Monetary Policy in Open Economies with Production Networks[1]

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June 2025

Abstract

This paper studies monetary policy design in small open economies with cross-border and input-output linkages and nominal rigidities. The monetary policy that closes the output gap is nearly optimal and can be implemented by stabilizing an aggregate inflation index that weights sectoral inflation according to each sector's role as a supplier to domestic and foreign demand and as a customer of domestic labor. To close the output gap, monetary policy should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (via downstream sectors) to domestic output. Disregarding cross-border linkages *overemphasizes* inflation in sectors that export directly and indirectly (via downstream sectors), and disregarding input-output linkages *underemphasizes* inflation in sectors that supply indirectly (via downstream sectors) to domestic and foreign demand. We quantify our theoretical results using the World Input-Output Database and show that the output-gap-stabilizing policy outperforms alternative policies that abstract from cross-border or input-output linkages.

Keywords: production networks, small open economy, monetary policy. *JEL*: C67, E52, F41.

1. Introduction

Modern production revolves around intricate input-output (IO) relations within domestic firms and between domestic and foreign firms, and the position and import-export intensity of each domestic firm along the production networks are critical for an economy's response to shocks and the efficacy of stabilizing

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economic policies. Disruptions to the global supply chain during trade tensions between China and the US since the "China Section 301-Tariff Actions" in 2018, the COVID-19 pandemic, and at the outset of the Trump administration in 2025 exemplify the primal role of international input-output linkages for the changes in output and prices and the stance of monetary policy.[3]

Yet, there is no systematic research focused on the design of monetary policy in open economies with both cross-border and input-output relations—despite two separate strands of literature providing distinct insights on the issue. On the one hand, in a one-sector small open economy (SOE) model with nominal price rigidities and without input-output relations as per Galí and Monacelli (2005), the optimal monetary policy stabilizes both domestic inflation and terms of trade. On the other hand, in a multi-sector closed economy with IO linkages as per La'O and Tahbaz-Salehi (2022), the monetary policy closing the output gap should target a weighted average of sectoral inflation with the weights proportional to Domar weights (i.e., sectoral sales-to-GDP ratio) to account for the propagation of sectoral distortions along input-output linkages.

In light of these separate findings, it remains unknown what the policy prescription should be for a monetary authority that operates in an open, multi-sector economy with both input-output and cross-border relations between firms. Our paper sheds light on this outstanding issue by revealing the separate roles of multi-sector structure, input-output linkages, and cross-border linkages in the design of monetary policy, and studying the pitfalls of monetary policy that disregards input-output or cross-border relations.

We study these issues by developing a multi-sector, small open economy model with production networks between domestic and foreign sectors that are subject to nominal rigidities. Our model combines the one-sector open economy framework in Galí and Monacelli (2005) with the multi-sector, production network framework similar to Ghassibe (2021b), La'O and Tahbaz-Salehi (2022), and Rubbo (2023).

In our multi-sector economy with nominal rigidities, inflation in the different sectors generates sectoral markup wedges that encapsulate the sectoral distortions that prevent attainment of allocations in the flexible-price equilibrium. The cross-border and input-output linkages further propagate these sectoral distortions throughout the economy, resulting in aggregate distortions. We show that, up to first-order approximation, the aggregate distortion is proportional to the aggregate output gap—defined as the difference between the aggregate output in the sticky-price and in the flexible-price equilibria. Thus, the monetary policy that closes the aggregate output gap eliminates the first-order aggregate distortions in the open economy with production networks. We refer to this policy as the output gap (OG) monetary policy.

To close the aggregate output gap, the OG monetary policy stabilizes the aggregate inflation index that is proportional to the aggregate output gap by weighting the inflation of different sectors. The weight assigned to inflation in each sector is the product of two components: (i) the degree of the sector's price rigidity that

[3]See Auray et al. (2024) and Bai et al. (2024, 2025) for discussions on the impacts of trade barriers and Covid-19 on output and monetary policy.

maps positive sectoral inflation into the negative sectoral markup wedge under nominal rigidities, similar to Rubbo (2023),[4] and (ii) the sector's OG weight that measures the contribution of the sectoral markup wedge to the aggregate output gap, which crucially depends on the interplay of cross-border and input-output linkages. The size of the sectoral OG weight is determined by three channels that rely on the distinct roles of the sector for the aggregate output in the network economy: (i) the *Consumer Price Index* (*CPI*), (ii) the *expenditure-switching*, and (iii) the *profit* channels. While the *CPI channel* is also present in closed economies, the *expenditure-switching* and *profit* channels are unique to open economies.[5]

In the *CPI channel*, a negative sectoral markup wedge leads to a lower CPI than in the flexible-price equilibrium, which raises the real wage and, thereby, the supply of domestic labor, hence generating a positive aggregate output gap. In the *expenditure-switching channel*, a negative sectoral markup wedge reduces the prices of domestic relative to foreign products and induces a switching of domestic and foreign expenditures from foreign to domestic goods, thereby increasing domestic income and generating a positive aggregate output gap. In the *profit channel*, the negative sectoral markup wedges raise domestic sectors' costs of imported inputs relative to sectoral sales, hence reducing domestic producers' profits and contributing negatively to the aggregate output gap.

The sizes of the three foregoing channels are determined by the different roles of the sector in the open-economy input-output network as a *supplier* of inputs to both domestic and foreign demand, as well as a *customer* for domestic labor inputs. Because the CPI is the price of aggregate output, the size of the *CPI channel* is determined by the sector's direct and indirect (via the *downstream* sectors) contribution to domestic aggregate output as a supplier of inputs—which we measure using *domestic supplier centrality*. The size of the *expenditure-switching channel*—measured by the sectoral *expenditure-switching centrality*—is proportional to two components: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on domestic sectors' prices; and (ii) the direct and indirect (via downstream sectors) impacts of the prices of domestic sectors—as input suppliers—on the domestic and foreign demand for domestic goods, and the consequential domestic income. Finally, the size of the *profit channel* is also proportional to two components: (i) the size of the sector measured by the sectoral *Domar weight*; and (ii) its direct and indirect (via upstream sectors) use of imported inputs, which is equal to the complement of the sector's direct and indirect use of domestic labor inputs as a customer—measured by the sectoral *customer centrality*.

Our centrality measures and OG weights encompass those in the closed economy framework with production networks á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), showing that the OG weight is equal to the domestic supplier centrality and the Domar weight in closed economies that abstract from cross-border linkages, where the expenditure-switching and profit channels are absent.

We study the role of two relevant concepts in the open-economy macroeconomic literature for the OG

[4]Under nominal rigidities, sticky-price firms cannot raise their prices in response to positive inflation in marginal costs, thus generating a lower sectoral markup in the sticky-price than in the flexible-price equilibria.

[5]The *expenditure-switching* channel is standard in the international macroeconomic literature. See Engel (2002) for a review.

monetary policy, namely, the exchange rate and the pricing currency. We show that in response to positive sectoral inflation, the domestic currency appreciates, which attenuates the increase in the aggregate output gap from the expenditure-switching and profit channels.[6] Our baseline model assumes producer-currency pricing. We show that under the alternative foreign-currency pricing—comprising the local-currency and dominant-currency pricing—the OG policy should target an aggregate inflation index that includes sectoral inflation of both domestic-market prices and export prices in the foreign market. In particular, while the CPI and profit channels remain dependent on domestic sectoral inflation, the expenditure-switching channel relies on inflation in domestic and export prices.

We compare our baseline OG policy to the monetary policy in one-sector SOEs á la Galí and Monacelli (2005), revealing the role of the multi-sector structure. We show that in the special case of the one-sector model, the optimal monetary policy simultaneously stabilizes domestic inflation and output gap, consistent with the "divine-coincidence" result in Galí and Monacelli (2005). In the one-sector SOE literature, the optimal policy is implemented by stabilizing domestic inflation of the Producer Price Index (PPI) that weights domestic sectors' inflation by their sectoral sales—which are proportional to sectoral Domar weights. We contribute to this literature by deriving the appropriate sectoral weights in the domestic aggregate inflation index to close output gap, which differ from the weights in the PPI and account for the interplay between multi-sector structure, cross-border linkages, and input-output linkages.

We then examine the pitfalls of two alternative monetary policies that adopt either: (i) the OG weights that close the output gap in multi-sector small open economies *without input-output linkages*, or (ii) the Domar weights that close the output gap in closed economies with input-output linkages, *abstracting from cross-border linkages*. The monetary policy that abstracts from input-output linkages under-emphasizes the relevance of a sector's inflation for the output gap by ignoring its *indirect* impacts as an input supplier in the network through both the CPI and expenditure-switching channels. It also over-emphasizes the relevance of sectoral inflation by ignoring the domestic sector's direct and indirect use of imported foreign factors and, thereby, overstating its contribution to domestic labor income.

The monetary policy that abstracts from cross-border linkages by adopting the Domar weights over-emphasizes the relevance of a domestic sector's inflation for the output gap for two reasons. First, the Domar weight in open economies is proportional to *total* sectoral sales that encapsulate the sector's direct and indirect (via downstream sectors) contribution to *foreign*, in addition to, *domestic* demand. Second, the Domar-weight policy assumes that the sector uses only domestic but no foreign factors and, therefore, over-emphasizes the sector's direct and indirect (via upstream sectors) contribution to domestic labor income as a customer. It also underestimates the relevance of a domestic sector's inflation by ignoring its impact on the domestic-to-foreign prices and the domestic and foreign demand for the sectoral goods (i.e., neglecting

[6]In Section 3.2, we show that the terms of trade—another relevant concept in the literature—plays a limited role for the OG policy in multi-sector small open economies.

the expenditure-switching channel).

Whether and to what extent monetary policy abstracting from cross-border or input-output linkages over- or under-emphasizes the relevance of a sector's inflation depends on the quantitative strength of the aforementioned countervailing sectoral forces. This theoretical possibility motivates our quantitative analysis, which calibrates the model to major economies based on data from the World Input-Output Database (WIOD), as discussed below.

We derive the welfare loss function, sectoral Phillips curves, and the resulting optimal monetary policy in our small open economies with production networks. We show that the welfare loss (up to the second-order approximation) comprises the losses from the output gap misallocation and the within- and across-sector misallocation—similar to those in closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023)—as well as the cross-border misallocation that is unique to the open economy. The sectoral Phillips curves include both output-gap and cost-push driven inflation, similar to those in closed economies with production networks. In particular, the slopes of the sectoral Phillips curves include a nominal wage channel that is standard in closed economies and a nominal exchange rate channel that is specific to open economies. The optimal monetary policy—which minimizes the welfare loss subject to the sectoral Phillips curves—cannot simultaneously stabilize the output gap, the within- and across-sector, and the cross-border misallocations, and thus needs to trade off among them. In other words, the "divine coincidence" that holds in one-sector SOEs á la Galí and Monacelli (2005) breaks down in our multi-sector SOEs.

Input-output and cross-border linkages enter the welfare loss function and the sectoral Phillips curves and, therefore, play an important role in optimal monetary policy. In multi-sector SOEs *without input-output linkages*, the across-sector and cross-border misallocations in the welfare loss that arise from the disproportional use of intermediate inputs—relative to sectoral output and between domestic and foreign inputs, respectively—are both absent. In multi-sector closed economies *without cross-border linkages*, the cross-border misallocation in the welfare loss function is absent. The nominal exchange rate channel in the slopes of the sectoral Phillips curves is absent in economies without cross-border *or* input-output linkages, as this channel influences sectoral inflation only through the import prices of foreign intermediate inputs.

To quantify the sizes of the different channels and the countervailing forces in our model, as well as determine the welfare differences across alternative monetary policies, we calibrate the model to the World Input-Output Database. The database comprises 43 countries with 56 major sectors for the year 2014. The variance decomposition of sectoral OG weights shows that the sizes of the CPI and expenditure-switching channels explain the bulk of the variation in the OG weight, with the importance of these two channels decreasing and increasing with the openness of the economy, respectively. We show that the Domar-weight policy that fails to account for cross-border linkages systemically overstates the contribution of sectoral inflation to the output gap, with the difference between the sectoral Domar and OG weights primarily driven by the sector's *export intensity*. This sectoral centrality measures the sector's direct and indirect (via downstream sectors) contribution to foreign demand. In contrast, the OG policy that fails to account for input-output linkages systemically understates the contribution of sectoral inflation to the output gap.

We use regression analysis to study the rule-of-thumb combinations of centrality measures to approximate the sectoral OG weights. We show that the sectoral OG weights can be well approximated using solely the *domestic supplier centrality*. The sectoral import intensity—measuring the direct and indirect (via IO linkages) impacts of import shares on sectoral demand—is an important determinant of domestic supplier centrality. It displays a larger explanatory power for the OG weights than the sector's import share, thereby supporting the relevant role of input-output linkages. Our regression analysis shows that the Domar-OG differences—capturing the pitfall in the monetary policy that disregards cross-border linkages, or equivalently, the PPI stabilization policy in the one-sector SOE literature—can be well approximated by a linear combination of *export intensity* and *customer centrality*.

Finally, we compare the welfare of alternative monetary policies, showing that the OG policy is close to the optimal monetary policy and outperforms three alternative monetary policies: (i) the monetary policy that targets the Domar-weighted inflation index (and therefore abstracts from the cross-border linkages), (ii) the monetary policy that accounts for the cross-border but abstracts from the input-output linkages, and (iii) the monetary policy that targets the CPI-weighted inflation index (and thus abstracts from both cross-border and input-output linkages).[7] For instance, in Mexico, the OG policy improves over the Domar-weight policy and the OG policy that ignores IO linkages by 67% and 99%, respectively, toward the optimal monetary policy. In the more open economy of Luxembourg, welfare improvements by the OG policy are larger at 95% and 99%, respectively. In the more closed economy of the US, however, there is little welfare difference between the OG and Domar-weight policies, hence indicating that the imports and exports play a limited role in the design of monetary policy in countries with a low degree of openness. Accordingly, our quantitative analysis further emphasizes the importance of considering both input-output and cross-border linkages in designing monetary policies in small open economies.

Related literature. Our paper is related to four separate strands of literature. First, we relate to literature on the design of monetary policy in closed economies with production networks. La'O and Tahbaz-Salehi (2022), Rubbo (2023), and Xu and Yu (2025) show that in closed economies, monetary policy that closes the output gap is nearly optimal, and weights inflation in the different sectors according to the sectoral Domar weights that account for the structure of the domestic production network. La'O and Tahbaz-Salehi (2025) study the optimal fiscal and monetary policies in a closed economy. Compared to these studies, we show that monetary policies in open economies need to account for the interplay between cross-border and input-output linkages.

Second, we relate to literature that investigates the aggregation of sectoral distortions and shocks. Chari et al. (2007) use labor and efficiency wedges to characterize the aggregation of disaggregated shocks and

[7]The Domar-weighted (vs. CPI-weighted) monetary policy targets the aggregate inflation index that weights each sector's inflation with the product of the sector's Domar (vs. CPI) weight and price rigidity.

distortions. Bigio and La'O (2020) extend that analysis to study a closed economy with production networks; they reveal that the efficiency wedge does not include first-order distortions and that only the labor wedge is critical to first-order economic efficiency. We generalize their results to an open economy with international production networks. Baqaee and Farhi (2024) study distortions in a global economy with interconnected countries and sectors. Elliott and Jackson (2024) study the propagation of supply chain disruption in an international production network. Compared to their work, we examine the distortions in small open economies and focus our analysis on the design of monetary policy.

Third, we relate to literature on the transmission of monetary policy in production networks. Ghassibe (2021a,b) and Afrouzi and Bhattarai (2023) develop an analytical characterization of the transmission mechanism of monetary policy in closed economies with production networks. Nakamura and Steinsson (2010) and Pasten et al. (2020) provide a numerical characterization of the effect of monetary policy on aggregate output and inflation. Silva (2024) explores how the production network alters the propagation of sectoral shocks into the consumer price index in small open economies. Kalemli-Ozcan et al. (2025) develop a New Keynesian open economy model incorporating global production networks and trade distortions to study the interaction between monetary policy and trade. Compared to these works, we focus on the design rather than the transmission of monetary policy in network economies.

Fourth, we link to the numerous studies on the design of monetary policies in small open economies without production networks. While earlier work focuses on one-sector small open economies (e.g., Galí and Monacelli, 2005; Soffritti and Zanetti, 2008; De Paoli, 2009), more recent studies—Matsumura (2022) and Wei and Xie (2020)—explore small open economy models with multiple sectors. Compared to these foregoing studies, we derive closed-form solutions for the output gap and optimal monetary policies and provide a comprehensive analysis of the design of monetary policies in small open economies with fully-fledged cross-border and input-output linkages.

Outline. The remainder of the paper is organized as follows. Section 2 describes our model of a small open economy with production networks. Section 3 studies the OG weights and characterizes the OG policy that eliminates the aggregate output gap. Section 4 derives the welfare loss function and the sectoral Phillips curves, and characterizes the optimal monetary policy. Section 5 quantifies the theoretical results using data and compares the welfare of alternative monetary policies. Section 6 concludes.

2. Small open economy with production networks

2.1. Environment

The static, small open economy is populated by a representative household consuming domestic and imported sectoral products and supplying labor in exchange for wage income, a government that levies sector-specific taxes and manages the aggregate demand by controlling the money supply, and producers that operate in $N \in \mathbb{N}_+$ different sectors, indexed by $i \in \{1, 2, \cdots, N\}$.

Each sector i comprises two types of producers: (i) a unit mass of monopolistically competitive firms indexed by $f \in [0, 1]$ that transform labor and intermediate inputs into differentiated goods, and (ii) a unit mass of perfectly competitive firms that pack the differentiated goods of each sector into a domestic sectoral product, which are both used domestically and exported to foreign countries. Each domestic sectoral product has a counterpart foreign sectoral product available for import. Consumption and intermediate inputs comprise domestic and foreign sectoral products.

2.2. Producers

Monopolistically competitive firms. Within each sector i, monopolistically competitive firms use a common constant-returns-to-scale production technology to transform labor and intermediate inputs into differentiated goods. The production technology of each firm f in sector i is

$$Y_{if} = A_i \cdot \left(\frac{L_{if}}{\alpha_i}\right)^{\alpha_i} \prod_{j=1}^{N} \left(\frac{X_{if,j}}{\omega_{i,j}}\right)^{\omega_{i,j}}, \tag{1}$$

where A_i is the sector-specific productivity shock, Y_{if} is the output of firm f in sector i, L_{if} is its labor input, and $X_{if,j}$ is the intermediate input acquired from sector j. Parameter α_i is the share of labor, and $\omega_{i,j}$ is the share of intermediate inputs from sector j. The collection of $\{\omega_{i,j}\}_{i,j}$ characterizes the input-output table. Constant returns-to-scale implies that $\alpha_i + \sum_{j=1}^{N} \omega_{i,j} = 1$.

The openness of the economy is reflected in the composition of $X_{if,j}$, which is aggregated from a domestic sectoral product $X_{Hif,Hj}$ and an imported foreign sectoral product $X_{Hif,Fj}$ according to the following constant-elasticity-of-substitution technology:

$$X_{if,j} = \left(v_{x,i,j}^{\frac{1}{\theta_j}} X_{Hif,Hj}^{\frac{\theta_j - 1}{\theta_j}} + (1 - v_{x,i,j})^{\frac{1}{\theta_j}} X_{Hif,Fj}^{\frac{\theta_j - 1}{\theta_j}}\right)^{\frac{\theta_j}{\theta_j - 1}}, \tag{2}$$

where θ_j is the elasticity of substitution between domestic and foreign sectoral products in intermediate input $X_{if,j}$. $v_{x,i,j}$ is the home bias parameter, which in equilibrium is equal to the steady-state expenditure share of $X_{Hif,Hj}$ in the composite intermediate input $X_{if,j}$.

The total cost of inputs used by the firm is

$$TC_{if} = WL_{if} + \sum_{j=1}^{N} (P_j X_{Hif,Hj} + S \cdot P_{IM,Fj}^{*} X_{Hif,Fj}), \tag{3}$$

where W is the nominal wage rate, P_j is the domestic sectoral price, $P_{IM,Fj}^{*}$ is the exogenous sectoral import price denominated in the foreign currency, and S is the nominal exchange rate. Given output Y_{if} and the production technology in equation (1), the firm optimally chooses labor and intermediate inputs to minimize TC_{if}, which yields the marginal cost of production that equals the average cost due to the constant-return-to-scale technology. Moreover, because all firms f in each sector i share the same production technology and face the same input prices, the marginal cost of production is identical across all firms in sector i, and we denote it by Φ_i.

We model nominal rigidity as a static Calvo-pricing friction, where only firms indexed by $f \leq \delta_i \in [0, 1]$ can choose their desired price $P_i^{\#}$ and the remaining firms maintain the price at the steady-state level. We refer to $(1 - \delta_i)/\delta_i$ as the price rigidity of sector i. In each sector i, firms operate in a monopolistically competitive market and pay a sectoral tax rate τ_i on sales. Those firms that can adjust their prices set the desired price to maximize profit.

In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods that the monopolistically competitive firms produce into a sectoral product using a constant-elasticity-of-substitution technology, with the within-sector elasticity of substitution between different firms' products equal to $\varepsilon_i > 1$. The price of the domestic sector i's products—denoted by P_i—is the selling price of its sectoral goods packer. We define the sectoral markup and the desired sectoral markup as $\mu_i \equiv P_i/\Phi_i$ and $\mu_i^{\#} \equiv P_i^{\#}/\Phi_i$, respectively. We further define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz*, $\ln(\mu_i) - \ln(\mu_i^{\#})$. Shown in Appendix A are the expressions for the nominal profit, demand function, and desired prices of the firms, as well as the sectoral product and price index.

2.3. Households

The preference of the representative household is described by the utility function defined over domestic aggregate consumption C and labor supply L:

$$u(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\varphi}}{1+\varphi}, \tag{4}$$

where σ is the degree of diminishing marginal utility of consumption, and φ is the inverse of the Frisch elasticity of labor supply. In our static model without investment, domestic aggregate consumption is equivalent to the (domestic) aggregate output; thus, we refer to C as the aggregate output throughout the paper.

The (domestic) aggregate output C combines sectoral consumption $\{C_i\}_i$ that comprises domestic and imported components, C_{Hi} and C_{Fi}, respectively, for each sector i, represented by:[8]

$$C = \prod_{i=1}^{N} \Big(\frac{C_i}{\beta_i}\Big)^{\beta_i}, \quad \text{where} \quad C_i = \Big(v_i^{\frac{1}{\theta_i}} C_{Hi}^{\frac{\theta_i - 1}{\theta_i}} + (1 - v_i)^{\frac{1}{\theta_i}} C_{Fi}^{\frac{\theta_i - 1}{\theta_i}}\Big)^{\frac{\theta_i}{\theta_i - 1}}. \tag{5}$$

Vector $\{\beta_i\}_i$ is the set of consumption shares satisfying $\sum_{i=1}^{N} \beta_i = 1$, and v_i is the home bias parameter for the consumption of sectoral products. Denote P_C as the price index of the aggregate output C—*viz*, the

[8] As we show in equation (36) of Proposition 4 in Section 4, the aggregate consumption gap ($\widehat{C}^{gap}$) drives sectoral inflation in the sectoral Phillips curves. For consistency with the terminology used in the optimal monetary policy literature, and with a slight abuse of notation, we will refer to $\widehat{C}^{gap}$ as the aggregate output gap, and to C as the aggregate output throughout the paper.

CPI. The budget constraint of the household is:

$$P_C C = \sum_{i=1}^{N} \left(P_i C_{Hi} + S \cdot P^*_{IM,i} C_{Fi}\right) \leq WL + \sum_{i=1}^{N} \int_0^1 \Pi_{if} df + T, \tag{6}$$

where Π_{if} is the profit from firm f in sector i, and T is the lump-sum transfer of the tax revenues to the household. To purchase the consumption goods, households demand the following amount of money as the medium of exchange: $M_d = P_C C$.

Cost minimization by the household yields the price index of aggregate output:

$$P_C = \prod_{i=1}^{N} \left(v_i P_i^{1-\theta_i} + (1-v_i)(S \cdot P^*_{IM,Fi})^{1-\theta_i}\right)^{\frac{\beta_i}{1-\theta_i}}. \tag{7}$$

2.4. International trade

In addition to the sales subsidy $\{\tau_i\}_i$, the government also imposes sector-specific export tax $\{\tau_{EX,i}\}_i$ on the products exported to foreign countries. The no-arbitrage condition implies that there is no difference between the prices that producers receive from exporting (i.e., $(1-\tau_{EX,i})P_{EX,i}$) or from selling domestically (i.e., P_i): $(1-\tau_{EX,i})P_{EX,i} = P_i, \quad \forall i \in \{1, 2, \cdots, N\}$.

The export demand for sector i's product is modeled as the reduced-form demand function:[9]

$$Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}, \tag{8}$$

where $D^*_{EX,Fi}$ is the exogenous component of foreign demand, $P_{EX,i}/S$ is the price of the exported domestic sector i goods in units of foreign currency, and the export demand is inversely related to domestic goods' export price, with $\theta_{F,i}$ as the price elasticity of export demand.

Trade is balanced in the static economy, which requires the value of exports to be exactly identical to the value of imports in the whole economy, resulting in the following:[10]

$$\sum_{i=1}^{N} P_{EX,i} Y_{EX,i} = S \sum_{i=1}^{N} P^*_{IM,Fi} \left(\sum_{j=1}^{N} \int_0^1 X_{Hjf,Fi} df + C_{Fi} \right). \tag{9}$$

This trade balance condition pins down the endogenous nominal exchange rate S in equilibrium. The trade balance condition is equivalent to the binding budget constraint of the households in the aggregate.

[9] In general, the export demand in equation (8) can be written as $Y_{EX,i} = [P_{EX,i}/(S \cdot P^*_{EX,Fi})]^{-\theta_{F,i}} D^*_{EX,Fi}$, where $P^*_{EX,Fi}$ is the exogenous price for foreign-produced sector i's product in foreign markets, $D^*_{EX,Fi}$ is the exogenous foreign demand given the prices. Therefore, $D^*_{EX,i}$ in equation (8) captures the effects of both $P^*_{EX,Fi}$ and $D^*_{EX,Fi}$ on export demand.

[10] Engel (2016) advocates using a balanced trade assumption instead of the risk sharing condition in the complete market.

2.5. Aggregate states

There are three types of exogenous sector-level states in the economy: productivity $\{A_i\}_i$, foreign demand $\{D^*_{EX,Fi}\}_i$, and import price $\{P^*_{IM,Fi}\}_i$. The aggregate state $\boldsymbol{\xi}$ collects the realized states:

$$\boldsymbol{\xi} \equiv \left\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\right\}_{i\in\{1,2,\ldots,N\}} \in \boldsymbol{\Xi} = \mathbb{R}^{3N}_{\geq 0}. \tag{10}$$

2.6. Government: fiscal and monetary policies

The government sets fiscal and monetary policies. Fiscal policy includes a pair of non-contingent sectoral sales and export taxes $\{\tau_i, \tau_{EX,i}\}_i$ that do not respond to changes in exogenous states. The lump-sum transfer T to the households satisfies a fiscal budget balance:

$$T = \sum_{i=1}^{N}\left(\tau_i \int_0^1 P_{if}Y_{if}df + \tau_{EX,i}P_{EX,i}Y_{EX,i}\right). \tag{11}$$

The monetary policy is a one-dimensional state-contingent money supply $M(\boldsymbol{\xi})$ contingent on the aggregate state $\boldsymbol{\xi}$. We investigate the design of this monetary policy, with a particular focus on the monetary policy that eliminates the aggregate output gap.

2.7. Equilibrium definition

The market clearing conditions for product, labor, and money markets are:

$$Y_i(\boldsymbol{\xi}) = C_{Hi}(\boldsymbol{\xi}) + \sum_{j=1}^{N}\int_0^1 X_{Hjf,Hi}(\boldsymbol{\xi})df + Y_{EX,i}(\boldsymbol{\xi}), \tag{12}$$

$$L(\boldsymbol{\xi}) = \sum_{i=1}^{N}\int_0^1 L_{if}(\boldsymbol{\xi})df, \qquad M(\boldsymbol{\xi}) = M_d(\boldsymbol{\xi}). \tag{13}$$

Definition 1. *A sticky-price equilibrium is a set of allocations, prices, and policies (i.e., $\{\tau_i, \tau_{EX,i}\}_i$ and $M(\boldsymbol{\xi})$) such that for any realized state $\boldsymbol{\xi} \in \boldsymbol{\Xi}$,*

(i) producers optimally choose inputs to minimize the cost of production;

(ii) monopolistically competitive firms $f \in [0, \delta_i]$ set prices to maximize profits subject to their demand functions, and the remaining firms $f \in (\delta_i, 1]$ do not adjust prices;

(iii) the representative household chooses consumption and labor to maximize utility subject to its budget constraint, and the total expenditure determines the money demand;

(iv) the government budget constraint is satisfied;

(v) all markets clear.

We define the *flexible-price equilibrium* as the special case of the *sticky-price equilibrium* in Definition 1 that involves no Calvo-pricing friction, as stated in the following definition:

Definition 2. *A flexible-price equilibrium is a set of allocations, prices, and policies satisfying all of the conditions stated in Definition 1, except that for any sector $i \in \{1, 2, \cdots, N\}$, $\delta_i = 1$,* viz*, all firms can adjust prices flexibly.*

While the *sticky-price equilibrium* is our focus, the allocation of the *flexible-price equilibrium* serves as a benchmark to define the distortions and welfare losses that nominal rigidities introduce.

2.8. Flexible-price equilibrium as reference equilibrium

As per Woodford (2003) and Galí (2015), we use non-contingent subsidies and taxes to eliminate domestic-market distortion while allowing domestic producers to exert their market power fully in the international market in the flexible-price equilibrium, as defined by the following assumption:[11]

Assumption 1. *The non-contingent tax rates for sales and exports are equal to*

$$\tau_i = -1/(\varepsilon_i - 1) \ \text{ and } \ \tau_{EX,i} = 1/\theta_{Fi}, \ \text{ respectively, for } \ \forall i \in \{1, ..., N\}. \tag{14}$$

Under Assumption 1, the *flexible-price equilibrium* yields the optimal allocation for the domestic social planner, as stated in the following lemma:

Lemma 1. *Under Assumption 1, the flexible-price equilibrium implements the optimal allocation for the domestic social planner.*

Proof: See Appendix G.2.

Lemma 1 allows use of *flexible-price equilibrium* as the reference equilibrium for our further analyses of the domestic country's aggregate distortion and welfare loss.

2.9. Notations

This section summarizes the notation in the model to facilitate the tracking of variables, vectors, and matrices.

Deviations from the steady state and flexible-price equilibrium. We define the steady state of the static economy as the equilibrium in which all exogenous states A_i, $P^*_{IM,Fi}$, and $P^*_{EX,Fi}$ are at the steady state. We denote with x^{ss} and x^{flex} the values for the variable x in the steady state and in the flexible-price

[11] In one-sector closed economies, Woodford (2003) and Galí (2015) show that a sales subsidy eliminates the monopoly distortion and makes the flexible-price equilibrium optimal for the social planner. La'O and Tahbaz-Salehi (2022) and Rubbo (2023) use sector-specific subsidies for the same purpose in a multi-sector closed economy. In small open economies, given that sales subsidies eliminate the monopoly distortion, the monopoly power of domestic producers on the international market needs to be retained for the domestic social planner to restore the optimality of the allocation in the flexible-price equilibrium. Therefore, we use sector-specific subsidies and export taxes to remove the monopoly distortion in the domestic market and exert the monopoly power in the international market, respectively, as in Matsumura (2022).

equilibrium, respectively. We express the log deviation of the variable x from the steady state x^{ss} and the *flexible-price equilibrium* x^{flex} as:

$$\widehat{x} \equiv \ln x - \ln x^{ss}, \quad \text{and} \quad \widehat{x}^{gap} \equiv \ln x - \ln x^{flex}, \tag{15}$$

respectively. We denote the vector that collects the sectoral inflation by $\widehat{\mathbf{P}} = (\widehat{P}_1, \widehat{P}_2, \cdots, \widehat{P}_N)^\top$.[12] We denote the aggregate output gap by $\widehat{C}^{gap}$. The sectoral markup wedge is $\ln(\mu_i) - \ln(\mu_i^{\#}) = \ln(\mu_i) - \ln(\mu_i^{ss}) \equiv \widehat{\mu}_i$ as the steady-state markup is equal to the desired markup.

