percentage deviation of the normalized Domar weight relative to the normalized OG weight is:*

$$\frac{\widetilde{\lambda}_i - \widetilde{\mathcal{M}}_{OG,i}}{\widetilde{\lambda}_i} = \frac{\kappa_\lambda}{\kappa_{OG}}\Bigg[\underbrace{\frac{\widetilde{\lambda}_{F,i}}{\lambda_i}}_{\text{export intensity}} \underbrace{-\kappa_{CPI}^{-1}\cdot\frac{\widetilde{\rho}_{ES,i}}{\lambda_i}}_{\text{expenditure switching}} \underbrace{+\kappa_{CPI}^{-1}\cdot\left((1-\widetilde{\alpha}_i) - \widetilde{\lambda}_{F,i}/\lambda_i\right)}_{\text{profit}}\Bigg] + \left(1 - \frac{\kappa_\lambda}{\kappa_{OG}}\right). \tag{38}$$

Proof. Straightforward result from Lemma 2 and Theorem 1.

Proposition 3 shows that the percentage deviation of the normalized Domar weight from the normalized OG weight is equal to the percentage deviation of the Domar from OG weights (in brackets) — rescaled by the ratio of the sums of (price-rigidity-adjusted) Domar to OG weights ($\kappa_\lambda/\kappa_{OG}$) — plus the sector-invariant constant $1 - \kappa_\lambda/\kappa_{OG}$. Proposition 3 demonstrates that the PPI targeting that fails to consider cross-border linkages and uses Domar weights can *either* overstate *or* understate the inflation of a domestic sector, depending on the magnitudes of *two main countervailing forces*.[33]

First, Domar weights in open economies capture domestic sectors' supply of inputs to foreign countries in addition to domestic output — summarized by the sectoral *export intensity*, which we define as the ratio of a sector's total content in exports to its Domar weight (i.e., the first component in the brackets on the RHS of Eq. (38)). Thus, the output gap targeting that disregards cross-border linkages and uses the Domar weights overemphasizes the contribution of the domestic sector to the domestic output as a supplier, thereby overstating the relative weights of sectors that export more directly and indirectly in the DC index.

Second, the normalized Domar weights — which disregard cross-border imports and exports — can understate the importance of a domestic sector's inflation by failing to consider its impact on the domestic-to-foreign prices and, in turn, the demand for domestic

[31] Our standard assumption of a Cobb–Douglas production function is crucial for establishing the equivalence between the sectoral *total content in domestic consumption* and the Domar weight, as discussed in Baqaee (2018).

[32] Similarly, throughout the paper, we refer to the price-rigidity-adjusted CPI targeting policy as "*CPI targeting*", which targets the aggregate inflation index using the normalized CPI weights, i.e., $\widetilde{\beta}_i \equiv (\beta_i(1-\delta_i)/\delta_i)/\kappa_\beta$, where $\kappa_\beta \equiv \sum_{i'} \beta_{i'}(1-\delta_i')/\delta_i'$.

[33] As we show in the quantitative Section 6.2, the magnitude of the profit channel is close to zero.

goods and labor — as captured by the negative *expenditure-switching component* (i.e., the second component in the brackets on the RHS of Eq. (38)). Accordingly, the output gap targeting that ignores openness understates the relative weights of sectors that face a large expenditure-switching effect and a large total content of domestic labor in the DC index.

Takeaways for relative sectoral weights in output gap targeting. Overall, we find that the output gap targeting policies used in the one-sector SOE literature and disregarding cross-border linkages both implement the PPI targeting — which uses normalized Domar instead of OG weights. The extent to which the PPI targeting over- or under-states the relevance of a sector's inflation depends on the *strengths* of the two major countervailing forces — namely, an overestimation from overstating the sector's total content in domestic output versus an underestimation from ignoring the expenditure-switching channel — for this sector *relative to* other sectors.[34]

5. Welfare loss and optimal monetary policy

In this section, we study the welfare loss function and optimal monetary policy. As in Woodford (2003) and Galí (2015), we derive the closed-form solution of the policy that minimizes welfare losses up to the second-order approximation.

Welfare loss. Under the assumption of non-contingent subsidy and tax rates in Lemma 1, the *flexible-price equilibrium* represents the optimal allocation for the domestic social planner. We define *welfare loss* as the utility gap of the representative household between the *sticky* and *flexible-price equilibria*, $u(\xi) - u^{flex}(\xi)$, and approximate it to the second order, as stated in the following proposition:

Proposition 4 (*Welfare Loss*). *Given the realized state $\xi \in \Xi$, the welfare loss can be decomposed as:*

$$u(\xi) - u^{flex}(\xi) = \underbrace{-\frac{1}{2}\left(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\right)\widehat{C}^{gap}(\xi)^2}_{\text{output-gap misallocation}} \underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\xi)^\top(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})\widehat{\mathbf{P}}(\xi)}_{\text{within- and across-sector, and cross-border misallocations}}, \tag{39}$$

where the within-sector, across-sector, and cross-border misallocations are equal to

$$-\frac{1}{2}\widehat{\mathbf{P}}(\xi)^\top\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\xi) = -\frac{1}{2}\sum_i \lambda_i\epsilon_i\frac{1-\delta_i}{\delta_i}\widehat{P}_i(\xi)^2, \tag{40}$$

