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FRANCESCO ZANETTI

Labor Market Frictions, Indeterminacy, and Interest Rate Rules

This paper studies the emergence of indeterminate equilibria in a standard New Keynesian model characterized by labor market frictions, under a policy rule that reacts strictly to inflation. Given labor market frictions, monetary policy may not be able to prevent aggregate fluctuations from being driven solely by self-fulfilling expectations. This is not, though, a result that holds under all circumstances: a monetary policy that reacts to some average measures of inflation or to the output gap may guarantee determinacy in the economy.

JEL codes: E4, E5
Keywords: labor market frictions, indeterminacy, interest rate rules.

A FUNDAMENTAL ISSUE in macroeconomics is whether a particular economic model is associated with a determinate equilibrium or not. If the equilibrium is indeterminate, business cycle fluctuations may be driven solely by self-fulfilling expectations, which could have disastrous effects on the welfare of the economy.[1] Monetary policy can play an active role in preventing an economy from being indeterminate, but its effectiveness depends on the structural economic features of the economy.

Following Taylor (1993), monetary models embed monetary policy through the nominal interest rate, which is set as an increasing linear function of current inflation and the output gap, with an inflation coefficient of about 1.5. Taylor and other studies[2] argue that an active monetary policy—that is, a Taylor-type policy rule

1. See Clarida, Galì, and Gertler (2000), Woodford (2003, chap. 2) and references therein for a recent discussion of the issue.

2. See Clarida, Galì, and Gertler (2000), Kerr and King (1996), and Taylor (1999).

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with an inflation coefficient higher than one—would stabilize the economy and guarantee local determinacy.

A number of recent studies cast doubt on this general conclusion and point out that the ability of an active monetary policy to ensure determinacy depends heavily on the specification of the economic environment. Carlstrom and Fuerst (2000) show that in New Keynesian models based on the assumption of Calvo staggered price setting, an active monetary policy can be effective only where it targets backward-looking inflation. An active forward-looking or current-looking rule generates indeterminacy. In a similar framework, Benhabib, Schmitt-Grohe, and Uribe (2001) and Carlstrom and Fuerst (2001) show that the preference specification is another crucial element that further limits the scope for a stabilizing rule. Different preference specifications fundamentally alter the results. Carlstrom and Fuerst (2005) show that even moderate assumptions on the structure of the economy, such as the presence of investment spending, may drastically affect results. In the presence of investment, an active current-looking or backward-looking policy guarantees determinacy, while a forward-looking policy always generates indeterminacy. Using a similar model, Weder (2005) shows that in presence of product market externalities, namely increasing returns to production, the effects of most of the policy proposals in Carlstrom and Fuerst (2005) are flipped on their head: strategies that should be chosen in perfect market environments in fact yield multiple rational expectations equilibria in alternative settings. Overall, this literature seems to suggest that it is hard to formulate general policy prescriptions and those depend on the underlined structure of the economy.

Recent work used search models to introduce labor market frictions to study how those drive an economy to indeterminacy. Giammarioli (2003) shows that increasing returns in the matching function generate indeterminacy in an otherwise determinate economy. Burda and Weder (2002) enrich a standard labor search model with different labor market distortions, such as taxes and associated policy interventions, to show how these features, together with search frictions, may generate indeterminacy. Krause and Lubik (2004) and Hashimzade and Ortigueira (2005) show that a standard model of labor search may suffer indeterminacy for empirically plausible parameter calibrations. Remarkably, none of these works studies indeterminacy in the context of monetary policy.

In this paper, we take on this task. We introduce labor market frictions to study indeterminacy in the context of monetary policy. The question we want to answer is whether labor market frictions change the conditions a monetary policy rule needs to guarantee a determinate equilibrium. To answer this question we employ a standard New Keynesian setting, where monetary policy is conducted with a Taylor-type rule, in which the nominal interest rate reacts to inflation without accounting for the output gap, as in the original formulation by Taylor. In this way, the model is analytically simple, and makes the results directly comparable with previous studies. In this paper, labor frictions are the sole departure from a standard New Keynesian setting and, therefore, the structural source for indeterminacy. While previous works employ search frictions, we use a simultaneous Nash bargaining

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over wages and employment between workers and firms. The advantage of this choice is that the outcome of the bargain is privately efficient—the choice over employment coincides with the one of a market without frictions—and the wage plays a distributive role.