Table 1: Notations of parameters and steady-state objects

Name	Expression
Consumption shares and home bias	$\boldsymbol{\beta} \equiv (\beta_1, \beta_2, \cdots, \beta_N)^\top \quad \& \quad \mathbf{v} \equiv (v_1, v_2, \cdots, v_N)^\top$
Labor shares	$\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \cdots, \alpha_N)^\top$
Intermediate input shares and home bias	$\boldsymbol{\Omega} \equiv \{\omega_{i,j}\}_{i,j \in \{1,2,\cdots,N\}} \quad \& \quad \mathbf{V}_x \equiv \{v_{x,i,j}\}_{i,j \in \{1,2,\cdots,N\}}$
Elasticity of home-foreign substitution	$\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \cdots, \theta_N)^\top \quad \& \quad \boldsymbol{\theta}_F \equiv (\theta_{F,1}, \theta_{F,2}, \cdots, \theta_{F,N})^\top$
Frequency of price adjustment	$\boldsymbol{\Delta} = diag(\delta_1, \delta_2, \cdots, \delta_N)$
Steady-state sectoral Domar weight	$\boldsymbol{\lambda} \equiv (\lambda_1, \lambda_2, \cdots, \lambda_N)^\top \equiv \left(\frac{P_1^{ss} Y_1^{ss}}{P_C^{ss} C^{ss}}, \frac{P_2^{ss} Y_2^{ss}}{P_C^{ss} C^{ss}}, \cdots, \frac{P_N^{ss} Y_N^{ss}}{P_C^{ss} C^{ss}}\right)^\top$
Steady-state sectoral export-to-GDP ratio	$\boldsymbol{\lambda}_{EX} \equiv (\lambda_{EX,1}, \cdots, \lambda_{EX,N})^\top \equiv \left(\frac{P_1^{ss} Y_{EX,1}^{ss}}{P_C^{ss} C^{ss}}, \cdots, \frac{P_N^{ss} Y_{EX,N}^{ss}}{P_C^{ss} C^{ss}}\right)^\top$
Steady-state economy-wise labor share	$\Lambda_L \equiv W^{ss} L^{ss} / P_C^{ss} C^{ss}$

Parameters and steady-state objects. Summarized in Table 1 are the key parameters and steady-state variables. Throughout the paper, for any variable x, we use bold fonts to denote the corresponding vector or matrix, i.e., $\boldsymbol{x} \equiv \{x_i\}_i$ or $\boldsymbol{x} \equiv \{x_{i,j}\}_{i,j}$. For expositional simplicity, the superscript "ss" to denote the steady state is omitted when there is no obvious confusion.

Definitions of upstream and downstream sectors in the open economy. We introduce the open-economy version of the Leontief-inverse matrix: $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1} = \{l_{vx,r,i}\}_{r,i}$, which defines the upstream and downstream relationships between sector pairs in an open economy with production networks.

Definition 3. *For a pair of domestic sectors $r \neq i$, r is a downstream sector of i if $l_{vx,r,i} > 0$; r is an upstream sector of i if $l_{vx,i,r} > 0$. Accordingly, we define $l_{vx,r,i}$ and $l_{vx,i,r}$ as the downstream and upstream Leontief inverse of domestic sector i, respectively.*

We decompose the downstream and upstream relationships between a pair of sectors r and i from the Leontief inverse into the direct impact component, and the direct and indirect downstream (vs. upstream)

[12]In our static model, inflation is identical to the log deviation of sectoral price from its steady-state level.

components as follows

$$l_{vx,r,i} = \underbrace{\mathbf{1}\left(r=i\right)}_{\text{direct impact}} + \underbrace{\omega_{r,i} v_{x,r,i} l_{vx,i,i}}_{\text{direct downstream}} + \underbrace{\sum\nolimits_{s \neq i} \omega_{r,s} v_{x,r,s} l_{vx,s,i}}_{\text{indirect downstream}}, \tag{16}$$

$$l_{vx,i,r} = \underbrace{\mathbf{1}\left(r=i\right)}_{\text{direct impact}} + \underbrace{\omega_{i,r} v_{x,i,r} l_{vx,r,r}}_{\text{direct upstream}} + \underbrace{\sum\nolimits_{s \neq i} \omega_{i,s} v_{x,i,s} l_{vx,s,r}}_{\text{indirect upstream}}, \tag{17}$$

which involve import shares (i.e., $v_{x,r,i}$) in the direct and indirect downstream (vs. upstream) components and indicate the interaction between the import structure of an open economy and the input-output linkages (i.e., $\omega_{r,i}$) in determining the upstream and downstream relationships.

3. Aggregate output gap and OG monetary policy

In this section, we study the design of monetary policy and the role of cross-border and input-output linkages for the output gap (OG) monetary policy that closes the aggregate output gap in small open economies with production networks. To study the contributions of sectoral distortions to the aggregate output gap, subsection 3.1 defines the centrality measures that describe the relative importance of each sector as a direct and indirect (via downstream or upstream sectors): (i) supplier of inputs to aggregate output, (ii) customer for domestic labor, and (iii) supplier for both domestic and foreign demand. Subsection 3.2 shows that the aggregate output gap is a weighted average of the sectoral distortions. The sectoral weights—which we refer to as output gap (OG) weights—comprise three distinct channels that are functions of the sector's centrality measures, which, in turn, depend on the cross-border and input-output linkages. Based on the sectoral OG weights, we derive an analytical solution for the OG monetary policy that closes the aggregate output gap. Subsection 3.3 studies the role of cross-border and input-output linkages for the OG monetary policy by investigating the pitfalls of the OG policies that ignore either cross-border or input-output linkages.

In Appendix B, we follow the business cycle accounting approach in Chari et al. (2007), using efficiency and labor wedges to characterize how sectoral shocks and distortions aggregate in our economy. We show that the efficiency wedge is a weighted average of exogenous sectoral shocks and is independent of sectoral markup wedges, up to a first-order approximation. In contrast, the labor wedge is a weighted average of sectoral markup wedges and is proportional to the aggregate output gap. Therefore, closing the aggregate output gap is the primary objective of monetary policy aimed at offsetting first-order distortions, which justifies our focus on OG monetary policy.

3.1. Centrality measures in an open economy with networks

To facilitate the study of the link between sectoral distortions and the aggregate output gap, we define the following sectoral centrality measures that represent the relevance of a sector in the economy across three different dimensions: (i) as a direct and indirect (via downstream sectors) supplier of inputs, (ii) as a direct and indirect (via upstream sectors) customer for domestic labor, and (iii) as a direct and indirect supplier for both domestic and foreign demand and the associated user of domestic labor. These centrality measures depend on the cross-border and input-output linkages of the economy.

Definition 4 (Centrality Measures). *For each domestic sector i, the* ***domestic supplier centrality*** $\widetilde{\lambda}_{D,i}$ *and the* ***foreign supplier centrality*** $\widetilde{\lambda}_{F,i}$ *are defined as:*

$$\widetilde{\lambda}_{D,i} \equiv \sum_r \beta_r v_r l_{vx,r,i} \quad \textit{and} \quad \widetilde{\lambda}_{F,i} \equiv \sum_r \lambda_{EX,r} l_{vx,r,i}, \quad \textit{respectively.} \tag{18}$$

The ***customer centrality*** $\widetilde{\alpha}_i$ *is:*

$$\widetilde{\alpha}_i = \sum_r l_{vx,i,r} \alpha_r. \tag{19}$$

The ***expenditure-switching centrality*** $\widetilde{\rho}_{ES,i}$ *is:*

$$\widetilde{\rho}_{ES,i} = \sum_r \left(\rho_{ES,r} \widetilde{\alpha}_r + \lambda_{EX,r}\right) l_{vx,r,i}, \quad \textit{where} \tag{20}$$

$$\rho_{ES,r} \equiv \underbrace{\left(\theta_{F,r} - 1\right) \lambda_{EX,r}}_{\textit{foreign expenditure}} + \underbrace{\left(\theta_r - 1\right) \left[\beta_r v_r \left(1 - v_r\right) + \textstyle\sum_s \lambda_s \omega_{s,r} v_{x,s,r} \left(1 - v_{x,s,r}\right)\right]}_{\textit{domestic expenditure}}. \tag{21}$$

The *domestic supplier centrality* $\widetilde{\lambda}_{D,i}$ (vs. *foreign supplier centrality* $\widetilde{\lambda}_{F,i}$) of a domestic sector i in equation (18) encapsulates the importance of the sector in the network economy as both a direct and an indirect supplier (via downstream sectors)—captured by the downstream Leontief inverse $l_{vx,r,i}$—for the (domestic) aggregate output (vs. foreign demand or exports). As a result, a sector's domestic supplier centrality decreases in the import shares of the sector and its downstream sectors, as shown in Proposition D.1 of Appendix D. The *customer centrality* of a domestic sector i in equation (19) summarizes the sector's role in the network economy as both a direct and an indirect customer (via upstream sectors)—captured by the upstream Leontief inverse $l_{vx,i,r}$—of domestic labor.

The *expenditure-switching centrality* $\widetilde{\rho}_{ES,i}$ of a domestic sector i in equation (20) summarizes the direct and indirect (via downstream sectors) switching of expenditures from foreign to domestic goods in response to sector i's deflation and the resulting increase in both domestic labor income and export taxes—captured by the customer centrality $\widetilde{\alpha}_r$ and export-to-GDP ratio $\lambda_{EX,r}$, respectively. In particular, the direct expenditure switching $\rho_{ES,r}$ in equation (21) shows that both foreign and domestic expenditures are redirected to domestic products in response to the sectoral deflation.

3.2. Aggregate output gap and OG monetary policy

In this subsection, we show that the aggregate output gap originates from sectoral distortions and can be expressed as a weighted average of sectoral markup wedges. The weight assigned to each sector—which we refer to as the sectoral OG weight—measures the contribution of the sector's markup wedge to the aggregate output gap. It is composed of three distinct channels: the *CPI*, the *expenditure-switching*, and the *profit channels*. The size of each of these channels in the OG weight is determined by the centrality measures defined in the previous subsection, thereby depending on the structure of cross-border and input-output linkages in the economy.

We further define the monetary policy that achieves the zero aggregate output gap (referring to it as the OG monetary policy). We show that the OG policy is implemented by setting the money supply to stabilize the aggregate inflation index that appropriately weights the sectoral inflation. Specifically, the weight of sectoral inflation in the OG monetary policy is the product of two components: (i) the sectoral price-rigidity that maps sectoral inflation into the sectoral markup wedge and (ii) the OG weight that maps the sectoral markup wedge into the aggregate output gap.

Sectoral distortions and the aggregate output gap. Under nominal rigidities, sectoral inflation generates negative sectoral markup wedges, as the fraction $(1-\delta_i)$ of sector i's firms cannot adjust prices in response to changes in marginal costs. As a result, sectoral markup wedges—encapsulating sectoral distortions—are linked to sectoral inflation through sectoral price rigidities as follows:[13]

$$\widehat{\mu}_i(\boldsymbol{\xi}) = -(1-\delta_i)/\delta_i \cdot \widehat{P}_i(\boldsymbol{\xi}). \tag{22}$$

These negative sectoral markup wedges resulting from sectoral inflation contribute to a positive aggregate output gap through the aforementioned three distinct channels, as outlined in the following theorem:

Theorem 1 (Aggregate output gap and sectoral distortions)**.** *In a sticky-price equilibrium, negative sectoral markup wedges $\{\widehat{\mu}_i(\boldsymbol{\xi})\}_i$ contribute to a positive aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ as follows:*

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot \widehat{\mu}_i(\boldsymbol{\xi}), \tag{23}$$

where the sectoral OG weight ($\mathcal{M}_{OG,i}$) is equal to:

$$\mathcal{M}_{OG,i} \equiv \underbrace{\widetilde{\lambda}_{D,i}}_{\textit{CPI channel}} + \underbrace{\kappa_S \cdot \widetilde{\rho}_{ES,i}}_{\textit{expenditure-switching channel}} \underbrace{-\kappa_S \cdot \lambda_i(1-\widetilde{\alpha}_i)}_{\textit{profit channel}}, \tag{24}$$

$$\kappa_S \equiv \frac{1-\sum_{i=1}^{N}\widetilde{\lambda}_{D,i}\alpha_i}{1-\sum_{i=1}^{N}\widetilde{\lambda}_{D,i}\alpha_i+\sum_{i=1}^{N}(\rho_{ES,i}\widetilde{\alpha}_i+\lambda_{EX,i})\widetilde{\alpha}_i}, \tag{25}$$

$$\kappa_C \equiv \kappa_S\Big(1-\sum\nolimits_{i=1}^{N}\widetilde{\lambda}_{D,i}\alpha_i\Big) + \Big[1-\kappa_S\Big(1-\sum\nolimits_{i=1}^{N}\widetilde{\lambda}_{D,i}\alpha_i\Big)\Big](\sigma+\varphi/\Lambda_L).$$

[13] Exogenous shocks to sectoral productivity, import prices, and export demand drive sectoral inflation in the sticky-price equilibrium. Appendix G.6 derives equation (22).

Proof: See Appendix H.6.

Equation (23) shows that negative sectoral markup wedges contribute to a positive aggregate output gap. The OG weight ($\mathcal{M}_{OG,i}$) in equation (24) measures the contribution of the markup wedge of each sector to the aggregate output gap, and its size is determined by three distinct channels: (i) the positive *CPI* channel, (ii) the positive *expenditure-switching* channel, and (iii) the negative *profit* channel.[14] We now describe each of the three channels in detail.

(i) CPI channel. The *CPI channel*—standard in closed economies with nominal rigidities—describes the impacts of sectoral markup wedges on the aggregate output through distorting the price of the aggregate output—i.e., the CPI. Specifically, negative sectoral markup wedges result in a lower CPI in the sticky-price relative to the flexible-price equilibrium. The lower CPI increases the real wage (W/P_C) and stimulates a higher supply of domestic labor, thereby generating a positive aggregate output gap.[15]

For a domestic sector in an open economy with production networks, the size of the CPI channel is determined by the sector's direct and indirect (via downstream sectors) contribution to domestic aggregate output as an input supplier, captured by the sector's domestic supplier centrality $\widetilde{\lambda}_{D,i}$—as introduced in equation (18) of Definition 4—in the OG weight $\mathcal{M}_{OG,i}$ of equation (24).

(ii) Expenditure-switching channel. The *expenditure-switching channel*—specific to the open economy—is standard in the international macroeconomic literature. It describes how domestic sectoral markup wedges affect domestic aggregate output, by changing the relative price of domestic-to-foreign products and generating a switch of domestic and foreign expenditures from foreign toward domestic products. To illustrate this channel, we log-linearize the trade balance condition around the flexible-price equilibrium, which yields:

$$\begin{aligned}(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\widehat{C}^{gap} &= \underbrace{-(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^\top\big(\widehat{\mathbf{P}}^{gap}-\mathbf{1}\widehat{S}^{gap}\big)}_{\text{expenditure-switching channel}} \\ &\quad+\underbrace{[\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})]^\top\widehat{\boldsymbol{\mu}}}_{\text{profit channel}}+(1-\widetilde{\boldsymbol{\lambda}}_D^\top\boldsymbol{\alpha})\big(\widehat{S}^{gap}-\widehat{P}_C^{gap}\big),\end{aligned} \tag{26}$$

where the log-linearized sectoral pricing around the flexible-price equilibrium satisfies:[16]

$$\widehat{\mathbf{P}}^{gap}-\mathbf{1}\widehat{S}^{gap}=\widetilde{\boldsymbol{\alpha}}\big(\widehat{W}^{gap}-\widehat{S}^{gap}\big)+\mathbf{L}_{vx}\widehat{\boldsymbol{\mu}}. \tag{27}$$

[14]The negative sign on the RHS of equation (23) indicates that negative sectoral markup wedges resulting from the positive sectoral inflation lead to a positive aggregate output gap.

[15]Equation (H.21) in Lemma H.1 of Appendix H.4 shows the link between the real wage gap and the aggregate output gap.

[16]Equation (27) is derived by rearranging the sectoral pricing equation (H.16)—viz, collating all sectoral inflation and nominal exchange rate terms to the LHS—and taking the difference of this equation between the sticky-price and flexible-price equilibria.

The unitary vector $\mathbf{1}$ in equations (26) and (27) indicates that a depreciation of domestic currency (i.e., an increase in $\widehat{S}^{gap}$) uniformly raises the prices of foreign products in units of domestic currency. Equation (27) shows that negative sectoral markup wedges—directly and indirectly via the Leontif inverse $\mathbf{L}_{vx}$—reduce the prices of domestic products relative to foreign products, which are captured by the difference between the sectoral inflation gap $\widehat{\mathbf{P}}^{gap}$ and the exchange rate gap $\mathbf{1}\widehat{S}^{gap}$.[17] This reduction in domestic-to-foreign goods prices generates a switch of both domestic and foreign expenditures from foreign to domestic products, thereby increasing exports and reducing imports, as evinced by the negative term $-(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top$ in equation (26). As a result, domestic income from international trade increases, leading to a positive aggregate output gap, as evinced by the expenditure-switching term in equation (26).

In a domestic sector of an open economy with production networks, the size of the expenditure-switching channel is determined by the magnitudes of two sub-channels: (i) the direct and indirect (via downstream sectors) impacts of sectoral markup wedges on sectoral domestic-to-foreign price gaps through input-output linkages—captured by $\mathbf{L}_{vx}$ in equation (27), and (ii) the direct and indirect (via downstream sectors) impacts of sectoral domestic-to-foreign price gaps on the domestic and foreign demand for domestic products and, in turn, the consequential domestic labor income and export taxes—captured by $(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top$ in equation (26). These two sub-channels are combined to yield the domestic sector's expenditure-switching centrality, $\widetilde{\rho}_{ES,i}$, in the OG weight of equation (24).

(iii) Profit channel. The *profit channel*—also specific to the open economy—contributes to the aggregate output gap through the costs of imported inputs and the profits of domestic producers. Specifically, negative sectoral markup wedges increase domestic sectors' costs of imported inputs relative to the sectoral sales, thereby reducing domestic producers' profits and contributing negatively to the aggregate output gap. For a domestic sector in an open economy with production networks, the size of the profit channel is determined by both the sector's size and its direct and indirect (via upstream sectors) use of imported inputs—captured by the product of the sectoral Domar weight (i.e., λ_i) and the complement of the sector's direct and indirect use of domestic labor inputs (i.e., $1 - \widetilde{\alpha}_i$) in equations (26) and (24). Accordingly, this channel applies only to sectors that directly or indirectly import foreign products as intermediate inputs and is absent in closed economies.

Characterization and implementation of OG monetary policy. Theorem 1 implies that a monetary policy that sets the weighted average of sectoral markup wedges to zero closes the output gap, as formalized in the following definition:

Definition 5. *The output gap monetary policy (OG policy for short) eliminates the aggregate output gap,*

[17] The nominal exchange rate (S) is the endogenous component in foreign goods prices (in units of the domestic currency) that can be affected by sectoral markup wedges and, therefore, monetary policy. Therefore, the exchange rate gap ($\mathbf{1}\widehat{S}^{gap}$) reflects the prices of foreign products in the log linearization of equilibrium conditions around the flexible-price equilibrium.

viz, $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$, *for any realized state* $\boldsymbol{\xi} \in \Xi$.[18]

To implement the OG policy, the monetary authority sets the money supply to stabilize the aggregate inflation index that appropriately weights the domestic sectoral inflation. The aggregate inflation index accounts for (i) the mapping from sectoral inflation into sectoral markup wedges, as shown in equation (22); and (ii) the contribution of sectoral markup wedges to the aggregate output gap, as shown in Theorem 1. The next proposition formally characterizes the implementation of the OG monetary policy.

Corollary 1. *The OG policy is implemented by setting the following aggregate inflation index to zero:*

$$\sum_{i=1}^{N} \mathcal{M}_{OG,i} \cdot (1 - \delta_i)/\delta_i \cdot \widehat{P}_i(\boldsymbol{\xi}) = 0, \tag{28}$$

for any realized state $\boldsymbol{\xi} \in \Xi$. *The OG monetary policy achieves zero labor wedge and aggregate output gap up to the first-order approximation,* viz,

$$[\sigma - 1 + (\varphi + 1)/\Lambda_L]^{-1} \kappa_C \widehat{\Gamma}_L(\boldsymbol{\xi}) = \kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = 0.$$

Proof: Straightforward substitution of equation (22) in equation (23) from Theorem 1.

Corollary 1 shows that the monetary authority implements the OG policy by choosing the money supply that makes the aggregate inflation index in equation (28) equal to zero.[19] Equation (28) reveals that the weight assigned to sector i in the aggregate inflation index is proportional to the sectoral price rigidity $(1-\delta_i)/\delta_i$. The OG policy assigns higher weights to sectors with high nominal rigidities, which is consistent with the results in closed economies (La'O and Tahbaz-Salehi, 2022; Rubbo, 2023). Important in an open economy, however, equation (28) indicates that the weight assigned to sector i is proportional to the OG weight $\mathcal{M}_{OG,i}$ defined in equation (24), which internalizes the structure of the domestic and cross-border input-output linkages as stated in Theorem 1. The next section studies the role of the structure of import-export and the production network for the OG monetary policy.

Role of the exchange rate in the OG policy. Exchange rate adjustments influence the OG monetary policy by attenuating the positive impacts of sectoral inflation on the output gap in the expenditure-switching and profit channels. Specifically, negative markup wedges arising from positive sectoral inflation improve domestic income and trade conditions through both expenditure-switching and profit channels, leading to an

[18]Lemma H.3 in Appendix H.10 shows that the monetary policy that controls the supply of money $M(\boldsymbol{\xi})$ uniquely determines the aggregate output gap. Therefore, our OG monetary policy is well-defined.

[19]The OG monetary policy can be achieved owing to two reasons. First, the aggregate output gap strictly increases in the amount of the money supply (see Lemma H.3 in Appendix H.10). Second, inflation in each sector strictly increases in the aggregate output gap as a result of the positive slopes of the sectoral Phillips curves (see equation 36 in Section 4).

appreciation of the domestic currency (i.e., a decrease in $\widehat{S}^{gap}$) needed to preserve the trade balance.[20] The appreciated domestic currency increases the domestic-to-foreign price gap by $\widetilde{\boldsymbol{\alpha}}$ in equation (27), which reduces the demand for domestic products and, in turn, lowers the aggregate output gap by $(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top$ in equation (26). Thus, the initial positive effect of sectoral inflation on output gap through expenditure-switching and profit channels is attenuated by endogenous exchange rate adjustments, reflected by the coefficient of these two channels κ_S being less than one with $(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}$ in its denominator.[21]

OG monetary policy under foreign-currency pricing. Our baseline model assumes producer-currency pricing (PCP)—where domestic producers set export prices in their own (i.e., domestic) currency. In Appendix C, we follow Engel (2011) to extend our model to study the foreign-currency pricing that comprises the alternatives local-currency pricing (LCP) and dominant-currency pricing (DCP).[22] Under LCP and DCP, domestic producers set sectoral exporting prices in foreign and dominant (e.g., US dollars) currencies, respectively, and can price discriminate among domestic and foreign markets, facing market-specific Calvo-pricing rigidities for the same sector. In particular, because our model summarizes the rest of the world as a single foreign economy and treats import prices of foreign products denominated in foreign currency as exogenous, local-currency pricing is equivalent to dominant-currency pricing. We show in Corollary C.1 of Appendix C that under foreign-currency pricing, the contribution of sectoral markup wedges to the aggregate output gap is equal to the sum of the OG weight in equation (23) and an extra export-related term that replaces domestic-market with foreign-market sectoral markup wedges.[23] Therefore, the OG monetary policy under foreign-currency pricing should target an aggregate inflation index that includes sectoral inflation of both domestic-market prices and export prices in the foreign market, as in equation (C.5) of Corollary C.1 in Appendix C). In particular, while the CPI and profit channels remain dependent on domestic sectoral inflation, the expenditure-switching channel relies on inflation in domestic and export prices.

Comparison to monetary policies in one-sector small open economy model. A well-established result in one-sector SOE models is that optimal monetary policy should stabilize domestic inflation (Galí and

[20] As we will show in Section 5 (e.g., see Figure 1), the expenditure-switching channel dominates the profit channel quantitatively, thereby allowing negative markup wedges to generate a positive aggregate output gap through international trade.

[21] In contrast, in our multi-sector small open economies, the terms of trade (an important concept in the SOE literature) has a limited role in the design of monetary policy. In the special case of one-sector small open economies á la Galí and Monacelli (2005), the terms of trade gap is proportional to the output gap, both of which are closed under the optimal policy of domestic inflation stabilization. In our multi-sector small open economies, as we show in Appendix H.7, the terms of trade gap is equal to a weighted average of sectoral sectoral domestic-to-foreign price gaps, i.e., $\left[(\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}))^\top \boldsymbol{\lambda}_{EX}\right]^{-1} (\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap})$. As shown in equation (26), the sectoral domestic-to-foreign price gaps are important components of expenditure-switching channel in the OG policy. However, their impacts on the output gap are captured by their sectoral weights in equation (26) rather than their weights in the terms of trade gap. As such, the relevance of the terms of trade for the monetary policy design in multi-sector open economies with production networks is limited.

[22] Dominant-currency pricing means the international trade is priced and invoiced in the dominant currency of a large economy, primarily the US dollar in reality. For a small open economy, the dominant currency is a foreign currency.

[23] In particular, the export-related term specific to foreign-currency pricing is proportional to the product of the sectoral export value and customer centrality.

Monacelli, 2005), which simultaneously closes the output gap, as well as the terms-of-trade gap. This result is consistent with our theoretical finding in the special case of the one-sector version of our model (Section 4). In the one-sector SOE literature, the optimal policy is typically implemented by stabilizing aggregate domestic PPI inflation, where sectoral inflation rates are weighted by sectoral sales, which are proportional to the sectoral Domar weights. In the next subsection 3.3, we show that implementing the optimal policy according to this prescription—rather than using our OG weights in equation (24)—coincides with the monetary policy using Domar weights. Thus, our result confirms the finding from one-sector SOE models that domestic inflation should be stabilized. We enrich extant findings by deriving the appropriate sectoral weights in the domestic aggregate inflation index necessary to close the output gap. These weights differ from those in the PPI and depend on the interplay between the multi-sector structure, cross-border linkages, and IO linkages.

3.3. Role of input-output and cross-border linkages for the OG monetary policy

In this section, we study how input-output linkages interact with cross-border linkages in multi-sector small open economies to determine the sectoral OG weights through centrality measures. We focus on two questions: (i) What is the pitfall in the monetary policy that closes the output gap in the multi-sector open economy without production networks, thereby disregarding the role of input-output linkages? (ii) What is the pitfall in the monetary policy that adopts the Domar weights—which close the output gap in the closed economy—while disregarding the roles of cross-border linkages?

Pitfall in the monetary policy that disregards input-output linkages. To investigate the relevance of input-output linkages for the monetary policy, we study the pitfalls in adopting the sectoral weights that close the output gap in multi-sector horizontal small open economies without accounting for the input-output linkages. The sectoral OG weights in such economies—which we refer to as the *OG weights without input-output linkages* and denote by $\mathcal{M}_{OG,i}^{NoIO}$—account for the direct impacts (rather than indirect impacts through IO linkages) of sectoral markup wedges on the aggregate output gap, as outlined in the following corollary.

Proposition 1 (Aggregate output gap in multi-sector SOEs without input-output linkages)**.** *In a multi-sector horizontal small open economy without input-output linkages, the sectoral OG weight in equation (24) is equal to:*[24]

$$\mathcal{M}_{OG,i}^{NoIO} \equiv \underbrace{\beta_i v_i}_{\textit{CPI channel}} + \underbrace{\kappa_S^{NoIO} \left[\theta_{F,i}\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1 - v_i)\right]}_{\textit{expenditure-switching channel}}, \tag{29}$$

$$\kappa_S^{NoIO} \equiv \frac{1 - \sum_{i=1}^N \beta_i v_i}{1 - \sum_{i=1}^N \beta_i v_i + \sum_{i=1}^N \left[\theta_{F,i}\lambda_{EX,i} + (\theta_i - 1)\,\beta_i v_i\,(1 - v_i)\right]}.$$

[24] In multi-sector small open economies without input-output linkages, the Leontief inverse reduces to a diagonal matrix, with $l_{vx,r,i} = 1$ for $r = i$ and $l_{vx,r,i} = 0$ for $r \neq i$, and sectoral labor cost shares reduce to $\alpha_i = 1$. It follows that the domestic supplier and customer centralities reduce to $\widetilde{\lambda}_{D,i} = \beta_i v_i$ and $\widetilde{\alpha}_i = 1$, respectively.

Proof: Straightforward result from Theorem 1.

Proposition 1 shows that, on the one hand, the monetary policy that disregards input-output linkages under-estimates the relevance of a sector's inflation for the aggregate output gap, as it fails to account for its *indirect* impacts as an input supplier in the network—through both the CPI (i.e., ignoring $\sum_{r\neq i} \beta_r \upsilon_r l_{vx,r,i}$) and the expenditure-switching channels (i.e., ignoring $\sum_{r\neq i} \rho_{ES,r} \widetilde{\alpha}_r l_{vx,r,i}$ and $\sum_s \lambda_s \omega_{s,i} \upsilon_{x,s,i} (1 - \upsilon_{x,s,r})$). On the other hand, a monetary policy that disregards input–output linkages fails to account for domestic sectors' direct and indirect use of imported foreign inputs as customers—as evidenced by the absence of customer centrality in the expenditure-switching channel and the omission of the profit channel in equation (29). As a result, such policies may also overstate the relevance of sectoral inflation for domestic labor income and, in turn, the aggregate output gap.

Whether and how much the monetary policy disregarding input-output linkages over- or under-emphasizes the relevance of a sector's inflation is indeterminate and depends on the quantitative strength of the countervailing sectoral forces outlined above. That is, an underestimation that arises from ignoring sectors' indirect impact as input suppliers in the CPI and expenditure-switching channels versus an overestimation that results from disregarding their imports of foreign inputs. In Section 5.1, we show that— quantitatively using the World Input-Output Database for major economies—a monetary policy that ignores input–output linkages uniformly underestimates the relevance of sectoral inflation. Moreover, the underestimation of the CPI channel (as opposed to the expenditure-switching channel) dominates in economies with low openness, whereas the expenditure-switching channel dominates in highly open economies.

Pitfall in the monetary policy that disregards cross-border linkages. To investigate the relevance of accounting for the degree of openness for the monetary policy, we study the pitfalls of adopting the sectoral weights that close the output gap in the closed economy instead of the OG weights and disregard the role of cross-border linkages.

As a first step, we derive the OG weight in closed economies. The closed economy is characterized by domestic demand only and, in turn, zero *foreign supplier centrality* (i.e., $\lambda_{EX,i} = 0, \ \forall i$), as goods are entirely supplied to the domestic market. Thus, *domestic supplier centrality* is the unique supplier centrality, and it is equal to the Domar weight.[25] Moreover, the *expenditure-switching* and *profit channels* are equal to zero. Thus, OG weights are equal to the Domar weights in closed economies, which is consistent with the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023) and summarized in the next lemma.

Lemma 2. *In a closed economy, the OG weight of any sector reduces to the Domar weight, i.e., $\mathcal{M}_{OG,i} = \lambda_i$ for each sector $i \in \{1, 2, \cdots, N\}$. In the open economy, the Domar weight of the sector i equals the sum of*

[25] Our standard assumption of a Cobb-Douglas production function is crucial for the equivalence between the *supplier centrality* and the Domar weight, as discussed in Baqaee (2018).

the sectoral domestic and foreign supplier centralities, i.e.,

$$\lambda_i = \widetilde{\lambda}_{D,i} + \widetilde{\lambda}_{F,i}. \tag{30}$$

Proof: See Appendix H.9.

Lemma 2 implies that the monetary policy that aims at closing the domestic output gap but fails to account for the cross-border linkages will adopt the Domar weight in place of the OG weight. Equation (30) in Lemma 2 further shows that, unlike in closed economies, the Domar weight in an open economy comprises not only the *domestic supplier centrality* ($\widetilde{\lambda}_{D,i}$), but also the *foreign supplier centrality* ($\widetilde{\lambda}_{F,i}$), because sectoral output in open economies is supplied to both domestic and foreign customers.

Combining Lemma 2 and Theorem 1 yields the percentage deviation of the closed-economy OG weight (i.e., the Domar weight) from the open-economy OG weight, as outlined in the following proposition:

Proposition 2. *The percentage deviation of the Domar weight from the OG weight is equal to*

$$\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i} = \underbrace{\frac{\widetilde{\lambda}_{F,i}}{\lambda_i}}_{\textit{export intensity}} + \underbrace{\kappa_S(1-\widetilde{\alpha}_i)}_{\textit{factor-import}} \underbrace{-\kappa_S \cdot \frac{\widetilde{\rho}_{ES,i}}{\lambda_i}}_{\textit{expenditure-switching}} . \tag{31}$$

Proof: Straightforward result from Lemma 2 and Theorem 1.

Proposition 2 demonstrates that the monetary policy aiming at closing the domestic aggregate output gap, yet failing to account for the cross-border linkages and using Domar weights, can *either* overstate *or* understate the inflation of a domestic sector, depending on the magnitudes of *three countervailing forces*. First, sectoral Domar weights in open economies capture domestic sectors' supply of inputs to foreign countries besides domestic output—summarized by the sectoral *export intensity* that we define as the sector's ratio of foreign supplier centrality to Domar weight (i.e., the first component on the RHS of equation 31). Therefore, the monetary policy that disregards cross-border linkages and uses the Domar weights overemphasizes the contribution of the domestic sector to domestic output as a supplier, thereby overstating the contribution of its inflation to domestic aggregate output gap.

Second, in open economies, domestic producers use imported foreign factors in addition to domestic labor in production—captured by $(1-\widetilde{\alpha}_i)$ (i.e., the second component on the RHS of equation 31, labeled "factor-import"). Therefore, the monetary policy that fails to consider cross-border linkages and assumes that the domestic sector uses only domestic factors overemphasizes the contribution of the domestic sector to domestic output as a customer of domestic factors, thereby also overstating the contribution of its inflation to domestic aggregate output gap.