$$-\frac{1}{2}\widehat{\mathbf{P}}(\xi)^\top\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\xi) = -\frac{1}{2}\sum_{i=1}^n \beta_i\left[\widehat{C}_i^{gap}(\xi) - \widehat{C}^{gap}(\xi)\right]^2 - \frac{1}{2}\sum_{i=1}^n \lambda_i\alpha_i\left[\widehat{L}_i^{gap}(\xi) - \widehat{Y}_i^{gap}(\xi)\right]^2 - \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \lambda_i\omega_{i,j}\left[\widehat{X}_{i,j}^{gap}(\xi) - \widehat{Y}_i^{gap}(\xi)\right]^2, \tag{41}$$

$$-\frac{1}{2}\widehat{\mathbf{P}}(\xi)^\top\boldsymbol{\mathcal{L}}^{cb}\widehat{\mathbf{P}}(\xi) = -\frac{1}{2}\sum_{i=1}^n \frac{\beta_i}{\theta_i}v_i(1-v_i)\left[\widehat{C}_{Hi}^{gap}(\xi) - \widehat{C}_{Fi}^{gap}(\xi)\right]^2 - \frac{1}{2}\sum_{i=1}^n\sum_{j=1}^n \frac{\lambda_i\omega_{i,j}}{\theta_j}v_{x,i,j}(1-v_{x,i,j})\left[\widehat{X}_{Hi,H_j}^{gap}(\xi) - \widehat{X}_{Hi,F_j}^{gap}(\xi)\right]^2 - \frac{1}{2}\sum_{i=1}^n \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\left[\frac{\theta_{F,i}-1}{\theta_{F,i}}\widehat{Y}_{EX,i}^{gap}(\xi)^2 - \Lambda_L\widehat{L}^{gap}(\xi)^2\right]. \tag{42}$$

Proof. See Appendix M.1.

Eq. (39) shows that, to a second-order approximation, *welfare loss* consists of the sum of losses from the output gap misallocation, within- and across-sector misallocation — similar to those in closed economies à la (La'O and Tahbaz-Salehi, 2022) and Rubbo (2023) — as well as the cross-border misallocation, which is specific to the open economy. Specifically, the within-sector misallocation is the sum of the misallocation arising from inflation dispersion in all sectors, which is similar to its counterpart in one-sector economies. The across-sector misallocation includes those arising from the disproportional sectoral consumption relative to aggregate consumption (first term on the RHS of Eq. (41)), as well as those arising from the disproportional use of sectoral labor and intermediate inputs relative to sectoral output across different sectors i (second and third terms on the RHS of Eq. (41), respectively). The cross-border misallocation includes distortions arising from the disproportional use of domestic versus foreign goods for both consumption and intermediate inputs (first and second terms on the RHS of Eq. (42)). The cross-border misallocation also includes distortions arising from disproportionate exports relative to the use of domestic labor, which cause domestic producers' monopoly power in international markets to deviate from the optimal level (the third term on the right-hand side of Eq. (42)).

[34] In Appendix F, we study the impacts of introducing input–output linkages into a multi-sector, horizontal SOE versus a multi-sector, horizontal closed economy on normalized sectoral OG weights. We show that the increase in the sectoral normalized OG weights due to the introduction of IO linkages is positively linked to the sector's share of the CPI channel in the OG weight.

Role of input–output and cross-border linkages in the welfare loss. In what follows, we analyze how input–output and cross-border linkages affect welfare loss using two special cases of our framework: (i) the workhorse model of the one-sector small open economy without IO linkages, as in Galí and Monacelli (2005), and (ii) the multi-sector closed economy without cross-border linkages à la (Rubbo, 2023).

In the one-sector small open economy, the welfare loss in Eq. (39) reduces to the sum of the output gap, within-sector misallocation, and cross-border misallocation, the latter two terms being proportional to the square of domestic inflation, as shown in Eq. (39). In addition, in the one-sector economy, the output gap is proportional to domestic inflation, as can be seen by substituting Eq. (20) into (21). As a result, the welfare loss is proportional to the square of domestic inflation, allowing the optimal monetary policy to achieve the first-best allocation by fully stabilizing domestic inflation (i.e., the divine coincidence) as in Galí and Monacelli (2005). In multi-sector closed economies with production networks, the welfare loss in Eq. (39) reduces to the cases in La'O and Tahbaz-Salehi (2022) and Rubbo (2023). In other words, the OG weights $\mathcal{M}_{OG}$ in the output-gap misallocation reduce to the Domar weight, and cross-border misallocation is absent.

Optimal monetary policy. Next, we provide an analytical characterization of the optimal monetary policy.

Definition 5 (*Optimal Monetary Policy*)**.** For any aggregate state $\xi \in \Xi$, the optimal monetary policy sets the money supply $M(\xi)$ — which is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\xi)$ in equilibrium — to minimize the welfare loss in Eq. (39) *subject to* the sectoral Phillips curves in Eq. (34).

Consistent with Definition 5, we derive the aggregate inflation index that the monetary authority should target to implement the optimal monetary policy, as stated in the following proposition:

Proposition 5 (*Implementation of the Optimal Monetary Policy*)**.** *The optimal monetary policy is implemented by setting the following aggregate inflation index to zero:*

$$\left\{\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right] \kappa_C^{-1} \mathcal{M}_{OG}^{\top}(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \mathcal{B}^{\top}(\mathcal{L}^{within} + \mathcal{L}^{across} + \mathcal{L}^{cb})\right\} \widehat{\mathbf{P}} = 0, \tag{43}$$

for any realized state $\xi \in \Xi$.

Proof. See Appendix M.2.