The main findings are that labor market frictions introduce indeterminacy in an otherwise determinate economy, and that monetary policy would not guarantee local determinacy under any rule that is restricted to target a measure of inflation. This may suggest that labor market frictions limit the ability of a monetary authority that reacts strictly to inflation to prevent self-fulfilling expectations to drive aggregate fluctuations. We show that this is not the case. In fact, it is sufficient for a monetary authority to react either to some average measures of inflation or to the output gap to guarantee determinacy in the economy.

The remainder of the paper is organized as follows: Section 1 describes the economic environment, Section 2 discusses results and, finally, Section 3 concludes.

1. ECONOMIC ENVIRONMENT

The model resembles those used by Carlstrom and Fuerst (2005) and Weder (2005), with the additional feature of labor market frictions in the form of a simultaneous Nash bargain over wages and employment between workers and firms. The set-up describes the behavior of a representative household, a production sector comprised of a representative finished goods-producing firm, and a continuum of intermediate goods-producing firms characterized by Calvo price setting, and a monetary authority. Since we focus on local determinacy, we can limit the analysis to a deterministic model. In fact, if the deterministic dynamics are not unique, then sunspot equilibria may be constructed in the economy. In what follows we describe the economic environment.

The infinitely lived representative household maximizes an expected utility function of the form

$$E_0\sum_{t=0}^{\infty}\beta^t U(C_t,M_{t+1}/P_t,1 - L_t)\,,$$

where β is the discount factor, C_t is consumption, M_{t+1}/P_t is real money holdings, and $1 - L_t$ is leisure. As in Carlstrom and Fuerst (2005), we assume that the utility function is linear in leisure so that $U(C_t,M_{t+1}/P_t,1 - L_t) = V(C_t,M_{t+1}/P_t) - N(L_t)$, where $N(L_t) = L_t$.[3] The representative household enters period t with bonds B_{t-1} and money M_t. Before entering the goods market, the representative household visits the financial market where trades bonds and receives a cash transfer of $M_t^s(G_t - 1)$ from the monetary authority, where M_t^s denotes per capita money supply

3. In this way, different from Benhabib, Schnitt-Grohe, and Uribe (2001), conditions for determinacy are independent from the sign of the cross-partial derivative of utility with respect to consumption and real balances, V_{cm}. See Carlstrom and Fuerst (2001) for a discussion of the issue.

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and G_t gross money growth rate. After the representative household engages in goods trading, she/he receives nominal profits F_t from the representative intermediate goods-producing firm, supplies L_t units of labor at the wage rate W_t, and K_t units of capital at the rental rate Q_t to each intermediate goods-producing firm $i \in [0,1]$. Capital depreciates at the rate δ. The representative household ends the period with the following budget constraint:

$$M_{t+1} = M_t + M_t^s(G_t - 1) + R_{t-1}B_{t-1} - B_t + F_t - P_tC_t + P_t\{[Q_t + (1 - \delta)]K_t + L_tW_t\} - P_tK_{t+1}\,. \tag{1}$$

Thus the representative household chooses $\{C_t, K_{t+1}, B_t, M_t\}_{t=0}^{\infty}$ to maximize her/his utility subject to the budget constraint (Equation 1) for all $t = 0,1,2,\ldots$. The first-order conditions for this problem are

$$U_c(t) = \beta\{U_c(t+1)[Q_{t+1} + (1-\delta)]\}\,, \tag{2}$$

$$U_c(t) = \beta R_t U_c(t+1)/\pi_{t+1}\,, \tag{3}$$

$$U_m(t)/U_c(t) = (R_t - 1)/R_t\,, \tag{4}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate at time $t + 1$, and $U_c(t)$ is the marginal utility of consumption at time t. The representative household faces the resource constraint $C_t + K_{t+1} - (1 - \delta)K_t = Y_t$. Equations (2), (3), (4), are standard Euler equations and describe the optimal path for capital, bonds, and money holdings, respectively.