Third, the Domar-weight policy—which disregards cross-border imports and exports—can understate the importance of a domestic sector's inflation by failing to consider its impact on the domestic-to-foreign prices and, in turn, the domestic and foreign demand for the sectoral products—as captured by the *expenditure-switching component* (i.e., the third component on the RHS of equation 31).

Because of the critical role of foreign supplier centrality (in the first component) and customer centrality (in the second and third components) in driving the Domar–OG difference, the inflation of sectors with high foreign supplier centrality and low customer centrality may be overstated under a Domar-weighted policy.[26]

Overall, the extent to which the monetary policy that disregards cross-border linkages over- or under-states the relevance of a sector's inflation is ambiguous and depends on the quantitative strength of the three countervailing forces discussed above. That is, an overestimation that results from overstating the sector's contribution to domestic consumption as an input supplier and to domestic labor income as a customer, versus an underestimation that results from ignoring the expenditure-switching channel. In Section 5.1, we calibrate our model using the WIOD data and show that the closed-economy Domar-weight policy, in general, overstates the relevance of sectoral inflation, with the export intensity component contributing to most of the over-estimation.

4. Welfare loss and optimal monetary policy

In this section, we study optimal monetary policy. As in Woodford (2003) and Galí (2015), we derive the closed-form solution of the policy that minimizes welfare losses up to the second-order approximation.

Welfare loss and sectoral Phillips curves. Under the assumption of non-contingent subsidy and tax rates that offset monopolistic distortions (Lemma 1), the *flexible-price equilibrium* represents the optimal allocation for the domestic social planner. We define *welfare loss* as the utility gap of the representative household between the *sticky* and *flexible-price equilibria*, $u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi})$, and approximate it to the second order, as stated in the following proposition.

Proposition 3 (Welfare loss)**.** *Given the realized state $\boldsymbol{\xi} \in \Xi$, the welfare loss can be decomposed as*

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{-\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi + 1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\text{output-gap misallocaton}} \underbrace{-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})\widehat{\mathbf{P}}(\boldsymbol{\xi})}_{\text{within- and across-sector, and cross-border misallocations}}, \tag{32}$$

where the within-sector, across-sector, and cross-border misallocations are equal to

$$-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = -\frac{1}{2}\sum_i \lambda_i\varepsilon_i\frac{1-\delta_i}{\delta_i}\widehat{P}_i(\boldsymbol{\xi})^2, \tag{33}$$

$$-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = -\frac{1}{2}\sum_{i=1}^n \beta_i\big[\widehat{C}_i^{gap}(\boldsymbol{\xi}) - \widehat{C}^{gap}(\boldsymbol{\xi})\big]^2 - \frac{1}{2}\sum_{i=1}^n \lambda_i\alpha_i\big[\widehat{L}_i^{gap}(\boldsymbol{\xi}) - \widehat{Y}_i^{gap}(\boldsymbol{\xi})\big]^2 \tag{34}$$

[26] The expenditure-switching centrality, $\widetilde{\rho}_{ES,i}$, increases in the sector's customer centrality, $\widetilde{\alpha}_i$. Therefore, disregarding cross-border imports of foreign factors also overstates a sector's contribution to domestic aggregate output through the expenditure-switching component. A prototypical sector of this type is the *export processing* sector, which primarily supplies inputs abroad rather than fulfilling domestic demand, and predominantly uses foreign inputs instead of domestic labor.

$$
\begin{aligned}
& -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\omega_{i,j}\big[\widehat{X}_{i,j}^{gap}(\boldsymbol{\xi})-\widehat{Y}_i^{gap}(\boldsymbol{\xi})\big]^2,\\
-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^\top\boldsymbol{\mathcal{L}}^{cb}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = & -\frac{1}{2}\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}v_i(1-v_i)\big[\widehat{C}_{Hi}^{gap}(\boldsymbol{\xi})-\widehat{C}_{Fi}^{gap}(\boldsymbol{\xi})\big]^2 \qquad (35)\\
& -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\lambda_i\omega_{i,j}}{\theta_j}v_{x,i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Hj}^{gap}(\boldsymbol{\xi})-\widehat{X}_{Hi,Fj}^{gap}(\boldsymbol{\xi})\big]^2\\
& -\frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_{EX,i}}{\theta_{F,i}-1}\Big[\frac{\theta_{F,i}-1}{\theta_{F,i}}\widehat{Y}_{EX,i}^{gap}(\boldsymbol{\xi})^2-\Lambda_L\widehat{L}^{gap}(\boldsymbol{\xi})^2\Big].
\end{aligned}
$$

Proof: See Appendix I.1.

Equation (32) shows that, to a second-order approximation, *welfare loss* consists of the sum of losses from output gap misallocation, within- and across-sector misallocation—similar to those in closed economies à la La'O and Tahbaz-Salehi (2022) and Rubbo (2023)—as well as cross-border misallocation, which is specific to the open economy. Specifically, the within-sector misallocation is the sum of the misallocation arising from inflation dispersion in all sectors, which is similar to its counterpart in one-sector economies. The across-sector misallocation includes those arising from the disproportional sectoral consumption relative to aggregate consumption (first term on the RHS of equation 34), as well as those arising from the disproportional use of sectoral labor and intermediate inputs relative to sectoral output across different sectors i (second and third terms on the RHS of equation 34, respectively). The cross-border misallocation includes distortions arising from the disproportional use of domestic versus foreign goods for both consumption and intermediate inputs (first and second terms on the RHS of equation 35). The cross-border misallocation also includes distortions arising from disproportionate exports relative to the use of domestic labor, which cause domestic producers' monopoly power in international markets to deviate from the optimal level (the third term on the right-hand side of equation 35).

To attain the optimal monetary policy analytically, we derive the sectoral Phillips curves that link the output gap and the exogenous sectoral shocks to sectoral inflation, as stated in the next Proposition.

Proposition 4 (Sectoral Phillips curves)**.** *In the* sticky-price equilibrium*, the following multi-sector Phillips curves hold:*

$$
\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \underbrace{\boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi})}_{\textit{output-gap-driven inflation}} + \underbrace{\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}}_{\textit{cost-push inflation}} + o(\|\widehat{\boldsymbol{\xi}}\|), \qquad (36)
$$

where $\widehat{\mathbf{P}}(\boldsymbol{\xi})$ is an N-by-1 vector with sectoral inflation, and parameters $\boldsymbol{\mathcal{B}}$ (an N-by-1 vector) and $\boldsymbol{\mathcal{V}}$ (an N-by-3N matrix) are the slopes of Phillips curves and the coefficients of exogenous shocks, respectively.

Proof: See Appendix I.2.

In Proposition 4, the slopes of the sectoral Phillips curves are equal to:[27]

$$\boldsymbol{\mathcal{B}} \equiv \boldsymbol{\Delta}_{\Phi}\Big\{ \underbrace{\boldsymbol{\alpha}\Gamma_{W,C}}_{\text{nominal wage channel}} + \underbrace{(\boldsymbol{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}})\,\mathbf{1}\Gamma_{S,C}}_{\text{nominal exchange rate channel}} \Big\}, \tag{37}$$

where $\Gamma_{W,C} \equiv (\Gamma_C + \sigma + \varphi/\Lambda_L)$ and $\Gamma_{S,C} \equiv [(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}]^{-1}(\Gamma_C + 1)$ capture the impacts of the aggregate output gap on sectoral inflation *via* the nominal wage and nominal exchange rate, respectively.[28] Both channels are positive, making the slopes of the sectoral Phillips curves positive for all sectors.

The nominal wage channel in equation (37) comprises two sub-effects. First, a positive aggregate output gap increases the CPI via the current account and nominal exchange rate, as captured by the term Γ_C in $\Gamma_{W,C}$. Second, a positive aggregate output gap increases the real wage via the labor supply, as captured by the term $(\sigma + \varphi/\Lambda_L)$. The nominal exchange rate channel in equation (37) functions as now described. A positive aggregate output gap raises the nominal expenditure and worsens the current account, generating a depreciation of the domestic currency and an increase in the nominal exchange rate, represented by the term $\Gamma_{S,C}$ in equation (37). The increase in the nominal exchange rate propagates into the costs of imported inputs and thus sectoral inflation, encapsulated by the term $(\boldsymbol{\Omega} \odot \mathbf{V}_{\mathbf{1}-\boldsymbol{x}})\,\mathbf{1}$. The nominal exchange rate channel is specific to open economies and becomes more prominent with greater openness—evidenced by the matrix of import shares, $\mathbf{V}_{1-x}$.

Optimal monetary policy. In the following definition, we analytically derive the optimal monetary policy and compare it to the OG policy outlined in Definition 5:

Definition 6 (Optimal monetary policy)**.** *For any realized aggregate state $\boldsymbol{\xi} \in \Xi$, the optimal monetary policy sets the money supply $M(\boldsymbol{\xi})$—which is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ in equilibrium—to minimize the welfare loss in equation (32)* subject to *the sectoral Phillips curves (36).*

Consistent with Definition 6, we derive the aggregate inflation index that the monetary authority should target to implement the optimal monetary policy, as stated in the next proposition.

Proposition 5 (Implementation of the optimal monetary policy)**.** *The optimal monetary policy is implemented by setting the following aggregate inflation index to zero:*

$$\big\{[\sigma - 1 + (\varphi+1)/\Lambda_L]\,\kappa_C^{-1}\boldsymbol{\mathcal{M}}_{OG}^\top(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^\top(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})\big\}\,\widehat{\mathbf{P}} = 0, \tag{38}$$

[27]The definitions and the interpretations of the matrix $\boldsymbol{\mathcal{V}}$, and the scalar Γ_C, vectors $\boldsymbol{\mathcal{M}}_{EX}$ and $\boldsymbol{\mathcal{M}}_{IM}$, and matrix $\boldsymbol{\Delta}_\Phi$ composing $\boldsymbol{\mathcal{B}}$ in equation (37) are presented in Appendix I.2. Specifically, Γ_C is defined by equation (I.28); $\boldsymbol{\mathcal{M}}_{EX}$ and $\boldsymbol{\mathcal{M}}_{IM}$ in equations (I.24) and (I.25) capture the impacts of export demand shocks and import price shocks on the current account, respectively.

[28]The unitary vector $\mathbf{1}$ in the nominal exchange rate channel indicates that changes in the nominal exchange rate uniformly affect the costs of imported inputs for all sectors.

for any realized state $\boldsymbol{\xi} \in \Xi$.

Proof: See Appendix I.3.

Comparing the optimal monetary policy in equation (38) with the OG policy in equation (28) shows that the optimal policy accounts for *both* the output gap misallocation—as evinced by the OG weights $\boldsymbol{\mathcal{M}}_{OG}^{\top}$ as the first term in the brackets—*and* the within- and across-sector, and cross-border misallocation generated by sectoral distortions—as manifested by the second term $\boldsymbol{\mathcal{B}}^{\top}(\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})$ in the curly brackets. In contrast, the OG policy that closes the output gap does not simultaneously eliminate the within- and across-sector, and cross-border misallocations, because the sectoral inflation underlying these misallocations is not proportional to the aggregate output gap according to sectoral Phillips curves (36). Therefore, the "divine coincidence"—which holds in the workhorse model of one-sector SOEs, as in Galí and Monacelli (2005)—fails to hold in our multi-sector open economies, similar to the case of the multi-sector closed economies in La'O and Tahbaz-Salehi (2022) and Rubbo (2023).

Role of multiple sectors, input-output linkages, and cross-border linkages. We investigate the role of multiple sectors, cross-border linkages, and input-output linkages for the welfare loss, sectoral Phillips curves, and the optimal monetary policy by studying the following three special cases of our framework: (i) the workhorse model of the one-sector small open economy, as in Galí and Monacelli (2005); (ii) the multi-sector closed economy (without cross-border linkages); and (iii) the multi-sector small open economy without input-output linkages.

In the one-sector small open economy, the welfare loss in equation (32) reduces to the sum of the output gap, within-sector, and cross-border misallocations, the latter two of which are proportional to the squares of domestic inflation according to equation (32). In addition, in the one-sector economy, the output gap is proportional to domestic inflation, which can be seen by substituting equation (22) into (23). As a result, the welfare loss is proportional to the squares of both the output gap and domestic inflation, and the optimal monetary policy in one-sector SOEs achieves the first-best allocation by fully stabilizing domestic inflation, consistent with the "divine coincidence," as in Galí and Monacelli (2005).

In multi-sector small open economies without input-output linkages, the optimal monetary policy differs from the counterpart in our baseline economy with input-output linkages for the differences in both the welfare loss and the sectoral Phillips curves. In the welfare loss of equation (32), the mapping from sectoral inflation into the aggregate output gap (i.e., the OG weights $\boldsymbol{\mathcal{M}}_{OG}$) reduces to those in equation (29), and the across-sector and cross-border misallocations arising from the disproportional use of intermediate inputs (i.e., the third and second terms in equations 34 and 35, respectively) are absent. In the sectoral Phillips curves, the nominal exchange rate channel is absent in the coefficients of the output gap because it operates through the importing price of foreign intermediate inputs, as discussed in Proposition 4.

Similarly, in multi-sector closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), the optimal monetary policy also differs from the counterpart in the baseline multi-sector open economies for the differences in both welfare loss and the sectoral Phillips curves. In the welfare loss of equation (32), the mapping from sectoral inflation into the aggregate output gap (i.e., the OG weights $\boldsymbol{\mathcal{M}}_{OG}$) reduces to the Domar weight, and the cross-border misallocation is absent. In the sectoral Phillips curves, the nominal exchange rate channel is absent in the coefficients of the output gap denoted by $\boldsymbol{\mathcal{B}}$ in equation (37), as we discussed in the paragraph following the equation.

Overall, our analysis demonstrates the important roles of multiple sectors, as well as cross-border and IO linkages, in the welfare loss, sectoral Phillips curves, and formulation of the optimal monetary policy.

5. Quantitative analysis

In this section, we quantify our theoretical results by calibrating the model to the input-output matrices of major economies in the WIOD. Subsection 5.1 studies the contribution of the different channels to sectoral OG weights using variance decomposition, as well as the differences of the OG weights from the corresponding weights in the monetary policy that ignores either cross-border or IO linkages, focusing on the contribution of the distinct components to these differences. Subsection 5.2 studies rule-of-thumb combinations of the centrality measures to approximate both sectoral OG weights and their differences from Domar weights, revealing the relevance of cross-border and IO linkages for approximating OG weights. Subsection 5.3 studies the welfare of alternative monetary policies, revealing the welfare enhancement achieved by the OG policy over alternative policies that ignore either cross-border or input-output linkages.

Our quantitative analysis uses the National Input-Output Tables (NIOTS) for 43 economies (28 EU and 15 OECD countries, each of them comprising 56 sectors) from the World Input-Output Database (WIOD) for the year 2014. We calibrate the input-output matrix and import and export shares of each country using its NIOTS sector-level data.[29] Shown in Table 2 is the calibration of the key parameters in our model. Appendix F.1 presents the full parameter calibration and provides additional details on the WIOD.

5.1. Size, variance decomposition, and difference of OG weights from misspecified weights

For each country, we compute the percentage contribution of each of the three components in the OG weights—namely, the *CPI*, the *expenditure-switching*, and the *profit channels*, respectively—to the variance of the OG weight in equation (24) using the following variance decomposition:

$$100\% = \frac{cov\big(\widetilde{\lambda}_{D,i}, \mathcal{M}_{OG,i}\big)}{var(\mathcal{M}_{OG,i})} + \frac{cov\big(\kappa_S \widetilde{\rho}_{ES,i}, \mathcal{M}_{OG,i}\big)}{var(\mathcal{M}_{OG,i})} + \frac{cov\big(-\kappa_S \lambda_i (1-\widetilde{\alpha}_i), \mathcal{M}_{OG,i}\big)}{var(\mathcal{M}_{OG,i})}. \tag{39}$$

[29]Data source: https://www.rug.nl/ggdc/valuechain/wiod/?lang=en. The release of the WIOD in 2016 provides information for the period 2000-2014. In our analysis, we use the latest available year—2014. The NIOTS provides each country's sector-level imports from (vs. exports to) the Rest of the World (RoW) and exports to the RoW, which are aggregates of the country's imports from (vs. exports to) all other countries, including those countries that are not listed in the WIOD. Using the NIOTS data, we calibrate each country individually as a small open economy relative to the rest of the world, rather than calibrating all countries simultaneously within a global equilibrium.

Table 2: Model calibration

Parameters	Data variables/moments used
Common across all countries	
Risk aversion, $\sigma = 2$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Labor supply elasticity, $\varphi = 1$	Business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019)
Elasticity of substitution (EOS) across varieties, $\varepsilon_i = 8$	Atkeson and Burstein (2008)
EOS. btw. domestic and foreign goods, $\theta_i = \theta_{Fi} = 5$	Head and Mayer (2014)
Sector-level frequency of price adjustment, δ_i	Pasten et al. (2024)
Frequency of wage adjustment, δ_0	Beraja et al. (2019) and Barattieri et al. (2014)
Country specific	
Input-output matrix, $\mathbf{\Omega}$	Sectoral gross output, intermediate goods from both domestic and foreign
Home bias for firms' import $\mathbf{V}_x$	Intermediate goods from both domestic and foreign
Labor share, $\boldsymbol{\alpha}$	Sectoral gross output, labor compensation
Export to foreign countries in steady state, $\mathbf{D}_H^{*,ss}$	Sectoral exports to foreign countries
Consumer consumption share, $\boldsymbol{\beta}$	Sectoral consumption from both domestic and foreign, GDP
Consumer consumption home bias, $\boldsymbol{v}$	Sectoral consumption from both domestic and foreign

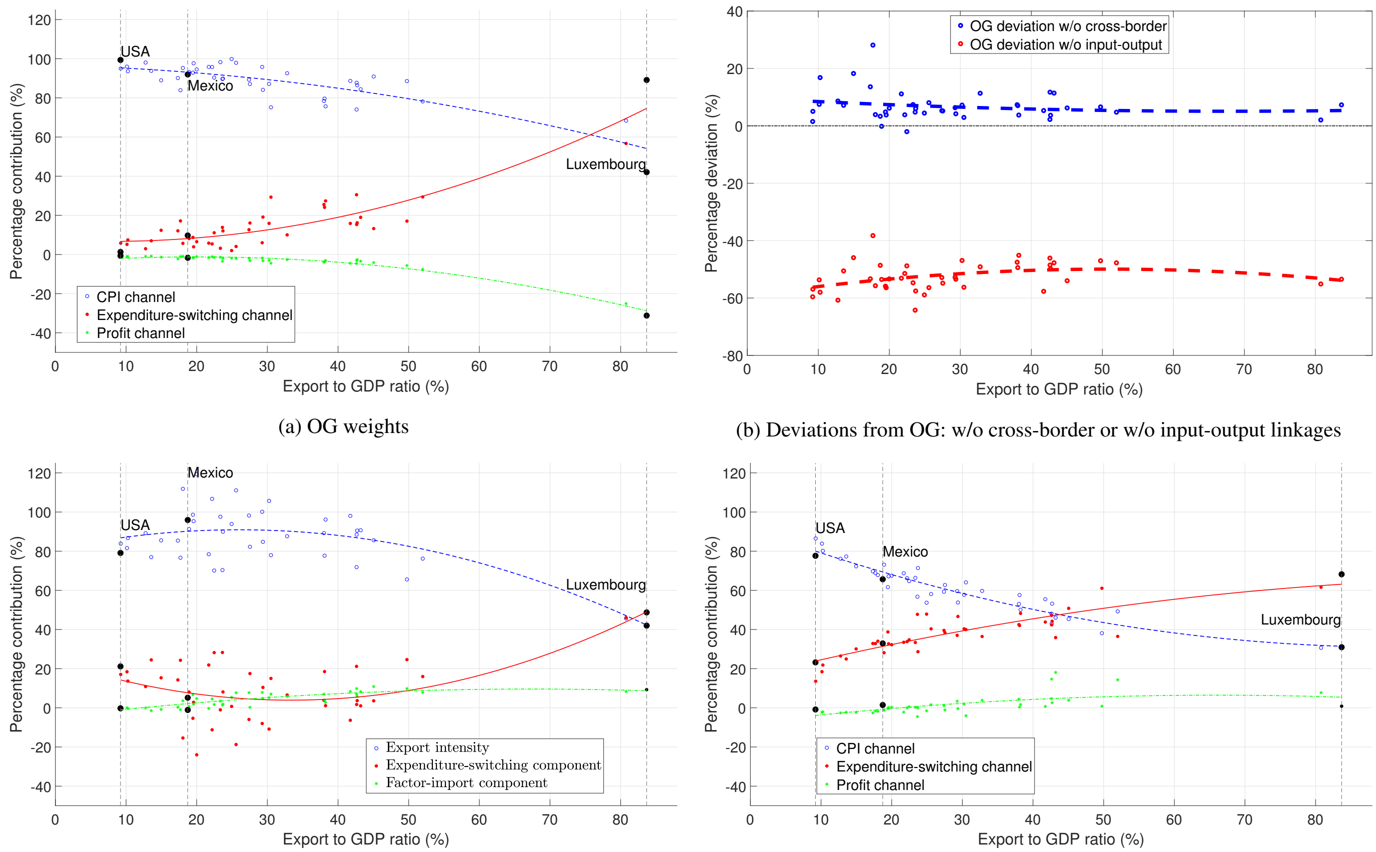
Panel (a) of Figure 1 plots the percentage contributions of each of the three channels to the total variation of OG weights for each country in the sample. Each set of the vertically aligned markers in blue circles, red dots, and green stars represents the contributions of the *CPI channel*, the *expenditure-switching channel*, and the *profit channel*, respectively, for a specific country. The vertical dashed lines show the cases for the U.S., Mexico, and Luxembourg, as representative economies with polar degrees of openness (relatively closed or fully open). The dashed-blue, solid-red, and dash-dotted-green lines show the fitted curves for each of the three channels across countries.

As shown in the figure, the *CPI channel* (blue circle) and *expenditure-switching channel* (red dot) explain the bulk of the variation in the sectoral OG weights across sectors for all countries. In contrast, the contribution of the *profit channel* (green star) is marginal except in economies with extremely large openness like Luxembourg, as evinced by the near zero dashed-dotted green line. Moreover, the percentage contribution of the *expenditure-switching channel* (*CPI channel*) increases (declines) with the openness of the country measured by the economy-wise export-to-GDP ratio, as shown by the rising solid-red line (the declining dashed-blue line).[30] For example, in Luxembourg—the most open economy in our sample with an economy-wise export-to-GDP ratio of 83%—the contribution of the *CPI channel* is inferior to the *expenditure-switching channel* (42% vs. 89%). In contrast, in Mexico—a moderately open economy with an export-to-GDP ratio of 19%—the contribution of the *CPI channel* to the OG weight is large compared to the *expenditure-switching channel* (92% vs. 9.7%). Finally, for the U.S.—a nearly closed economy with an export-to-GDP ratio of 9%—the *CPI channel* contributes to almost the entire variation in OG weights (99%) while the contribution of the *expenditure-switching channel* is minimal (1.3%).

Quantifying the pitfalls of monetary policy that disregards cross-border linkages. To quantify the pitfalls of the Domar-weight monetary policy that disregards cross-border linkages, Panel (b) of Figure 1 shows the country-level average of the percentage deviations of the sectoral Domar weights from the OG weights (i.e., $(\lambda_i - \mathcal{M}_{OG,i})/\lambda_i$) (blue circle). The Domar-weight policy over-estimates the contribution of sectoral

[30] The patterns are robust to the alternative measurement of the degree of openness using the economy-wise import-to-GDP ratio and the ratio of total trade volume to GDP.

Figure 1: Size and variance decomposition of sectoral OG weights and their deviations from misspecified weights



(a) OG weights

(b) Deviations from OG: w/o cross-border or w/o input-output linkages

(c) Difference between Domar and OG weights

(d) Difference between OG weights w/ and w/o input-output linkages

Notes: Shown in the scatter plots in Panels (a), (c), and (d) are the percentage contributions of each of the three channels (or components) to the OG weight, the percentage deviation of Domar from OG weights, and the percentage deviation of the OG weights without from with input-output linkages, respectively, for each country (y-axis) against the country's economy-wise export-to-GDP ratio (x-axis). In Panels (a) and (d), the CPI, the net export income, and the net profit income channels are denoted by blue circles, red dots, and green stars, respectively. In Panel (c), the export intensity, factor-import, and expenditure-switching components are denoted by blue circles, red dots, and green stars, respectively. The dashed-blue, solid-red, and dash-dotted-green lines are the fitted curves for the three channels (or components), respectively. Shown in Panel (b) are the country-wise averages of the percentage deviations of the sectoral Domar weights from the OG weights (blue circles) and the sectoral OG weights without from with input-output linkages (red dots), with the dashed-blue and dashed-red lines being the fitted curves.

inflation to the aggregate output gap across almost all economies, as evinced by the blue circles that are above the zero horizontal line. The degree of over-estimation vis-à-vis the percentage deviation of Domar from OG weights averages around 8.5% across all economies and can be as large as 28%.

Panel (c) of Figure 1 further shows the percentage contribution of each of the three components—namely, the *export intensity*, *factor-import*, and *expenditure-switching components*, respectively—to the variance of the difference between Domar and OG weights using the following variance decomposition:[31]

$$100\% = \frac{cov\big(\frac{\widetilde{\lambda}_{F,i}}{\lambda_i}, \frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)}{var\big(\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)} + \frac{cov\big(\kappa_S(1-\widetilde{\alpha}_i), \frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)}{var\big(\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)} + \frac{cov\big(-\kappa_S \frac{\widetilde{\rho}_{ES,i}}{\lambda_i}, \frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)}{var\big(\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i}\big)}. \tag{40}$$

[31]Equation (40) is derived using the bilinearity of covariance and $\frac{\lambda_i - \mathcal{M}_{OG,i}}{\lambda_i} = \frac{\widetilde{\lambda}_{F,i}}{\lambda_i} + \kappa_S(1-\widetilde{\alpha}_i) - \kappa_S \frac{\widetilde{\rho}_{ES,i}}{\lambda_i}$ in equation (31).

As shown in the figure, the sectoral Domar-OG difference is predominantly driven by the export intensity (blue circle) component with an average percentage contribution of 87%. Therefore, the pitfalls of the OG policy that disregards cross-border linkages arise from overlooking the sector's contribution to foreign demand as an input supplier, particularly for economies with a medium degree of openness (i.e., 20% to 30%). In contrast, the contributions of the *expenditure-switching* (red dot) and *factor-import components* (green star) are small, except in economies with extremely large openness like Luxembourg, where the *expenditure-switching component* contributes to a substantial percentage of the Domar-OG difference that is even greater than the *factor-import component*. These results imply that, to correct for the pitfalls of closed-economy Domar-weight policies—which coincide with the PPI-stabilizing policy used in one-sector SOE literature—the monetary authority mainly needs to adjust the weights of sectors downward by their direct and indirect exports, with larger downward adjustments on those sectors with large export intensity.

Quantifying the pitfalls of the OG policy that disregards IO linkages. To quantify the pitfalls of the monetary policy that disregards input-output linkages, Panel (b) of Figure 1 shows the country-level average of the percentage deviation of sectoral OG weights without IO linkages from those with IO linkages—i.e., $(\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i})/\mathcal{M}_{OG,i}$ (red dot). As shown in the figure, the Domar-weight policy consistently under-estimates the contribution of sectoral inflation to the aggregate output gap by ignoring their indirect impacts, as evinced by the red dots that are below the zero horizontal line. The degree of under-estimation—vis-à-vis the percentage deviation of the OG weights without IO linkages from those with IO linkages—averages around -56% across economies and can be as negative as -64%.

Panel (d) of Figure 1 further shows the percentage contributions of each of the three channels—namely, the *CPI*, the *expenditure-switching*, and the *profit channels*, respectively—to the variance of the difference between the OG weights *without and with IO linkages* using the following variance decomposition:[32]

$$100\% = \frac{cov\left(\frac{\beta_i v_i - \widetilde{\lambda}_{D,i}}{\mathcal{M}_{OG,i}}, \frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)}{var\left(\frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)} + \frac{cov\left(\frac{\kappa_S^{NoIO}\left[\theta_{F,i}\lambda_{EX,i} + (\theta_i - 1)\beta_i v_i (1 - v_i)\right] - \kappa_S \widetilde{\rho}_{ES,i}}{\mathcal{M}_{OG,i}}, \frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)}{var\left(\frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)}$$
$$+ \frac{cov\left(\frac{\kappa_S \lambda_i (1 - \widetilde{\alpha}_i)}{\mathcal{M}_{OG,i}}, \frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)}{var\left(\frac{\mathcal{M}_{OG,i}^{NoIO} - \mathcal{M}_{OG,i}}{\mathcal{M}_{OG,i}}\right)}. \tag{41}$$

As shown in the figure, the differences between the OG weights with and without input-output linkages are primarily attributed to the CPI channel (blue circle) in economies with a low degree of openness, with the contribution declining with the openness of the economy, as evinced by the downward-sloping dashed-blue line. In economies with large openness, however, these differences are mainly attributed to the expenditure-switching channel (red dot), with the contribution increasing with the openness of the economy, as evinced by the upward-sloping solid-red line. Compared to the above two channels, the contribution of

[32]Equation (41) is derived by applying the linearity of covariance in each of its arguments and the decompositions of OG weights with and without IO linkages in equations (24) and (29), respectively.

the profit channel is limited across all economies, as evinced by the near-zero dashed-dotted green line. Because both the CPI and the expenditure-switching channels lead to under-estimates of OG weights—as shown by Proposition 1—our results imply that the monetary authority should adjust sectoral OG weights upward to correct for the pitfalls of monetary policies that disregard input-output linkages. In relatively closed economies (i.e., those with a low export-to-GDP ratio in Figure 1d), such upward adjustments should be larger in sectors with a larger indirect contribution (via downstream sectors) to domestic consumption through the CPI channel. In relatively open economies (i.e., those with a high export-to-GDP ratio in Figure 1d), such upward adjustments should be larger for sectors with a large indirect contribution (via downstream sectors) to both domestic and foreign demand through the expenditure-switching channel.

5.2. Approximation of sectoral OG weights and Domar-OG differences

In this section, we use panel regressions to study the rule-of-thumb combinations of centrality measures to approximate the sectoral OG weights and the difference between the Domar and OG weights. The Domar weights correspond to the OG weights in closed economies, which coincide with the sectoral weights in the PPI-stabilizing policy used in one-sector SOE literature, as discussed in Section 3.2. We show that the sectoral OG weights and the Domar-OG differences can be approximated by the single measure of *domestic supplier centrality*, and the linear combination of *export intensity* and *customer centrality*, respectively. We also show that ignoring input-output linkages leads to an inaccurate approximation of sectoral OG weights.

We study the combinations of centrality measures to approximate the sectoral OG weights and Domar-OG differences using the following regressions:[33]

$$y_{c,i} = \mathbf{X}_{c,i}^{\top}\boldsymbol{\beta} + \eta_c + \epsilon_{c,i}, \quad \text{with} \quad y_{c,i} \in \{\mathcal{M}_{OG,c,i}, (\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}\}, \tag{42}$$

where the dependent variable $y_{c,i}$ is either the level of the OG weight ($\mathcal{M}_{OG,c,i}$), or the percentage difference between the Domar and OG weights ($(\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}$) for sector i and country c. The variable $\boldsymbol{X}_{c,i}$ includes our centrality measures for the regressions (see Tables 3 and 4), and η_c is the country fixed effect.

Approximation of sectoral OG weights. Shown in Table 3 are the estimates of the equation with the level of sectoral OG weight ($\mathcal{M}_{OG,c,i}$) as the dependent variable. Column (2) shows that the domestic supplier centrality is positively related to the OG weight of the sector with a coefficient equal to 1.02. More importantly, the domestic supplier centrality *alone* provides an accurate approximation of the sectoral OG weights, with an R-square of 0.93.

In contrast, Column (1) shows that the Domar weight—which is the nearly the optimal OG weight in closed economies á la Rubbo (2023) and La'O and Tahbaz-Salehi (2022)—has a weaker explanatory power

[33] We focus on a subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Results are robust, albeit less strong, for less open economies. We do not include sectoral fixed effects in the regression, as our main purpose is to explore the variations in OG weights across different sectors.

Table 3: Centrality measures and the OG weight in the data

	(1)	(2)	(3)	(4)	(5)	(6)
Domar weight	0.620***					
	(0.0894)					
Domestic supplier centrality		1.022***			0.974***	
		(0.0134)			(0.00365)	
Import share			-0.000152*			
			(8.06e-05)			
Import intensity				-0.0633***		
				(0.00494)		
Customer centrality					0.00464***	
					(0.000787)	
Expenditure-switching (ES) centrality					0.105***	
					(0.00748)	
Domestic supplier centrality w/o input-output						1.074***
						(0.0196)
ES centrality w/o input-output						0.0590***
						(0.00545)
Observations	601	601	601	601	601	601
R-square	0.739	0.927	0.010	0.322	0.994	0.880
Country FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (42), which regresses the level of the sectoral OG weight over the centrality measures defined in Section 3.1. The analysis includes the subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

than domestic supplier centrality, with a smaller R-square of 0.74. The comparison between Columns (1) and (2) illustrates the importance of considering openness in approximating sectoral OG weights.