Eq. (43) shows that the optimal policy accounts for *both* the output gap misallocation — as evinced by the OG weights $\mathcal{M}_{OG}^{\top}$, as the first term in the brackets — *and* the within- and across-sector, and cross-border misallocation generated by sectoral distortions —as captured by the second term $\mathcal{B}^{\top}(\mathcal{L}^{within} + \mathcal{L}^{across} + \mathcal{L}^{cb})$ in the curly brackets. In contrast, output gap targeting closes the output gap, but it does not simultaneously eliminate the within- and across-sector, and cross-border misallocations, because the sectoral inflation underlying these misallocations is not proportional to the output gap according to sectoral Phillips curves (34). Therefore, the "divine coincidence" — which holds in the workhorse model of one-sector SOEs, as in Galí and Monacelli (2005) — fails to hold in our multi-sector open economies, similar to the case of the multi-sector closed economies in La'O and Tahbaz-Salehi (2022) and Rubbo (2023). However, as we show in Section 6.3, the optimal monetary policy is well approximated by output gap targeting in terms of welfare loss, as in the case of multi-sector closed economies à la (Rubbo, 2023).

Takeaways for welfare loss and optimal monetary policy. Overall, our analysis demonstrates that treating the economy as a one-sector SOE and as a closed economy with production networks ignores the across-sector and cross-border distortions, respectively, in welfare losses and formulating optimal monetary policy.

6. Quantitative analysis

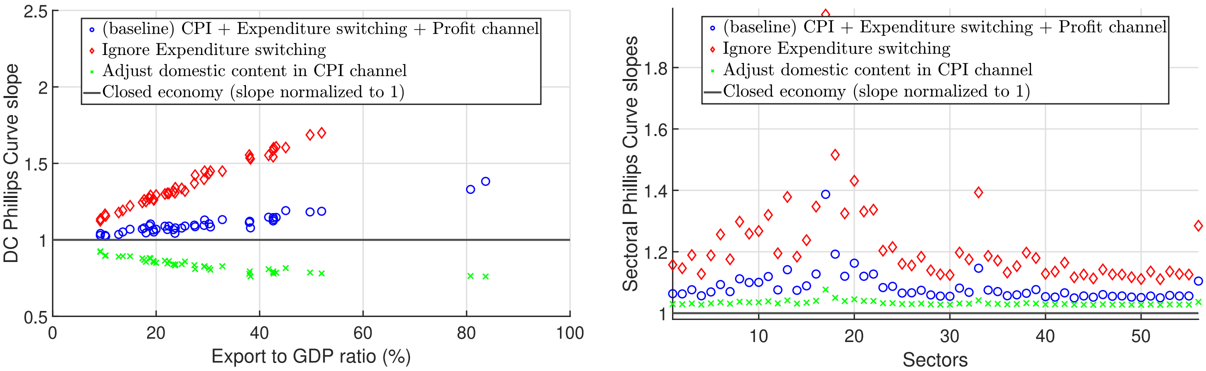
In this section, we quantify our theoretical results by calibrating the model to the input–output matrices of major economies in the WIOD. Section 6.1 studies the slopes of the DC and sectoral Phillips curves in open economies relative to closed economies, focusing on the relevance of different channels for the differences between open-economy and closed-economy slopes. Section 6.2 examines the relevance of the different channels for the normalized OG weights and for the differences between normalized Domar and OG weights. It then uses the rule-of-thumb combinations of sectoral relevance metrics to approximate the normalized OG weights and Domar-OG differences, revealing the relevance of cross-border and IO linkages for the normalized OG weights. Section 6.3 investigates the welfare of alternative monetary policies, showing that output gap targeting is close to the optimal policy, and enhances welfare over alternative policies that ignore cross-border or input–output linkages.

Our quantitative analysis uses the National Input–Output Tables (NIOTS) for 43 economies (28 EU and 15 OECD countries, each of them comprising 56 sectors) from the WIOD for the year 2014. We calibrate the input–output matrix and import and export shares of each country using its NIOTS sector-level data.[35] Table 2 shows the calibration of the key parameters in our model. Appendix I.1Figure I.1 in Appendix I.2 presents the calibration of the full set of parameters and provides additional details on the WIOD.

[35] Data source: https://www.rug.nl/ggdc/valuechain/wiod/?lang=en. The release of the WIOD in 2016 provides information for the period 2000–2014. In our analysis, we use the latest available year, 2014. The NIOTS provides each country's sector-level imports from the Rest of the World (RoW) and exports to the RoW, which are aggregates of the country's imports from and exports to all other countries, respectively, including those countries that are not listed in the WIOD. Using the NIOTS data, we calibrate each country individually as a small open economy relative to the rest of the world, rather than calibrating all countries simultaneously within a global equilibrium.

Table 2
Model calibration.

Parameters	Data variables/moments used
Common across all countries	
Risk aversion, $\sigma = 2$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Labor supply elasticity, $\varphi = 1$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Elasticity of substitution (EOS) across varieties, $\epsilon_i = 8$	Atkeson and Burstein (2008)
EOS. btw. domestic and foreign goods, $\theta_i = \theta_{Fi} = 5$	Head and Mayer (2014)
Sector-level frequency of price adjustment, δ_i	Pasten et al. (2024)
Frequency of wage adjustment, δ_0	Beraja et al. (2019) and Barattieri et al. (2014)
Country specific	
Input–output matrix, $\boldsymbol{\Omega}$	Sectoral gross output, intermediate goods from both domestic and foreign
Home bias for firms' import, $\mathbf{V}_x$	Intermediate goods from both domestic and foreign
Labor share, $\boldsymbol{\alpha}$	Sectoral gross output, labor compensation
Export to foreign countries in steady state, $D^*_{EX,Fi}$	Sectoral exports to foreign countries
Consumer consumption share, $\boldsymbol{\beta}$	Sectoral consumption from both domestic and foreign, and GDP
Consumer consumption home bias, $\boldsymbol{v}$	Sectoral consumption from both domestic and foreign