The production sector is comprised of a representative finished goods-producing firm and a continuum of intermediate goods-producing firms indexed by $i \in [0,1]$. This sector is modeled as in Yun (1996), where the representative intermediate goods-producing firm sets prices as in Calvo (1983). During each period $t = 0,1,2,\ldots$, the representative finished goods-producing firm uses $y_t(i)$ units of each intermediate good $i \in [0,1]$, purchased at the nominal price $P_t(i)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology described by

$$Y_t = \int_0^1 \left[y_t(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}$$

The firm acts to maximize its profits; the first-order conditions for this problem are

$$Y_t(i) = [P_t(i)/P_t]^{-\eta}Y_t$$

for all $i \in [0,1]$ and $t = 0,1,2,\ldots$. Competition drives the finished goods-producing firm's profits to zero in equilibrium, determining P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\eta} di\right]^{\frac{1}{1-\eta}} \tag{5}$$

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for all $t = 0,1,2, \ldots$. During each period $t = 0,1,2, \ldots$, the representative intermediate goods-producing firm hires $L_t(i)$ units of labor and $K_t(i)$ units of capital from the representative household to manufacture $Y_t(i)$ units of the intermediate good i according to the constant return to scale Cobb–Douglass technology $f(K_t(i), L_t(i)) = K_t^{\alpha}(i)L_t^{1-\alpha}(i)$. Imperfect competition implies that factor payments are distorted so that, with z_t representing marginal costs, $Q_t = z_t f_k(K_t, L_t)$ and $W_t = z_t f_l(K_t, L_t)$. The representative intermediate goods-producing firm sets prices as in Calvo (1983). During each period $t = 0,1,2, \ldots$, a fraction $(1 - \nu)$ of intermediate goods-producing firms sets a new price, while the remaining fraction ν charges the previous period's price time steady-state inflation. The probability of a price change is constant across time and independent of the firm's price history. Hence, firm i that sets a new price $P_t(i)$ in time t maximizes

$$E_0 \sum_{j=0}^{\infty} (\nu\beta)^j (\Lambda_{t+j}/\Lambda_t)\{[P_t(i)/P_t]^{-\eta} Y_t [P_t(i)/P_t - z_t]\} ,$$

where $\beta^j(\Lambda_{t+j}/\Lambda_t)$ is the rate at which the firm discounts its earnings at time $t + j$.[4] First-order conditions for this problem are

$$P_t(i) = \frac{\eta \sum_{j=0}^{\infty} (\nu\beta)^j \Lambda_{t+j} P_{t+j}^{\eta} Y_{t+j} z_{t+j}}{(\eta - 1)\sum_{j=0}^{\infty} (\nu\beta\pi)^j \Lambda_{t+j} P_{t+j}^{\eta-1} Y_{t+j}} . \tag{6}$$

During each period $t = 0,1,2, \ldots$, the representative intermediate goods-producing firm bargains with the representative household over the wage and the level of employment. We assume a simultaneous Nash bargaining, the parties choose the wage and the level of employment that maximize the weighted product of the surpluses from employment:

$$(W_t L_t/P_t - N(t)/\Lambda_t)^{\theta} (K_t^{\alpha} L_t^{1-\alpha} - W_t L_t/z_t)^{1-\theta} ,$$

where the first and the second term in brackets represent the representative household and the representative intermediate goods-producing firm surpluses from employment, respectively. The parameter θ reflects the parties' relative bargaining power. Since it is a contemporaneous bargain, the outcome is privately efficient. In fact, as shown below, the optimal level of employment is the same as in a model without bargaining, while the wage is set to split the surpluses from employment.

The wage W_t chosen by the parties satisfies

$$W_t L_t/z_t = \theta K_t^{\alpha} L_t^{1-\alpha} + (1 - \theta)(N(t)/z_t U_c(t)) , \tag{7}$$

which shows how the wage is composed, for the faction θ, by the firm revenues and, for the fraction $1 - \theta$, by the disutility of being unemployed. Note that in a

4. The variable Λ_t is the non-negative Lagrange multiplier on the representative household budget constraint (Equation 1). Formally, $\Lambda_t = U_c(t)/P_t$ from the representative household's problem.