To validate the negative impact of the import shares on the sectoral domestic supplier centrality and OG weights—as discussed in Section 3.1 and Appendix D—we define the *import intensity* of a sector i as $1 - \widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$, where $\widetilde{\lambda}_{D,i}/\widetilde{\lambda}_{All,D,i}$ captures the domestic demand for i's goods in the baseline economy with imports ($\widetilde{\lambda}_{D,i}$) relative to that without imports ($\widetilde{\lambda}_{All,D,i}$).[34] Accordingly, the import intensity of a sector measures the impact of the entire economy's import shares on the domestic demand for this sector's goods. As shown in Column (4), a sector's OG weight significantly decreases with the import intensity, thereby validating the negative impact of the direct and indirect import shares of a sector on its OG weight.

Approximation of the pitfalls in OG weights that ignore openness. Presented in Table 4 are the results for the version of the regression in equation (42) with the percentage difference between the Domar and OG weights ($(\lambda_{c,i} - \mathcal{M}_{OG,c,i})/\lambda_{c,i}$) as the dependent variable. Columns (1)-(3) show that the Domar-OG percentage difference increases with the export share and intensity of the sector, and it decreases with customer centrality. The export intensity is positively related to the Domar-OG difference with a coefficient

[34]The term $\widetilde{\lambda}_{All,D,i}$ is the i-th entry of the vector $\boldsymbol{\beta}^{\top}(\mathbf{I} - \boldsymbol{\Omega})^{-1}$ and captures the domestic demand that reaches the domestic sector i directly and indirectly via downstream sectors *if* the entire economy—including sector i and its downstream sectors—does not import from abroad (i.e., $v_r = 1$ for all r and $v_{x,r,s} = 1$ for all r and s).

Table 4: Centrality measures and the difference between Domar and OG weights

	(1)	(2)	(3)	(4)	(5)	(6)
Export Intensity	0.722***			0.599***		0.972***
	(0.0123)			(0.0130)		(0.0117)
Export share		0.569***			0.465***	
		(0.00925)			(0.0125)	
Customer centrality			-1.070***	-0.345***	-0.361***	-0.142***
			(0.0412)	(0.0259)	(0.0317)	(0.0169)
Expenditure-switching centrality						-0.103***
						(0.00258)
Observations	601	601	601	601	601	601
R-square	0.891	0.868	0.591	0.924	0.904	0.979
Country FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Shown in the table are regression results based on equation (42), which regresses the sectoral Domar-OG percentage difference $(\lambda_i - \mathcal{M}_{OG,i})/\lambda_i$ over the centrality measures defined in Section 3.1 and the interaction terms between the centrality measures. The analysis includes the subsample of 11 relatively open economies—in terms of the economy-wise export-to-GDP ratio—out of all 43 economies. Country fixed effects are controlled. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

of 0.72 (Column 1), which is consistent with our theoretical results in Proposition 2. Furthermore, it has the largest explanatory power among all single explanatory variables of the Domar-OG difference, as evinced by the largest R-square of 0.89.

Column (4) shows that, conditional on the export intensity, customer centrality is negatively related to the Domar-OG difference with a coefficient of -0.35. Customer centrality and export intensity explain most of the variations in the Domar-OG difference, as evinced by the large R-square of 0.92. Therefore, the sector-level pitfalls in the monetary policy that ignore cross-border linkages, in turn, can be well approximated by the rule-of-thumb, linear combination of sectoral export intensity and customer centrality.[35] Thus, the results in Table 4 support Proposition 2, demonstrating that monetary policy adopting Domar weights—which coincides with the PPI-stabilizing policy used in the one-sector SOE literature—overstates inflation in sectors with either large export intensity or limited customer centrality. The monetary authority should assign smaller weights to these sectors to account for the openness of the economy in the closed-economy, Domar-weight policy.

Relevance of input-output linkages for approximating OG weights. Our theoretical analysis in Section 3.3 shows that input-output linkages are important drivers of the centrality measures that underpin the sectoral OG weights. In a small open economy without production networks, domestic sectors are only direct rather than indirect suppliers for domestic and foreign demand, causing import and export intensities to simplify

[35]The combination of export intensity, customer centrality, and expenditure-switching centrality explains the largest variation of 98% of Domar-OG differences, as shown by the R-square in Column (6) of Table 4. However, the complex structure of expenditure-switching centrality makes such a combination less appealing than the simple combination of export intensity and customer centrality (Column 4) as a rule-of-thumb approximation of the pitfalls in closed-economy Domar-weight policies.

to the import and export shares, respectively.

Comparing Columns (3) and (4) in Table 3 shows that import intensity explains more variation in OG weights than the direct import share, as evinced by the larger R-square for import intensity (0.32) and the almost negligible R-square for import share (0.01). Moreover, Column (5) in Table 3 shows that the three centrality measures—namely, the domestic supplier, customer, and expenditure-switching centralities—in the baseline OG weights in equation (24) explains 99% of the variation in the sectoral OG weights. In contrast, the counterfactual centrality measures that ignore input-output linkages in equation (29) explain only 88% of the variation in the correct, baseline sectoral OG weights (Column 6 of Table 3). Thus, indirect imports via both upstream and downstream sectors—as captured by import intensity rather than import shares, and centrality measures with IO linkages rather than without—are important for the approximation of sectoral OG weights, hence supporting the relevance of input-output linkages for monetary policy.

Comparing Columns (1) and (2) in Table 4 indicates that export intensity explains a larger variation in the sectoral Domar-OG difference compared to the direct export share (with R-squares of 0.89 vs. 0.87, respectively). Thus, the joint explanatory power of export intensity and customer centrality is larger than that of export share and customer centrality, as evinced by the larger R-squares of 0.92 in Column (4) than 0.9 in Column (5). Therefore, indirect exports via downstream sectors—as captured by export intensity rather than export shares—are important for the approximation of sectoral Domar-OG difference, again underlying the relevance of input-output linkages for monetary policy.

We conclude that the structure of input-output linkages interplays with the imports and exports of the small open economy—and, in turn, the centrality measures—to determine the weights of the OG policy that closes the aggregate output gap. Ignoring production networks results in a poor approximation of the correct sectoral weights in the monetary policy.

5.3. Welfare comparison of alternative monetary policies

In this section, we quantitatively compare the welfare losses of the economy—using equation (32) in Proposition 3 of Section 4—under alternative monetary policies, and show that the OG policy performs closely to the optimal monetary policy, and outperforms the policies that ignore either the cross-border or input-output linkages of the economy.[36]

Specifically, we compare the welfare loss under the following five alternative monetary policies: the optimal policy, the OG policy, the Domar-weight policy, the OG policy without input-output linkages, and

[36]The welfare loss represents the expected welfare loss in the remaining part of Subsection 5.3. For each economy, we compute welfare losses under different monetary policies using the same simulations of log-normal shocks to the import prices of all sectors. For simplicity, we assume that the shocks to different sectors have the same mean. We set the mean of sectoral shocks to generate an average CPI inflation of 2% for each economy to compare—under the same aggregate level of inflation—the welfare losses across different economies with different openness and structures of input-output linkages. The variance-covariance matrix of these shocks is calibrated on Mexico. We simulate the shocks 100,000 times to compute the expected welfare loss under each of the alternative monetary policies. In Appendix F.2, we show that the main patterns of our results are similar under shocks to import prices of only manufacturing sectors and under productivity shocks to all sectors.

the CPI-weight policy. The Domar-weight (CPI-weight) policy targets an aggregate inflation index where the Domar weight λ_i (consumption share β_i)—after adjusting for sectoral price rigidities (i.e., multiply by $(1-\delta_i)/\delta_i$)—is used as the weight for each sector i's inflation. The OG policy without IO linkages weights sectoral inflation with the OG weights that ignore IO linkages (i.e., $\mathcal{M}_{OG,i}^{NoIO}$ in equation 29) and sectoral price rigidities. We study the Domar-weight policy because it is a widely used policy that targets PPI inflation, which ignores the openness of the economy and coincides with the policy used in one-sector small open economy literature. We study the CPI-weight policy because it is another widely used policy that targets CPI inflation, which ignores both the openness and the input-output linkages. In addition, we study the OG policy without IO linkages to evaluate the relevance of input-output linkages for the welfare implications of monetary policy.

Table 5: Welfare loss under different monetary policies

	(1) Optimal	(2) OG	(3) Domar	(4) OG w/o IO	(5) CPI
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-1.859	-1.879	-1.922	-4.948	-4.968
Improvement by OG policy towards optimal			67.1%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.002	-0.385	-0.388
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.024	0.000	-0.041	-2.684	-2.701
— policy-irrelevant	-1.879	-1.879	-1.879	-1.879	-1.879
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-7.742	-7.777	-8.504	-11.551	-10.675
Improvement by OG policy towards optimal			95.4%	99.1%	98.8%
Output gap misallocation	-0.006	0.000	-0.089	-0.569	-0.427
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.041	0.000	-0.638	-3.205	-2.471
— policy-irrelevant	-7.777	-7.777	-7.777	-7.777	-7.777
U.S. Export-to-GDP ratio: 9%					
Total welfare loss	-1.400	-1.472	-1.476	-6.546	-6.757
Improvement by OG policy towards optimal			5.4%	98.6%	98.6%
Output gap misallocation	-0.011	0.000	0.000	-0.596	-0.623
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.083	0.000	-0.004	-4.478	-4.662
— policy-irrelevant	-1.472	-1.472	-1.472	-1.472	-1.472

Notes: Shown in the table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the OG policy without IO linkages, and the CPI-weight policy, respectively. The sectoral weights in all five policies adjust for sectoral price rigidities. Appendix F.2 outlines the sectoral weights adopted by the alternative monetary policies.

Presented in Table 5 is the total welfare loss expressed as a percentage of the steady-state consumption under the alternative monetary policies. We consider the welfare loss for Mexico, Luxembourg, and the U.S., as these countries represent those with medium, large, and small degrees of openness, respectively—as measured by the economy-wise export-to-GDP ratio (19%, 83%, and 9%). Using equation (32), we decompose the welfare loss into the *output gap misallocation* and the *within- and across-sector, and cross-border misallocation*. We further use equation (E.1) to decompose the latter misallocation into two sub-components: (i) an output-gap-related term and (ii) a policy-irrelevant term.

As shown in Table 5, the OG monetary policy yields a welfare loss that is close to the optimal policy and significantly outperforms the Domar-weight policy (Column 3), the OG policy without IO linkages (Column 4), and the CPI-weight policy (Column 5)—which ignore cross-border linkages, IO linkages, and *both* cross-border *and* IO linkages, respectively. For Mexico, the difference in the welfare loss between the optimal and the OG policies is very small and equal to 0.020 percent of the steady-state consumption (-1.859 vs. -1.879), thereby establishing that the OG policy is nearly optimal. Important to our analysis, the OG policy improves the welfare loss over the Domar-weight policy by 0.043 percent of the steady-state consumption, and it generates an even larger improvement over the OG policy that ignores IO linkages and the CPI-weight policy (-1.879 vs. -1.922 vs. -4.948 vs. -4.968). The welfare improvement of the OG policy over the Domar-weight policy (vs. OG policy without IO linkages) corresponds to 67.1% (vs. 99.3%) of the welfare difference between the optimal and the Domar-weight policy (vs. OG policy without IO linkages), thereby exhibiting welfare enhancement if the design of monetary policy accounts for openness and the input-output linkages of the economy.[37] The welfare improvement of the OG policy over the Domar-weight policy—which is equivalent to the PPI-stabilizing policy used in one-sector SOE literature—also shows the importance of considering multi-sector and input-output linkages in designing monetary policies in SOEs.

Decomposing the total welfare loss into different components illustrates why the OG policy is closer to the optimal policy and improves over policies that ignore cross-border and input-output linkages. Because the OG policy closes the aggregate output gap, it eliminates the welfare losses arising from the output gap misallocation and from the output-gap-related components in the within- and across-sector, and cross-border misallocation. Quantitatively, Table 5 shows that these two components related to the aggregate output gap generate large welfare losses in Mexico for the Domar-weight policy (-0.002 and -0.041), and even larger losses for the OG policy without IO linkages (-0.385 and -2.684) and the CPI-weight policy (-0.388 and -2.701). These results support the adoption of the OG policy that considers both the cross-border and input-output linkages to enhance welfare in small open economies.

Finally, we examine the welfare loss under alternative monetary policies for two additional economies: namely, Luxembourg and the U.S., which represent the polar cases of open and closed economies, respectively. In the most open economy of Luxembourg (the middle panel of Table 5), the OG policy improves over the Domar-weight policy by a large 95.4%, compared to a more limited 67.1% for Mexico. The same qualitative results outlined for Mexico hold for Luxembourg and are stronger quantitatively. The bottom panel of Table 5 presents the welfare loss for the nearly closed economy of the US, showing that the OG and

[37] In Appendix F.3, we show that our results are robust to alternative shocks, including shocks to the import prices of only manufacturing sectors (with sector IDs from 6 to 24 in Table F.1) and shocks to sectoral productivity.

Domar-weight policies yield similar welfare loss and they are equally close to the optimal policy, echoing the results of La'O and Tahbaz-Salehi (2022) and Rubbo (2023) in closed economies. Therefore, we conclude that the difference between the OG and the Domar-weight policies is significant for open economies, but its importance diminishes in relatively closed economies like the U.S.

6. Conclusion

This paper investigates the design of monetary policy in small open economies with cross-border and input-output linkages and nominal rigidities. Aggregate distortions are proportional to the aggregate output gap, which can be expressed as a weighted average of sectoral markup wedges that encapsulate the inefficiency in each sector. Monetary policy can close the output gap and offset the sectoral distortions by stabilizing the aggregate index of inflation that weights inflation in each sector based on the degree of nominal rigidities and the centrality of the sector as a supplier of inputs to both domestic and foreign demand and as a customer of domestic labor factor within the international production networks. To close the output gap, monetary policy should assign larger weights to inflation in sectors that supply more inputs directly or indirectly (i.e., via the downstream sectors) to domestic output. Disregarding cross-border linkages overstates inflation in sectors that export intensively directly and indirectly. Disregarding input-output linkages understates inflation in sectors that supply indirectly to domestic and foreign demand intensively, both generating quantitatively significant welfare losses.

We derive the closed-form solution for the optimal monetary policy that minimizes the welfare losses up to the second-order approximation, as well as calibrate our model to the WIOD to quantify our theoretical results. We show that the OG policy generates welfare losses quantitatively close to the optimal policy, and outperforms alternative monetary policies using the Domar weights that abstract from cross-border linkages or the OG weights that ignore input-output linkages. Overall, our analysis demonstrates that cross-border and input-output linkages are jointly important for the conduct of monetary policy in small open economies with international production networks.

Our study suggests several interesting avenues for future research. First, the analysis could be extended by relaxing the assumption of financial autarky and studying the interplay between the incompleteness of the financial market and the production networks for the design of monetary policy. Second, the analysis could be extended to cases in which fiscal policy fails to offset the first-order distortions with non-contingent subsidies. Such contexts lead to a sub-optimal flexible-price equilibrium for the domestic social planner, as in Baqaee and Farhi (2024), such that the monetary policy needs to account for the interaction between the supply-side effect of monetary policy and the openness of the economy to improve efficiency. Third, the analysis could be extended to consider large open economies where monetary policy would need to account for feedback effects from the responses of foreign economies to the domestic policy—which may interplay with international product networks to determine the impact of the domestic monetary policy. Finally, the analysis could be extended to models incorporating endogenous adjustments in domestic and cross-border input-output linkages, as in Xu et al. (2025). We plan to investigate some of these issues in future work.

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Online Appendix

Monetary Policy in Open Economies with Production Networks

(Zhesheng Qiu, Yicheng Wang, Le Xu, and Francesco Zanetti)

A. Firm profit, sectoral goods packer, and Calvo pricing

Given the firm's price P_{if} and the sectoral tax (or subsidy if negative) rate τ_i on sales, the nominal profit of firm f in sector i equals:

$$\Pi_{if} = (1-\tau_i)P_{if}Y_{if} - \Phi_i \cdot Y_{if}. \tag{A.1}$$

Sectoral goods packers. In each sector i, the perfectly competitive and identical sectoral goods packers transform the differentiated goods produced by the monopolistically competitive firms into a sectoral product using the following constant-elasticity-of-substitution technology:

$$Y_i = \Big(\int_0^1 Y_{if}^{\frac{\varepsilon_i-1}{\varepsilon_i}} df\Big)^{\frac{\varepsilon_i}{\varepsilon_i-1}}, \tag{A.2}$$

where the within-sector elasticity of substitution between different firms' products is $\varepsilon_i > 1$. The cost minimization of the goods packers yields the following sectoral price index and demand function for the firms:

$$P_i = \Big(\int_0^1 P_{if}^{1-\varepsilon_i} df\Big)^{\frac{1}{1-\varepsilon_i}} \quad \text{and} \quad Y_{if} = \Big(\frac{P_{if}}{P_i}\Big)^{-\varepsilon_i} Y_i. \tag{A.3}$$

Nominal rigidity and sectoral markup wedges. Denote $P_i^{\#}$ the price that maximizes the firm's profit in equation (A.1) subject to the demand function in equation (A.3), which is equal to the following:

$$P_i^{\#} = \frac{1}{1-\tau_i}\frac{\varepsilon_i}{\varepsilon_i-1}\Phi_i \equiv \mu_i^{\#} \cdot \Phi_i, \tag{A.4}$$

where $\mu_i^{\#}$ denotes the desired sectoral (gross) markup. Nominal rigidity is modeled as static Calvo-pricing friction, in which only firms indexed by $f \le \delta_i \in [0,1]$ are allowed to choose their desired price $P_i^{\#}$ and the remaining firms maintain their price at the steady-state level. We refer to $(1-\delta_i)/\delta_i$ as the price rigidity of sector i. The sectoral markup $\mu_i \equiv P_i/\Phi_i$ differs from the desired markup $\mu_i^{\#}$ if the price rigidity of sector i is strictly positive, *viz*, $(1-\delta_i)/\delta_i > 0$. We define the *sectoral markup wedge* for domestic sector i as the log deviation of the sectoral markup from the desired markup, *viz*, $\ln(\mu_i) - \ln(\mu_i^{\#})$.

B. Aggregate wedges and aggregate output gap

We follow the approach in Chari et al. (2007) to define the efficiency and labor wedges in the multi-sector, small open economy.

Definition B.1 (Aggregate wedges)**.** *The two wedges $A_{agg} : \Xi \mapsto \mathbb{R}_+$ and $\Gamma_L : \Xi \mapsto \mathbb{R}_+$ allow equilibrium* *aggregate consumption and labor inputs to satisfy the following equations:*[1]

$$C(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})}, \quad \forall \boldsymbol{\xi} \in \Xi, \tag{B.1}$$

$$\frac{u_L(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))}{-u_C(C(\boldsymbol{\xi}), L(\boldsymbol{\xi}))} = \Gamma_L(\boldsymbol{\xi}) \frac{\partial C}{\partial L}(\boldsymbol{\xi}), \quad \forall \boldsymbol{\xi} \in \Xi, \tag{B.2}$$

in the economy. We refer to $A_{agg}(\boldsymbol{\xi})$ as the efficiency wedge, or aggregate TFP, and $\Gamma_L(\boldsymbol{\xi})$ as the labor wedge, respectively, for any realized state $\boldsymbol{\xi} \in \Xi$.

The equilibrium of the economy is summarized by the aggregate production function in equation (B.1) and the intratemporal condition between aggregate consumption and labor supply in equation (B.2). The aggregate production function describes the transformation of labor inputs into aggregate consumption, where the transformation ratio equals the economy-wide share of domestic labor inputs in aggregate consumption expenditure in the flexible-price equilibrium ($\Lambda_L^{flex}(\boldsymbol{\xi})$). In our open economy, domestic consumption comprises foreign goods, which are supplied through exports of domestically produced goods in exchange for imports of foreign products. Because the marginal revenue of export strictly decreases with the export quantity and its use of domestic labor inputs, domestic labor supplies foreign products (through exports) in a *decreasing-return-to-scale* way, leading to a lower-than-unitary transformation ratio in the open economy. In contrast, the transformation ratio is equal to one in a closed economy—as in Bigio and La'O (2020)—because all domestic consumption uses domestic products instead of imported foreign goods that are exchanged using exports. The efficiency wedge $A_{agg}(\boldsymbol{\xi})$ captures the shifts in the aggregate production function or the aggregate TFP.

The intratemporal condition in equation (B.2) relates the marginal product of labor for aggregate output (i.e., $\partial C / \partial L$) to the marginal rate of substitution between consumption and labor (i.e., $-u_L/u_C$), and the labor wedge $\Gamma_L(\boldsymbol{\xi})$ encapsulates the distortions that make the marginal product of labor different from the marginal rate of substitution.

Based on the definition of the efficiency wedge in Definition B.1, we establish the following open-economy version of Hulten's theorem:[2]

Lemma B.1 (The open-economy version of Hulten's theorem)**.** *Up to a first-order approximation, the deviation of the efficiency wedge from the steady state is a weighted average of sectoral shocks as follows:*

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}(\boldsymbol{\xi}) \tag{B.3}$$

[1] Appendix H.2 shows that the marginal product of labor is $(\partial C/\partial L)(\boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \Lambda_L^{flex} L(\boldsymbol{\xi})^{\Lambda_L^{flex}(\boldsymbol{\xi})-1}$.

[2] In closed economies with production networks, Bigio and La'O (2020) define a prototype economy and the corresponding efficiency and labor wedges. They also show that Hulten's theorem holds and that sectoral distortions have no first-order effect on the efficiency wedge. While Baqaee and Farhi (2024) decompose the real GDP of an open economy into the efficiency wedge and the labor wedge in an inter-connected global production network, our Lemma B.1 shows the decomposition for small open economies under the assumption of nominal rigidities.

$$= \boldsymbol{\lambda}^\top \widehat{\mathbf{A}} - \Big\{ \underbrace{[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top}_{\textit{imported consumption}} + \underbrace{\boldsymbol{\lambda}^\top (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})}_{\textit{imported interm. inputs}} \Big\} \widehat{\mathbf{P}}^*_{IM,F} + \underbrace{[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top}_{\textit{profits from exports}} \widehat{\mathbf{D}}^*_{EX,F}.$$

Proof: See Appendix H.1.

Equation (B.3) shows that deviation of the efficiency wedge from the steady state is linked to the deviations of exogenous sectoral productivity ($\widehat{\mathbf{A}}$), import prices ($\widehat{\mathbf{P}}^*_{IM,F}$), and foreign demand ($\widehat{\mathbf{D}}^*_{EX,F}$) from the steady state. The elasticity of the efficiency wedge to the sectoral productivity is the Domar weight of the sector ($\boldsymbol{\lambda}$), as in a closed economy (Hulten, 1978; Bigio and La'O, 2020). In an open economy, however, the elasticities of the efficiency wedge to import prices and foreign demand depend on the linkages between the domestic and foreign economies. The elasticity of the efficiency wedge to a sector's import price shock equals the share of the sector's imports of consumption goods and intermediate inputs in aggregate output. Such elasticity is negative, as imported inflation materializes as a negative supply shock. The elasticity of the efficiency wedge to a shock to the sector's foreign demand equals the share of the sector's profits from exports in aggregate output. Such elasticity is positive because an increase in the foreign demand for domestic goods raises export profits, which increases domestic income and consumption for a given amount of domestic labor.

Lemma B.1 implies that—similar to the closed economy case—sectoral distortions have no first-order impact on the efficiency wedge in a small open economy with production networks. Therefore, the labor wedge encapsulates sectoral distortions entirely, as stated in the following proposition:

Proposition B.1 (Sectoral distortion, efficiency and labor wedges, and the aggregate output gap)**.** *Up to the first-order approximation, the efficiency wedge in the sticky-price equilibrium is the same as the efficiency wedge in the flexible-price equilibrium:*

$$\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}^{flex}_{agg}(\boldsymbol{\xi}) = \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L \cdot \widehat{L}^{gap}(\boldsymbol{\xi}) = 0. \tag{B.4}$$

The labor wedge, though, deviates from the flexible-price level, and the deviation is proportional to the aggregate output gap:[3]

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}^{flex}_L(\boldsymbol{\xi}) = \widehat{\Gamma}_L(\boldsymbol{\xi}) = [\sigma - 1 + (\varphi + 1)/\Lambda_L]\, \widehat{C}^{gap}(\boldsymbol{\xi}). \tag{B.5}$$

Proof: See Appendix H.2.

Proposition B.1 shows that up to the first-order approximation, the efficiency wedge is unaffected by sectoral distortions, but that the labor wedge is different from zero and it summarizes the distortions at

[3]The deviation of the labor wedge from the flexible-price equilibrium equals the deviation of the labor wedge from the steady state. This is because the labor wedge equals one in the flexible-price equilibrium for any realized state $\boldsymbol{\xi}$, including the steady state.

the aggregate level. In particular, the deviation of the labor wedge from the flexible-price equilibrium is proportional to the aggregate output gap. Therefore, to study the first-order inefficiencies and the monetary policy needed to eliminate them, we can focus on the impacts of nominal rigidities and inflation on aggregate output gap, which we pursue in Section 3.2.

C. OG monetary policy under foreign-currency pricing

Our baseline model assumes producer-currency pricing (PCP)—in which domestic producers set exporting prices in producers' (i.e., domestic) currencies. In this Appendix, we follow Engel (2011) to extend our model to the alternative settings in the literature, i.e., the foreign-currency pricing (FCP) that includes both local-currency pricing (LCP) and dominant-currency pricing (DCP). Under local-currency and dominant-currency pricing, domestic producers set sectoral exporting prices in foreign and dominant (e.g., US dollars) currencies, respectively, and can price discriminate among domestic and foreign markets, facing different Calvo-pricing rigidities in these two markets. In particular, because our model summarizes the rest of the world using a single foreign country and treats the import prices of foreign products denominated in foreign currency as exogenous, local-currency pricing is equivalent to dominant-currency pricing in our setting.

We show that under foreign-currency pricing, the contribution of sectoral markup wedges to the aggregate output gap is equal to the sum of the OG weight in equation (23) and an extra export-related term that replaces domestic-market with foreign-market sectoral markup wedges. Therefore, the OG monetary policy under foreign-currency pricing should target an aggregate inflation index that includes sectoral inflation of both domestic-market prices and export prices in the foreign market. In particular, while the CPI and profit channels remain dependent on domestic sectoral inflation, the expenditure-switching channel relies on inflation in domestic and export prices.

C.1. Extension of baseline model to foreign-currency pricing

In this subsection, we describe the changes in our extended model with foreign-currency pricing, compared to the baseline model with producer-currency pricing. In each domestic sector i, we assume that there are two types of monopolistically competitive firms, each of which has a unit mass: the first type of firms only sell their products to domestic customers, which we denote by type DM; the second type of firms only export their products to foreign customers, which we denote by type EX. In comparison, our baseline model includes only one type of firms that sell products to both domestic and foreign customers in each sector. In each sector i, the two types of firms share the same production function and, therefore, the same marginal cost of production Φ_i. While the selling prices of type DM firms are denominated in the domestic currency and denoted by P_{if}, the exporting prices of type EX firms are denominated in the foreign currency and denoted by $P^*_{EX,if}$.

For the monopolistically competitive firms of type DM in each sector i, there are perfectly competitive and identical sectoral goods packers that transform their differentiated goods into a *sectoral domestic-market* *product*, which is sold only to domestic customers, using the following constant-elasticity-of-substitution technology:

$$Y_{DM,i} = \left(\int_0^1 Y_{DM,if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \qquad \text{and} \qquad P_i = \left(\int_0^1 P_{if}^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}}, \tag{C.1}$$

where the within-sector elasticity of substitution between different firms' products is equal to $\varepsilon_i > 1$, and P_i denotes the price of the sectoral domestic-market product of sector i.

Similarly, for the monopolistically competitive firms of type EX in each sector i, there are perfectly competitive and identical sectoral export goods packers located in the foreign country that transform their differentiated goods into a *sectoral foreign-market product* that is only exported to foreign customers, using the following constant-elasticity-of-substitution technology:

$$Y_{EX,i} = \left(\int_0^1 Y_{EX,if}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} df \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \qquad \text{and} \qquad P^*_{EX,i} = \left(\int_0^1 \left(P^*_{EX,if}\right)^{1-\varepsilon_i} df \right)^{\frac{1}{1-\varepsilon_i}}, \tag{C.2}$$

where the within-sector elasticity of substitution between different firms' products is also equal to ε_i, and $P^*_{EX,i}$ denotes the price of the sectoral foreign-market product of sector i. We denote the total quantity of sector i products by $Y_i \equiv Y_{DM,i} + Y_{EX,i}$.

The two types of firms face separate Calvo-pricing friction. Type DM firms face the same Calvo-pricing friction as in the baseline model, with domestic-market price rigidity of sector i equal to $(1-\delta_i)/\delta_i$. Among EX firms, only firms indexed by $f \leq \delta^*_{EX,i} \in [0,1]$ are allowed to choose their desired price $P^{*,\#}_{EX,if}$ and the remaining firms maintain their price at the steady-state level $P^{*,SS}_{EX,i}$. We refer to $(1-\delta^*_{EX,i})/\delta^*_{EX,i}$ as the foreign-market price rigidity of sector i. Facing the sectoral sales tax rate τ_i, type DM firms that can adjust prices choose the desired price to maximize the following nominal profits:

$$\begin{aligned} &\max_{P^*_{EX,if}} (1-\tau_i) P_{if} Y_{DM,if} - \Phi_i Y_{DM,if}, \\ &s.t. \quad Y_{DM,if} = \left(\frac{P_{if}}{P_i} \right)^{-\varepsilon_i} Y_{DM,i}. \end{aligned}$$

Facing both the sectoral sales tax rate τ_i and the sectoral export tax rate $\tau_{EX,i}$, type EX firms that can adjust prices choose the desired price to maximize the following nominal profits:

$$\begin{aligned} &\max_{P^*_{EX,if}} (1-\tau_i)(1-\tau_{EX,i}) S P^*_{EX,if} Y_{EX,if} - \Phi_i Y_{EX,if}, \\ &s.t. \quad Y_{EX,if} = \left(\frac{P^*_{EX,if}}{P^*_{EX,i}} \right)^{-\varepsilon_i} Y_{EX,i}, \end{aligned}$$

where the foreign demand for sector i's foreign-market products is the same as in the baseline model and equal to:

$$Y_{EX,i} = \left(P^*_{EX,i}\right)^{-\theta_{F,i}} D^*_{EX,Fi}. \tag{C.3}$$

We keep Assumption 1—i.e., $\tau_i = -1/(\varepsilon_i - 1)$ and $\tau_{EX,i} = 1/\theta_{Fi}$—from the baseline model such that in both the steady state and the flexible-price equilibrium,

$$P_i^{ss} = \Phi_i \qquad \text{and} \qquad P^{*,ss}_{EX,i} = \frac{\Phi_i}{S}\frac{\theta_{F,i}}{\theta_{F,i} - 1},$$

which are the same as those in the baseline model. In this way, the within-sector distortion due to monopolistic competition is removed, and the monopoly power of exporting firms on the international market is retained—in both the steady state and the flexible-price equilibrium. Thus, the allocation in the flexible-price equilibrium is equivalent to the solution to the optimization problem of the domestic social planner, as in the baseline model.[4]

We define the sectoral markups of domestic-market and foreign-market products as $\mu_i \equiv P_i/\Phi_i$ and $\mu^*_{EX,i} \equiv SP^*_{EX,i}/\Phi_i$, respectively. Under Assumption 1, in the sticky-price equilibrium and outside the steady state, the desired prices of sectoral domestic-market and foreign-market products are equal to:

$$\mu_i^{\#} \equiv \frac{P_i^{\#}}{\Phi_i} = 1 \qquad \text{and} \qquad \mu^{*,\#}_{EX,i} \equiv \frac{SP^{*,\#}_{EX,i}}{\Phi_i} = \frac{\theta_{F,i}}{\theta_{F,i} - 1},$$

respectively. We further define the sectoral markup wedges of domestic and export products as $\widehat{\mu}_i \equiv \ln(\mu_i) - \ln(\mu_i^{\#}) = \ln(\mu_i) - \ln(\mu_i^{ss})$ and $\widehat{\mu}^*_{EX,i} \equiv \ln(\mu^*_{EX,i}) - \ln(\mu^{*,\#}_{EX,i}) = \ln(\mu^*_{EX,i}) - \ln(\mu^{*,ss}_{EX,i})$, respectively, because the steady-state markups are equal to the desired markups.

Finally, we follow the baseline model to assume the import prices of foreign products denominated in foreign currency to be exogenous, and sectoral markups of imported foreign products are completely embedded in such exogenous sectoral import prices. As a result, sectoral markups of imported foreign products—where under our baseline PCP or under the FCP—will not emerge in the expressions of domestic output gap and, therefore, the OG monetary policy.

