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## The Impact of the Volatility of Monetary Policy Shocks

This paper studies the impact of the volatility of monetary policy using a structural vector autoregression (SVAR) model enriched along two dimensions. First, it allows for time-varying variance of monetary policy shocks via a stochastic volatility specification. Second, it allows a dynamic interaction between the level of the endogenous variables in the VAR and the time-varying volatility. The analysis establishes that the nominal interest rate, output growth, and inflation fall in reaction to an increase in the volatility of monetary policy. The analysis also develops a dynamic stochastic general equilibrium model enriched with stochastic volatility to monetary policy that generates similar responses and provides a theoretical underpinning of these findings.

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A VAST BODY OF EMPIRICAL research has focused on estimating the impact of structural economic shocks on the economy, with a large proportion of papers studying the impact of monetary policy shocks. Structural vector autoregression (SVAR) models, originally proposed by Sims (1980), have featured prominently in this literature as they offer a flexible data-driven approach to modeling the transmission mechanism. Results from these models have been used as a benchmark for the performance of more structural economic models such as dynamic stochastic general equilibrium (DSGE) models.

While the transmission mechanism of monetary policy shocks has been studied extensively, the role played by changes in the volatility of these shocks has been

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largely ignored in the SVAR literature. Most of the adopted SVAR models assume homoskedastic shocks. Studies that do allow for time-varying shock volatility (see, e.g., Primiceri 2005, Sims and Zha 2006) do not incorporate a direct impact of the shock variance on the endogenous variables. This paper proposes an extension to the monetary SVAR model by (i) allowing for time-varying variance of monetary policy shocks via a stochastic volatility specification, and (ii) allowing a dynamic interaction between the level of the endogenous variables in the VAR and the time-varying volatility. This extended VAR model can therefore be used to not only gauge the effect of shocks such as the monetary policy shock but also the impact of changes in the volatility of the shock in question.

Changes in the volatility of monetary policy are important because of three considerations. First, a growing number of studies have demonstrated that the volatility of structural shocks such as monetary policy, supply, and demand has fluctuated substantially in industrialized countries. For example, the estimated volatility of the (U.S.) monetary policy shock in Primiceri (2005) increases by more than 100% during the early 1980s. Second, the recent financial crisis has highlighted the fact that volatility cannot be regarded as a “pre–Great Moderation phenomenon.” In other words, fluctuations in volatility and their potential impact is a relevant concern for policymakers. Third, there is a growing body of theoretical work that has identified channels through which changes in volatility can affect the real economy. For example, Bloom (2009) presents simulations from a model where higher uncertainty causes firms to pause their hiring and investment, leading to a drop in real activity. Using a nonlinear small open economy DSGE model, Fernandez-Villaverde et al. (2011) suggest a channel through which changes in real interest rate volatility can affect open economies that use foreign debt to smooth consumption and to hedge against idiosyncratic productivity shocks. As real interest rate volatility increases and as countries are increasingly exposed to variations in marginal utility, they reduce the level of foreign debt by cutting consumption. Investment falls as foreign debt becomes a less attractive hedge for productivity shocks leading to a fall in real activity.<sup>1</sup>

The contribution of this paper is twofold. First, it develops a data-driven approach to estimating the impact of volatility shocks. These methods therefore complement the more structural analyses in Bloom (2009) and Fernandez-Villaverde et al. (2011). The advantage of the extended SVAR is that it retains the flexibility of the standard homoskedastic SVAR—that is, it is applicable to a variety of identified shocks and has the potential to fit the data better than a DSGE model that is subject to stringent theoretical restrictions. In other words (in an analogous manner to standard VARs), the extended SVAR can be used to provide estimates of the impact of changes in structural shock volatility without making strong assumptions about the driving force behind such effects. This flexibility and potential for better data fit, of course, comes with the cost that the model has little to say about the various channels of shock transmission. Hence, the second contribution of the paper is to develop a

1. Other recent studies that focus on the role of policy uncertainty are those by Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2012).

nonlinear DSGE model solved by a third-order perturbation around the steady state in order to rationalize the effect of an increase in volatility of monetary policy on macroeconomic aggregates.

The results of the analysis show that an increase in the volatility of the monetary policy shock generates a fall in the nominal interest rate, inflation, and output growth. The findings are robust to perturbations to the benchmark VAR specification, such as the addition of extra lags and the inclusion of additional endogenous variables. The analysis also shows that the estimated volatility of monetary policy shocks reaches its peak during the Great Inflation of the mid-1970s and the Volker's experiment of targeting nonborrowed reserves at the end of the 1970s. As expected, the Great Moderation period, starting from the mid-1980s, was associated with less volatile monetary policy shocks. However, high volatility of monetary policy characterizes the most recent recessions in 2000 and 2009.