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labor market without bargaining, the wage would equal the marginal rate of substitution between consumption and leisure.

The employment L_t chosen by the parties satisfies the optimal condition

$$\theta(W_t - N_l(t)/\Lambda_t)(K_t^{\alpha}L_t^{1-\alpha} - W_tL_t/z_t)$$
$$= (1 - \theta)(W_t/z_t - MPL_t)(W_tL_t/P_t - N(t)/\Lambda_t)\,,$$

where $MPL_t = (1 - \alpha)(K_t/L_t)^{\alpha}$. If we substitute Equation (7) into this last expression, it yields

$$N_l(t)/U_c(t) = z_tMPL_t\,, \tag{8}$$

which is the familiar equilibrium condition in which the marginal product of labor is equal to the marginal rate of substitution between consumption and leisure. As stressed, the optimal condition for employment is the same as in a labor market without frictions, while the wage plays a distributive role.[5]

Each period the monetary authority conducts monetary policy through changes of the nominal interest rate R_t in response to changes in inflation, following the Taylor-type rule:

$$\ln(R_t/R) = \tau\ln(\pi_{t+i}/\pi)\,. \tag{9}$$

Here, R and π are the steady-state values of the nominal interest rate and inflation, respectively. We consider three specifications of this rule: where $i = 1$ it is a forward-looking rule, where $i = 0$ it is a current-looking rule, and where $i = -1$ it is a backward-looking rule.

2. RESULTS

In this section we focus on how a simultaneous Nash bargain on wages and the level of employment affects the response of a monetary policy that aims at assuring determinacy. Equation system (2)–(9) does not have an analytical solution. Instead, we characterize the model's dynamics by log-linearizing the relevant first-order conditions around the steady-state following King, Plosser, and Rebelo (1988). As in Carlstrom and Fuerst (2005), the conditions for determinacy are independent from the functional form of $V(t)$. If we let $x_t = L_t/K_t$ represent capital deepening, Equation (8) yields $U_c(t) = x_t^{\alpha}/z_t(1 - \alpha)$. Substituting out $U_c(t)$ and $Q_t = z_tf_k(K_t,L_t)$ into Equations (2) and (3) and log-linearizing the results around the steady-state yields:

$$\alpha\hat{x}_t - \hat{z}_t = [1 - \beta(1 - \delta)(1 - \alpha)]\hat{x}_{t+1} - \beta(1 - \delta)\hat{z}_{t+1}\,, \tag{10}$$

5. Note that if $\theta \rightarrow 1$, i.e. there is no longer any real bargaining, then $W_t = N_l(t)/\Lambda_t$. So this model nests the standard neoclassical labor market similar to the one in Carlstrom and Fuerst (2005).

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and

$$\alpha\hat{x}_t - \hat{z}_t = \hat{R}_t + \alpha\hat{x}_{t+1} - \hat{\pi}_{t+1} - \hat{z}_{t+1}, \tag{11}$$

where a hat on a variable denotes the logarithmic deviation from its steady state. As shown in Yun (1996), Equations (5) and (6) produce the log-linerarized Phillips curve

$$\hat{\pi}_t = \lambda\hat{z}_t + \beta\hat{\pi}_{t+1}, \tag{12}$$

where $\lambda = (1 - \nu)(1 - \nu\beta)/\nu$. The log-linearized economy constraint is

$$\hat{k}_{t+1} = (1 + c/k)\hat{k}_t + [(1 - \alpha)y/k]\hat{x}_t - (c/k)\hat{c}_t. \tag{13}$$

Equations (11)–(13) represent a model as in Carlstrom and Fuerst (2005). Their analysis shows that a forward-looking rule has limited chances to avoid indeterminacy, while an active current-looking or a backward-looking policy would guarantee determinacy.

The incorporation of labor market frictions through union bargaining rules out all these determinate equilibria. The introduction of a representative union enriches the model with the wage equation (Equation 7), whose log-linearized version is

$$\hat{w}_t = \hat{z}_t - \alpha\hat{x}_t. \tag{14}$$

This is a condition that relates wages to marginal costs and capital deepening. The addition of this equation to the system always produces a zero eigenvalue. In fact, as stressed in Carlstrom and Fuerst (2005), since an equation like Equation (14) contains only time t variables, this immediately suggests a zero eigenvalue.