C.2. OG monetary policy under foreign-currency pricing

In the extended model with foreign-currency pricing, we show that the aggregate output gap under foreign-currency pricing is attributed to both domestic-market (i.e., $\widehat{\mu}_i$) and foreign-market sectoral markup

[4]Intuitively, we can think of type EX firms in each sector as constituting an extreme sector in the baseline model that only exports to foreign countries and supplies no goods to domestic customers. Therefore, the same Assumption 1 as in the baseline model is needed and sufficient to make the allocation in the flexible-price equilibrium equivalent to the solution to the optimization problem of the domestic social planner.

wedges (i.e., $\widehat{\mu}^*_{EX,i}$) of domestic products, as outlined in the following corollary of Theorem 1:[5]

Corollary C.1 (Aggregate output gap and OG monetary policy under foreign-currency pricing)**.** *Under local-currency pricing, in a sticky-price equilibrium, negative sectoral markup wedges in both domestic market $\{\widehat{\mu}_i(\boldsymbol{\xi})\}_i$ and foreign market $\{\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi})\}_i$ contribute to a positive aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ as follows:*

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\sum_{i=1}^{N}\left[\mathcal{M}_{OG,i}\cdot\widehat{\mu}_i(\boldsymbol{\xi}) + \kappa_S\cdot(\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i)\cdot\left(\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi}) - \widehat{\mu}_i(\boldsymbol{\xi})\right)\right], \tag{C.4}$$

where the sectoral OG weight ($\mathcal{M}_{OG,i}$), κ_S, κ_C are the same as those in Theorem 1. Accordingly, the OG monetary policy under LCP is implemented by setting the following aggregate inflation index to zero:

$$\sum_{i=1}^{N}\left[(\mathcal{M}_{OG,i} - \kappa_S\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i)\cdot(1-\delta_i)/\delta_i\cdot\widehat{P}_i(\boldsymbol{\xi}) + \kappa_S\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i\cdot(1-\delta_{EX,i})/\delta_{EX,i}\cdot\widehat{P}^*_{EX,i}(\boldsymbol{\xi})\right] = 0. \tag{C.5}$$

Proof: See Appendix H.11

Corollary C.1 shows that under foreign-currency pricing, sectoral markup wedges contribute to the aggregate output gap in a very similar fashion to that under producer-currency pricing as in equation (23). Under foreign-currency pricing, however, exports are determined by foreign-market sectoral inflation and markup wedges instead of domestic ones, thus leading to the extra export-related term on the RHS of equation (C.4) that replaces domestic-market with foreign-market sectoral markup wedges. Specifically, negative foreign-market sectoral markup wedges—caused by price rigidities under foreign-market sectoral inflation—reduce domestic products' exporting prices relative to the foreign products' prices in the foreign market. As a result, the foreign expenditure switches from foreign to domestic products, increasing domestic labor income from international trade and leading to a positive aggregate output gap—as summarized by the coefficient of the foreign-market sectoral markup wedge $\kappa_S(\theta_{F,i}\lambda_{EX,i}\widetilde{\alpha}_i)$. The existence of both domestic- and foreign-market markup wedges in the output gap of equation (C.4) implies that, under foreign-currency pricing, the OG monetary policy should target an aggregate inflation index that includes sectoral inflation of both domestic-market prices and export prices in the foreign market, as shown in equation (C.5). In particular, while the CPI and profit channels remain dependent on domestic sectoral inflation, the expenditure-switching channel relies on inflation in both domestic and export prices.

D. Import shares and OG weights

Our definitions of centralities in equations (18), (19), and (20) include the Leontief inverse that depends on the import shares and input-output matrix. Therefore, by combining the equations of centralities and the

[5]Similar to equation (22), foreign-market sectoral markup wedges in exporting prices are linked to domestic producers' exporting prices as follows: $\widehat{\mu}^*_{EX,i}(\boldsymbol{\xi}) = -(1-\delta_{EX,i})/\delta_{EX,i}\cdot\widehat{P}^*_{EX,i}(\boldsymbol{\xi})$.

decomposition equations of the Leontief inverse (16) and (17), respectively, we determine how the import structure of the economy influences our centrality measures and the sectoral OG weights, as summarized by the following proposition.

Proposition D.1. *Domestic supplier centrality of the domestic sector i (i.e., $\widetilde{\lambda}_{D,i}$) strictly decreases in its import share of consumption $(1-v_i)$ if and only if $\beta_i > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases in its direct downstream sector r's import share of sector i goods (i.e., $\omega_{r,i} v_{x,r,i} > 0$) if and only if $\widetilde{\lambda}_{D,r} > 0$; $\widetilde{\lambda}_{D,i}$ strictly decreases in its indirect downstream sector s' import share of sector r goods if and only if $\widetilde{\lambda}_{D,s} > 0$, $\omega_{s,r} > 0$, and $\ell_{vx,r,i} > 0$.*

Proof: See Appendix H.8.

Proposition D.1 shows that the domestic supplier centrality of a domestic sector i decreases in sector i's import share of foreign goods as consumption, as well as sector i's direct and indirect downstream sectors' import shares (of intermediate inputs). Intuitively, more direct and indirect imports reduce the sector's contribution to the domestic aggregate output, thereby reducing the size of the CPI channel and resulting in a smaller OG weight. This implies that monetary policy *should* assign higher weights to inflation in domestic sectors with small direct and indirect (via downstream sectors) import shares.[6]

E. Additional results of welfare and the optimal monetary policy

Welfare loss as a function of the aggregate output gap. We substitute the sectoral Phillips curves (equation 36) in Proposition 4 into the welfare loss (equation 32) in Proposition 3 to re-write the welfare loss as a function of the aggregate output gap and exogenous shocks, yielding the following:

$$\begin{aligned} u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = & \underbrace{-\frac{1}{2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{C}^{gap}(\boldsymbol{\xi})^2}_{\textit{output gap misallocation}} \\ & \underbrace{\underbrace{-\frac{1}{2}\boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}} \cdot \widehat{C}^{gap}(\boldsymbol{\xi})^2 - (\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}} \cdot \widehat{C}^{gap}(\boldsymbol{\xi})}_{\text{output-gap-related}} \underbrace{-\frac{1}{2}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})^\top \boldsymbol{\mathcal{L}}(\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}})}_{\text{policy-irrelevant}}}_{\textit{within- and across-sector, and cross-border misallocation}} + o(\|\widehat{\boldsymbol{\xi}}\|^2), \end{aligned} \quad \text{(E.1)}$$

where $\boldsymbol{\mathcal{L}} \equiv (\boldsymbol{\mathcal{L}}^{within} + \boldsymbol{\mathcal{L}}^{across} + \boldsymbol{\mathcal{L}}^{cb})$.

Equation (E.1) shows that the welfare loss depends on the *output gap misallocation* (the first line on the RHS of equation E.1, as already shown in equation 32), as well as the *within- and across-sector, and cross-border misallocation* (the second line of equation E.1). This second component is further decomposed

[6] Our model with a fully-fledged production network and analytical solutions allows us to identify three channels determining the sectoral weights in the monetary policy. The net export centrality in our analysis encompasses the export share of upstream sector that Wei and Xie (2020) outline by numerical simulations in the special case of a vertical network.

into two sub-components: (i) the output-gap-related component, and (ii) the policy-irrelevant component of exogenous shocks that cannot be influenced by monetary policy.

Equation (E.1) shows that closing the output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$) eliminates the output gap misallocation and the output-gap-related component of the within- and across-sector, and cross-border misallocation, but it is unable to eliminate the misallocation arising from the policy-irrelevant sectoral shocks.

Optimal monetary policy as a function of the aggregate output gap. To further study the difference between the optimal and the OG monetary policies, we relate the optimal monetary policy to the aggregate output gap by noticing that the optimal monetary policy is equivalent to choosing the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$ that minimizes welfare loss in equation (E.1).

Proposition E.1 (Aggregate output gap in the optimal monetary policy)**.** *The optimal monetary policy satisfies the first-order condition of equation (E.1) with respect to the aggregate output gap $\widehat{C}^{gap}(\boldsymbol{\xi})$, i.e.,*

$$\left[\sigma - 1 + (\varphi + 1)/\Lambda_L + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\mathcal{B}}\right] \widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}} = 0. \tag{E.2}$$

Proof: See Appendix I.3.

Proposition E.1 highlights that the OG policy—which closes the aggregate output gap (i.e., $\widehat{C}^{gap}(\boldsymbol{\xi}) = 0$)—does not satisfy condition (E.2) for the optimal monetary policy. In multi-sector economies, those sector-specific cost-push components in sectoral Phillips curves do not comove with the one-dimensional aggregate output gap (i.e., $\boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}} \neq \mathbf{0}$ in equation 36), thus making the OG policy unable to simultaneously minimize the within- and across-sector, and cross-border misallocation (as captured by $\boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}}$ in equation E.2). Proposition E.1 shows that the "divine coincidence" in multi-sector open economies breaks down as in multi-sector closed economies: the OG policy that closes the output gap does not simultaneously minimize the within- and across-sector, and cross-border misallocation and is therefore suboptimal.

F. Quantitative analysis

F.1. Data and calibration

We calibrate our model of a small open economy with production networks using the World Input-Output Database. The WIOD covers 28 EU countries and 15 other major countries in the world from 2000 to 2014 and provides information for 56 major sectors.[7] Specifically, we calibrate our model using the National Input-Output Tables from the WIOD in 2014 for each country. The NIOTS provides each country's sector-level imports from (vs. exports to) the Rest of the World (RoW) and exports to the RoW, which are aggregates of the country's imports from (vs. exports to) all other countries, including those countries that are not listed in the WIOD. For each sector in each country, the NIOTS reports the following

[7] We use the version of Release 2016 of the World Input-Output Database. Shown in Table F.1 is the list of sectors.

sectoral values: (i) intermediate goods expenditures on goods from different domestic and foreign sectors, (ii) labor compensation, (iii) gross output, (iv) value-added, and (v) exports to foreign countries. Using the NIOTS data, we calibrate each country one at a time as a small open economy against the RoW, instead of simultaneously calibrating all countries at once in a global equilibrium.

For each country, we calibrate the parameters as follows: (i) the (i, j) element of the input-output matrix $\mathbf{\Omega}$ is calibrated using the share of customer sector i's intermediate goods expenditure on the supplier sector j (the sum of expenditures on the domestic and foreign sector j) in the customer i's gross output, (ii) the (i, j) element of the home bias in intermediate inputs $\mathbf{V}_x$ is calibrated using the ratio of customer sector i's intermediate goods expenditure on the domestic supplier sector j to the sum of expenditures on the domestic and foreign sector j's goods; (iii) the sectoral labor share of $\boldsymbol{\alpha}$ is calibrated using the share of sectoral labor compensation in sectoral gross output for each sector; (iv) the steady-state values of sectoral demand from foreign countries $\mathbf{D}_H^{*,ss}$ are calibrated such that the sectoral export-to-GDP ratios in the model matches the sector's export-to-GDP ratios in the data; (v) the i-th element of the consumption shares $\boldsymbol{\beta}$ is calibrated using the ratio of the sum of domestic households' and government's consumption expenditures on sector i goods to the value added of sector i; and (vi) the i-th element of the home bias in consumption $\mathbf{v}$ is calibrated using the ratio of the sum of domestic household's and government's consumption expenditures on the domestic sector i's goods to the sum of expenditures on the domestic and foreign sector i's goods.

We calibrate the values of other parameters to their standard levels in the literature. The risk aversion parameter and the inverse of the labor supply elasticity of the households are calibrated to $\sigma = 2$ and $\varphi = 1$, respectively, following the business cycle literature (e.g., Corsetti et al., 2010; Arellano et al., 2019). We follow Atkeson and Burstein (2008) and calibrate the within-sector elasticity of substitution to $\varepsilon_i = 8$ for all sectors i. We calibrate the elasticity of substitution between domestic and foreign goods to 5 for both domestic and foreign households and firms—*viz*, $\theta_i = \theta_{Fi} = 5$ for all sectors i, following Head and Mayer (2014). We calibrate the sector-level parameters of price rigidity δ_i using the sector-level price rigidities from Pasten et al. (2024).[8] With the calibrated sector-level price rigidities, the average quarterly frequency of price adjustment across all sectors equals 0.49. We follow Rubbo (2023) and La'O and Tahbaz-Salehi (2022) to introduce wage stickiness by adding a labor sector 0; it uses domestic labor to produce the product of "labor" that is supplied to all other sectors as inputs. We follow Beraja et al. (2019) and Barattieri et al. (2014) to calibrate the parameter of wage rigidity δ_0 such that the quarterly frequency of wage adjustment equals 0.25. Summarized in Table 2 in Section 5.1 is the calibration of different parameters.

Last, we calibrate the exogenous shocks as now described. We calculate the growth rates of sectoral import prices and productivity using the social economic accounts in the WIOD. We compute the covariance matrix between different sectors' import price series and use it to calibrate the covariance matrix of import prices used in the simulation of the model. We use the same method to calibrate the covariance matrix for

[8]We thank Michael Weber for kindly providing the sector-level price rigidities.

the sectoral productivity.

Table F.1: Industry classifications in World Input-Output Database

ID	Industry code	Description	ID	Industry code	Description
1	A01	Crop and animal production, hunting and related service	29	G46	Wholesale trade, except of motor vehicles and motorcycles
2	A02	Forestry and logging	30	G47	Retail trade, except of motor vehicles and motorcycles
3	A03	Fishing and aquaculture	31	H49	Land transport and transport via pipelines
4	B	Mining and quarrying	32	H50	Water transport
5	C10-C12	Manufacture of food products, beverages and tobacco products	33	H51	Air transport
6	C13-C15	Manufacture of textiles, wearing apparel and leather products	34	H52	Warehousing and support activities for transportation
7	C16	Manufacture of wood products, plaiting materials	35	H53	Postal and courier activities
8	C17	Manufacture of paper and paper products	36	I	Accommodation and food service activities
9	C18	Printing and reproduction of recorded media	37	J58	Publishing activities
10	C19	Manufacture of coke and refined petroleum products	38	J59_J60	Motion picture, video, and television
11	C20	Manufacture of chemicals and chemical products	39	J61	Telecommunications
12	C21	Manufacture of basic pharmaceutical products	40	J62_J63	Computer programming, consultancy and related activities; information service activities
13	C22	Manufacture of rubber and plastic products	41	K64	Financial service activities, except insurance and pension funding
14	C23	Manufacture of other non-metallic mineral products	42	K65	Insurance, reinsurance and pension funding, except compulsory social security
15	C24	Manufacture of basic metals	43	K66	Activities auxiliary to financial services and insurance activities
16	C25	Manufacture of fabricated metal products	44	L68	Real estate activities
17	C26	Manufacture of computer, electronic and optical products	45	M69_M70	Legal and accounting activities; activities of head offices; management consultancy activities
18	C27	Manufacture of electrical equipment	46	M71	Architectural and engineering activities; technical testing and analysis
19	C28	Manufacture of machinery and equipment n.e.c.	47	M72	Scientific research and development
20	C29	Manufacture of motor vehicles, trailers and semi-trailers	48	M73	Advertising and market research
21	C30	Manufacture of other transport equipment	49	M74_M75	Other professional, scientific and technical activities; veterinary activities
22	C31_C32	Manufacture of furniture; other manufacturing	50	N	Administrative and support service activities
23	C33	Repair and installation of machinery and equipment	51	O84	Public administration and defence; compulsory social security
24	D35	Electricity, gas, steam and air conditioning supply	52	P85	Education
25	E36	Water collection, treatment and supply	53	Q	Human health and social work activities
26	E37-E39	Sewerage; waste management services	54	R_S	Other service activities
27	F	Construction	55	T	Activities of households as employers
28	G45	Wholesale and retail trade,repair motor vehicles	56	U	Activities of extraterritorial organizations and bodies

F.2. Sectoral weights under alternative monetary policies

All of the alternative monetary policies we study in Section 5.3 are implemented by setting the following aggregate inflation index to zero:

$$\boldsymbol{\chi}^\top \cdot (\boldsymbol{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}}(\widehat{\boldsymbol{\xi}}) = 0, \tag{F.1}$$

where the sectoral weights $\boldsymbol{\chi}$ are equal to the following:

$$\begin{aligned}
\text{optimal monetary policy:} \quad & \boldsymbol{\chi}^\top = \left\{ [\sigma - 1 + \varphi + 1)/\Lambda_L] \, \kappa_C^{-1} \boldsymbol{\mathcal{M}}_{OG}^\top + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}} \boldsymbol{\Delta} (\mathbf{I} - \boldsymbol{\Delta})^{-1} \right\}; \\
\text{OG monetary policy:} \quad & \boldsymbol{\chi}^\top = \boldsymbol{\mathcal{M}}_{OG}^\top; \\
\text{Domar-weight policy:} \quad & \boldsymbol{\chi}^\top = \boldsymbol{\lambda}^\top; \\
\text{CPI-weight policy:} \quad & \boldsymbol{\chi}^\top = \boldsymbol{\beta}^\top; \\
\text{OG policy w/o IO linkages:} \quad & \boldsymbol{\chi}^\top = (\boldsymbol{\mathcal{M}}_{OG}^{NoIO})^\top,
\end{aligned}$$

where $\boldsymbol{\mathcal{M}}_{OG}$ and $\boldsymbol{\mathcal{M}}_{OG}^{NoIO}$ are the vectors of the OG weights with and without IO linkages in equations (24) and (29), respectively. Combining the monetary policy rule in equation (F.1) with the sectoral Phillips curves in equation (36), yields the aggregate output gap as a function of the specific policy weights $\boldsymbol{\chi}$ and the parameters of the sectoral Phillips curves, *viz*:

$$\widehat{C}^{gap}(\widehat{\boldsymbol{\xi}}) = -\frac{\boldsymbol{\chi}^\top (\boldsymbol{\Delta}^{-1} - \mathbf{I}) \boldsymbol{\mathcal{V}} \widehat{\boldsymbol{\xi}}}{\boldsymbol{\chi}^\top (\boldsymbol{\Delta}^{-1} - \mathbf{I}) \boldsymbol{\mathcal{B}}}. \tag{F.2}$$

Substituting equation (F.2) into the welfare loss function in equation (E.1) of Section 4, we obtain the welfare loss under the alternative monetary policy with policy weights χ and any realized state $\widehat{\xi}$.

F.3. *Welfare loss under alternative shocks*

Table F.2: Welfare loss under different monetary policies: Shocks to import prices of only manufacturing sectors

	(1) Optimal	(2) OG	(3) Domar	(4) OG w/o IO	(5) CPI
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-3.334	-3.357	-3.428	-6.669	-6.620
Improvement by OG policy towards optimal			75.8%	99.3%	99.3%
Output gap misallocation	-0.003	0.000	-0.004	-0.415	-0.408
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.026	0.000	-0.068	-2.898	-2.855
— policy-irrelevant	-3.357	-3.357	-3.357	-3.357	-3.357
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-1.595	-1.604	-1.678	-3.012	-4.545
Improvement by OG policy towards optimal			89.2%	99.4%	99.7%
Output gap misallocation	-0.002	0.000	-0.007	-0.220	-0.481
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.011	0.000	-0.067	-1.189	-2.460
— policy-irrelevant	-1.604	-1.604	-1.604	-1.604	-1.604
U.S. Export-to-GDP ratio: 9.2%					
Total welfare loss	-2.634	-2.734	-2.740	-9.248	-9.816
Improvement by OG policy towards optimal			5.5%	98.5%	98.6%
Output gap misallocation	-0.015	0.000	0.000	-0.758	-0.832
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.115	0.000	-0.006	-5.757	-6.250
— policy-irrelevant	-2.734	-2.734	-2.734	-2.734	-2.734

Notes: Reported in this table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the OG policy that ignores the IO linkages, and the CPI-weight policy, respectively. The sectoral weights in all of the five policies adjust for sectoral price rigidities.

Table F.3: Welfare loss under different monetary policies: Shocks to sectoral productivity

	(1) Optimal	(2) OG	(3) Domar	(4) OG w/o IO	(5) CPI
Mexico Export-to-GDP ratio: 19%					
Total welfare loss	-0.744	-0.754	-0.755	-1.527	-1.529
Improvement by OG policy towards optimal			6.6%	98.7%	98.7%
Output gap misallocation	-0.001	0.000	0.000	-0.092	-0.093
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.011	0.000	-0.001	-0.681	-0.682
— policy-irrelevant	-0.754	-0.754	-0.754	-0.754	-0.754
Luxembourg Export-to-GDP ratio: 83%					
Total welfare loss	-3.057	-3.061	-3.213	-3.833	-3.459
Improvement by OG policy towards optimal			97.4%	99.5%	99.0%
Output gap misallocation	-0.001	0.000	-0.022	-0.123	-0.061
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.005	0.000	-0.130	-0.648	-0.337
— policy-irrelevant	-3.061	-3.061	-3.061	-3.061	-3.061
U.S. Export-to-GDP ratio: 9.2%					
Total welfare loss	-1.208	-1.216	-1.216	-2.047	-2.056
Improvement by OG policy towards optimal			2.3%	99.1%	99.1%
Output gap misallocation	-0.001	0.000	0.000	-0.103	-0.104
Within- and across-sector, and cross-border misallocation					
— output-gap-related	0.009	0.000	0.000	-0.729	-0.737
— policy-irrelevant	-1.216	-1.216	-1.216	-1.216	-1.216

Notes: Reported in this table is the welfare loss—expressed in units of percent of steady-state consumption—under different monetary policy designs. Columns (1) to (5) show the welfare losses under the optimal policy, the OG policy, the Domar-weight policy, the OG policy that ignores the IO linkages, and the CPI-weight policy, respectively. The sectoral weights in all of the five policies adjust for sectoral price rigidities.

G. Basic results of the model

This section derives some basic results of the model, thus preparing for the proofs of our main theoretical results in Sections 3 and 4.

G.1. Feasible allocation

The feasible allocation of the economy can be defined at the sector level with the help of an additional variable ι_i that captures the within-sector output dispersion in each sector i, as stated in the following definition:

Definition G.1 (Feasible allocation). *Denote the use of labor and intermediate inputs of each sector i and j by*

$$\big(L_i, X_{i,j}, X_{Hi,Hj}, X_{Hi,Fj}\big) \equiv \int_0^1 \big(L_{if}, X_{if,j}, X_{Hif,Hj}, X_{Hif,Fj}\big) df.$$

A feasible allocation is a state-contingent allocation of C, $\{C_i\}_i$, $\{Y_i\}_i$, $\{L_i\}_i$, $\{X_{i,j}\}_{i,j}$, $\{C_{Hi}\}_i$, $\{C_{Fi}\}_i$, $\{X_{Hi,Hj}\}_{i,j}$, $\{X_{Hi,Fj}\}_{i,j}$, L, $\{Y_{EX,i}\}_i$, and $\{\iota_i\}_i$ that satisfies the following equations (G.1)-(G.8) for each $i, j \in \{1, 2, \cdots, N\}$ and any realized state $\boldsymbol{\xi} \equiv \left\{A_i, D^*_{EX,Fi}, P^*_{IM,Fi}\right\}_i \in \boldsymbol{\Xi}$:

$$\text{(consumption basket)} \qquad C = \mathcal{C}\big(\{C_i\}_i\big), \tag{G.1}$$

$$\text{(production function)} \qquad Y_i = A_i \cdot \iota_i \cdot F_i\big(L_i, \{X_{i,j}\}_j\big), \tag{G.2}$$

$$\text{(consumption with import)} \qquad C_i = \mathcal{C}_i\big(C_{Hi}, C_{Fi}\big), \tag{G.3}$$

$$\text{(intermediate inputs with import)} \qquad X_{i,j} = \mathcal{X}_{i,j}\big(X_{Hi,Hj}, X_{Hi,Fj}\big), \tag{G.4}$$

$$\text{(labor market clearing)} \qquad L = \sum_i L_i, \tag{G.5}$$

$$\text{(goods market clearing)} \qquad Y_i = C_{Hi} + \sum_j X_{Hj,Hi} + Y_{EX,i}, \tag{G.6}$$

$$\text{(balance of trade)} \qquad EX \equiv \sum_i \big(D^*_{EX,Fi}\big)^{\frac{1}{\theta_{F,i}}} Y_{EX,i}^{\frac{\theta_{F,i}-1}{\theta_{F,i}}} = \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big), \tag{G.7}$$

$$\text{(within-sector output dispersion)} \qquad \iota_i \equiv Y_i \Big/ \Big(\int_0^1 Y_{if} df\Big), \tag{G.8}$$

where the aggregators $F_i = (L_{if}/\alpha_i)^{\alpha_i} \prod_{j=1}^N (X_{i,j}/\omega_{i,j})^{\omega_{i,j}}$ following equation (1), $\{\mathcal{X}_{i,j}\}_{i,j}$ is defined in equation (2), and $\mathcal{C}$ and $\{\mathcal{C}_i\}_i$ are defined in equation (5).

For sector-level conditions in equations (G.1) to (G.8) to summarize the feasible allocation of the economy at the firm level, all firms within each sector must share the same marginal product of inputs, which happens to hold in the *first-best allocation*, the *sticky-price equilibrium*, and the *flexible-price equilibrium* under our model setup.

G.2. Proof of Lemma 1: Efficient flexible-price equilibrium

To prove Lemma 1, we define the *first-best allocation* (Definition G.2), present the conditions for it (Lemma G.1), and show that these conditions coincide with those for the *flexible-price equilibrium* when Assumption 1 holds.

The *first-best allocation* is the feasible allocation that solves the social planner's problem, as outlined in the following definition.

Definition G.2 (First-best allocation)**.** *The first-best allocation is a feasible allocation that maximizes the representative household's utility $u(C, L)$—i.e., it solves the following social planner's problem:*

$$\max_{\{\iota_i, L_i, \{X_{Hi,Hj}, X_{Hi,Fj}\}_j, C_{Hi}, C_{Fi}\}_i} u(C, L)$$
$$\text{s.t.} \quad \textit{equations } (G.1) \textit{ to } (G.7) \textit{ and } \iota_i \in [0,1] \textit{ for all } i.$$

Substituting equations (G.1), (G.3), and (G.5) into the utility function $u(C, L)$ yields the following:

$$u(C, L) = u\Big(\mathcal{C}\big(\{\mathcal{C}_i(C_{Hi}, C_{Fi})\}_i\big), \sum_i L_i\Big). \tag{G.9}$$

Substituting equations (G.2), (G.4), and (G.6) into equation (G.7) yields the consolidated constraint of the social planner's problem in the following:

$$\sum_i \left(D^*_{EX,Fi}\right)^{\frac{1}{\theta_{F,i}}} \Big[A_i \iota_i F_i\big(\{L_i, \mathcal{X}_{i,j}(X_{Hi,Hj}, X_{Hi,Fj})\}_j\big) - C_{Hi} - \sum_j X_{Hj,Hi}\Big]^{\frac{\theta_{F,i}-1}{\theta_{F,i}}}$$
$$= \sum_i P^*_{IM,Fi}\Big(C_{Fi} + \sum_j X_{Hj,Fi}\Big). \tag{G.10}$$

As a result, the first-best allocation is the feasible allocation that maximizes the utility function in equation (G.9)—subject to the constraint in equation (G.10)—which, in turn, satisfies the optimality conditions outlined in Lemma G.1.

Lemma G.1 (First-best allocation)**.** *The first-best allocation satisfies the following optimality conditions:*

$$\iota_i = 1, \tag{G.11}$$
$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}}} = A_i \frac{\partial F_i}{\partial L_i}, \tag{G.12}$$
$$\frac{\partial \mathcal{C}/\partial \mathcal{C}_j}{\partial \mathcal{C}/\partial \mathcal{C}_i}\frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}} = A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}\frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}, \tag{G.13}$$
$$\frac{\partial \mathcal{C}_i/\partial C_{Fi}}{\partial \mathcal{C}_i/\partial C_{Hi}} = P^*_{IM,Fi} \cdot \frac{\theta_{F,i}}{\theta_{F,i}-1}\left(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\right)^{\frac{1}{\theta_{F,i}}}, \tag{G.14}$$

$$\frac{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Hj}} = P^*_{IM,Fj} \cdot \frac{\theta_{F,j}}{\theta_{F,j}-1}\Big(\frac{Y_{EX,j}}{D^*_{EX,Fj}}\Big)^{\frac{1}{\theta_{F,j}}}. \tag{G.15}$$

Proof of Lemma G.1. To eliminate distortions and maximize welfare, the social planner would close the within-sector dispersion in output—i.e., choosing $\iota_i = 1$. Furthermore, denote κ the multiplier for the constraint (G.10) of the social planner's problem, the first-order conditions w.r.t. L_i, $X_{Hi,Hj}$, $X_{Hi,Fj}$, C_{Hi}, and C_{Fi} are

$$\begin{aligned}
0 &= \frac{\partial u}{\partial L} + \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial L_i}, \\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}} \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}} - \frac{\theta_{F,j}-1}{\theta_{F,j}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fj}}\Big)^{-\frac{1}{\theta_{F,j}}}, \\
0 &= \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}} A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}} \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Fj}} - P^*_{IM,Fj}, \\
0 &= \frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}} - \kappa \cdot \frac{\theta_{F,i}-1}{\theta_{F,i}}\Big(\frac{Y_{EX,i}}{D^*_{EX,Fi}}\Big)^{-\frac{1}{\theta_{F,i}}}, \\
0 &= \frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Fi}} - \kappa \cdot P^*_{IM,Fi}.
\end{aligned}$$

Rearranging the above first-order conditions and eliminating the multiplier κ yields equations (G.12)-(G.15) of Lemma G.1. ☐

Proof of Lemma 1. Under $\tau_i = -1/(\varepsilon_i - 1)$ of Assumption 1, in the *flexible-price equilibrium*, combining the optimal pricing conditions of the firms that maximize profits in equation (A.1)—subject to demand function in equation (A.3)—with the cost minimization conditions that minimize the total costs in equation (3)—subject to the production technology in equations (1) and (2)—yields the following two conditions:

$$A_i \frac{\partial F_i}{\partial L_i}(\boldsymbol{\xi}) = \frac{W^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}, \tag{G.16}$$

$$A_i \frac{\partial F_i}{\partial \mathcal{X}_{i,j}}(\boldsymbol{\xi}) \frac{\partial \mathcal{X}_{i,j}}{\partial X_{Hi,Hj}}(\boldsymbol{\xi}) = \frac{P_j^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{G.17}$$

Under $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1, combining the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ with the no-arbitrage condition $(1 - \tau_{EX,i})P_{EX,i} = P_i$, yields the following equation:

$$\frac{\theta_{F,i}}{\theta_{F,i}-1}\Big(\frac{Y^{flex}_{EX,i}(\boldsymbol{\xi})}{D^*_{EX,Fi}}\Big)^{\frac{1}{\theta_{F,i}}} = \frac{S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{G.18}$$

Furthermore, for the households' problem that maximizes utility function (4)—subject to the consumption aggregator (5) and budget constraint (6)—combining the first-order conditions with respect to L and C_{Hi} yields condition (G.19), combining the first-order conditions with respect to C_{Hj} and C_{Hi} yields condition

(G.20), and combining the first-order conditions with respect to C_{Fi} and C_{Hi} yields condition (G.21). For the firm's cost minimization problem that minimizes the total costs in equation (3) subject to the production technology in equations (1) and (2), combining the first-order conditions with respect to $X_{Hi,Fj}$ and $X_{Hi,Hj}$, yields condition (G.22).

$$-\frac{\partial u/\partial L}{\frac{\partial u}{\partial C}\frac{\partial \mathcal{C}}{\partial C_i}\frac{\partial \mathcal{C}_i}{\partial C_{Hi}}}(\boldsymbol{\xi}) = \frac{W^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}, \tag{G.19}$$

$$\frac{\partial \mathcal{C}/\partial C_j}{\partial \mathcal{C}/\partial C_i}(\boldsymbol{\xi})\frac{\partial \mathcal{C}_j/\partial C_{Hj}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\boldsymbol{\xi}) = \frac{P_j^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})} \tag{G.20}$$

$$\frac{\partial \mathcal{C}_i/\partial C_{Fi}}{\partial \mathcal{C}_i/\partial C_{Hi}}(\boldsymbol{\xi}) = \frac{P^*_{IM,Fi}S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{G.21}$$

$$\frac{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Fj}}{\partial \mathcal{X}_{i,j}/\partial X_{Hi,Hj}}(\boldsymbol{\xi}) = \frac{P^*_{IM,Fj}S^{flex}(\boldsymbol{\xi})}{P_i^{flex}(\boldsymbol{\xi})}. \tag{G.22}$$

Substituting equations (G.16)-(G.18) into equations (G.19)-(G.22) to eliminate all of the equilibrium prices $W^{flex}(\boldsymbol{\xi})$, $S^{flex}(\boldsymbol{\xi})$, and $\{P_i^{flex}(\boldsymbol{\xi})\}_i$, yields *exactly the same* conditions for the *flexible-price equilibrium as* the conditions (G.12)-(G.15) for the first-best allocation in Lemma G.1, thereby proving the efficiency of the *flexible-price equilibrium*. □

The role of export taxes $\{\tau_{EX,i}\}_i$. In closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), non-contingent sector-specific subsidies $\tau_i = -1/(\varepsilon_i - 1)$ eliminate sectoral distortions due to monopolistic competition and, therefore, are sufficient to make the *flexible-price equilibrium* efficient. In open economies, however, it is welfare-enhancing for the social planner of the small open economy to exploit fully the monopoly powers of the domestic producers in the international market. As a result, the non-contingent sector-specific subsidies that eliminate the sectoral distortions due to monopolistic competition *alone* are no longer optimal in small open economies, and an additional non-contingent export tax $\tau_{EX,i} = 1/\theta_{F,i}$ is required to retain the monopoly powers of the domestic producers in the international market and make the *flexible-price equilibrium* efficient. Under such export taxes, the sectoral export prices become:

$$P_{EX,i} = \frac{1}{1-\tau_{EX,i}}P_i = \frac{\theta_{F,i}}{\theta_{F,i}-1}P_i, \quad \forall i \in \{1, 2, \cdots, N\}.$$

G.3. Steady-state Domar weights and sectoral export-to-GDP ratios

Lemma G.2 (Steady-state Domar weights and sectoral export-to-GDP ratios)**.** *The steady-state Domar weights* $\boldsymbol{\lambda}$ *and sectoral export-to-GDP ratios* $\boldsymbol{\lambda}_{EX}$ *are functions of parameters as in the following equations:*

$$\begin{aligned}\boldsymbol{\lambda}^\top = &\left\{\boldsymbol{\beta}\odot\mathbf{v} + (1-\boldsymbol{\beta}^\top\mathbf{v})[(\boldsymbol{\theta}_F - 1)\oslash\boldsymbol{\theta}_F\odot\mathbf{v}^*_H]\right\}^\top \\ &\cdot\left\{\mathbf{I} - \boldsymbol{\Omega}\odot\mathbf{V}_x - (\boldsymbol{\Omega}\odot\mathbf{V}_{1-x})\mathbf{1}[(\boldsymbol{\theta}_F-1)\oslash\boldsymbol{\theta}_F\odot\mathbf{v}^*_H]^\top\right\}^{-1},\end{aligned} \tag{G.23}$$

$$\boldsymbol{\lambda}_{EX}^{\top} = \boldsymbol{\lambda}^{\top}(\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x) - (\boldsymbol{\beta} \odot \mathbf{v})^{\top}, \tag{G.24}$$

where $\mathbf{v}_H^*$ *is the vector of the steady-state shares of sectoral exports in the value of the aggregate exports, with the* i*-th element* v_{Hi}^* *equal to:*

$$v_{Hi}^* \equiv \frac{\left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{1-\theta_{F,i}} D_{EX,Fi}^{*,ss}}{\sum_{i'} \left(\frac{\theta_{F,i'}}{\theta_{F,i'}-1}\right)^{1-\theta_{F,i'}} D_{EX,Fi'}^{*,ss}}.$$

Proof of Lemma G.2. In the steady state, the nominal exchange rate S^{ss} and the sectoral prices P_i^{ss} are both normalized to 1. As a result, for each sector i, the export price $P_{EX,i}^{ss}$ is equal to $\theta_{F,i}/(\theta_{F,i} - 1)$, and the foreign demand for domestic sector i's product in terms of quantity and value are equal to

$$Y_{EX,i}^{ss} = \left(\frac{P_{EX,i}^{ss}}{S^{ss}}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss} = \left(\frac{\theta_{F,i}}{\theta_{F,i} - 1}\right)^{-\theta_{F,i}} D_{EX,Fi}^{*,ss}, \tag{G.25}$$

and

$$\frac{\theta_{F,i}}{\theta_{F,i} - 1} Y_{EX,i}^{ss} = v_{Hi}^* \cdot \sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'} - 1} Y_{EX,i'}^{ss}, \tag{G.26}$$

respectively. In the steady state, the import price $P_{IM,Fi}^{*,ss}$ is also normalized to 1, which yields the steady-state balance of trade condition $\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1} Y_{EX,i'}^{ss} = \sum_{i'} (C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss})$. Combining this steady-state balance of trade condition with equation (G.26), yields the following equation of the quantity of foreign demand:

$$Y_{EX,i}^{ss} = \frac{\theta_{F,i} - 1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \left(C_{Fi'}^{ss} + \sum_j X_{Hj,Fi'}^{ss}\right). \tag{G.27}$$

Substituting equation (G.27) into the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ and dividing both sides by the steady-state aggregate output C^{ss} yields:

$$\lambda_i = \beta_i v_i + \sum_j \lambda_j \omega_{j,i} v_{x,j,i} + \frac{\theta_{F,i} - 1}{\theta_{F,i}} v_{Hi}^* \sum_{i'} \left[\beta_{i'}(1 - v_{i'}) + \sum_j \lambda_j \omega_{j,i'}(1 - v_{x,j,i'})\right],$$

which has equation (G.23) as its matrix form.