We conclude the analysis by developing a DSGE model that resembles those by Ireland (2004) and Sargent and Surico (2011), but solved by a third-order perturbation around the steady state, which therefore encompasses the effect of stochastic volatility to monetary policy. We then use the framework to investigate the transmission mechanism of an increase in the volatility of the monetary policy. We establish that the qualitative responses to an increase in the volatility of monetary policy are similar to those in the SVAR model, as the nominal interest rate, inflation, and output growth fall in the aftermath of the shock. The use of a DSGE model advances our understanding of the mechanism that induces the contemporaneous fall in these variables. In particular, higher monetary policy uncertainty that generates a greater interest rate spread implies higher dispersion of inflation, which, due to Jensen's inequality, as detailed in the paper, reduces its expected value. Hence, the rate at which bonds gain value due to movements in inflation increases on average with the level of uncertainty. As a result, holders of nominal assets demand a smaller compensation to keep bonds in their portfolio when uncertainty rises. Thus, the nominal interest rate falls in response to an increase in the volatility of monetary policy. If the monetary authority follows the Taylor rule, which dictates that the nominal interest rate adjusts to movements in inflation and output, the fall in the interest rate must be associated with a decrease in either inflation, or output, or both of them. Since the Phillips curve generates a positive relation between fluctuations in inflation and output, both variables must fall in response to a decrease in the nominal interest rate. Thus, an increase in monetary policy uncertainty causes the nominal interest rate, inflation and output growth to fall.

The remainder of the paper is structured as follows. Section 1 sets up the SVAR model with stochastic volatility. Section 2 details the estimation procedure and the identification of the monetary policy shock. Section 3 reports the estimated variables' volatilities and impulse responses to the monetary policy volatility. Section 4 develops a DSGE model with stochastic volatility shocks to study the transmission mechanism of an increase in the volatility of monetary policy. Section 5 concludes the paper.

## 1. EXTENDED SVAR MODEL WITH STOCHASTIC VOLATILITY

The VAR model with stochastic volatility is given by the following equation:

$$Z_t = c + \sum_{j=1}^P \beta_j Z_{t-j} + \sum_{j=0}^P \gamma_j \tilde{h}_{t-j} + \Omega_t^{1/2} e_t, e_t \sim N(0, 1), \quad (1)$$

where

$$\Omega_t = A^{-1} H_t A^{-1'}, \quad (2)$$

and  $Z_t$  denotes the  $N$  macroeconomic variables, while  $\tilde{h}_t = [h_{1t}, h_{2t} \dots h_{Nt}]$  refers to the log volatility of the  $N$  structural shocks in the VAR. This latter feature can be seen more clearly by considering an example where  $N = 3$ , as in our application. The structure of  $H_t$  in equation (2) is

$$H_t = \begin{pmatrix} \exp(h_{1t}) & 0 & 0 \\ 0 & \exp(h_{2t}) & 0 \\ 0 & 0 & \exp(h_{3t}) \end{pmatrix}. \quad (3)$$

The structure of the  $A$  matrix is chosen to model the contemporaneous relationship among the reduced-form shocks, and the transition equation for the stochastic volatility is given by

$$\tilde{h}_t = \theta \tilde{h}_{t-1} + \eta_t, \eta_t \sim N(0, Q), E(e_t, \eta_t) = 0, \quad (4)$$

where  $\theta$  is a diagonal matrix implying that each element of  $\tilde{h}_t$  follows an AR(1) process.

There are three noteworthy features about the complete system defined by equations (1), (2), and (4). First, equation (1) allows the volatility of the *structural* shocks  $\tilde{h}_t$  to have an impact on the endogenous variables  $Z_t$ .<sup>2</sup> Second, the structure of the matrix  $A$  in equation (2) determines the interpretation of structural shocks and hence their volatility  $\tilde{h}_t$ . The ability to place an economic interpretation on some or all of the shocks is important as it allows the model to tackle the analysis of the impact of volatility in a theoretically consistent manner. Third, the system proxies the reduced form of a DSGE model approximated to the third order around the steady state, since it involves a direct effect of the volatility of monetary policy shocks on the level of the endogenous variables, as shown in Fernandez-Villaverde et al. (2011). Hence, the SVAR model is an approximation of the reduced form of the DSGE model, which abstracts from terms that involve the cross-product of the shock to volatility with the endogenous variables.