(a) The DC Phillips curve (b) Sectoral Phillips curves (Mexico)

Fig. 2. Slopes of Phillips curves (relative to a closed economy).
Notes: Shown in panel (a) is the ratio of the slope of the DC Phillips curve in open economies to its counterpart in closed economies, for each country in the sample. Shown in panel (b) are the ratios of the slopes of the sectoral Phillips curves in open economies to their counterparts in closed economies, for Mexico. The closed-economy slopes are normalized to one, corresponding to the bold black horizontal line at one. In both panels, the blue circles indicate the baseline open-economy Phillips curve slopes, the red diamonds indicate the counterfactual case that ignores the expenditure-switching channel, and the green crosses indicate the counterfactual slope that replaces the open-economy sectoral content in domestic consumption with its closed-economy counterpart in the CPI channel (i.e., in panel a, change $\widetilde{\lambda}_D$ to λ; in panel b, change $\boldsymbol{\beta} \odot \mathbf{v}$ to $\boldsymbol{\beta} \odot \mathbf{1}$, $\boldsymbol{\beta}^\top \mathbf{v}$ to 1, and $\mathbf{V}_x$ to $\mathbf{1}_{N \times N}$ in $\boldsymbol{\Delta}_{\boldsymbol{\Phi}}$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

6.1. The slopes of Phillips curves in open versus closed economies

Shown in panels (a) and (b) in Fig. 2 are the ratios of the slopes of the DC Phillips curve and the sectoral Phillips curves in open economies to their counterparts in closed economies à la (Rubbo, 2023), for each country in the sample and for Mexico, respectively. In both panels, we consider the following three cases: (i) the baseline open-economy DC and sectoral Phillips curves in Eqs. (33) and (35), respectively (blue circles); (ii) the counterfactual case that ignores the expenditure-switching channel, keeping only the CPI channel (red diamonds); and (iii) the counterfactual case that replaces the open-economy content in domestic consumption with its closed-economy counterpart in the CPI channel — as shown on the RHS of equation (D.2) (green crosses).[36] In both panels, because we plot the ratios of the slopes of the Phillips curves to their counterparts in the closed economy, the slopes of the closed-economy Phillips curves are indicated by the bold black horizontal line at one; a value above (vs. below) one indicates that the Phillips curve is steeper (vs. flatter) in open than in closed economies.

As shown in panels (a) and (b) of Fig. 2, the slopes of both the DC and sectoral Phillips curves in open economies are generally *steeper* than those in closed economies — as shown by the blue circles above the black horizontal line of unity — which result from the combination of two countervailing forces. On the one hand, the smaller content of domestic goods in domestic consumption in open than in closed economies generates a smaller elasticity of the output gap to domestic sectoral inflation and, therefore, *steeper*

[36] The slopes of the DC and the sectoral Phillips curves for the closed economy and the counterfactual cases (ii) and (iii) are shown in Appendix D.1.

slopes of the DC and sectoral Phillips curves. This is captured by the blue circles for the baseline open-economy case (i) lying *above* the counterfactual case (iii) of green crosses — where the open-economy content in domestic consumption in the CPI channel is replaced by the closed-economy counterpart. On the other hand, the positive, open-economy-specific channel of expenditure switching increases the elasticity of the output gap to domestic sectoral inflation, thereby *flattening* the slopes of the DC and sectoral Phillips curve in open relative to closed economies. This is shown by the blue circles for the baseline open-economy case (i) lying *below* the counterfactual case (ii) of red diamonds — where the open-economy-specific channel of expenditure switching is absent. Quantitatively, the force of the smaller content of domestic goods in domestic consumption in the open-economy CPI channel dominates the countervailing force that arises from the positive expenditure-switching channel, thus leading to *steeper* slopes of both the DC and sectoral Phillips curves in open economies than in closed economies, consistent with our analyses in Section 4.2.

6.2. Quantifying and approximating normalized OG weights and Domar-OG differences

In this section, we quantify and approximate the normalized OG weights and the differences between normalized Domar and OG weights. The normalized Domar weights correspond to the normalized OG weights in closed economies and coincide with the sectoral weights in the PPI targeting policy used in one-sector SOE literature, as discussed in Section 4.3. We show that the *CPI* and *expenditure-switching channels* explain the bulk of the variation in normalized sectoral OG weights across sectors for all countries, and that the contribution of the *CPI* (vs. *expenditure-switching*) channel decreases (vs. increases) with the openness of the economy (panel a of Figure I.1 in Appendix I.2). In contrast, the contribution of the *profit channel* is marginal. We also show that the pitfalls in the output gap targeting ignoring openness — measured by the differences between the normalized Domar and OG weights — are mostly driven by the *export intensity* and the *expenditure switching* components, and that the contribution of the *export intensity* (vs. *expenditure switching*) components decreases (vs. increases) with the openness of the economy (panel b of Figure I.1).[37]

Then, we use panel regressions to study the rule-of-thumb combinations of the sectoral relevance metrics in Definition 3 to approximate the normalized sectoral OG weights and the difference between the normalized Domar and OG weights. We show that the normalized OG weights and the normalized Domar-OG differences can be approximated by the linear combination of *total content in domestic consumption* and *generalized expenditure-switching elasticity* and the linear combination of *export intensity* and the *ratio of generalized expenditure-switching elasticity to Domar weight*, respectively. We also show that ignoring IO linkages leads to an inaccurate approximation of normalized sectoral OG weights.