Dividing both sides of the steady-state goods market clearing condition $Y_i^{ss} = C_{Hi}^{ss} + \sum_j X_{Hj,Hi}^{ss} + Y_{EX,i}^{ss}$ by the steady-state aggregate output C^{ss} and substituting in the definition of the sectoral export-to-GDP ratio

$\lambda_{EX,i} \equiv (P_i^{ss} Y_{EX,i}^{ss})/(P_C^{ss} C^{ss})$ with normalized $P_i^{ss} = P_C^{ss} = 1$ yields the following equation:

$$\lambda_{EX,i} = \lambda_i - \Big(\beta_i \upsilon_i + \sum_j \lambda_j \omega_{j,i} \upsilon_{x,j,i}\Big), \tag{G.28}$$

which has equation (G.24) as its matrix form. □

G.4. Goods market clearing condition up to the first-order approximation

Lemma G.3 (Goods market clearing condition)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*.*

$$\begin{aligned}\big[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}(\boldsymbol{\xi})\big)\big]^\top = {} & \widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})\big) \\ & - \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) + \big[\boldsymbol{\lambda}_{EX} \widehat{S}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{ES} \odot \big(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\big)\big]^\top \mathbf{L}_{vx} \\ & + \big\{\boldsymbol{\lambda}_{EX} \odot \widehat{\mathbf{D}}_{EX,F}^* + \big[\boldsymbol{\rho}_{ES} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}\big] \odot \widehat{\mathbf{P}}_{IM,F}^*\big\}^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{G.29}$$

Proof of Lemma G.3. The goods market clearing condition (G.6) multiplied by the sectoral price P_i is

$$P_i Y_i = P_i C_{Hi} + P_i \sum_j X_{Hj,Hi} + P_i Y_{EX,i}. \tag{G.30}$$

Denote $P_{c,i}$ as the price index of the sectoral consumption goods from sector i and $P_{x,j,i}$ as the price index of the intermediate inputs purchased by sector j from sector i—both of which are weighted averages of domestic price P_i and import price $S \cdot P_{IM,Fi}^*$. Minimizing the costs of purchasing $\mathcal{C}$, $\{F_i\}_i$, $\{\mathcal{C}_i\}_i$, $\{\mathcal{X}_{i,j}\}_{i,j}$ yields the following quantity of the demand for consumption and intermediate inputs as functions of prices:

$$\begin{aligned} C_{Hi} &= \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} \upsilon_i C_i = \left(\frac{P_i}{P_{c,i}}\right)^{-\theta_i} \frac{\upsilon_i \beta_i P_C C}{P_{c,i}}, \\ X_{Hj,Hi} &= \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} \upsilon_{x,j,i} X_{j,i} = \left(\frac{P_i}{P_{x,j,i}}\right)^{-\theta_i} \frac{\upsilon_{x,j,i} \omega_{j,i} P_j Y_j}{P_{x,j,i} \mu_j}. \end{aligned}$$

Substituting equation (2.4) into equation (8) yields the export demand as follows:

$$Y_{EX,i} = \left(\frac{P_{EX,i}}{S}\right)^{-\theta_{F,i}} D_{EX,Fi}^* = \left(\frac{\theta_{F,i}}{\theta_{F,i} - 1}\right)^{-\theta_{F,i}} \left(\frac{P_i}{S}\right)^{-\theta_{F,i}} D_{EX,Fi}^*.$$

Substituting the quantity of consumption, intermediate inputs, and export demand above back to the goods market-clearing condition in equation (G.30) yields:

$$P_i Y_i = \left(\frac{P_i}{P_{c,i}}\right)^{1-\theta_i} \upsilon_i \beta_i P_C C + \sum_j \left(\frac{P_i}{P_{x,j,i}}\right)^{1-\theta_i} \frac{\upsilon_{x,j,i} \omega_{j,i} P_j Y_j}{\mu_j}$$

$$+\left(\frac{\theta_{F,i}}{\theta_{F,i}-1}\right)^{-\theta_{F,i}}\left(\frac{P_i}{S}\right)^{1-\theta_{F,i}} S\cdot D^*_{EX,Fi}. \tag{G.31}$$

Log-linearizing equation (G.31) yields:

$$\begin{aligned}\lambda_i(\widehat{P}_i+\widehat{Y}_i) &= \beta_i v_i\Big[(\theta_i-1)(\widehat{P}_{c,i}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big]\\ &\quad+\sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(\widehat{P}_{x,j,i}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big]\\ &\quad+\lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big]+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{G.32}$$

Log-linearizing the price indices $P_{c,i}$ and $P_{x,j,i}$ yields:

$$\widehat{P}_{c,i} = v_i\widehat{P}_i+(1-v_i)\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big)+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{G.33}$$
$$\widehat{P}_{x,j,i} = v_{x,j,i}\widehat{P}_i+(1-v_{x,j,i})\big(\widehat{S}+\widehat{P}^*_{IM,Fi}\big)+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{G.34}$$

which implies the following relative prices:

$$\begin{aligned}\widehat{P}_{c,i}-\widehat{P}_i &= (1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+o(\|\widehat{\boldsymbol{\xi}}\|),\\ \widehat{P}_{x,j,i}-\widehat{P}_i &= (1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

Substituting these relative prices into equation (G.32) yields:

$$\begin{aligned}\lambda_i(\widehat{P}_i+\widehat{Y}_i) &= \beta_i v_i\Big[(\theta_i-1)(1-v_i)(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_C+\widehat{C}\Big]\\ &\quad+\sum_j \lambda_j\omega_{j,i}v_{x,j,i}\Big[(\theta_i-1)(1-v_{x,j,i})(\widehat{S}+\widehat{P}^*_{IM,Fi}-\widehat{P}_i)+\widehat{P}_j+\widehat{Y}_j-\widehat{\mu}_j\Big]\\ &\quad+\lambda_{EX,i}\Big[(\theta_{F,i}-1)(\widehat{S}-\widehat{P}_i)+\widehat{S}+\widehat{D}^*_{EX,Fi}\Big]+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

Rearranging the above equation and substituting in the definition of the expenditure-switching elasticity $\rho_{ES,i}$ in equation (21) yield the following:

$$\begin{aligned}\lambda_i(\widehat{P}_i+\widehat{Y}_i)-\sum_j\lambda_j\omega_{j,i}v_{x,j,i}(\widehat{P}_j+\widehat{Y}_j) &= \beta_i v_i(\widehat{P}_C+\widehat{C})-\sum_j\lambda_j\omega_{j,i}v_{x,j,i}\widehat{\mu}_j+\lambda_{EX,i}\widehat{S}-\rho_{ES,i}(\widehat{P}_i-\widehat{S})\\ &\quad+\lambda_{EX,i}\widehat{D}^*_{EX,Fi}+[\rho_{ES,i}-(\theta_{F,i}-1)\lambda_{EX,i}]\widehat{P}^*_{IM,Fi}+o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned}$$

which has the following matrix form as in equation (G.29) in Lemma G.3:

$$\begin{aligned}\big[\boldsymbol{\lambda}\odot(\widehat{\boldsymbol{P}}+\widehat{\boldsymbol{Y}})\big]^\top &= \widetilde{\boldsymbol{\lambda}}_D^\top(\widehat{P}_C+\widehat{C})-(\boldsymbol{\lambda}\odot\widehat{\boldsymbol{\mu}})^\top(\mathbf{L}_{vx}-\mathbf{I})+\big[\boldsymbol{\lambda}_{EX}\widehat{S}-\boldsymbol{\rho}_{ES}\odot(\widehat{\mathbf{P}}-\mathbf{1}\widehat{S})\big]^\top\mathbf{L}_{vx}\\ &\quad+\big\{\boldsymbol{\lambda}_{EX}\odot\widehat{\mathbf{D}}^*_{EX,F}+\big[\boldsymbol{\rho}_{ES}-(\boldsymbol{\theta}_F-\mathbf{1})\odot\boldsymbol{\lambda}_{EX}\big]\odot\widehat{\mathbf{P}}^*_{IM,F}\big\}^\top\mathbf{L}_{vx}+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

□

G.5. Household's budget constraint up to first-order approximation

Lemma G.4 (Household's budget constraint)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\begin{aligned}\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi}) = \left[\boldsymbol{\lambda} \odot \left(\widehat{\boldsymbol{P}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}(\boldsymbol{\xi})\right)\right]^\top \boldsymbol{\alpha} + \left(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\right)^\top (\mathbf{1} - \boldsymbol{\alpha}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S}(\boldsymbol{\xi}) \\ - \boldsymbol{\lambda}_{EX}^\top \left(\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})\right) + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{G.35}$$

Proof of Lemma G.4. Substituting the profit, total cost of inputs, and lump-sum transfer in equations (A.1), (3), and (11) into the household budget constraint in equation (6) yields

$$\begin{aligned}P_C C &= WL + \sum_i \int_0^1 \Pi_{if} df + T \\ &= WL + \sum_i \left[(1-\tau_i)P_i Y_i - W L_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] \\ &\quad + \sum_i \left(\tau_i P_i Y_i + \tau_{EX,i} P_{EX,i} Y_{EX,i}\right) \\ &= \sum_i \left[P_i Y_i - \sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right)\right] + \sum_i \tau_{EX,i} P_{EX,i} Y_{EX,i}.\end{aligned} \tag{G.36}$$

Under the Cobb-Douglas production functions, $\sum_j \left(P_j X_{Hi,Hj} + S \cdot P^*_{IM,Fj} X_{Xi,Fj}\right) = P_i Y_i (1-\alpha_i)/\mu_i$. Therefore, substituting the export tax rate $\tau_{EX,i} = 1/\theta_{F,i}$, the export price $P_{EX,i} = P_i/(1-\tau_{EX,i})$, and the export demand $Y_{EX,i} = (P_{EX,i}/S)^{-\theta_{F,i}} D^*_{EX,Fi}$ into equation (G.36) yields

$$P_C C = \sum_i P_i Y_i \left(1 - \frac{1-\alpha_i}{\mu_i}\right) + \sum_i \left(\frac{S}{\theta_{F,i}}\right)^{\theta_{F,i}} \left(\frac{P_i}{\theta_{F,i}-1}\right)^{1-\theta_{F,i}} D^*_{EX,Fi}. \tag{G.37}$$

In the steady state, the sectoral markups, prices, and nominal exchange rate are normalized to $\mu_i^{ss} = P_i^{ss} = S^{ss} = 1$. As a result, equation (G.37) becomes

$$1 = \sum_i \lambda_i \alpha_i + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}. \tag{G.38}$$

Log-linearizing equation (G.37) around the steady state yields

$$\widehat{P}_C + \widehat{C} = \sum_i \lambda_i \alpha_i \left(\frac{1-\alpha_i}{\alpha_i}\widehat{\mu}_i + \widehat{P}_i + \widehat{Y}_i\right) + \sum_i \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\left[\widehat{S} - (\theta_{F,i}-1)(\widehat{P}_i - \widehat{S}) + \widehat{D}^*_{EX,Fi}\right] + o(\|\widehat{\boldsymbol{\xi}}\|),$$

which has the following matrix form as in equation (G.35) in Lemma G.4:

$$\widehat{P}_C + \widehat{C} = \left[\boldsymbol{\lambda} \odot (\widehat{\boldsymbol{P}} + \widehat{\boldsymbol{Y}})\right]^\top \boldsymbol{\alpha} + (\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}})^\top (\mathbf{1} - \boldsymbol{\alpha})$$
$$+ (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S} - \boldsymbol{\lambda}_{EX}^\top (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S}) + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

G.6. *Sectoral markup wedges and sectoral inflation*

Lemma G.5 (Sectoral markup wedges and sectoral inflation)**.** *Up to the first-order approximation, the following condition holds in the* sticky-price equilibrium*:*

$$\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = -(\boldsymbol{\Delta}^{-1} - \mathbf{I})\widehat{\mathbf{P}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{G.39}$$

Proof of Lemma G.5. Under static Calvo-pricing, the vector of sectoral inflation is a function of the sectoral frequency of price adjustment $\boldsymbol{\Delta}$ and the vector of sectoral nominal marginal costs $\boldsymbol{\Phi}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{G.40}$$

On the other hand, the definition of the sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ yields:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}). \tag{G.41}$$

Combining the above two conditions to eliminate $\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi})$ yields equation (G.39). □

H. Proofs of the theoretical results in Section 3

This appendix derives the theoretical results associated with the aggregate output gap and the OG policy in Section 3. These theoretical results are all up to the first-order approximation around the efficient steady state under Assumption 1.

H.1. *Proof of Lemma B.1: The open economy version of Hulten's theorem*

Hulten's theorem in Hulten (1978) characterizes the first-order impact of disaggregated productivity shocks on the aggregate TFP in an efficient closed economy (e.g., Baqaee and Farhi, 2019). Our paper extends the closed-economy version of Hulten's theorem into a small open economy with international trade, exchange rate adjustments, and sector-specific shocks to import prices and export demand besides sectoral productivity.

Under $\tau_i = -1/(\varepsilon_i - 1)$ and $\tau_{EX,i} = 1/\theta_{F,i}$ of Assumption 1 and with all of the prices but $P^{ss}_{EX,i}$ and W^{ss} normalized to 1, the first-order approximation of the conditions in Lemma G.1 around the efficient steady state yields the following:

$$C^{ss}\widehat{C}(\boldsymbol{\xi}) = \sum_i C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.1}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.2}$$

$$C_i^{ss}\widehat{C}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.3}$$

$$X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi}) = X_{Hi,Hj}^{ss}\widehat{X}_{Hi,Hj}(\boldsymbol{\xi}) + X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.4}$$

$$L^{ss}\widehat{L}(\boldsymbol{\xi}) = \sum_i L_i^{ss}\widehat{L}_i(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.5}$$

$$Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) = C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) + \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) + Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.6}$$

$$\begin{aligned}
EX_i^{ss}\widehat{EX}_i(\boldsymbol{\xi}) &= \sum_i Y_{EX,i}^{ss}\Big[(\theta_{F,i}-1)^{-1}\widehat{D}_{EX,Fi}^* + \widehat{Y}_{EX,i}(\boldsymbol{\xi})\Big] \\
&= \sum_i \Big[C_{Fi}^{ss}\big(\widehat{P}_{IM,Fi}^* + \widehat{C}_{Fi}(\boldsymbol{\xi})\big) + \sum_j X_{Hj,Fi}^{ss}\big(\widehat{P}_{IM,Fi}^* + \widehat{X}_{Hj,Fi}(\boldsymbol{\xi})\big)\Big] + o(\|\widehat{\boldsymbol{\xi}}\|).
\end{aligned} \tag{H.7}$$

Then, we combine equations (H.1)-(H.7) to prove Lemma B.1. Rearranging the balance of trade condition (H.7) to move all endogenous terms to the LHS and all exogenous ones to the RHS yields the following:

$$\begin{aligned}
LHS &\equiv \sum_i \Big(Y_{EX,i}^{ss}\widehat{Y}_{EX,i}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hi,Fj}^{ss}\widehat{X}_{Hi,Fj}(\boldsymbol{\xi})\Big) \\
&= \sum_i \Big(C_{Fi}^{ss}\widehat{P}_{IM,Fi}^* + \sum_j X_{Hj,Fi}^{ss}\widehat{P}_{IM,Fi}^* - \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^*\Big) + o(\|\widehat{\boldsymbol{\xi}}\|) \equiv RHS.
\end{aligned} \tag{H.8}$$

Combined with the goods market clearing condition in equation (H.6), the LHS of equation (H.8) becomes:

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - C_{Hi}^{ss}\widehat{C}_{Hi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Hi}^{ss}\widehat{X}_{Hj,Hi}(\boldsymbol{\xi}) - C_{Fi}^{ss}\widehat{C}_{Fi}(\boldsymbol{\xi}) - \sum_j X_{Hj,Fi}^{ss}\widehat{X}_{Hj,Fi}(\boldsymbol{\xi})\Big).$$

Further combined with the aggregators in equations (H.1), (H.3), and (H.4), the LHS becomes:

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{Y}_i(\boldsymbol{\xi}) - \sum_j X_{i,j}^{ss}\widehat{X}_{i,j}(\boldsymbol{\xi})\Big) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the production function in equation (H.2),

$$LHS = \sum_i \Big(Y_i^{ss}\widehat{A}_i + W^{ss}L_i^{ss}\widehat{L}_i(\boldsymbol{\xi})\Big) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Combined with the labor market clearing condition in equation (H.5),

$$LHS = \sum_i Y_i^{ss}\widehat{A}_i + W^{ss}L^{ss}\widehat{L}(\boldsymbol{\xi}) - C^{ss}\widehat{C}(\boldsymbol{\xi}).$$

Substituting LHS back into equation (H.8) yields:

$$\begin{aligned} &C^{ss}\widehat{C}(\boldsymbol{\xi}) - W^{ss}L^{ss}\widehat{L}(\boldsymbol{\xi}) \\ &= \sum_i \left(Y_i^{ss}\widehat{A}_i - C_{Fi}^{ss}\widehat{P}_{IM,Fi}^* - \sum_j X_{Hj,Fi}^{ss}\widehat{P}_{IM,Fi}^* + \frac{Y_{EX,i}^{ss}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^* \right) + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{H.9}$$

In the steady state, the sectoral output prices and the CPI are normalized to 1. Therefore, dividing both sides of equation (H.9) by the steady-state aggregate output C^{ss} yields the following:

$$\begin{aligned} \widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) = \sum_i \Bigg\{ &\lambda_i\widehat{A}_i + \frac{\lambda_{EX,i}}{\theta_{F,i}-1}\widehat{D}_{EX,Fi}^* \\ &- \Big[\beta_i(1-v_i) + \sum_j \lambda_j\omega_{j,i}(1-v_{x,j,i})\Big]\widehat{P}_{IM,Fi}^* \Bigg\} + o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{H.10}$$

H.2. Proof of Proposition B.1: Efficiency and labor wedges

Efficiency wedge. Log-linearizing the efficiency wedge $A_{agg}(\boldsymbol{\xi})$ in Definition B.1 around the steady state yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L^{flex}(\boldsymbol{\xi})\widehat{L}(\boldsymbol{\xi}).$$

Substituting $\Lambda_L^{flex}(\boldsymbol{\xi}) = \Lambda_L + O(\|\widehat{\boldsymbol{\xi}}\|)$ into the above equation yields

$$\widehat{A}_{agg}(\boldsymbol{\xi}) = \widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

where $\widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi})$ are functions of only exogenous shocks up to the first-order approximation, as shown in equation (H.10) of Appendix H.1. Therefore, taking the difference of equation (H.10) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields the following:

$$\begin{aligned} \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi}) &= \left(\widehat{C}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) - \left(\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{flex}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|)\right) \\ &= \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L\widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = o(\|\widehat{\boldsymbol{\xi}}\|). \end{aligned} \tag{H.11}$$

In sum, sectoral markup wedges under price rigidities have no first-order impact on the efficiency wedge.

Labor wedge. Consider a prototype economy similar to the closed economy á la Chari et al. (2007), except that the aggregate production function defined on domestic labor inputs has state-contingent aggregate TFP and returns-to-scale, as in the following equation:

$$F(L, \boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})},$$

where $\Lambda_L^{flex}(\boldsymbol{\xi})$ is the economy-wise labor share in the *flexible-price equilibrium* of the small open economy that is contingent on the states of exogenous shocks. According to Definition B.1, $C(\boldsymbol{\xi}) = F(L(\boldsymbol{\xi}), \boldsymbol{\xi})$ and, therefore, the labor wedge $\Gamma_L(\boldsymbol{\xi})$ satisfies:

$$-\frac{\partial u/\partial L}{\partial u/\partial C}(C(\boldsymbol{\xi}), L(\boldsymbol{\xi})) = \Gamma_L(\boldsymbol{\xi}) \cdot \frac{\partial F}{\partial L}(L(\boldsymbol{\xi}), \boldsymbol{\xi}), \tag{H.12}$$

where the marginal product of labor in the *sticky-price equilibrium* is equal to:

$$\frac{\partial F}{\partial L}(L(\boldsymbol{\xi}), \boldsymbol{\xi}) = A_{agg}(\boldsymbol{\xi}) \cdot \Lambda_L^{flex}(\boldsymbol{\xi}) \cdot L^{\Lambda_L^{flex}(\boldsymbol{\xi})-1} \equiv \frac{\partial C}{\partial L}(\boldsymbol{\xi}).$$

Therefore, substituting the utility function in equation (4) into equation (H.12) and log-linearizing it around the steady state yields:

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}) - \widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{\Lambda}_L^{flex}(\boldsymbol{\xi}) - \big(\Lambda_L^{flex}(\boldsymbol{\xi}) - 1\big)\widehat{L}(\boldsymbol{\xi}). \tag{H.13}$$

Taking the difference of equation (H.13) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned} \widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}_L^{flex}(\boldsymbol{\xi}) = {} & \sigma \widehat{C}^{gap}(\boldsymbol{\xi}) + \varphi \widehat{L}^{gap}(\boldsymbol{\xi}) \\ & - \big(\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi})\big) - (\Lambda_L - 1)\widehat{L}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) \end{aligned} \tag{H.14}$$

Combining equation (H.14) with equation (H.11) yields the labor wedge as follows:

$$\widehat{\Gamma}_L(\boldsymbol{\xi}) - \widehat{\Gamma}_L^{flex}(\boldsymbol{\xi}) = \left(\sigma - 1 + \frac{\varphi + 1}{\Lambda_L}\right) \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

H.3. Impacts of sectoral markup wedges on CPI

Under the production technology and the total cost of inputs in equations (1), (2), and (3), deriving the sectoral nominal marginal costs $\boldsymbol{\Phi}(\boldsymbol{\xi})$ from the producers' cost minimization problem and log-linearizing it around the steady state, yields the following:

$$\widehat{\boldsymbol{\Phi}}(\boldsymbol{\xi}) = \boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^*_{IM,F}\big) - \widehat{\mathbf{A}} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.15}$$

which, substituted into equation (G.41), yields:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^{*}_{IM,F}\big) - \widehat{\mathbf{A}} + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.16}$$

Taking the difference of equation (H.16) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks, yields:

$$\begin{aligned}\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) &= \boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).\\ &= \mathbf{L}_{vx}\big(\boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\alpha}\widehat{S}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big) + \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned} \tag{H.17}$$

where the second equality is derived using the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1}$ and the identity $\boldsymbol{\alpha} = \mathbf{1} - \boldsymbol{\Omega}\mathbf{1}$.

Log-linearizing the CPI in equation (7) around the steady state yields:

$$\widehat{P}_C(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^{\top}\widehat{\mathbf{P}}(\boldsymbol{\xi}) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^{\top}\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}^{*}_{IM,F}\big) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.18}$$

Taking the difference of equation (H.18) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* to eliminate the exogenous shocks yields:

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^{\top}\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^{\top}\mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.19}$$

Substituting equation (H.17) into equation (H.19) and using the identity $(\boldsymbol{\beta} \odot \mathbf{v})^{\top}\mathbf{L}_{vx} = \widetilde{\boldsymbol{\lambda}}_D^{\top}$ yields:

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha}\widehat{W}^{gap}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi}) + \widetilde{\boldsymbol{\lambda}}_D^{\top}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

which can be rearranged to highlight the real wage and real exchange rate:

$$\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha}\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\big) + (1 - \widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha})\big(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\big) = -\widetilde{\boldsymbol{\lambda}}_D^{\top}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.20}$$

In Lemma H.1 and Lemma H.2 below, we further relate the real wage gap and real exchange rate gap in equation (H.20) to the aggregate output gaps. □

H.4. Real wage gap and aggregate output gap

Lemma H.1 (Real wage gap and aggregate output gap)**.** *Up to the first-order approximation, the real wage gap is proportional to the aggregate output gap as in the following equation:*

$$\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) = \sigma\widehat{C}^{gap}(\boldsymbol{\xi}) + \varphi\widehat{L}^{gap}(\boldsymbol{\xi}) = \big(\sigma + \varphi/\Lambda_L\big)\widehat{C}^{gap}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.21}$$

Proof of Lemma H.1. For the households' problem that maximizes utility function (4) subject to the budget constraint (6), combining the first-order conditions with respect to L and C and log-linearizing it yield:

$$\widehat{W}(\boldsymbol{\xi}) - \widehat{P}_C(\boldsymbol{\xi}) = \sigma \widehat{C}(\boldsymbol{\xi}) + \varphi \widehat{L}(\boldsymbol{\xi}). \tag{H.22}$$

Taking the difference of equation (H.22) in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields the first equality in equation (H.21). Further substituting in equation (H.11) from Section H.2 yields the second equality in equation (H.21). □

Interpreting Lemma H.1. Equation (H.21) shows that the lower CPI in the sticky-price equilibrium than in the efficient, flexible-price equilibrium (i.e., $\widehat{P}_C^{gap} < 0$ on the LHS) increases the real wage (i.e., $\widehat{W}^{gap} - \widehat{P}_C^{gap}$) and induces a higher supply of domestic labor (i.e., $\widehat{L}^{gap} > 0$ in the middle), thereby fostering production and generating a positive aggregate output gap (i.e., $\widehat{C}^{gap} > 0$ on the RHS).

H.5. Real exchange rate gap and aggregate output gap

Lemma H.2 (Real exchange rate gap and aggregate output gap)**.** *Up to the first-order approximation, the real exchange rate gap is a linear function of the aggregate output gap, real wage gap, and sectoral markup wedges, as reflected in the following equation:*

$$\begin{aligned}(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\widehat{C}^{gap}(\boldsymbol{\xi}) = &-\widetilde{\boldsymbol{\rho}}_{ES}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \left[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\right]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) \\ &+ \left[(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) + (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}\right]\big(\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\big) \\ &- (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widetilde{\boldsymbol{\alpha}}\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{H.23}$$

Proof of Lemma H.2. Taking the difference of equation (G.29) from Lemma G.3 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned}\left[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}^{gap}(\boldsymbol{\xi})\big)\right]^\top = &\,\widetilde{\boldsymbol{\lambda}}_D^\top \big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) - \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{L}_{vx} - \mathbf{I}) \\ &+ \left[\boldsymbol{\lambda}_{EX} \widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\rho}_{ES} \odot \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}(\boldsymbol{\xi})^{gap}\big)\right]^\top \mathbf{L}_{vx} + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{H.24}$$

Taking the difference of equation (G.35) from Lemma G.4 in the *sticky-price equilibrium* and in the *flexible-price equilibrium* yields:

$$\begin{aligned}\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) = &\left[\boldsymbol{\lambda} \odot \big(\widehat{\boldsymbol{P}}^{gap}(\boldsymbol{\xi}) + \widehat{\boldsymbol{Y}}^{gap}(\boldsymbol{\xi})\big)\right]^\top \boldsymbol{\alpha} + \big(\boldsymbol{\lambda} \odot \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})\big)^\top (\mathbf{1} - \boldsymbol{\alpha}) \\ &+ (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi}) - \boldsymbol{\lambda}_{EX}^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{H.25}$$

Substituting equation (H.24) into equation (H.25) and using the identity equations $\widetilde{\boldsymbol{\alpha}} = \mathbf{L}_{vx}\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\lambda}}_F = \boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx}$ yield

$$\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}\big(\widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{C}^{gap}(\boldsymbol{\xi})\big) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + (1 - \boldsymbol{\lambda}^\top \boldsymbol{\alpha} + \widetilde{\boldsymbol{\lambda}}_F^\top \boldsymbol{\alpha})\widehat{S}^{gap}(\boldsymbol{\xi})$$
$$- (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{H.26}$$

Rearranging it and using $\boldsymbol{\lambda} = \widetilde{\boldsymbol{\lambda}}_D + \widetilde{\boldsymbol{\lambda}}_F$ from equation (30) in Lemma 2 yields

$$(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\widehat{C}^{gap}(\boldsymbol{\xi}) = [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big(\widehat{S}^{gap}(\boldsymbol{\xi} - \widehat{P}_C^{gap}(\boldsymbol{\xi}))\big) \tag{H.27}$$
$$- (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \big(\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi})\big) + o(\|\widehat{\boldsymbol{\xi}}\|),$$

which is exactly equation (26) in Section 3.2.

Combining equation (H.17) in Section H.3 and the identity $\widetilde{\boldsymbol{\alpha}} = \mathbf{L}_{vx}\boldsymbol{\alpha}$ yields:

$$\widehat{\mathbf{P}}^{gap}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{gap}(\boldsymbol{\xi}) = \widetilde{\boldsymbol{\alpha}}\big(\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi}) + \widehat{P}_C^{gap}(\boldsymbol{\xi}) - \widehat{S}^{gap}(\boldsymbol{\xi})\big) + \mathbf{L}_{vx}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{H.28}$$

which is exactly equation (27) in Section 3.2.