Note that equation (4) makes the simplifying assumption that the shocks to the volatility equation  $\eta_t$  and the observation equation  $e_t$  are uncorrelated and  $Q$  is a

2. In our specification the log volatility enters the VAR equations rather than its level. This is primarily because the former specification proved to be substantially more computationally stable than the latter in our experiments. In particular, the level specification is sensitive to the scaling of the variables with the possibility of overflow whenever the scale of the variables is somewhat large.

diagonal matrix. With these assumptions in place, one can interpret an innovation in an element of  $\eta_t$  as a shock to the volatility of the structural shock of interest and then calculate the response of  $\tilde{h}_t$  and  $Z_t$ . On the other hand, if these assumptions are relaxed, further identifying restrictions are required to distinguish among the volatility shocks and to separate the innovation to the volatility from the innovation to the level. Note that in this more general scenario (i.e., with a full covariance matrix among the volatility and level innovations), identification of the volatility shocks is substantially more involved. In particular, there is no simple way to assign elements of  $\tilde{h}_t$  to a particular structural shock (as done in the proposed model above) and the researcher has to take a stand on the restrictions to place on the contemporaneous relationships among the volatilities. In contrast, the assumptions in equation (4) allow the use of standard identification schemes (that apply to the contemporaneous relationships among the *levels* of the reduced form shocks rather than their volatility). To retain this ease of interpretation of  $\tilde{h}_t$  we incorporate the assumption of a diagonal  $Q$  and no correlation between  $e_t$  and  $\eta_t$  in the proposed empirical model.

The model proposed above is related to a number of recent contributions. In particular, the structure of the stochastic volatility model used above closely resembles the formulations used in time-varying VAR models (see Primiceri 2005). Our model differs from these studies in that it allows the volatilities to have a direct impact on the levels of the endogenous variables. The model proposed above can be thought of as a multivariate extension of the stochastic volatility in mean model proposed in Koopman and Uspensky (2002) and applied in Berument, Yalcin, and Yildirim (2009), Kwiatkowski (2010), and Lemoine and Mougin (2010). In addition, our model has similarities with the stochastic volatility models with leverage studied in Asai and McAleer (2009). However, unlike these contributions, the model proposed in this study is formulated with the aim of characterizing the dynamic effects of the volatility of *structural* shocks.

## 2. ESTIMATION

### 2.1 The Gibbs Sampling Algorithm

The nonlinear state space model consisting of the observation equation (1) and transition equation (4) is estimated using a Gibbs sampling algorithm. Appendix A presents details of the priors and the conditional posterior distributions while a summary of the algorithm is presented below.

The Gibbs sampling algorithm proceeds in the following steps:

- (i) Conditional on a draw for the stochastic volatility  $\tilde{h}_t$ , and the matrix  $A$ , equation (1) represents a VAR model with heteroskedastic disturbances. We rewrite the VAR as a state space model and draw from the conditional distribution of  $\Gamma = [\beta, \gamma]$  using the Carter and Kohn (2004) algorithm.
- (ii) Conditional on a draw for  $\tilde{h}_t$  and  $\Gamma$ , the elements of the matrix  $A$  can be drawn using a series of linear regression models among the elements of the residual matrix  $v_t = \Omega_t^{1/2} e_t$  as shown in Cogley and Sargent (2005). Conditional on

$\tilde{h}_t$ , the autoregressive parameters  $\theta$  and variances in  $Q$  can be drawn using standard results for linear regressions.

- (iii) Conditional on  $\Gamma$ ,  $A$ ,  $\theta$ , and  $Q$ , the stochastic volatilities are simulated using a date by date independence Metropolis step as described in Jacquier, Polson, and Rossi (2004) (see also Carlin, Polson, and Stoffer 1992).

We use 1,000,000 replications and base our inference on the last 10,000 replications. The recursive means of the retained draws (see Appendix A) show little fluctuation, providing support for convergence of the algorithm.

## 2.2 Model Specification and the Identification of the Monetary Policy Shock

In our application, the vector of endogenous variables  $Z_t$  contains U.S. quarterly data on the 3-month Treasury bill rate, real quarterly GDP growth, and quarterly CPI inflation over the period 1957Q1 to 2011Q4.<sup>3</sup> We use the following benchmark VAR specification:

$$Z_t = c + \sum_{j=1}^2 \beta_j Z_{t-j} + \sum_{j=0}^1 \gamma_j \tilde{h}_{t-j} + \Omega_t^{1/2} e_t. \quad (5)$$

The lag length of the endogenous variables is set at two, reflecting convention in studies employing similar VAR models to quarterly data (see Cogley and Sargent 2005, Primiceri 2005). In our benchmark model, the contemporaneous and the lagged values of  $\tilde{h}_t$  are allowed to affect  $Z_t$ . Since we employ quarterly data, we allow the possibility of an impact of  $\tilde{h}_t$  within a 3-month period. We show in the sensitivity analysis below that the benchmark results are not affected if longer lags of volatility are included in the mean equations.