We study the combinations of sectoral relevance metrics to approximate the normalized OG weights and Domar-OG differences using the following regressions[38]:

$$y_{c,i} = \mathbf{X}_{c,i}^{\top}\boldsymbol{\beta} + \eta_c + \epsilon_{c,i}, \quad \text{with} \quad y_{c,i} \in \{\widetilde{\mathcal{M}}_{OG,c,i}, (\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}\}, \tag{44}$$

where the dependent variable $y_{c,i}$ is either the level of the normalized OG weight ($\widetilde{\mathcal{M}}_{OG,c,i}$) or the percentage difference between the normalized Domar and OG weights ($(\widetilde{\lambda}_{c,i} - \widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$) for sector i and country c. The variable $\boldsymbol{X}_{c,i}$ includes our sectoral relevance metrics for the regressions (see Tables 3 and 4), and η_c is the country fixed effect.

Approximation of relative sectoral weights in the DC index. Shown in Table 3 are the estimates of Eq. (44) with the normalized sectoral OG weight ($\widetilde{\mathcal{M}}_{OG,c,i}$) as the dependent variable; the sectoral relevance metrics on the RHS are multiplied by the sectoral price rigidities ($(1-\delta_i)/\delta_i$) to align with the normalized OG weight on the LHS. As shown in column (1), the price-rigidity-adjusted total content in domestic consumption is positively related to the normalized OG weight of the sector with a coefficient equal to 0.48 and an R-squared of 0.84. As shown in column (4), the price-rigidity-adjusted generalized expenditure-switching elasticity is positively related to the normalized OG weight of the sector, with a coefficient of 0.175 and a medium-sized R-squared of 0.24.

To validate the negative impact of the import shares on the sectoral total content in domestic consumption and normalized OG weights — as discussed in Section 2.10 and Appendix G — we define the *import intensity* of a sector i as $1 - \widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$, where $\widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$ captures the domestic demand for i's goods in the baseline economy with imports ($\widetilde{\lambda}_{D,i}$) relative to that without imports ($\widetilde{\lambda}_{All,D,i}$).[39] Accordingly, the import intensity of a sector measures the impact of the entire economy's import shares on the domestic demand for this sector's goods. As shown in column (3), a sector's normalized OG weight significantly decreases with the import intensity, thereby validating the negative impact of the direct and indirect import shares of a sector on its normalized OG weight.

As shown in column (5), the linear combination of the price-rigidity-adjusted total content in domestic consumption and generalized expenditure-switching elasticity provides a precise approximation of the normalized sectoral OG weights, with a large R-squared of 0.99 that is significantly higher than those with each of the two sectoral metrics in columns (1) and (4). As shown in column (6), the total content of domestic labor is significantly and positively related to the normalized OG weight, but adding it to the total content in domestic consumption does not improve the approximation accuracy — as shown by the R-squared of 0.847 that is very close to the 0.841 in column (1). Comparing columns (6) and (5) shows that the expenditure switching effect is more

[37] Presented in Appendix I.2 are the details of the method, figures, and results of the variance decomposition analysis.

[38] We focus on a subsample of 11 relatively open economies — in terms of the economy-wise export-to-GDP ratio — out of all 43 economies. Results are robust, albeit less strong, for less open economies. We do not include sectoral fixed effects in the regression, as our main purpose is to explore the variations in normalized OG weights across different sectors.

[39] The term $\widetilde{\lambda}_{All,D,i}$ is the ith entry of the vector $\boldsymbol{\beta}^{\top}(\mathbf{I}-\boldsymbol{\Omega})^{-1}$ and captures the domestic demand that reaches the domestic sector i directly and indirectly (via downstream sectors) *if* the entire economy — including sector i and its downstream sectors — does not import from abroad (i.e., $v_r = 1$ for all r and $v_{x,r,s} = 1$ for all r and s).

Table 3
Sectoral relevance metrics and the normalized OG weights in the data.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Total content in dom. consumption	0.483***				0.460***	0.468***		
	(0.0121)				(0.00663)	(0.0122)		
Import share		−0.000109*						
		(5.79e−05)						
Import Intensity			−0.000431***					
			(4.20e−05)					
Generalized ES elasticity				0.175***	0.139***			
				(0.00856)	(0.00452)			
Total content of dom. labor						0.00309***		
						(0.000627)		
Domar weight							0.409***	
							(0.0226)	
Total content in dom. consumption w/o IO								0.712***
								(0.0333)
Generalized ES elasticity w/o IO								0.546***
								(0.0992)
Observations	601	601	601	601	601	601	601	601
R-squared	0.841	0.005	0.251	0.236	0.988	0.847	0.899	0.658
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (44), which regresses the level of the normalized sectoral OG weight $\widetilde{\mathcal{M}}_{OG,c,i}$ over the sectoral relevance metrics characterized by Definition 3 in Section 2.10. Total content in domestic consumption, generalized expenditure-switching (ES) elasticity, total content of domestic labor, and the Domar weight are multiplied by the sectoral price rigidities, $(1-\delta_i)/\delta_i$. The analysis includes the subsample of 11 relatively open economies — in terms of the economy-wise export-to-GDP ratio — out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 4
Sectoral relevance metrics and the difference between normalized Domar and OG weights.