Substituting equation (H.28) into equation (H.27), we obtain equation (H.23). □

H.6. *Proof of Theorem 1: aggregate output gap and sectoral markup wedges*

Substituting equation (H.21) from Appendix H.4 and equation (H.23) from Appendix H.5 into equation (H.20) from Appendix H.3 to eliminate the real wage gap $\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$ and real exchange rate gap $\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$, yields

$$\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big[\kappa_S + (1 - \kappa_S)(\sigma + \varphi/\Lambda_L)\big]\widehat{C}^{gap}(\boldsymbol{\xi})$$
$$= -\widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) - \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{ES}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \kappa_S \cdot \big[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big]^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

Using the following notation of grouped parameter

$$\kappa_C \equiv \kappa_S(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}) + \big[1 - \kappa_S(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big](\sigma + \varphi/\Lambda_L)$$

yields the following matrix form of equation (23) of Theorem 1:

$$\kappa_C \cdot \widehat{C}^{gap}(\boldsymbol{\xi}) = -\big\{\widetilde{\boldsymbol{\lambda}}_D + \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{ES} - \kappa_S \cdot \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big\}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|) = -\boldsymbol{\mathcal{M}}_{OG}^\top \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|).$$

□

H.7. Deriving terms of trade gap

Due to the trade balance, we can denote the steady-state share of sector i's exports in total exports and share of sector i's imports in total imports by

$$EX\_Share_i \equiv \frac{\theta_{F,i}}{\theta_{F,i}-1}\lambda_{EX,i} \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1}\lambda_{EX,i'} \Big), \quad \text{and}$$
$$IM\_Share_i \equiv \Big[\big(\sum_j \lambda_j \omega_{j,i} \left(1 - v_{x,j,i}\right) \big) + \beta_i \left(1 - v_i\right) \Big] \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1}\lambda_{EX,i'} \Big),$$

respectively. Then, we can define the terms of trade as:

$$ToT \equiv \frac{\prod_i P_{EX,i}^{EX\_Share_i}}{\prod_i \left(SP^*_{IM,Fi}\right)^{IM\_Share_i}}.$$

The terms of trade gap is equal to:

$$\begin{aligned}
\widehat{ToT}^{gap} &= \sum_i EX_Share_i \widehat{P}_i^{gap} - \sum_i IM_Share_i \widehat{S}^{gap} \\
&= \sum_i EX_Share_i \widehat{P}_i^{gap} - \sum_i EX_Share_i \widehat{S}^{gap} \\
&= \Big[\sum_i \frac{\theta_{F,i}}{\theta_{F,i}-1}\lambda_{EX,i} \left(\widehat{P}_i^{gap} - \widehat{S}^{gap} \right) \Big] \Big/ \Big(\sum_{i'} \frac{\theta_{F,i'}}{\theta_{F,i'}-1}\lambda_{EX,i'} \Big) \\
&= \left[(\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}))^\top \boldsymbol{\lambda}_{EX} \right]^{-1} (\boldsymbol{\theta}_F \oslash (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}}^{gap} - \mathbf{1}\widehat{S}^{gap}),
\end{aligned}$$

where the second quality comes from the trade balance.

H.8. Proof of Propositions D.1: Centralities and import shares

As preparation, we derive the partial derivatives of the Leontief inverse matrix $\mathbf{L}_{vx}$ with respect to the home bias in intermediate inputs, as in the following equation:

$$\frac{\partial \mathbf{L}_{vx}}{\partial v_{x,r,s}} = -\mathbf{L}_{vx} \frac{\partial \mathbf{L}_{vx}^{-1}}{\partial v_{x,r,s}} \mathbf{L}_{vx} = -\mathbf{L}_{vx} \frac{\partial (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)}{\partial v_{x,r,s}} \mathbf{L}_{vx} = \left\{ \ell_{vx,j,r} \omega_{r,s} \ell_{vx,s,i} \right\}_{j,i},$$

where $\left\{ \ell_{vx,j,r} \omega_{r,s} \ell_{vx,s,i} \right\}_{j,i}$ is the (j,i)-th element of the partial derivative matrix.

Because $\mathbf{L}_{vx} = (\mathbf{I} - \boldsymbol{\Omega} \odot \mathbf{V}_x)^{-1} = \mathbf{I} + \sum_{n=1}^{+\infty} (\boldsymbol{\Omega} \odot \mathbf{V}_x)^n$, $\omega_{j,i} \geq 0$ and $v_{x,j,i} \geq 0$ for all j and i, we have:

$$\ell_{vx,j,i} \begin{cases} > 0 & \forall\, j = i, \\ \geq 0 & \forall\, j \neq i. \end{cases}$$

Proof of Proposition D.1. According to $\widetilde{\boldsymbol{\lambda}}_D^\top \equiv (\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx}$ in equation (18) of Definition 4, the partial derivatives of the domestic supplier centrality in sector i ($\widetilde{\lambda}_{D,i}$) with respect to the import shares of consumption goods and intermediate inputs are as follows:

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_j)} = -\beta_j \ell_{vx,j,i}, \qquad \forall\, j, \tag{H.29}$$

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,s})} = -\Big(\sum_j \beta_j v_j \ell_{vx,j,r}\Big)\omega_{r,s}\ell_{vx,s,i} = -\widetilde{\lambda}_{D,r}\omega_{r,s}\ell_{vx,s,i}, \qquad \forall\, r, s. \tag{H.30}$$

Equation (H.29) implies that the domestic supplier centrality of sector i strictly decreases in its own import share of consumption—*viz*, $\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_i)} < 0$—if and only if $\beta_i > 0$, because $\ell_{vx,i,i} > 0$.

Equation (H.30) implies that, the domestic supplier centrality of sector i strictly decreases in its direct downstream sector r's import share of sector i's goods (i.e., $\omega_{r,i} > 0$ and $v_{x,r,i} > 0$), if and only if sector r, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,r} > 0$); that is,

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,r,i})} = -\widetilde{\lambda}_{D,r}\omega_{r,i}\ell_{vx,i,i} < 0.$$

Equation (H.30) also implies that, the domestic supplier centrality of sector i strictly decreases in its indirect downstream sector s's import share of sector r goods if and only if both of the following two conditions hold: (i) sector s, directly and indirectly, supplies to domestic aggregate output (i.e., $\sum_j \beta_j v_j \ell_{vx,j,s} > 0$); and (ii) sector i indirectly supplies inputs to sector s via sector r (i.e., $\omega_{s,r} > 0$ and $\ell_{vx,r,i} > 0$); that is,

$$\frac{\partial \widetilde{\lambda}_{D,i}}{\partial(1-v_{x,s,r})} = -\widetilde{\lambda}_{D,s}\omega_{s,r}\ell_{vx,r,i} < 0.$$

$\square$

H.9. Proof of Lemma 2: OG reduces to Domar weight in closed economies

Recall the expression of OG weights (23) in Theorem 1 in the following:

$$\boldsymbol{\mathcal{M}}_{OG} = \widetilde{\boldsymbol{\lambda}}_D + \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{ES} - \kappa_S \cdot \boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}}).$$

The centrality measures reduce to the following values in closed economies:

$$\widetilde{\boldsymbol{\lambda}}_D = \boldsymbol{\lambda}, \qquad \widetilde{\boldsymbol{\rho}}_{ES} = \mathbf{0}, \qquad \widetilde{\boldsymbol{\lambda}}_F = \mathbf{0}, \qquad \widetilde{\boldsymbol{\alpha}} = \mathbf{1},$$

which, substituted into the OG weights in equation (23) of Theorem 1, yields $\boldsymbol{\mathcal{M}}_{OG} = \boldsymbol{\lambda}$.

Multiplying both sides of equation (G.24) in Lemma G.2 by the Leontief inverse matrix $\mathbf{L}_{vx} \equiv (\mathbf{I} - \boldsymbol{\Omega} \odot$

$\mathbf{V}_x)^{-1}$, yields the following:

$$\boldsymbol{\lambda}_{EX}^\top \mathbf{L}_{vx} = \boldsymbol{\lambda}^\top - (\boldsymbol{\beta} \odot \mathbf{v})^\top \mathbf{L}_{vx} \qquad \Longleftrightarrow \qquad \widetilde{\boldsymbol{\lambda}}_F^\top = \boldsymbol{\lambda}^\top - \widetilde{\boldsymbol{\lambda}}_D^\top,$$

where the last equality holds due to definitions of domestic and foreign supplier centralities in equation (18). □

H.10. Output strictly increases in money supply

Lemma H.3 (aggregate output increases in money supply)**.** *In the* sticky-price equilibrium *where $\delta_i > 0$ for all $i \in \{1, 2, \cdots, N\}$, for any realized state $\boldsymbol{\xi} \in \Xi$, a rise in $\widehat{M}$ strictly increases $\widehat{C}(\boldsymbol{\xi})$ up to the first-order approximation.*

Proof of Lemma H.3. Up to the first-order approximation, given the shock to the money supply $\widehat{M}$, we have the following five conditions: (i) decomposition of CPI in equation (H.18):

$$\widehat{P}_C = (\boldsymbol{\beta} \odot \mathbf{v})^\top \widehat{\mathbf{P}} + [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})]^\top \mathbf{1} \widehat{S} + \boldsymbol{\Upsilon}_1^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(ii) the determination of the exchange rate in equation (H.27):

$$(1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})(\widehat{P}_C - \widehat{S} + \widehat{C}) = [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \widehat{\boldsymbol{\mu}} - (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top (\widehat{\mathbf{P}} - \mathbf{1}\widehat{S}) + \boldsymbol{\Upsilon}_2^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iii) the sectoral Phillips curves in equation (I.31):

$$\widehat{\mathbf{P}} = \boldsymbol{\mathcal{B}} \widehat{C} + \boldsymbol{\Upsilon}_3 \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|);$$

(iv) the relationship of sectoral markup wedges and inflation in equation (G.39):

$$\widehat{\boldsymbol{\mu}} = -(\boldsymbol{\Delta}^{-1} - \mathbf{I}) \widehat{\mathbf{P}} + o(\|\widehat{M}\|);$$

(v) the money demand equation (2.3):

$$\widehat{M} = \widehat{P}_C + \widehat{C}.$$

Combining the above five equations yields the following:

$$\widehat{C} = \frac{\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_P^\top \mathbf{1}}{(1 + \boldsymbol{\mathcal{M}}_P^\top \mathbf{1})[1 + (\boldsymbol{\beta} \odot \mathbf{v})^\top \boldsymbol{\mathcal{B}}] + (1 - \boldsymbol{\beta}^\top \mathbf{v}) \left[(\boldsymbol{\Delta}^{-1} - \mathbf{I}) \frac{\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})}{1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}} + \boldsymbol{\mathcal{M}}_P \right]^\top \boldsymbol{\mathcal{B}}} \widehat{M}$$
$$+ \boldsymbol{\Upsilon}^\top \widehat{\boldsymbol{\xi}} + o(\|\widehat{M}\|), \tag{H.31}$$

where vector $\boldsymbol{\Upsilon}$ is a linear combination of $\{\boldsymbol{\Upsilon}_i\}_{i=1,2,3}$ and $\boldsymbol{\mathcal{M}}_p \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})$. In particular, we need $\delta_i > 0$ for all $i \in \{1, 2, \cdots, N\}$ to ensure that the slopes $\boldsymbol{\mathcal{B}}$ of the sectoral Phillips curves will be finite. $\square$

H.11. Proof of Corollary C.1 under foreign-currency pricing

In this proof, we only show equilibrium equations under foreign-currency pricing that differ from those in the baseline model under producer-currency pricing.

With the definition of sectoral markup wedges of foreign-market products, we have the following pricing equation of sectoral foreign-market products:

$$\widehat{P}^*_{EX,i} = (\widehat{\Phi}_i - \widehat{S}) + \widehat{\mu}^*_{EX,i}. \tag{H.32}$$

Thus, the log deviation of the export demand function in equation (C.3) is equal to:

$$\widehat{Y}_{EX,i} = -\theta_{F,i}(\widehat{\Phi}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}. \tag{H.33}$$

Under our Calvo-pricing friction, the price of sectoral foreign-market product satisfies:

$$\widehat{P}^*_{EX,i} = \delta^*_{EX,i}(\widehat{\Phi}_i - \widehat{S}),$$

which, combined with equation (H.32), yields:

$$\widehat{\mu}^*_{EX,i} = -\frac{1 - \delta^*_{EX,i}}{\delta^*_{EX,i}} \widehat{P}^*_{EX,i}. \tag{H.34}$$

Under Assumption 1, the sectoral goods market clearing condition under foreign-currency pricing is the same as in the baseline model under PCP as follows:

$$P_i Y_i = \left(\frac{P_i}{P_{c,i}}\right)^{1-\theta_i} v_i \beta_i P_C C + \sum_j \left(\frac{P_i}{P_{x,j,i}}\right)^{1-\theta_i} \frac{v_{x,j,i}\omega_{j,i} P_j Y_j}{\mu_j} + P_i Y^*_{EX,i}. \tag{H.35}$$

However, log-linearizing equation (H.35) and combining it with the log linearization of the demand and pricing equations (H.32) and (H.33) of sectoral foreign-market products yields the following condition:

$$\begin{aligned}
\lambda_i(\widehat{P}_i + \widehat{Y}_i) = {}& \beta_i v_i \big[(\theta_i - 1)(1 - v_i)(\widehat{S} + \widehat{P}^*_{IM,Fi} - \widehat{P}_i) + \widehat{P}_C + \widehat{C}\big] \\
& + \sum_j \lambda_j \omega_{j,i} v_{x,j,i} \big[(\theta_i - 1)(1 - v_{x,j,i})(\widehat{S} + \widehat{P}^*_{IM,Fi} - \widehat{P}_i) + \widehat{P}_j + \widehat{Y}_j - \widehat{\mu}_j\big] \\
& + \lambda_{EX,i}\big[\widehat{P}_i - \theta_{F,i}(\widehat{P}_i - \widehat{\mu}_i - \widehat{S} + \widehat{\mu}^*_{EX,i}) + \widehat{D}^*_{EX,Fi}\big] + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}$$

which can be re-arranged and stacked into the following matrix form:

$$\begin{aligned}\left[\boldsymbol{\lambda}\odot(\widehat{\mathbf{P}}+\widehat{\mathbf{Y}})\right]^{\top} &= \widetilde{\boldsymbol{\lambda}}_D^{\top}(\widehat{P}_C+\widehat{C})-(\boldsymbol{\lambda}\odot\widehat{\boldsymbol{\mu}})^{\top}(\mathbf{L}_{vx}-\mathbf{I})+\left\{\boldsymbol{\lambda}_{EX}\widehat{S}-\left[\boldsymbol{\rho}_{ES}\odot(\widehat{\mathbf{P}}-\mathbf{1}\widehat{S})\right]\right\}^{\top}\mathbf{L}_{vx}\\ &\quad+\left\{\boldsymbol{\lambda}_{EX}\odot\widehat{\mathbf{D}}^{*}_{EX,F}+\left[\boldsymbol{\rho}_{ES}-(\boldsymbol{\theta}_F-\mathbf{1})\odot\boldsymbol{\lambda}_{EX}\right]\odot\widehat{\mathbf{P}}^{*}_{IM,F}\right\}^{\top}\mathbf{L}_{vx}\\ &\quad-\left[\boldsymbol{\theta}_F\odot\boldsymbol{\lambda}_{EX}\odot(\widehat{\boldsymbol{\mu}}^{*}_{EX}-\widehat{\boldsymbol{\mu}})\right]^{\top}\mathbf{L}_{vx}+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{H.36}$$

Equation (H.36) differs from its counterpart in the model under PCP (equation G.29 in Appendix G.4) in the last term $-\left[\boldsymbol{\theta}_F\odot\boldsymbol{\lambda}_{EX}\odot(\widehat{\boldsymbol{\mu}}^{*}_{EX}-\widehat{\boldsymbol{\mu}})\right]^{\top}\mathbf{L}_{vx}$ that represents exports.

Under foreign-currency pricing, the household's budget constraint is equal to:

$$\begin{aligned}P_C C &= WL+\Pi+T\\ &=\sum_i\Big\{\left[P_iY_{DM,i}-\Phi_iY_{DM,i}(1-\alpha_i)\right]+\left[SP^{*}_{EX,i}Y^{*}_{EX,i}-\Phi_iY^{*}_{EX,i}(1-\alpha_i)\right]\Big\}\\ &=\sum_i\Big\{\left[P_iY_{DM,i}-P_iY_{DM,i}(1-\alpha_i)/\mu_i\right]+\left[SP^{*}_{EX,i}Y^{*}_{EX,i}-P_iY^{*}_{EX,i}\left(1-\alpha_i\right)/\mu_i\right]\Big\}\\ &=\sum_i\left[P_iY_i\Big(1-\frac{1-\alpha_i}{\mu_i}\Big)+SP^{*}_{EX,i}Y^{*}_{EX,i}-P_iY^{*}_{EX,i}\right],\end{aligned}$$

the log-linearization of which—combined with equations (H.32) and (H.33)—yields:

$$\begin{aligned}\widehat{P}_C+\widehat{C} &=\sum_i\lambda_i\alpha_i\Big(\frac{1-\alpha_i}{\alpha_i}\widehat{\mu}_i+\widehat{P}_i+\widehat{Y}_i\Big)+\sum_i\lambda_{EX,i}\frac{\theta_{F,i}}{\theta_{F,i}-1}\left[\widehat{S}+(1-\theta_{F,i})(\widehat{\Phi}_i-\widehat{S}+\widehat{\mu}^{*}_{EX,i})+\widehat{D}^{*}_{EX,Fi}\right]\\ &\quad-\sum_i\lambda_{EX,i}\left[\widehat{P}_i-\theta_{F,i}(\widehat{\Phi}_i-\widehat{S}+\widehat{\mu}^{*}_{EX,i})+\widehat{D}^{*}_{EX,Fi}\right]+o(\|\widehat{\boldsymbol{\xi}}\|)\\ &=\sum_i\lambda_i\alpha_i\Big(\frac{1-\alpha_i}{\alpha_i}\widehat{\mu}_i+\widehat{P}_i+\widehat{Y}_i\Big)+\sum_i\lambda_{EX,i}\left[\frac{1}{\theta_{F,i}-1}\widehat{S}+\big(\widehat{S}-\widehat{P}_i\big)+\frac{1}{\theta_{F,i}-1}\widehat{D}^{*}_{EX,Fi}\right]+o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned}$$

which is the same as its counterpart in the baseline model (equation G.35 in Appendix G.5). Taking its matrix form, combining it with equation (H.36), and taking the difference of it between the sticky-price and flexible-price equilibria, yields the following log-linearization of the trade balance condition under foreign-currency pricing:

$$\begin{aligned}\big(1-\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha}\big)\widehat{C}^{gap} &=-\left(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX}\right)^{\top}\left(\widehat{\mathbf{P}}^{gap}-\mathbf{1}\widehat{S}^{gap}\right)+\left[\boldsymbol{\lambda}\odot(\mathbf{1}-\widetilde{\boldsymbol{\alpha}})\right]^{\top}\widehat{\boldsymbol{\mu}}\\ &\quad+\big(1-\widetilde{\boldsymbol{\lambda}}_D^{\top}\boldsymbol{\alpha}\big)\big(\widehat{S}^{gap}-\widehat{P}_C^{gap}\big)-\left(\boldsymbol{\theta}_F\odot\boldsymbol{\lambda}_{EX}\odot\widetilde{\boldsymbol{\alpha}}\right)^{\top}\left(\widehat{\boldsymbol{\mu}}^{*}_{EX}-\widehat{\boldsymbol{\mu}}\right)+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{H.37}$$

Substituting equations (H.28) into equation (H.37) to eliminate domestic-to-foreign price gaps,[9] and further substituting it and equation (H.21) from Appendix H.4 into equation (H.20) from Appendix H.3 to

[9] As in the baseline, in the derivations we have used the definition $\widetilde{\boldsymbol{\rho}}_{ES}\equiv(\boldsymbol{\rho}_{ES}\odot\widetilde{\boldsymbol{\alpha}}+\boldsymbol{\lambda}_{EX})^{\top}\mathbf{L}_{vx}$ to simplify the coefficients of sectoral markup wedges of domestic products $\widehat{\boldsymbol{\mu}}$.

eliminate the real wage gap $\widehat{W}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$ and real exchange rate gap $\widehat{S}^{gap}(\boldsymbol{\xi}) - \widehat{P}_C^{gap}(\boldsymbol{\xi})$, yields:

$$\begin{aligned}&\widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha}(\sigma + \varphi/\Lambda_L)\widehat{C}^{gap} + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})\big[\kappa_S + (1-\kappa_S)(\sigma + \varphi/\Lambda_L)\big]\widehat{C}^{gap} \\ &= -\widetilde{\boldsymbol{\lambda}}_D^\top \widehat{\boldsymbol{\mu}} - \kappa_S \cdot \widetilde{\boldsymbol{\rho}}_{ES}^\top \widehat{\boldsymbol{\mu}} + \kappa_S \cdot \big[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})\big]^\top \widehat{\boldsymbol{\mu}} - (\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}})^\top (\widehat{\boldsymbol{\mu}}_{EX}^* - \widehat{\boldsymbol{\mu}}) + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned}$$

which is exactly equation (C.4) in Corollary C.1. Substituting equations (G.39) and (H.34) into equation (C.4), we obtain equation (C.5) in Corollary C.1.

Throughout the above proof of Corollary C.1, only $-(\boldsymbol{\theta}_F \odot \boldsymbol{\lambda}_{EX} \odot \widetilde{\boldsymbol{\alpha}})^\top (\widehat{\boldsymbol{\mu}}_{EX}^* - \widehat{\boldsymbol{\mu}})$ in equation (H.36) deviates from the baseline model under PCP—i.e., the quantity of demand for sectoral exports depends on the markup wedge of sectoral foreign-market rather than domestic-market products in the expenditure-switching channel. Therefore, compared to the three channels in the baseline OG weight in equation (24), only the expenditure-switching channel changes under foreign-currency pricing, while the CPI and the profit channels remain dependent on sectoral domestic-market markup wedges and inflation as in the baseline model under PCP.

I. Proofs of the theoretical results in Section 4

This appendix derives the welfare loss up to the second-order approximation and the sectoral Phillips curves, from which we derive the analytical solution of the optimal monetary policy by solving a linear-quadratic programming problem.

I.1. Proof of Proposition 3: welfare loss up to the second-order approximation

Step 1*: Decompose the welfare loss into labor wedge and efficiency wedge components.* Approximating the utility function around the *flexible-price equilibrium* up to the second-order approximation yields:

$$u - u^{flex} = u_C^{flex} C^{flex}\Big[\widehat{C}^{gap} - \frac{\sigma - 1}{2}(\widehat{C}^{gap})^2\Big] + u_L^{flex} L^{flex}\Big[\widehat{L}^{gap} + \frac{\varphi + 1}{2}(\widehat{L}^{gap})^2\Big] + o(\|\widehat{\boldsymbol{\xi}}\|^2). \quad \text{(I.1)}$$

Substituting into equation (I.1) the optimality condition of labor supply $-u_L^{flex}/u_C^{flex} = W^{flex}/P_C^{flex}$, the approximation of labor share $\Lambda_L^{flex} \equiv (W^{flex}L^{flex})/(P_C^{flex}C^{flex}) = \Lambda_L + O(\|\widehat{\boldsymbol{\xi}}\|)$, and the approximation of the coefficient $u_C^{flex}C^{flex} = \big(C^{flex}\big)^{1-\sigma} = 1 + O(\|\widehat{\boldsymbol{\xi}}\|)$ under normalization $C^{ss} = 1$, yields:

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \widehat{C}^{gap}(\boldsymbol{\xi}) - \Lambda_L^{flex}(\boldsymbol{\xi})\widehat{L}^{gap}(\boldsymbol{\xi}) - \frac{1}{2}\big[(\sigma - 1)\widehat{C}^{gap}(\boldsymbol{\xi})^2 + \Lambda_L(\varphi + 1)\widehat{L}^{gap}(\boldsymbol{\xi})^2\big] + o(\|\widehat{\boldsymbol{\xi}}\|^2). \quad \text{(I.2)}$$

Combined with Definition B.1 and Proposition B.1 on efficiency and labor wedges, equation (I.2) becomes

$$u(\boldsymbol{\xi}) - u^{flex}(\boldsymbol{\xi}) = \underbrace{\widehat{A}_{agg}(\boldsymbol{\xi}) - \widehat{A}_{agg}^{flex}(\boldsymbol{\xi})}_{\text{efficiency wedge component}} \underbrace{- \frac{1}{2}\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]^{-1}\widehat{\Gamma}_L(\boldsymbol{\xi})^2}_{\text{labor wedge component}} + o(\|\widehat{\boldsymbol{\xi}}\|^2).$$

***Step 2**: Derive the second-order approximation of the labor wedge and introduce the equivalent economy.* Combining equation (B.5) in Proposition B.1 and equation (23) in Theorem 1 yields a quadratic form of the labor wedge component in terms of markup wedge $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$:

$$\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]^{-1}\widehat{\Gamma}_L(\boldsymbol{\xi})^2 = \kappa_C^{-2}\left[\sigma - 1 + (\varphi+1)/\Lambda_L\right]\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{M}}_{OG}^\top \boldsymbol{\mathcal{M}}_{OG}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(\|\widehat{\boldsymbol{\xi}}\|^2). \quad \text{(I.3)}$$

To facilitate the derivation of the efficiency wedge component, we construct an *equivalent economy* with sectoral markup wedges. For the *sticky-price equilibrium* under realized shocks $\widehat{\boldsymbol{\xi}}$, the *equivalent economy* satisfies all of the equilibrium conditions in Definition 1 except that in condition (ii), the markups of sticky-price firms, μ_{if}, are derived from $1-\delta_i+\delta_i\mu_{if}^{1-\theta_i} = \mu_i(\boldsymbol{\xi})^{1-\theta_i}$, where $\widehat{\mu}_i(\boldsymbol{\xi})$ is the markup wedge of sector i in the *sticky-price equilibrium*. Therefore, the constructed economy has *identical allocations, prices, and welfare loss* as the *sticky-price equilibirum* for any realized shock $\widehat{\boldsymbol{\xi}}$, and thus we refer to it as the *equivalent economy*. With slight abuse of notation, in the remainder of this subsection, we express the utility and other sector-level allocations and prices in the *equivalent economy* as functions of $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ and $\widehat{\boldsymbol{\xi}}$, using the same function names as in the *sticky-price equilibirum* (e.g., $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}})$ and $C(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}})$).

The *equivalent economy* enables us to express the welfare loss of the original economy as a function of only sectoral markup wedges, using the following lemma.

Lemma I.1. *Let $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ be the sectoral markup wedges in the* sticky-price equilibrium *under realized shocks $\widehat{\boldsymbol{\xi}}$. Up to the second-order approximation, the welfare loss in the* sticky-price equilibrium *under any shock $\widehat{\boldsymbol{\xi}}$ is equal to the welfare loss in the equivalent economy under the same sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ but absent of all shocks,* viz*,*

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}}) - u(\mathbf{0},\widehat{\boldsymbol{\xi}}) = u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0}) - u(\mathbf{0},\mathbf{0}) + o(||\widehat{\boldsymbol{\xi}}||^2) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(I.4)}$$

which is, therefore, a function of only sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$.

To prove Lemma I.1, consider the following second-order approximation of the welfare loss:

$$u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}}) - u(\mathbf{0},\widehat{\boldsymbol{\xi}}) = \widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\mu}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) + \widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\xi}\widehat{\boldsymbol{\xi}} + o(||\widehat{\boldsymbol{\xi}}||^2). \quad \text{(I.5)}$$

Because the allocation in the *flexible-price equilibrium* is the solution to the domestic social planner's problem, the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ and $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}}) \le u(\mathbf{0},\widehat{\boldsymbol{\xi}})$ for any realized shocks $\widehat{\boldsymbol{\xi}}$. First, because the welfare is maximized at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$, the derivative of the RHS of equation (I.5) with respect to $\widehat{\boldsymbol{\mu}}$ equals $\mathbf{0}$ at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = 0$ for any realized shocks $\widehat{\boldsymbol{\xi}}$, requiring $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$. Second, we also have $\boldsymbol{\mathcal{L}}^u_{\xi\xi} = \mathbf{0}$. Otherwise, there exists some realized shocks $\widehat{\boldsymbol{\xi}}$ such that the RHS of equation (I.5) is strictly positive or negative at $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}) = \mathbf{0}$ (i.e., $|\widehat{\boldsymbol{\xi}}^\top \boldsymbol{\mathcal{L}}^u_{\xi\xi}\widehat{\boldsymbol{\xi}}| > 0$), which contradicts $u(\mathbf{0},\widehat{\boldsymbol{\xi}}) - u(\mathbf{0},\widehat{\boldsymbol{\xi}}) = 0$. Therefore, we conclude that $\boldsymbol{\mathcal{L}}^u_{\xi\mu} = \mathbf{0}$ and $\boldsymbol{\mathcal{L}}^u_{\xi\xi} = \mathbf{0}$, and the RHS of equation (I.5) degenerates to $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})^\top \boldsymbol{\mathcal{L}}^u_{\mu\mu}\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$, which proves the second equality in equation (I.5) of Lemma I.1.

Based on Lemma I.1, we derive the original welfare loss $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}})-u(\mathbf{0},\widehat{\boldsymbol{\xi}})$ by deriving the equivalent $u(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-u(\mathbf{0},\mathbf{0})$ with the sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ resulting from shocks $\widehat{\boldsymbol{\xi}}$ in the *sticky-price equilibrium*. Particularly, because both the welfare loss in equation (I.4) and the labor wedge component in equation (I.3) are quadratic functions of only sectoral markup wedges $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$, the efficiency wedge—as the remaining component of the welfare loss—is also a quadratic form of only sectoral markup wedges. Therefore, we arrive at the following:

$$\begin{aligned}\widehat{A}_{agg}(\boldsymbol{\xi})-\widehat{A}^{flex}_{agg}(\boldsymbol{\xi})&=\widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\widehat{\boldsymbol{\xi}})-\widehat{A}_{agg}(\mathbf{0},\widehat{\boldsymbol{\xi}})=\widehat{A}_{agg}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\widehat{A}_{agg}(\mathbf{0},\mathbf{0})+o(||\widehat{\boldsymbol{\xi}}||^2)\\&=\widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\Lambda^{flex}_L(\mathbf{0})\widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})+o(||\widehat{\boldsymbol{\xi}}||^2)=\widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})+o(||\widehat{\boldsymbol{\xi}}||^2),\end{aligned}$$

where the first equality holds because the allocation in the sticky-price equilibrium that is free of markup wedges is equivalent to those in the flexible-price equilibrium, under the same exogenous shocks—i.e., $\widehat{A}^{flex}_{agg}(\boldsymbol{\xi})=\widehat{A}_{agg}(\mathbf{0},\widehat{\boldsymbol{\xi}})$.

For simplicity of notation, in the remainder of this subsection, we denote $\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi})$ by $\widehat{\boldsymbol{\mu}}$ and ignore the entry of $\mathbf{0}$ for any function in the *equivalent economy* with sectoral markup wedges $\widehat{\boldsymbol{\mu}}$ but no realized shocks—e.g., $\widehat{C}(\widehat{\boldsymbol{\mu}})-\Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}})\equiv\widehat{C}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})$.

With the above simplifying notation, for any variable x, we have $\widehat{x}(\mathbf{0})=\mathbf{0}$ when all sectoral markup wedges are set to zero to represent both the *flexible-price equilibrium* and the steady state, leading to $\widehat{x}(\widehat{\boldsymbol{\mu}})=\widehat{x}(\widehat{\boldsymbol{\mu}})-\widehat{x}(\mathbf{0})\equiv\widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\widehat{x}(\mathbf{0},\mathbf{0})$. Up to the first-order approximation, $\widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\mathbf{0})-\widehat{x}(\mathbf{0},\mathbf{0})=\widehat{x}(\widehat{\boldsymbol{\mu}}(\boldsymbol{\xi}),\boldsymbol{\xi})-\widehat{x}(\mathbf{0},\boldsymbol{\xi})+o(||\widehat{\boldsymbol{\xi}}||)=\widehat{x}^{gap}(\boldsymbol{\xi})+o(||\widehat{\boldsymbol{\xi}}||)$. Thus, in the remaining proof of this section, we replace $\widehat{x}^{gap}(\boldsymbol{\xi})$ with $\widehat{x}(\widehat{\boldsymbol{\mu}})$ whenever only first-order approximation is used.

We also introduce, for any variable x, the notation of $\widehat{\Delta}x$ that denotes the percentage deviation of x from its steady state, compared to the log deviation of x from its steady state $\widehat{x}$.