In order to identify the monetary policy shock we consider the following structure for  $\tilde{A} = A^{-1}$ :

$$\tilde{A} = \begin{pmatrix} 1 & 0 & 0 \\ \tilde{a}_1^{(-)} & 1 & 0 \\ \tilde{a}_2^{(-)} & \tilde{a}_3 & 1 \end{pmatrix}, \quad (6)$$

where the superscript  $(-)$  denotes the fact this element is restricted to be less than zero. Given the ordering of the endogenous variables (3-month Treasury bill rate, real GDP growth, and CPI inflation), this structure for  $\tilde{A}$  implies that an increase in interest rates is assumed to lead to a contemporaneous fall in inflation and GDP growth. These sign restrictions are implied by standard DSGE models including the model we discuss in Section 4.<sup>4</sup> We impose these sign restrictions via rejection

3. The data are from the FRED database. The FRED mnemonics are as follows: real GDP: GDPC96, CPI: CPIAUCSL, and 3-month Treasury bill rate: TB3MS. The bottom row in Figure 1 reports the data series used in the estimation.

4. An appendix that presents evidence on the sign restrictions from the theoretical model is available upon request from the authors.

sampling. In particular, we repeat the draw of the  $A$  matrix in the second step of the algorithm outlined above until the restrictions on  $\bar{A}$  are (jointly) satisfied.<sup>5</sup>

### 3. EMPIRICAL RESULTS: MONETARY POLICY SHOCK UNCERTAINTY AND THE MACROECONOMY

This section uses the empirical model to produce estimated volatilities of shocks to monetary policy, GDP growth, and inflation, and studies the effect of an increase in the volatility of monetary policy on the endogenous variables.

#### 3.1 *Estimated Volatility*

The first column in Figure 1 presents the estimated volatility of the monetary policy shock. The evolution of the volatility is similar to the estimate in Benati and Mumtaz (2007), with large increases during the Great Inflation of the mid-1970s and then during Paul Volcker's experiment of targeting nonborrowed reserves at the end of the 1970s. Note that the Great Moderation period—starting from the mid-1980s—is associated, on the whole, with less volatile policy shocks. Two exceptions to this stability are the recessions of 2000 and 2009. The second and third columns in Figure 1 present the standard deviation of the shocks to the GDP growth and inflation equation in the VAR—that is, shocks that we do not place a direct economic interpretation on. The volatility of the GDP growth equation shock is highest in the pre-1985 period, reaching its peak during the early 1980s. The post-1985 period contains smaller peaks at the time of the first Gulf war during the early 1990s, the recession of 2000 and then toward the end of the sample coinciding with the recent financial crisis. The profile for the volatility of the shock to the inflation equation is similar with the highest variance concentrated in the pre-1985 sample. One noticeable feature, however, is the substantial increase in the volatility of this shock during the recent crisis.

Figure 2 plots the estimated posterior densities for the parameters of the AR(1) transition equation for the stochastic volatilities (see equation (4)). It is interesting to note that the volatility associated with the monetary policy shock is estimated to be the most persistent, as the mean estimate of  $\theta_1$  is equal to 0.98 approximately. In contrast, the posterior for  $\theta_2$  (the AR parameter associated with the stochastic volatility of the shock to GDP growth) is centered around a lower mean. Finally, the posterior estimates for the variance of the shock to the volatility of the structural shocks,  $Q_i$ , show that the estimates associated with the interest rate is higher than the corresponding number associated with inflation and GDP growth.<sup>6</sup>

5. Note that the shocks in front of the GDP growth and inflation equation are not of direct interest in our application and a recursive structure is assumed for the block of  $\bar{A}$  that applies to these shocks. Note that having a triangular structure for  $A$  implies that step 2 of the Gibbs algorithm is simplified and each row of the  $A$  matrix can be drawn separately.

6. Note that this reflects a scaling effect—the interest rate enters the model as an annual percentage while inflation and GDP growth are quarterly percentages.