	(1)	(2)	(3)	(4)
Export intensity	0.387***		0.878***	0.127***
	(0.0244)		(0.00863)	(0.0247)
Generalized ES elasticity over Domar		−0.0318***	−0.255***	
		(0.00813)	(0.00380)	
Total content of domestic labor				−0.730***
				(0.0501)
Observations	601	601	601	601
R-squared	0.366	0.043	0.939	0.570
Country FE	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (44), which regresses the normalized sectoral Domar-OG percentage difference $(\widetilde{\lambda}_{c,i}-\widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$ over the sectoral relevance metrics characterized by Definition 3 in Section 2.10. The generalized ES elasticity over Domar is the ratio of the sectoral generalized expenditure-switching elasticity to the Domar weight. The analysis includes the subsample of 11 relatively open economies — in terms of the economy-wise export-to-GDP ratio — out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

important than the domestic labor content for the generalized expenditure-switching elasticity to approximate the normalized OG weights.

As shown in column (7), the normalized Domar weight — which is the nearly optimal normalized OG weight in closed economies à la (Rubbo, 2023; La'O and Tahbaz-Salehi, 2022) — has a weaker explanatory power than the linear combination of the price-rigidity-adjusted total content in domestic consumption and generalized expenditure-switching elasticity in column (5), with a smaller R-squared of 0.90. The comparison between columns (5) and (7) illustrates the importance of considering openness in approximating the normalized sectoral OG weights in the DC index.

The results in Table 3 imply that the output gap targeting should assign larger weights to sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption and that face a large expenditure-switching effect.

Approximation of pitfalls in the output gap targeting abstracting from openness. Presented in Table 4 are the results for the version of the regression in Eq. (44) with the percentage difference between the normalized Domar and OG weights ($(\widetilde{\lambda}_{c,i}-\widetilde{\mathcal{M}}_{OG,c,i})/\widetilde{\lambda}_{c,i}$) as the dependent variable, which measures the pitfalls in the policy of output gap targeting that ignores openness, that is, the PPI targeting policy. As shown in column (1), the export intensity is positively related to the normalized Domar-OG difference with a coefficient of 0.39 and a medium-sized R-squared of 0.37. As shown in column (2), the ratio of the generalized expenditure-switching elasticity to the Domar weight is negatively related to the normalized Domar-OG difference with a coefficient of −0.03 and a small R-squared of 0.04.

Despite the limited explanatory power of each individual component shown in columns (1) and (2), column (3) shows that the linear combination of the export intensity and the ratio of the generalized expenditure-switching elasticity to the Domar weight explains most of the variations in the normalized Domar-OG difference, as evinced by the large R-squared of 0.94. This R-squared is significantly larger than those in columns (1) and (2), revealing the complementarity between the two components in approximating the normalized Domar-OG difference. As shown in column (4), adding the total content of domestic labor to the export intensity only partially improves the approximation accuracy — as evinced by the R-squared of 0.57, which is larger than 0.37 in column (1) but much smaller than 0.94 in column (3). This implies that the expenditure-switching mechanism in the generalized expenditure-switching elasticity plays an important role in explaining the normalized Domar-OG difference.

The results in Table 4 support Proposition 3 and show that the sector-level pitfalls in the output gap targeting that abstract from cross-border linkages can be well approximated by the rule-of-thumb, linear combination of the export intensity and the ratio of the generalized expenditure-switching elasticity to the Domar weight. These results demonstrate that monetary policy adopting normalized Domar weights, or the policy that targets PPI inflation in the one-sector SOE literature, overstates inflation in sectors with either large export intensity or limited generalized expenditure-switching elasticity. To account for the openness of the economy in the closed-economy PPI targeting policy, the monetary authority should assign smaller weights to sectors that export intensively directly and indirectly, and assign larger weights to sectors that face large expenditure-switching effects.

Relevance of input–output linkages for relative sectoral weights in the DC index. Our theoretical analysis in Section 2.10 shows that IO linkages are important drivers of the sectoral relevance metrics that underpin the sectoral OG weights. In an SOE without production networks, domestic sectors are only direct, rather than indirect, suppliers to domestic and foreign demand, so import intensity simplifies to the import share.

Comparing columns (2) and (3) in Table 3 shows that import intensity explains more variation in normalized OG weights than the import share, as evinced by the larger R-squared for import intensity (0.25) than the almost negligible R-squared for import share (0.01). Moreover, as shown in column (8) in Table 3, the counterfactual sectoral metrics of total content in domestic consumption and generalized expenditure-switching elasticity that abstract from IO linkages explain only 66% of the variation in the normalized sectoral OG weights, which is much smaller than that considering IO linkages in column (5).[40] Thus, indirect imports via both upstream and downstream sectors — as captured by sectoral relevance metrics with IO linkages rather than without — are important for the approximation of normalized OG weights, hence supporting the relevance of input–output linkages for monetary policy.

We conclude that the structure of input–output linkages interplays with the cross-border linkages of the small open economy to determine the metrics of sectoral relevance in Definition 3 and, in turn, the relative sectoral weights in the DC index for output gap targeting. Ignoring production networks results in an imprecise approximation of the correct sectoral weights required to close the output gap.