Step 3*: Derive the second-order approximation of the efficiency wedge component.* In the *equivalent economy* without realized shocks, we express the efficiency wedge component in terms of the percentage deviations of different variables from their steady states as follows:

$$\widehat{C}(\widehat{\boldsymbol{\mu}})-\Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}})=\widehat{\Delta}C(\widehat{\boldsymbol{\mu}})-\Lambda_L\widehat{\Delta}L(\widehat{\boldsymbol{\mu}})+\frac{1}{2}\Lambda_L(1-\Lambda_L)\widehat{L}(\widehat{\boldsymbol{\mu}})^2+o(||\widehat{\boldsymbol{\mu}}||^2). \tag{I.6}$$

The equivalent economy satisfies the conditions (G.1)-(G.7) of the *feasible allocation* in Definition G.1. Therefore, the terms in equation (I.6) satisfy the following equations up to the second-order approximation:

$$\widehat{\Delta}C(\widehat{\boldsymbol{\mu}})=\sum_{i=1}^n\beta_i\widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}})-\frac{1}{2}\sum_{i=1}^n\beta_i\big(\widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}})-\widehat{\Delta}C(\widehat{\boldsymbol{\mu}})\big)^2+o(||\widehat{\boldsymbol{\mu}}||^2), \tag{I.7}$$

$$\widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}})=\widehat{\iota}_i(\widehat{\boldsymbol{\mu}})+\alpha_i\widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}})+\sum_{j=1}^n\omega_{i,j}\widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) \tag{I.8}$$

$$- \frac{1}{2}\Big[\alpha_i\big(\widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}})\big)^2 + \sum_{j=1}^{n}\omega_{i,j}\big(\widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}})\big)^2\Big] + o(||\widehat{\boldsymbol{\mu}}||^2),$$

$$\widehat{\Delta}C_i(\widehat{\boldsymbol{\mu}}) = v_i\widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) + (1-v_i)\widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}}) - \frac{v_i(1-v_i)}{2\theta_i}\big(\widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}})\big)^2 + o(||\widehat{\boldsymbol{\mu}}||^2), \tag{I.9}$$

$$\widehat{\Delta}X_{i,j}(\widehat{\boldsymbol{\mu}}) = v_{x,i,j}\widehat{\Delta}X_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) + (1-v_{x,i,j})\widehat{\Delta}X_{Hi,Fj}(\widehat{\boldsymbol{\mu}}) - \frac{v_{x,i,j}(1-v_{x,i,j})}{2\theta_j}\big(\widehat{\Delta}X_{Hi,Hj}(\widehat{\boldsymbol{\mu}}) - \widehat{\Delta}X_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big)^2 + o(||\widehat{\boldsymbol{\mu}}||^2), \tag{I.10}$$

$$\Lambda_L\widehat{\Delta}L(\widehat{\boldsymbol{\mu}}) = \sum_{i=1}^{n}\lambda_i\alpha_i\widehat{\Delta}L_i(\widehat{\boldsymbol{\mu}}), \tag{I.11}$$

$$\lambda_i\widehat{\Delta}Y_i(\widehat{\boldsymbol{\mu}}) = \beta_i v_i\widehat{\Delta}C_{Hi}(\widehat{\boldsymbol{\mu}}) + \sum_{j=1}^{n}\lambda_j\omega_{j,i}v_{x,j,i}\widehat{\Delta}X_{Hj,Hi}(\widehat{\boldsymbol{\mu}}) + \lambda_{EX,i}\widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}}), \tag{I.12}$$

$$\begin{aligned}\lambda_{EX}\widehat{\Delta}EX(\widehat{\boldsymbol{\mu}}) &= \sum_{i=1}^{n}\lambda_{EX,i}\widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}}) - \frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_{EX,i}}{\theta_{F,i}}\widehat{\Delta}Y_{EX,i}(\widehat{\boldsymbol{\mu}})^2 + o(||\widehat{\boldsymbol{\mu}}||^2) \\ &= \sum_{i=1}^{n}\Big[\beta_i(1-v_i)\widehat{\Delta}C_{Fi}(\widehat{\boldsymbol{\mu}}) + \sum_{j=1}^{n}\lambda_j\omega_{j,i}(1-v_{x,j,i})\widehat{\Delta}X_{Hj,Fi}(\widehat{\boldsymbol{\mu}})\Big].\end{aligned} \tag{I.13}$$

Combining equations (I.6)-(I.13) eliminates all first-order terms following the same proof of Proposition B.1, and further applying equality $\widehat{\Delta}x(\widehat{\boldsymbol{\mu}}) = \widehat{x}(\widehat{\boldsymbol{\mu}}) + o(||\widehat{\boldsymbol{\mu}}||)$ to all square terms yields:

$$\begin{aligned}\widehat{C}(\widehat{\boldsymbol{\mu}}) - \Lambda_L\widehat{L}(\widehat{\boldsymbol{\mu}}) = &\left.-\sum_{i=1}^{n}\lambda_i\widehat{\iota}_i(\widehat{\boldsymbol{\mu}})\right\}\text{within-sector misallocation} \\ &\left.\begin{aligned}&-\frac{1}{2}\sum_{i=1}^{n}\beta_i\big[\widehat{C}_i(\widehat{\boldsymbol{\mu}}) - \widehat{C}(\widehat{\boldsymbol{\mu}})\big]^2 \\ &-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\alpha_i\big[\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2 \\ &-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\omega_{i,j}\big[\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2\end{aligned}\right\}\text{across-sector misallocation}\end{aligned} \tag{I.14}$$

$$
\left.\begin{aligned}
&-\frac{1}{2}\sum_{i=1}^{n}\frac{\beta_i}{\theta_i}v_i(1-v_i)\big[\widehat{C}_{Hi}(\widehat{\boldsymbol{\mu}})-\widehat{C}_{Fi}(\widehat{\boldsymbol{\mu}})\big]^2\\
&-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\lambda_i\omega_{i,j}}{\theta_j}v_{x,i,j}(1-v_{x,i,j})\big[\widehat{X}_{Hi,Hj}(\widehat{\boldsymbol{\mu}})-\widehat{X}_{Hi,Fj}(\widehat{\boldsymbol{\mu}})\big]^2\\
&-\frac{1}{2}\sum_{i=1}^{n}\frac{\lambda_{EX,i}}{\theta_{F,i}}\widehat{Y}_{EX,i}(\widehat{\boldsymbol{\mu}})^2\\
&+\frac{1}{2}\Lambda_L(1-\Lambda_L)\widehat{L}(\widehat{\boldsymbol{\mu}})^2
\end{aligned}\right\}\text{cross-border misallocation}
$$

$$
+\,o(\|\widehat{\boldsymbol{\mu}}\|^2).
$$

The within-sector misallocation has the same expression as in Rubbo (2023)—i.e.

$$
-\sum_{i=1}^{n}\lambda_i\widehat{\iota}_i(\widehat{\boldsymbol{\mu}})=-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2+o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{I.15}
$$

Replacing $\widehat{x}(\widehat{\boldsymbol{\mu}})$ with $\widehat{x}^{gap}(\boldsymbol{\xi})$ for all variables x in equation (I.14) and combining it with equations (I.15), (G.38), and (G.39) yield the RHS of equations (33), (34), and (35) in Proposition 3, which completes the main part of the proof.

***Step 4**: Express the efficiency wedge component in square terms of sectoral inflation.* Combining equation (H.20) with equation (H.21) in Lemma H.4 yields:

$$
\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})=\frac{(\sigma+\varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi})+\sum_{k=1}^{n}\widetilde{\lambda}_{D,k}\widehat{\mu}_k(\boldsymbol{\xi})}{1-\sum_k\widetilde{\lambda}_{D,k}\alpha_k}+o(\|\widehat{\boldsymbol{\xi}}\|).
$$

The scalar form of equation (27) implies that:

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})=\widetilde{\alpha}_i\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}\ell_{vx,i,k}\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.16}
$$

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})=-(1-\widetilde{\alpha}_i)\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}\ell_{vx,i,k}\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.17}
$$

$$
\widehat{P}_i^{gap}(\boldsymbol{\xi})-\widehat{P}_j^{gap}(\boldsymbol{\xi})=(\widetilde{\alpha}_i-\widetilde{\alpha}_j)\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+\sum_{k=1}^{n}(\ell_{vx,i,k}-\ell_{vx,j,k})\widehat{\mu}_k(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|). \tag{I.18}
$$

Denote the consumer price of sector i goods by

$$
P_{ci}\equiv\Big(v_iP_i^{1-\theta_i}+(1-v_i)(S\cdot P^*_{IM,Fi})^{1-\theta_i}\Big)^{\frac{1}{1-\theta_i}}.
$$

Using equation (H.21) in Lemma H.4, the difference between the gaps of the consumer price of sector i goods and the CPI is equal to:

$$\begin{aligned}\widehat{P}_{ci}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi})&=v_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(1-v_i)\big(\widehat{S}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi}))+o(\|\widehat{\boldsymbol{\xi}}\|)\\&=v_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(1-v_i)\big(\widehat{S}^{gap}(\boldsymbol{\xi})-\widehat{W}^{gap}(\boldsymbol{\xi})\big)+(\sigma+\varphi/\Lambda_L)\widehat{C}^{gap}(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned} \tag{I.19}$$

We can replace the terms of allocation gaps on the RHS of equations (34) and (35) in Proposition 3 with the relative price gaps on the LHS of equations (I.16)-(I.19) according to the following equations:

$$\begin{aligned}\widehat{C}_{i}^{gap}(\boldsymbol{\xi})-\widehat{C}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{P}_{ci}^{gap}(\boldsymbol{\xi})-\widehat{P}_{C}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{L}_{i}^{gap}(\boldsymbol{\xi})-\widehat{Y}_{i}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{W}^{gap}(\boldsymbol{\xi})-\widehat{P}_{i}^{gap}(\boldsymbol{\xi})\big)-\widehat{\mu}_i(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{X}_{i,j}^{gap}(\boldsymbol{\xi})-\widehat{Y}_{i}^{gap}(\boldsymbol{\xi})&=-\big(\widehat{P}_{j}^{gap}(\boldsymbol{\xi})-\widehat{P}_{i}^{gap}(\boldsymbol{\xi})\big)-\widehat{\mu}_i(\boldsymbol{\xi})+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{C}_{Hi}^{gap}(\boldsymbol{\xi})-\widehat{C}_{Fi}^{gap}(\boldsymbol{\xi})&=-\theta_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{X}_{Hi,Hj}^{gap}(\boldsymbol{\xi})-\widehat{X}_{Hi,Fj}^{gap}(\boldsymbol{\xi})&=-\theta_i\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|),\\\widehat{Y}_{EX,i}^{gap}(\boldsymbol{\xi})&=-\theta_{Fi}\big(\widehat{P}_{i}^{gap}(\boldsymbol{\xi})-\widehat{S}^{gap}(\boldsymbol{\xi})\big)+o(\|\widehat{\boldsymbol{\xi}}\|).\end{aligned}$$

Further combining the above equation with equation (B.4) in Proposition B.1, equation (23) in Theorem 1, and equation (G.39), we can express each of the RHS of equations (33), (34), and (35) in Proposition 3 as a square term of sectoral inflation—i.e., $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^{\top}\boldsymbol{\mathcal{L}}^{within}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^{\top}\boldsymbol{\mathcal{L}}^{across}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, and $-\frac{1}{2}\widehat{\mathbf{P}}(\boldsymbol{\xi})^{\top}\boldsymbol{\mathcal{L}}^{cb}\widehat{\mathbf{P}}(\boldsymbol{\xi})$, respectively, which are the LHS of equations (33), (34), and (35) in Proposition 3. □

Efficiency wedge component of welfare loss in closed economies. In closed economies á la La'O and Tahbaz-Salehi (2022) and Rubbo (2023), $v_i=v_{x,i,j}=\Lambda_L=1$, $\lambda_{EX,i}=0$, $\mathcal{M}_{OG,i}=\widetilde{\lambda}_{D,i}=\lambda_i$, and $\ell_{vx,i,j}$ reduces to $\ell_{i,j}$. The cross-border misallocation disappears, and equation (I.14) reduces to the following expression:

$$\begin{aligned}\widehat{C}(\widehat{\boldsymbol{\mu}})-\widehat{L}(\widehat{\boldsymbol{\mu}})=&-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\varepsilon_i\frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2-\frac{1}{2}\sum_{i=1}^{n}\beta_i\big[\widehat{C}_i(\widehat{\boldsymbol{\mu}})-\widehat{C}(\widehat{\boldsymbol{\mu}})\big]^2\\&-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\alpha_i\big[\widehat{L}_i(\widehat{\boldsymbol{\mu}})-\widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda_i\omega_{i,j}\big[\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}})-\widehat{Y}_i(\widehat{\boldsymbol{\mu}})\big]^2+o(\|\widehat{\boldsymbol{\mu}}\|^2).\end{aligned}$$

The mappings from sectoral markup wedges into allocations in the *equivalent economy* reduce to:

$$\widehat{C}_i(\widehat{\boldsymbol{\mu}})-\widehat{C}(\widehat{\boldsymbol{\mu}})=\widehat{P}_C(\widehat{\boldsymbol{\mu}})-\widehat{P}_i(\widehat{\boldsymbol{\mu}})=\sum_{k=1}^{n}(\lambda_k-\ell_{i,k})\widehat{\mu}_k+o(\|\widehat{\boldsymbol{\mu}}\|),$$

$$\widehat{L}_i(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = \widehat{P}_i(\widehat{\boldsymbol{\mu}}) - \widehat{W}(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_i = \sum_{k=1}^{n} \ell_{i,k}\widehat{\mu}_k - \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|),$$
$$\widehat{X}_{i,j}(\widehat{\boldsymbol{\mu}}) - \widehat{Y}_i(\widehat{\boldsymbol{\mu}}) = \widehat{P}_i(\widehat{\boldsymbol{\mu}}) - \widehat{P}_j(\widehat{\boldsymbol{\mu}}) - \widehat{\mu}_i = \sum_{k=1}^{n} (\ell_{i,k} - \ell_{j,k})\widehat{\mu}_k - \widehat{\mu}_i + o(\|\widehat{\boldsymbol{\mu}}\|).$$

Accordingly, we derive the same efficiency wedge component of welfare loss for closed economies as in Rubbo (2023)—i.e.,

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}}) = -\frac{1}{2}\sum_{i=1}^{n} \lambda_i \varepsilon_i \frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 - \frac{1}{2}\sum_{i=1}^{n} \lambda_i \widehat{\mu}_i^2 - \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \ell_{i,j}\widehat{\mu}_i\widehat{\mu}_j + \frac{1}{2}\Big(\sum_{i=1}^{n} \lambda_i \widehat{\mu}_i\Big)^2 + o(\|\widehat{\boldsymbol{\mu}}\|^2). \tag{I.20}$$

We further follow La'O and Tahbaz-Salehi (2022) to introduce the pricing error $\{\overline{e}_i\}_i$ and link them to sectoral markup wedges as below:

$$\overline{e}_i = \sum_{j=1}^{n} \ell_{i,j}\widehat{\mu}_j, \qquad \widehat{\mu}_i = \overline{e}_i - \sum_{j=1}^{n} \omega_{i,j}\overline{e}_j, \qquad \sum_j \beta_j \overline{e}_j = \sum_{i=1}^{n} \lambda_i \widehat{\mu}_i. \tag{I.21}$$

Combining equations (I.21) and (I.20), we derive the same efficiency wedge component of welfare loss for closed economies as in Rubbo (2023)—i.e.,

$$\widehat{C}(\widehat{\boldsymbol{\mu}}) - \widehat{L}(\widehat{\boldsymbol{\mu}}) = -\frac{1}{2}\sum_{i=1}^{n} \lambda_i \varepsilon_i \frac{\delta_i}{1-\delta_i}\widehat{\mu}_i^2 - \frac{1}{2} xvar_0(\overline{\mathbf{e}}) + \frac{1}{2}\sum_{i=1}^{n} \lambda_i \, xvar_i(\overline{\mathbf{e}}) + o(\|\widehat{\boldsymbol{\mu}}\|^2),$$

where $xvar_0(\overline{\mathbf{e}})$ and $xvar_i(\overline{\mathbf{e}})$ are the same short-hand notations as in La'O and Tahbaz-Salehi (2022)—i.e.,

$$xvar_0(\overline{\mathbf{e}}) = \sum_{j=1}^{n} \beta_j \overline{e}_j^2 - \Big(\sum_{j=1}^{n} \beta_j \overline{e}_j\Big)^2, \quad \text{and}$$
$$xvar_i(\overline{\mathbf{e}}) = \sum_{j=1}^{n} \omega_{i,j} \overline{e}_j^2 - \Big(\sum_{j=1}^{n} \omega_{i,j} \overline{e}_j\Big)^2, \quad \text{for } i \in \{1, 2, \cdots, n\}.$$

I.2. Proof of Proposition 4: Sectoral Phillips curves

Step 1: Derive $\widehat{S}$ and $\widehat{P}_C$ as functions of $\{\widehat{C}, \widehat{\mathbf{P}}, \widehat{\boldsymbol{\xi}}\}$. Following every step in the proof of Lemma H.2 in Appendix H.5—except for the *sticky-price equilibrium* instead of for the difference between the *sticky-price* and *flexible-price equilibria*—yields:

$$\begin{aligned}
&\big[1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha} + (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \mathbf{1}\big] \widehat{S}(\boldsymbol{\xi}) \\
&= (\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + [\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})]^\top \boldsymbol{\Delta}^{-1}(\mathbf{I} - \boldsymbol{\Delta})\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})(\widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi})) \\
&- [\widetilde{\boldsymbol{\lambda}}_F \odot \boldsymbol{\alpha} + \boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} - (\boldsymbol{\rho}_{IM} \odot \widetilde{\boldsymbol{\alpha}})^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned} \tag{I.22}$$

where $\boldsymbol{\rho}_{IM} \equiv \boldsymbol{\rho}_{ES} - (\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$ is the elasticity of sectoral imports to import price shocks, which is equal to the net export elasticity $\boldsymbol{\rho}_{EX}$ diminished by the export component $(\boldsymbol{\theta}_F - \mathbf{1}) \odot \boldsymbol{\lambda}_{EX}$.

Rearranging equation (I.22) and introducing shorthand notations yield:

$$\begin{aligned}(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}\widehat{S}(\boldsymbol{\xi}) = \widehat{P}_C(\boldsymbol{\xi}) + \widehat{C}(\boldsymbol{\xi}) + (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu)^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) \\ - (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top \widehat{\mathbf{D}}^*_{EX,F} - \boldsymbol{\mathcal{M}}_{IM}^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),\end{aligned} \tag{I.23}$$

where the shorthand notations are as follows:

$$\boldsymbol{\mathcal{M}}_{EX} \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}[\widetilde{\boldsymbol{\lambda}}_F \odot \boldsymbol{\alpha} + \boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})] \odot \boldsymbol{\theta}_F, \tag{I.24}$$

$$\boldsymbol{\mathcal{M}}_{IM} \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{IM} \odot \widetilde{\boldsymbol{\alpha}}), \tag{I.25}$$

$$\boldsymbol{\mathcal{M}}_p \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\rho}_{ES} \odot \widetilde{\boldsymbol{\alpha}} + \boldsymbol{\lambda}_{EX})$$

$$\boldsymbol{\mathcal{M}}_\mu \equiv (1 - \widetilde{\boldsymbol{\lambda}}_D^\top \boldsymbol{\alpha})^{-1}(\boldsymbol{\Delta}^{-1} - \mathbf{I})[\boldsymbol{\lambda} \odot (\mathbf{1} - \widetilde{\boldsymbol{\alpha}})].$$

According to equation (G.38) in Appendix G.5, $1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1} = (\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}$.

Substituting equation (H.18) in Appendix H.3 into equation (I.23), yield the following $\widehat{S}$ and $\widehat{P}_C$ as functions of $\{\widehat{C}, \widehat{\mathbf{P}}, \widehat{\boldsymbol{\xi}}\}$:

$$\widehat{S}(\boldsymbol{\xi}) = \Gamma_{S,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,P}^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{S,EX}^\top \widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\Gamma}_{S,IM}^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.26}$$

$$\widehat{P}_C(\boldsymbol{\xi}) = \Gamma_C\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_P^\top \widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{EX}^\top \widehat{\mathbf{D}}^*_{EX} + \boldsymbol{\Gamma}_{IM}^\top \widehat{\mathbf{P}}^*_{IM} + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{I.27}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\Gamma_{S,C} &\equiv (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}, \\
\boldsymbol{\Gamma}_{S,P} &\equiv \Gamma_{S,C} \cdot (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu) + \Gamma_{S,C} \cdot (\boldsymbol{\beta} \odot \mathbf{v}), \\
\boldsymbol{\Gamma}_{S,EX} &\equiv -\Gamma_{S,C} \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F), \\
\boldsymbol{\Gamma}_{S,IM} &\equiv -\Gamma_{S,C} \cdot \boldsymbol{\mathcal{M}}_{IM} + \Gamma_{S,C} \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})], \\
\Gamma_C &\equiv \Gamma_{S,C} \cdot (1 - \boldsymbol{\beta}^\top \mathbf{v}) = (\boldsymbol{\beta}^\top \mathbf{v} + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1})^{-1}(1 - \boldsymbol{\beta}^\top \mathbf{v}), \\
\boldsymbol{\Gamma}_P &\equiv \Gamma_C \cdot (\boldsymbol{\mathcal{M}}_p + \boldsymbol{\mathcal{M}}_\mu) + \Gamma_{S,C} \cdot (\boldsymbol{\beta} \odot \mathbf{v})(1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}), \\
\boldsymbol{\Gamma}_{EX} &\equiv -\Gamma_C \cdot (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F), \\
\boldsymbol{\Gamma}_{IM} &\equiv -\Gamma_C \cdot \boldsymbol{\mathcal{M}}_{IM} + \Gamma_{S,C} \cdot [\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v})](1 + \boldsymbol{\mathcal{M}}_p^\top \mathbf{1}).
\end{aligned} \tag{I.28}$$

In particular, we have $\Gamma_{S,C} = [(\boldsymbol{\mathcal{M}}_{EX} + \boldsymbol{\mathcal{M}}_{IM})^\top \mathbf{1}]^{-1}(\Gamma_C + 1)$.

Step 2: Derive $\widehat{W}$ as a function of $\{\widehat{\mathbf{P}}, \widehat{C}, \widehat{\boldsymbol{\xi}}\}$. Substituting $\widehat{P}_C$ in equation (I.27) and $\widehat{L}$ in equation (H.10) into the labor supply equation (H.22), yields:

$$\widehat{W}(\boldsymbol{\xi}) = \Gamma_{W,C}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{W,P}^{\top}\widehat{\mathbf{P}}(\boldsymbol{\xi}) + \boldsymbol{\Gamma}_{W,A}^{\top}\widehat{\mathbf{A}} + \boldsymbol{\Gamma}_{W,EX}^{\top}\widehat{\mathbf{D}}_{EX,F}^{*} + \boldsymbol{\Gamma}_{W,IM}^{\top}\widehat{\mathbf{P}}_{IM,F}^{*} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.29}$$

where the shorthand notations are

$$\begin{aligned}
\Gamma_{W,C} &\equiv \sigma + \frac{\varphi}{\Lambda_L} + \Gamma_C, \qquad \boldsymbol{\Gamma}_{W,P} \equiv \boldsymbol{\Gamma}_P, \qquad \boldsymbol{\Gamma}_{W,A} \equiv -\frac{\varphi}{\Lambda_L}\boldsymbol{\lambda}, \\
\boldsymbol{\Gamma}_{W,EX} &\equiv \boldsymbol{\Gamma}_{EX} - \frac{\varphi}{\Lambda_L}[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})], \\
\boldsymbol{\Gamma}_{W,IM} &\equiv \boldsymbol{\Gamma}_{IM} + \frac{\varphi}{\Lambda_L}\big[\boldsymbol{\beta} \odot (\mathbf{1} - \mathbf{v}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})^{\top}\boldsymbol{\lambda}\big].
\end{aligned}$$

Step 3: Substitute $\widehat{W}$ and $\widehat{S}$ into sectoral pricing equation. Substituting the sectoral marginal costs in equation (H.15) into the sectoral inflation in equation (G.40) yields the following pricing equation:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\Delta}\big[\boldsymbol{\alpha}\widehat{W}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_x)\widehat{\mathbf{P}}(\boldsymbol{\xi}) + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big(\mathbf{1}\widehat{S}(\boldsymbol{\xi}) + \widehat{\mathbf{P}}_{IM,F}^{*}\big) - \widehat{\mathbf{A}}\big] + o(\|\widehat{\boldsymbol{\xi}}\|). \tag{I.30}$$

Substituting $\widehat{W}$ and $\widehat{S}$ in equations (I.29) and (I.26) into the pricing equation (I.30) yields the following sectoral Phillips curves in terms of $\widehat{C}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{C,EX}\widehat{\mathbf{D}}_{EX,F}^{*} + \boldsymbol{\mathcal{V}}_{C,IM}\widehat{\mathbf{P}}_{IM,F}^{*} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.31}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\boldsymbol{\mathcal{B}} &\equiv \boldsymbol{\Delta}_{\Phi}\big[\boldsymbol{\alpha}\Gamma_{W,C} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\Gamma_{S,C}\big], \\
\boldsymbol{\mathcal{V}}_{C,A} &\equiv \boldsymbol{\Delta}_{\Phi}\big(\boldsymbol{\alpha}\boldsymbol{\Gamma}_{W,A}^{\top} - \mathbf{1}\big), \\
\boldsymbol{\mathcal{V}}_{C,EX} &\equiv \boldsymbol{\Delta}_{\Phi}\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}_{W,EX}^{\top} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}_{S,EX}^{\top}\big], \\
\boldsymbol{\mathcal{V}}_{C,IM} &\equiv \boldsymbol{\Delta}_{\Phi}\big[\boldsymbol{\alpha}\boldsymbol{\Gamma}_{W,IM}^{\top} + (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}_{S,IM}^{\top}\big], \\
\boldsymbol{\Delta}_{\Phi} &\equiv \big[\boldsymbol{\Delta}^{-1} - \boldsymbol{\Omega} \odot \mathbf{V}_x - \boldsymbol{\alpha}\boldsymbol{\Gamma}_{W,P}^{\top} - (\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\mathbf{1}\boldsymbol{\Gamma}_{S,P}^{\top}\big]^{-1}.
\end{aligned}$$

To derive further the sectoral Phillips curves in terms of the aggregate output gap $\widehat{C}^{gap}$, we need to solve for the log deviation of the aggregate output in the *flexible-price equilibrium* from the steady state, denoted by $\widehat{C}^{flex}(\boldsymbol{\xi})$. To do so, we derive the flexible-price version of equations (I.30), (H.22), (I.23), (H.10), and (H.18) by setting $\boldsymbol{\Delta} = \mathbf{I}$, which yields the following equations, respectively:

$$\begin{aligned}
\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi}) &= \widetilde{\boldsymbol{\alpha}}(\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi})) - \mathbf{L}_{vx}\widehat{\mathbf{A}} + \mathbf{L}_{vx}(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\widehat{\mathbf{P}}_{IM,F}^{*} + o(\|\widehat{\boldsymbol{\xi}}\|), \\
\widehat{W}^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) &= \widehat{P}_C^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \sigma\widehat{C}^{flex}(\boldsymbol{\xi}) + \varphi\widehat{L}^{flex}(\boldsymbol{\xi}), \\
\widehat{P}_C^{flex}(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) + \widehat{C}^{flex}(\boldsymbol{\xi}) &= -\boldsymbol{\mathcal{M}}_P^{\top}(\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^{\top}\widehat{\mathbf{D}}_{EX,F}^{*} + \boldsymbol{\mathcal{M}}_{IM}^{\top}\widehat{\mathbf{P}}_{IM,F}^{*} + o(\|\widehat{\boldsymbol{\xi}}\|),
\end{aligned}$$

$$\widehat{C}^{flex}(\boldsymbol{\xi}) - \Lambda_L \widehat{L}^{flex}(\boldsymbol{\xi}) = \boldsymbol{\lambda}^\top \widehat{A} + [\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \widehat{\mathbf{D}}^*_{EX,F} - \big[\boldsymbol{\beta}^\top \odot (\mathbf{1}-\mathbf{v})^\top + \boldsymbol{\lambda}^\top(\boldsymbol{\Omega} \odot \mathbf{V}_{1-x})\big] \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|),$$
$$\widehat{P}^{flex}_C(\boldsymbol{\xi}) - \widehat{S}^{flex}(\boldsymbol{\xi}) = (\boldsymbol{\beta} \odot \mathbf{v})^\top (\widehat{\mathbf{P}}^{flex}(\boldsymbol{\xi}) - \mathbf{1}\widehat{S}^{flex}(\boldsymbol{\xi})) + [\boldsymbol{\beta} \odot (\mathbf{1}-\mathbf{v})]^\top \widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|).$$

Combining the above five equations yields:

$$\widehat{C}^{flex}(\boldsymbol{\xi}) = \boldsymbol{\Gamma}^{flex}_{C,A}\widehat{\mathbf{A}} + \boldsymbol{\Gamma}^{flex}_{C,EX}\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\Gamma}^{flex}_{C,IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.32}$$

where the shorthand notations are as follows:

$$\begin{aligned}
\boldsymbol{\Gamma}^{flex}_{C,A} &\equiv (\Delta^{flex}_C)^{-1}\big(\boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}\boldsymbol{\lambda}^\top \varphi/\Lambda_L + \boldsymbol{\mathcal{M}}_L^\top \mathbf{L}_{vx}\big),\\
\boldsymbol{\Gamma}^{flex}_{C,IM} &\equiv -(\Delta^{flex}_C)^{-1}\Big\{\boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}\big[(\Lambda_L+\varphi)\boldsymbol{\beta}^\top \odot (\mathbf{1}-\mathbf{v})^\top + \boldsymbol{\lambda}^\top(\boldsymbol{\Omega}\odot \mathbf{V}_{1-x})\big]/\Lambda_L\\
&\qquad + \boldsymbol{\mathcal{M}}_L^\top \mathbf{L}_{vx}(\boldsymbol{\Omega}\odot \mathbf{V}_{1-x}) - \boldsymbol{\mathcal{M}}_{IM}^\top + \boldsymbol{\beta}^\top \odot (\mathbf{1}-\mathbf{v})^\top\Big\},\\
\boldsymbol{\Gamma}^{flex}_{C,EX} &\equiv (\Delta^{flex}_C)^{-1}\big\{\boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}[\boldsymbol{\lambda}_{EX} \oslash (\boldsymbol{\theta}_F - \mathbf{1})]^\top \varphi/\Lambda_L + (\boldsymbol{\mathcal{M}}_{EX} \oslash \boldsymbol{\theta}_F)^\top\big\},\\
\boldsymbol{\mathcal{M}}_L^\top &\equiv (\boldsymbol{\mathcal{M}}_P + \boldsymbol{\beta}\odot \mathbf{v})^\top\big[\mathbf{I} - \widetilde{\boldsymbol{\alpha}}(\boldsymbol{\beta}\odot\mathbf{v})^\top\big]^{-1},\\
\Delta^{flex}_C &\equiv 1 + \boldsymbol{\mathcal{M}}_L^\top \widetilde{\boldsymbol{\alpha}}(\sigma + \varphi/\Lambda_L).
\end{aligned}$$

Combining equations (I.31) and (I.32), yields the following sectoral Phillips curves in terms of the aggregate output gap $\widehat{C}^{gap}$:

$$\widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}_A\widehat{\mathbf{A}} + \boldsymbol{\mathcal{V}}_{EX}\widehat{\mathbf{D}}^*_{EX,F} + \boldsymbol{\mathcal{V}}_{IM}\widehat{\mathbf{P}}^*_{IM,F} + o(\|\widehat{\boldsymbol{\xi}}\|), \tag{I.33}$$

where the matrices of coefficients of exogenous shocks are as follows:

$$\begin{aligned}
\boldsymbol{\mathcal{V}}_A &\equiv \boldsymbol{\mathcal{V}}_{C,A} + \boldsymbol{\mathcal{B}}\cdot \boldsymbol{\Gamma}^{flex}_{C,EX},\\
\boldsymbol{\mathcal{V}}_{EX} &\equiv \boldsymbol{\mathcal{V}}_{C,EX} + \boldsymbol{\mathcal{B}}\cdot \boldsymbol{\Gamma}^{flex}_{C,EX},\\
\boldsymbol{\mathcal{V}}_{IM} &\equiv \boldsymbol{\mathcal{V}}_{C,IM} + \boldsymbol{\mathcal{B}}\cdot \boldsymbol{\Gamma}^{flex}_{C,IM}.
\end{aligned}$$

I.3. Proof of Propositions 5 and E.1: The optimal monetary policy

The optimal monetary policy maximizes the welfare loss (up to the second-order approximation) in equation (32) subject to the sectoral Phillips curves (up to the first-order approximation) in equation (36):

$$\begin{aligned}
\max_{\widehat{C}^{gap},\widehat{\mathbf{P}}} &\Big\{-\frac{1}{2}\Big(\sigma - 1 + \frac{\varphi+1}{\Lambda_L}\Big)\widehat{C}^{gap}(\boldsymbol{\xi})^2 - \frac{1}{2}\widehat{\mathbf{P}}^\top \boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}\Big\}\\
\text{s.t.}\qquad & \widehat{\mathbf{P}}(\boldsymbol{\xi}) = \boldsymbol{\mathcal{B}}\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}}.
\end{aligned}$$

Denote $\boldsymbol{\eta}$ the vector of multipliers for the constraint of sectoral Phillips curves. The first-order conditions with respect to $\widehat{C}^{gap}$ and $\widehat{\mathbf{P}}$, respectively, are:

$$-\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\eta}^\top \boldsymbol{\mathcal{B}} = 0, \tag{I.34}$$

$$-\boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) - \boldsymbol{\eta} = 0. \tag{I.35}$$

Substituting equation (I.35) into equation (I.34) to eliminate $\boldsymbol{\eta}$ yields:

$$\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0. \tag{I.36}$$

Substituting equations (23) and (22) from Section 3.2 into equation (I.36) yields:

$$\Big\{\big[\sigma - 1 + (\varphi + 1)/\Lambda_L\big]\kappa_C^{-1}\boldsymbol{\mathcal{M}}_{OG}^\top(\boldsymbol{\Delta}^{-1} - \mathbf{I}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\Big\}\widehat{\mathbf{P}}(\boldsymbol{\xi}) = 0.$$

Substituting the sectoral Phillips curves in equation (36) in equation (I.36) yields:

$$\big[\sigma - 1 + (\varphi + 1)/\Lambda_L + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{B}}\big]\widehat{C}^{gap}(\boldsymbol{\xi}) + \boldsymbol{\mathcal{B}}^\top \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{V}}\widehat{\boldsymbol{\xi}} = 0.$$