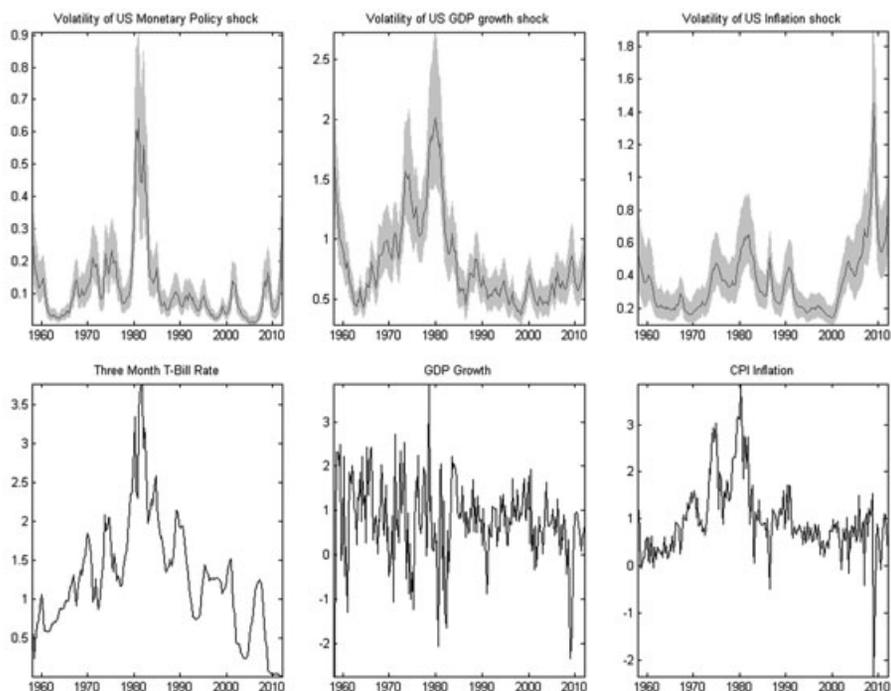


FIG. 1. Estimated Standard Deviation of the Structural Shocks.

NOTES: The top row shows volatility estimates of a monetary policy shock, GDP growth shock and inflation shock. Each entry shows the median, and the 68% confidence band. The bottom row shows the time series used in the estimation.

### 3.2 Impulse Response to Monetary Policy Shock Volatility

Figure 3 plots the impulse response to a 100% increase in the variance of the monetary policy shock. This was approximately the estimated increase in monetary policy shock volatility during the recent financial crisis. The response of the volatility is fairly persistent and dissipates after about 90 quarters. The 3-month Treasury bill rate falls in response to this shock, reaching a trough of  $-0.4\%$  at around the 5-year horizon. CPI inflation follows a similar pattern, falling by  $0.1\%$  at the 1- to 2-year horizon with the decrease persisting over a fairly long time period.<sup>7</sup> There is a sharp fall in GDP growth in response to the shock with a decline of around  $0.15\%$ . The estimated 68% error bands indicate that these responses are different from zero over horizons considered by policymakers. In Appendix B, we present impulse responses to this volatility shock using a version of the benchmark model that restricts the

7. The persistence in these impulse responses is largely driven by the fact that  $\theta_1$  is estimated to be close to 1. If the impulse responses are calculated fixing a smaller value for  $\theta_1$  (while keeping the other coefficients fixed at the estimated values) the persistence of the responses drops dramatically. The results of this counterfactual experiment are available on request.

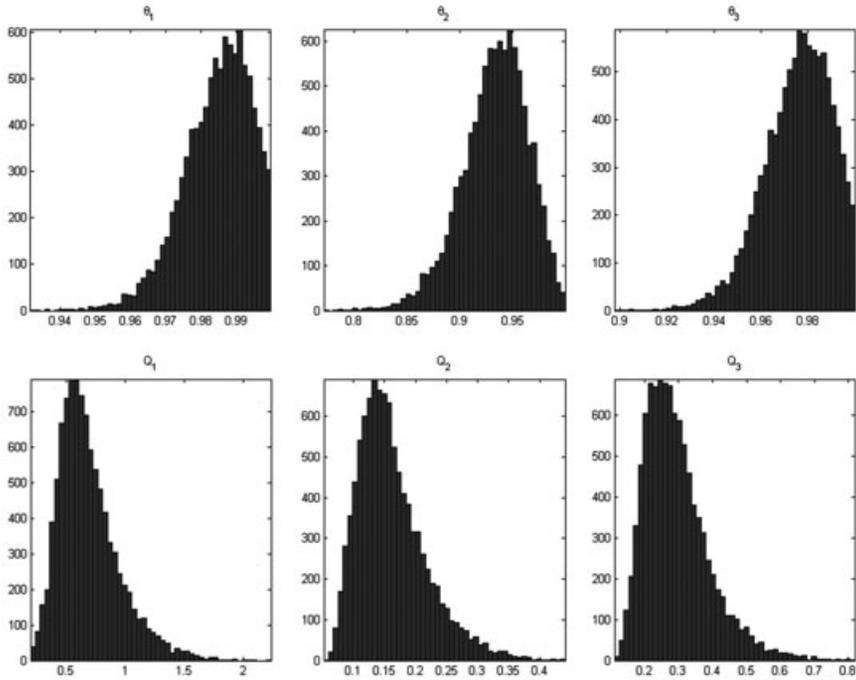


FIG. 2. Estimates of the Posterior Densities.