6.3. Welfare comparison of alternative monetary policies

In this section, we quantitatively compare the welfare losses of the economy —using Eq. (39) in Proposition 4 of Section 5 — under alternative monetary policies, and show that output gap targeting performs close to the optimal monetary policy, as in closed economies à la (Rubbo, 2023), and outperforms policies that ignore either cross-border or input–output linkages.[41]

Specifically, we compare the welfare loss under the following five alternative monetary policies: the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI targeting. The policy of PPI (vs. CPI) targeting targets an aggregate inflation index where the Domar weight λ_i (vs. consumption share β_i) — after adjusting for sectoral price rigidities (i.e., multiply by $(1-\delta_i)/\delta_i$) and normalized by the sum of sectoral weights — is used as the weight for each sector i's inflation. The output gap targeting without IO linkages weights sectoral inflation with the normalized OG weights that ignore IO linkages.[42] We study the PPI and CPI targeting because they are both widely used policies. While the PPI targeting ignores the openness of the economy and coincides with output gap targeting used in closed economies à la (Rubbo, 2023) and in one-sector small open economy literature, the CPI targeting ignores both openness and IO linkages. In addition, we examine the output gap targeting without IO linkages to evaluate the relevance of input–output linkages for the welfare implications of monetary policy.

Presented in Table 5 is the welfare loss expressed as a percentage of the steady-state consumption under the alternative monetary policies. We consider the welfare loss for Mexico, Luxembourg, and the US, as these countries represent those with medium, large, and small degrees of openness, respectively — as measured by the economy-wise export-to-GDP ratio (19%, 83%, and 9%,

[40] In multi-sector small open economies without IO linkages, the Leontief inverse reduces to a diagonal matrix, with $l_{vx,r,i} = 1$ for $r = i$ and $l_{vx,r,i} = 0$ for $r \neq i$, and sectoral labor cost shares reduce to $\alpha_i = 1$. Thus, the total content in domestic consumption and the generalized expenditure-switching elasticity reduce to $\widetilde{\lambda}_{D,i} = \beta_i v_i$ and $\widetilde{\rho}_{ES,i} = (\theta_{F,i} - 1)\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1 - v_i)$, respectively.

[41] The welfare loss represents the expected welfare loss in the remaining part of Section 6.3. For each case, we compute welfare losses under different monetary policies using the same simulations of log-normal shocks to the import prices of all sectors. For simplicity, we assume that the shocks to different sectors have the same mean. We set the mean of sectoral shocks to generate an average CPI inflation of 2% for each economy to compare — under the same aggregate level of inflation — the welfare losses across different economies with different openness and structures of input–output linkages. The variance–covariance matrix of these shocks is calibrated on Mexico. We simulate the shocks 100,000 times to compute the expected welfare loss under each of the alternative monetary policies. In Appendix I.4, we show that our results are similar under shocks to import prices in the manufacturing sectors and under aggregate productivity shocks.

[42] Specifically, output gap targeting without IO linkages targets the alternative DC inflation index that replaces the $\mathcal{M}_{OG,i}$ in the normalized OG weights — including the normalizer — in Eq. (31) with $\beta_i v_i + \kappa_{CPI}^{-1}[(\theta_{F,i} - 1)\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1 - v_i)]$.

Table 5
Welfare loss under different monetary policies.

	(1) Optimal	(2) Output gap targeting	(3) PPI targeting	(4) Output gap targeting w/o IO	(5) CPI targeting
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	−1.859	−1.879	−1.922	−4.948	−4.968
Improvement by OG targeting towards optimal			67.1%	99.3%	99.3%
Output gap misallocation	−0.003	0.000	−0.002	−0.385	−0.388
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.024	0.000	−0.041	−2.684	−2.701
— policy-irrelevant	−1.879	−1.879	−1.879	−1.879	−1.879
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	−7.742	−7.777	−8.504	−11.551	−10.675
Improvement by OG targeting towards optimal			95.4%	99.1%	98.8%
Output gap misallocation	−0.006	0.000	−0.089	−0.569	−0.427
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.041	0.000	−0.638	−3.205	−2.471
— policy-irrelevant	−7.777	−7.777	−7.777	−7.777	−7.777
U.S. Export-to-GDP ratio: 9%					
Total welfare loss	−1.400	−1.472	−1.476	−6.546	−6.757
Improvement by OG targeting towards optimal			5.4%	98.6%	98.6%
Output gap misallocation	−0.011	0.000	0.000	−0.596	−0.623
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.083	0.000	−0.004	−4.478	−4.662
— policy-irrelevant	−1.472	−1.472	−1.472	−1.472	−1.472

Notes: Shown in the table is the welfare loss — expressed in units of percent of steady-state consumption — under different monetary policy designs. Shown in columns (1) to (5) are the welfare losses under the optimal policy, output gap targeting, PPI targeting, output gap targeting without IO linkages, and CPI targeting policy, respectively. Outlined in Appendix I.3 are the relative sectoral weights of sectoral inflation adopted by the alternative monetary policies.

respectively). Using Eq. (39), we decompose the welfare loss into the *output gap misallocation* and the *within- and across-sector, and cross-border misallocation*. We further use equation (H.1) to decompose the within- and across-sector, and cross-border misallocation into two sub-components: (i) an output-gap-related term and (ii) a policy-irrelevant term.