PROPOSITION 1: *Suppose monetary policy is given by the log-linearized expression of the Taylor-type interest rate rule (9),* $\hat{R}_t = \tau\hat{\pi}_{t+i}$. *In the presence of labor market frictions, a backward-looking interest rate rule* $(i = -1)$, *a current-looking interest rate rule* $(i = 0)$, *and a forward-looking interest rate rule* $(i = 1)$ *all generate real indeterminacy for any value of* τ.

Proposition 1 is proved in Appendix A.

How do labor market frictions generate such a striking difference? As stressed in Woodford (2003, chap. 2), in an economy without labor frictions, an active monetary policy would prevent indeterminacy. Under such a rule, a rise in inflation brings about an increase in the real interest rate which reduces inflationary pressures, bringing the economy back towards the equilibrium. This would not happen in an economy with labor frictions. Consider a sunspot increase in expected inflation. An active monetary policy rule leads to an increase in the real interest rate. This reduces current consumption, investment, and output. Due to the reduction in investment, capital decreases and, since it is a state variable and cannot jump, labor also decreases in line with the output decrease. In presence of labor market frictions, a decrease in labor would put upward pressures on wages which, through the Phillips curve, would increase inflation. This mechanism validates the initial assumption of sunspot-driven inflation expectations. Analytically, the key element is the wage equation (Equation 14), which, as mentioned, is not intertemporal but entirely intratemporal.

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Consider instead the instance of a forward-looking rule ($i = 1$) so that the log-linearization of Equation (9) reads $\hat{R}_t = \tau\hat{\pi}_{t+1}$. If we use this relationship in Equation (11) and substitute the outcome into Equation (10), it yields

$$a_1\hat{z}_{t+1} + a_1(1 - \alpha)\hat{x}_{t+1} + (1 - \tau)\hat{\pi}_{t+1} = 0\,, \tag{15}$$

where $a_1 = 1 - \beta(1 - \delta)$. If we use Equation (14) to solve for $\hat{z}_t$ and $\hat{z}_{t+1}$, and substitute the outcome into Equations (11), (12), and (15), it yields

$$\hat{w}_{t+1} + (1 - \tau)\hat{\pi}_{t+1} = \hat{w}_t\,, \tag{16}$$

$$\beta\hat{\pi}_{t+1} = \hat{\pi}_t - \lambda\hat{w}_t - \alpha\lambda\hat{x}_t\,, \tag{17}$$

and

$$a_1\hat{w}_{t+1} + a_1\hat{x}_{t+1} + (1 - \tau)\hat{\pi}_{t+1} = 0. \tag{18}$$

Together with the resource constraint (Equation 13) and the wage equation (Equation 14), this describes the dynamics of the system. This set of equations provides five restrictions on the equilibria. The system has five unknowns, one of them, (capital, k_t), being a state variable. Hence, a necessary and sufficient condition for determinacy is to have four eigenvalues outside the unit circle. As in Carlstrom and Fuerst (2005), the eigenvalue associated with Equation (13) is equal to $1 + c/k$, while the one associated with Equation (18) is equal to zero. For determinacy, the remaining three eigenvalues have to be outside the unit circle. Here, the eigenvalue associated with Equation (14) is equal to zero, so that the system is indeterminate for any value of τ.[6]

Proposition 1 may suggest that if the structure of the economy is the one captured by the theoretical model, a monetary authority would have no power to prevent self-fulfilling expectations from driving aggregate fluctuations. This is not the case. In fact, it is sufficient for the monetary authority to react to some average measures of inflation to prevent this from happening. For instance, if the monetary authority sets the nominal interest rate in response to some average of future and current inflation following the Taylor-type rule

$$\ln(R_t/R) = \tau[\gamma\ln(\pi_{t+1}/\pi) + (1 - \gamma)\ln(\pi_t/\pi)]\,, \tag{19}$$

then the economy would be determinate.[7] In contrast to a policy rule that reacts strictly to inflation, Equation (19) implies that the arbitrage relationship between bonds and capital expressed by Equation (18) does depend upon time-t variable (namely, π_t). Hence, the eigenvalue associated with this equation is no longer equal

6. It is interesting to note that even if prices become completely flexible (i.e. $\nu \rightarrow 0$) and the economy is characterized by labor market frictions, the results in Proposition 1 would still hold. In fact, also in this instance, the wage equation would not contain expectations of future variables and, hence, would have a zero eigenvalue.