NOTES: Each entry shows the estimated posterior density for the elements of the transition equation (4). The top row shows the estimated posterior densities for the parameters of the AR(1) transition equation for the stochastic volatilities. The bottom row shows the posterior estimates for the variance of the volatility of the structural shocks.

coefficients on the volatility terms,  $\gamma_j$ , to be equal to zero. Therefore, this restricted model allows only an indirect impact of volatility through the (nonlinear) interaction between the covariance matrix  $\Omega_t$  and the structural shocks (see equation (2)).<sup>8</sup> The figure in Appendix B shows that this indirect effect is negligible, highlighting the importance of allowing for a direct impact of volatility in the VAR equations.

In Figure 4, we show the estimated impulse responses from the following extended version of the benchmark VAR model:

$$Z_t = c + \sum_{j=1}^2 \beta_j Z_{t-j} + \sum_{j=0}^4 \gamma_j \tilde{h}_{t-j} + \Omega_t^{1/2} e_t. \tag{7}$$

In this extended specification  $\tilde{h}_t$  and its four lags are allowed to enter the mean equations. This model has a larger number of parameters and subsequently the impulse responses are less precisely estimated. However, the results support those from the

8. From a numerical point of view, this indirect impact can be calculated via Monte Carlo integration. See Koop, Pesaran, and Potter (1996) for details.

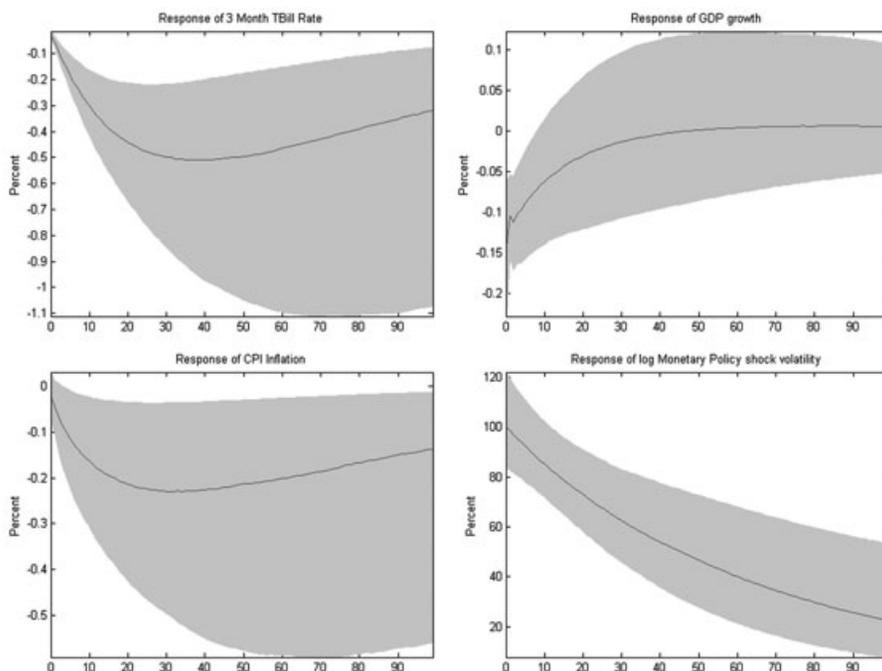


FIG. 3. Impulse Responses to a 100% Increase in the Variance of the Monetary Policy Shock.

NOTES: The entries show the response to a 100% increase in the variance of the monetary policy shock. Each plot shows the median response (solid line) and 68% confidence band (shaded area).

benchmark specification. In particular, the responses of inflation and the 3-month Treasury bill rate are negative and persistent. Similarly, GDP growth declines by about 0.2% and returns to base at around the 10-quarter horizon.

The top panel of Figure 5 presents the impulse responses from an expanded system that includes M2 growth and the growth of the Standard & Poor's Composite Index.<sup>9</sup> These variables are included in order to better proxy the central bank's information set. As argued in Leeper and Roush (2003), having a measure of money helps in the identification of the monetary policy shock. Similarly, the change in stock prices proxies forward-looking information and expectations. In this expanded VAR system we retain the same identification scheme—that is, requiring the policy shock that increases interest rates to have a contemporaneous negative impact on CPI inflation and GDP growth. The figure shows that an increase in monetary policy shock volatility leads to a fall in M2 growth of around 0.3% at the 3-year horizon. The response of stock price growth is more volatile with a decline of 1% after the shock. The response of the Treasury bill rate is similar to the benchmark case, with a decline of around 0.3% at the 3-year horizon. Similarly, GDP growth declines by around 0.15%, as in

9. The data for M2 are taken from the FRED database (FRED code M2SL). The data for the Standard & Poor's index are also obtained from FRED (FRED code SP 500).