As shown in Table 5, output gap targeting yields a welfare loss that is close to the optimal policy and significantly outperforms the PPI targeting (column 3), output gap targeting without IO linkages (column 4), and CPI targeting (column 5) — which ignore cross-border linkages, IO linkages, and *both* cross-border *and* IO linkages, respectively. For Mexico, the difference in the welfare loss between the optimal policy and output gap targeting is very small and equal to 0.020 percent of the steady-state consumption (−1.859 vs. −1.879), thereby establishing that output gap targeting is nearly optimal. Important to our analysis, output gap targeting improves the welfare loss over the PPI targeting by 0.043 percent of the steady-state consumption, and it generates an even larger improvement over output gap targeting that ignores IO linkages and the CPI targeting (−1.879 vs. −1.922 vs. −4.948 vs. −4.968). The welfare improvement of output gap targeting over the PPI targeting (vs. output gap targeting without IO linkages) corresponds to 67.1% (vs. 99.3%) of the welfare difference between the optimal and the PPI targeting policy (vs. output gap targeting without IO linkages), thereby exhibiting welfare enhancement if the design of monetary policy accounts for openness and IO linkages of the economy. The welfare improvement of output gap targeting over the PPI targeting — which is the policy used in one-sector SOE literature — also shows the importance of considering input–output linkages in designing monetary policies in SOEs.

Decomposing the total welfare loss into different components illustrates why output gap targeting is close to the optimal policy and improves over policies that ignore cross-border and input–output linkages. Output gap targeting eliminates the welfare losses arising from the output gap misallocation and from the output-gap-related components in the within- and across-sector, and cross-border misallocation. Quantitatively, Table 5 shows that these two components related to the output gap generate large welfare losses in Mexico for the PPI targeting (−0.002 and −0.041), and even larger losses for output gap targeting without IO linkages (−0.385 and −2.684) and the CPI targeting (−0.388 and −2.701). These results support the adoption of output gap targeting that considers both the cross-border and input–output linkages to enhance welfare in small open economies.

Finally, we examine the welfare loss under alternative monetary policies for two additional economies, namely Luxembourg and the US, which represent the polar cases of open and closed economies, respectively. In the most open economy of Luxembourg (the middle panel of Table 5), output gap targeting improves over the PPI targeting by a large 95.4%, compared to a more limited 67.1% for Mexico. The same qualitative results outlined for Mexico hold for Luxembourg and are stronger quantitatively. The bottom panel of Table 5 presents the welfare loss for the nearly closed economy of the US, showing that the output gap and PPI targeting yield similar welfare loss and are equally close to the optimal policy, echoing the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023) in closed economies. Therefore, we conclude that the difference between the output gap and PPI targeting is significant for open economies, but its importance diminishes in relatively closed economies like the US.[43]

[43] In Appendix I.4, we examine the welfare losses under alternative monetary policies that are optimal when considering and ignoring IO linkages, respectively, in both multi-sector SOEs and closed economies. As shown in Table I.4, the counterpart optimal monetary policies for the multi-sector economies without IO

7. Conclusion

This paper investigates the design of monetary policy in small open economies with cross-border and input–output linkages and nominal rigidities. The output gap can be expressed as a weighted average of sectoral markup wedges that encapsulate the inefficiency in each sector, with each sector's weight represented by the sectoral OG weight. We derive the divine coincidence Phillips curve, which links the aggregate inflation of the divine coincidence index to the output gap and allows for the simultaneous stabilization of inflation and the output gap. The relative sectoral OG weights determine the share of each sector's inflation in the DC index, and the sum of the sectoral OG weights determines the slope of the DC Phillips curve. We show that both the DC and sectoral Phillips curves are steeper in open than in closed economies.

The DC Phillips curve implies that the monetary policy of output gap targeting can be implemented by targeting the DC index to zero. The relative sectoral weights in the DC index for output-gap targeting, in turn, depend on the sector's relevance as a supplier of inputs to both domestic and foreign demand, and as a customer of domestic labor within international production networks. We show that output gap targeting should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic consumption and face larger expenditure-switching effects. Disregarding openness or treating the economy as a one-sector SOE overstates inflation in sectors that export intensively directly and indirectly, and understates inflation in sectors that face larger expenditure-switching effects.

We derive the closed-form solution for the optimal monetary policy that minimizes the welfare losses, as well as calibrate our model to the WIOD to quantify our theoretical results. We show that the policy of output gap targeting generates welfare losses quantitatively close to the optimal policy as in closed economies, and outperforms alternative monetary policies of the PPI targeting that abstract from cross-border linkages or the output gap targeting that ignores input–output linkages. Overall, our analysis demonstrates that cross-border and input–output linkages are important for the conduct of monetary policy in small open economies with international production networks.

Our study suggests several interesting extensions for future research. First, one could relax the assumption of financial autarky and study the interplay between the incompleteness of the financial market and the production networks for the design of monetary policy. Second, the analysis could consider the cases in which fiscal policy fails to offset the first-order distortions with non-contingent subsidies. Such contexts lead to a sub-optimal flexible-price equilibrium for the domestic social planner, as in Baqaee and Farhi (2024), such that the monetary policy needs to account for the interaction between the supply-side effect of monetary policy and the openness of the economy to improve efficiency. Third, the subsequent efforts might explore large open economies where monetary policy would need to account for feedback effects from the responses of foreign economies to the domestic policy — which may interplay with international production networks to determine the impact of the domestic monetary policy. Finally, the analysis could be extended to models incorporating endogenous adjustments in domestic and cross-border input–output linkages, as in Xu et al. (2025). We plan to investigate some of these issues in future work.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmoneco.2026.103918.

Data availability

Data will be made available on request.

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linkages lead to a larger welfare loss than the baseline optimal monetary policies that consider IO linkages. This additional welfare loss of the optimal monetary policy from ignoring IO linkages is larger in open than in closed economies, especially for countries with a large degree of openness, illustrating that IO and cross-border linkages interplay to determine the welfare losses associated with monetary policies.

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