7. See Appendix B for details. Note that this is only one example of the possible average measures of inflation that would guarantee determinacy.

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to zero. Another natural candidate to target would be the output gap, such that the monetary authority would set the nominal interest rate in response to changes in both inflation and the output gap (with a coefficient of ω). The determinacy analysis proceeds as before. The system is more complex, so here we simply report some numerical results for the case of a forward-looking rule. Plausible values for ω are in the range of $0.1 - 2$. The other parameter values are $\beta = 0.99$, $\alpha = 1/3$, $\lambda = 1/3$, $\delta = 0.02$, so that $a_1 \approx 0.03$. For the case of a low response to the output gap, $\omega = 0.1$, the determinacy region is $\tau > 0.87$. As we increase ω, the lower bound of τ increases. Some illustrative results for the determinacy region that corresponds to a given output gap response coefficient are given by (ω, τ_{Lower}): (0.1, 0.87), (0.5, 0.9), (1.5, 0.91), (2, 0.92). This numerical analysis is in line with Weder (2005) and suggests that Taylor rules incorporating the output gap can pre-empt sunspot equilibria that arise from real market imperfections.

3. CONCLUSION

In this paper, we incorporated labor market frictions based on a simultaneous Nash bargain into a standard New Keynesian monetary model to study their influence on the determinacy of the economy. The main finding is that introducing labor market frictions may allow aggregate fluctuations to be driven solely by changes in agents' expectations about the future path of the economy. This means that an interest rate rule based solely on current-looking or active forward-looking reaction to inflation is not sufficient to generate local stability, even though it would do in a setting without labor frictions.

Christiano, Eichenbaum, and Evans (2005) show that incorporating labor market frictions is crucial to improving the performance of New Keynesian models. This paper points out that such frictions may have a nontrivial impact for the design of monetary policy.

APPENDIX A

We prove Proposition 1 for the case of a forward-looking Taylor-type rule, where $i = 1$ in Equation (9). The instances for a backward-looking, $i = -1$, and a current-looking, i = 0, policy can be proved similarly.

PROPOSITION 1: *Suppose monetary policy is given by the log-linearized expression of the forward-looking Taylor-type rule $\hat{R}_t = \tau\hat{\pi}_{t+1}$. In the presence of labor market frictions, such a rule generates real indeterminacy for any value of τ.*

PROOF: Using $MPL_t = (1 - \alpha)(K_t / L_t)^{\alpha}$ and the fact that $U(t)$ is linear in leisure, we can write Equation (8) as $U_c(t) = x_t^{\alpha}/(1 - \alpha)z_t$. If we use this last equation together with the firms optimal condition for capital, $Q_t = z_t f_k(K_t, L_t)$, into Equation (2),

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it yields $x_t^{\alpha}/z_t = \beta\{x_{t+1}^{\alpha}/z_{t+1}[\alpha x_{t+1}^{1-\alpha}z_{t+1} + (1-\delta)]\}$. The log-linear approximation of this last expression is Equation (10), it can be more generally expressed as

$$\hat{x}_{t+1} = F(\hat{x}_t,\hat{z}_{t+1},\hat{z}_t)\,. \tag{A1}$$

The money demand Equation (4) implies that real money balances depend only on R_t, z_t, and x_t. If we use the policy rule, we can then rewrite the log-linearized resource constraint (Equation 13) as

$$\hat{k}_{t+1} = G(\hat{k}_t,\hat{x}_t,\hat{\pi}_{t+1},\hat{z}_t)\,. \tag{A2}$$

Together with the policy rule, the log-linear version of the Euler equation (11) can be written as

$$\hat{z}_{t+1} = H(\hat{z}_t,\hat{\pi}_{t+1},\hat{x}_t,\hat{x}_{t+1})\,. \tag{A3}$$