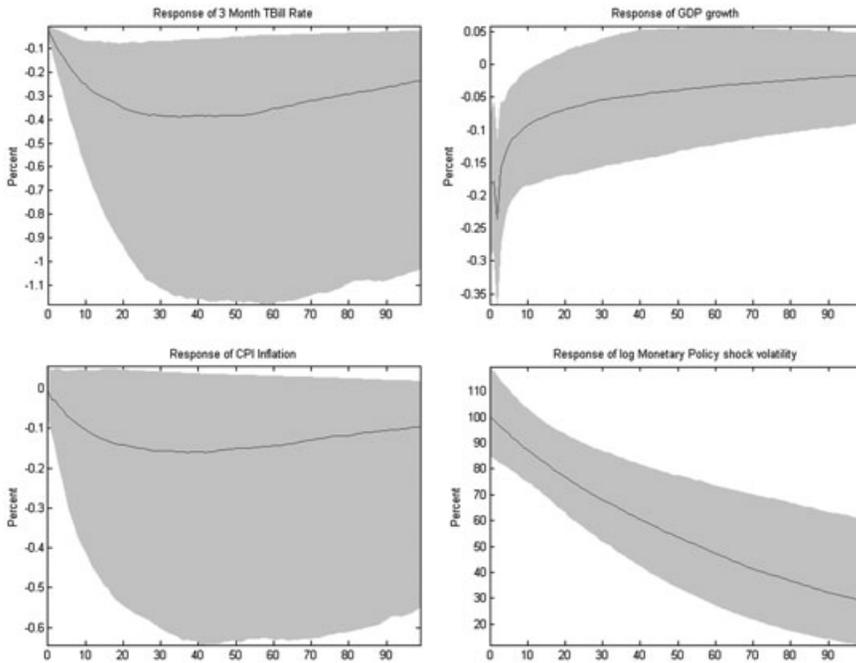


FIG. 4. Impulse Responses to a 100% Increase in the Variance of the Monetary Policy Shock, Four Lags.

NOTES: The entries show the response to a 100% increase in the variance of the monetary policy shock when four lags of volatility are included in the VAR model. Each plot shows the median response (solid line) and 68% confidence band (shaded area).

the benchmark case. In contrast to the benchmark results, however, the response of CPI inflation is imprecise with the error bands including zero almost immediately after the shock.

The bottom panel of Figure 5 shows the impulse responses from a version of the VAR model that includes the growth rate of government spending to nominal GDP as an additional variable to proxy the role of fiscal policy and its volatility (see Fernandez-Villaverde et al. 2011 for evidence on the role of fiscal volatility shocks).<sup>10</sup> As above, we retain the same identification scheme in this alternative specification. The response of the interest rate and GDP growth to a monetary policy volatility shock in this expanded system is very close to the benchmark case. There is a persistent estimated decline in the interest rate after this shock. GDP growth is estimated to decline by 0.15%. Inflation declines but the response is imprecisely estimated.

Overall, these estimates represent strong evidence that an increase in the volatility of monetary policy shocks leads to a fall in the short-term interest rate and GDP growth. They also provide some evidence to suggest that this negative impact also extends to CPI inflation.

10. Government spending is defined as government consumption expenditures and gross investment (Bureau of Economic Analysis, Table 1.15, line 21) deflated by nominal GDP (FRED code GDP).

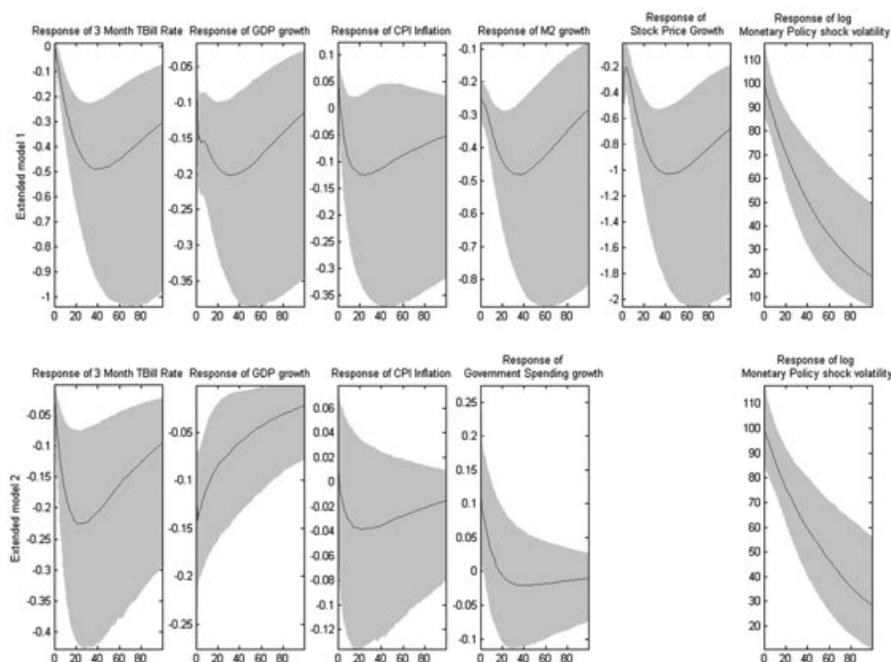


Fig. 5. Impulse Responses to a 100% Increase in the Variance of the Monetary Policy Shock, Expanded System.