The log-linearized Phillips curve (Equation 12) can be expressed as

$$\hat{\pi}_{t+1} = P(\hat{\pi}_t,\hat{z}_t)\,. \tag{A4}$$

Finally, the log-linearized wage equation (Equation 14) can be expressed as

$$\hat{w}_t = T(\hat{x}_t,\hat{z}_t)\,. \tag{A5}$$

Equations (A1)–(A5) represent the log-linearized equilibrium conditions of the model. If we denote the vector $S_t = [\hat{x}_t,\hat{z}_t,\hat{\pi}_t,\hat{w}_t,\hat{k}_t]$ we can write Equations (A1)–(A5) as $AS_{t+1} = BS_t$, where A and B are 5×5 matrices. If we invert A we have $A^{-1}B$, which has five eigenvalues. Since there is only one state variable (capital, k_t), we must have four eigenvalues outside the unit circle in order to guarantee determinacy. From Equation (A5) it appears that one eigenvalue equals zero. If we use Equation (A.3) into Equation (A.1) it also appears that another eigenvalue of the system equals zero. □

APPENDIX B

We show that a monetary authority that sets the nominal interest rate as in Equation (19) may prevent self-fulfilling expectations to drive aggregate fluctuations.

If we log-linearize Equation (19) around the steady state, it yields $\hat{R}_t = \tau[\gamma\hat{\pi}_{t+1} + (1-\gamma)\hat{\pi}_t]$. If we substitute this expression into Equation (11), we obtain $\hat{z}_{t+1} = (\tau\gamma - 1)\hat{\pi}_{t+1} + \tau(1-\gamma)\hat{\pi}_t - \hat{z}_t - \alpha\hat{x}_t + \alpha\hat{x}_{t+1}$. This last expression into Equation (10) yields $a_1\hat{z}_{t+1} + a_1(1-\alpha)\hat{x}_{t+1} + (1-\tau\gamma)\hat{\pi}_{t+1} = \tau(1-\gamma)\hat{\pi}_t$. These last two equations together with Equations (12)–(14) represent the model's dynamics. If we denote the vector $S_t = [\hat{x}_t,\hat{z}_t,\hat{\pi}_t,\hat{w}_t,\hat{k}_t]$ we can write the five system equations as $AS_{t+1} = BS_t$, where A and B are 5×5 matrices. If we invert A we have $A^{-1}B$, which has five eigenvalues. Since there is only one state variable (capital, k_t), we must have four eigenvalues outside the unit circle in order to guarantee determinacy. The eigenvalue associated with Equation (13) equals $1 + c/k$, while the one associated with Equation (14) equals zero. Hence, a necessary and sufficient

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condition for determinacy is that the remaining three roots be outside the unit circle. The relevant quadratic equation is given by $J_3q^3 + J_2q^2 + J_1q + J_0$, where $J_3 = a_1\beta$, $J_2 = a_1(\beta - 1) - \lambda[a_1(1 - \alpha) + \alpha](1 - \tau)$, $J_1 = - a_1 + a_1\lambda\tau(1 - \alpha)(1 - \gamma) + \alpha\lambda(\tau - 1)$, and $J_0 = \alpha\lambda\tau(1 - \gamma)$. This implies $J(0) = \alpha\lambda\tau(1 - \gamma) > 0 \forall \tau > 0$. If $\tau > 1 + \{2(1 - \beta)a_1/\lambda[a_1 + \alpha(2 - a_1)]\}$ then $J(1) > 0$.[8] This means that if $J(-1) > 0$ then the three roots, either real or complex, are outside the unit circle. Note that $J(-1) = a_1\lambda(1 - \alpha)[1 + \tau(1 - 2\gamma)]$ and if $\gamma < (1 + \tau)/2\tau$ then $J(-1) > 0$. Hence, if the monetary authority sets $\tau > 1 + \{2(1 - \beta)a_1/\lambda[a_1 + \alpha(2 - a_1)]\}$ and $\gamma < (1 + \tau)/2\tau$ the three roots must be outside the unit circle. □

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8. Note that $J(1) = 2a_1(\beta - 1) + (\tau - 1)\{\lambda[a_1(1 - \alpha) + 2\alpha]\}$.

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