NOTES: The entries show the response to a 100% increase in the variance of the monetary policy shock when M2 growth and the growth of the Standard & Poor's Composite Index and government spending are included in the VAR model. Each plot shows the median response (solid line) and 68% confidence band (shaded area).

#### 4. THE DSGE MODEL WITH MONETARY POLICY VOLATILITY SHOCKS

This section develops a DSGE model to investigate how changes in the volatility of monetary policy translate into movements in output growth, inflation, and the nominal interest rate, in order to rationalize the responses in the SVAR model. To implement the analysis, a standard New Keynesian model that resembles those by Ireland (2004) and Sargent and Surico (2011) is enriched with time-varying volatility to monetary policy. In addition, the model is solved by a third-order perturbation around the steady state, since, as documented by Fernandez-Villaverde et al. (2011), models with volatility shocks are inherently nonlinear and therefore linear methods cannot be applied to compute them. The way we embed uncertainty about monetary policy in the model complements the alternative approaches in the literature based on regime switching and learning. In these settings, uncertainty about future policy affects agents' expectations or learning process such that expected or perceived changes in policy are able to generate fluctuations in nominal and real variables. Recent examples of studies that emphasize these channels are those by Bianchi (Forthcoming) and Milani (2007).

The model economy consists of a representative household, a production sector that comprises a representative goods-producing firm and a continuum of intermediate-goods-producing firms indexed by  $i \in [0, 1]$ , and a monetary authority. The next subsections describe the agents' tastes, technologies, and the monetary policy rule in detail.

#### 4.1 The Representative Household

The representative household maximizes the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln C_t - (1/\eta)h_t^\eta], \tag{8}$$

where the variable  $C_t$  is consumption,  $h_t$  is units of labor,  $\beta$  is the discount factor  $0 < \beta < 1$ , and  $\eta$  is the inverse of the Frisch intertemporal elasticity,  $\eta > 0$ . The representative household enters period  $t$  with bonds  $B_{t-1}$ . At the beginning of the period, the household receives a lump-sum nominal transfer,  $T_t$ , from the monetary authority and nominal profits  $F_t$  from the intermediate-goods-producing firms. The household supplies  $h_t$  units of labor at the wage rate,  $W_t$ , to each intermediate-goods-producing firm  $i \in [0, 1]$  during period  $t$ . Then, the household's bonds mature, providing  $B_{t-1}$  additional units of currency. The household uses part of this additional currency to purchase  $B_t$  new bonds at nominal cost  $B_t/R_t$ , where  $R_t$  represents the gross nominal interest rate between  $t$  and  $t + 1$ . The household uses its income for consumption,  $C_t$ , and carries  $B_t$  bonds into period  $t + 1$ , subject to the budget constraint

$$P_t C_t + B_t/R_t = B_{t-1} + W_t h_t + F_t + T_t, \tag{9}$$

for all  $t = 0, 1, 2, \dots$ . Thus the household chooses  $\{C_t, h_t, B_t\}_{t=0}^{\infty}$  to maximize its utility (8) subject to the budget constraint (9) for all  $t = 0, 1, 2, \dots$ . Letting  $\pi_t = P_t/P_{t-1}$  denote the gross inflation rate, the first-order conditions for this problem yield

$$h_t^{\eta-1} = (1/C_t)W_t/P_t \tag{10}$$

and

$$1/C_t = \beta R_t E_t [(1/C_{t+1})(1/\pi_{t+1})]. \tag{11}$$

Equations (10) and (11) are the standard household's labor supply equation and the Euler equation that describes the optimal consumption decision, respectively.

#### 4.2 The Production Sector

The production sector comprises a representative goods-producing firm and a continuum of intermediate-goods-producing firms indexed by  $i \in [0, 1]$ . The representative finished-goods-producing firm uses  $Y_t(i)$  units of each intermediate good

$i \in [0, 1]$ , purchased at nominal price  $P_t(i)$ , to manufacture  $Y_t$  units of the finished product at constant returns to scale technology  $Y_t = [\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}$ , where  $\theta > 1$ . Hence, the finished-goods-producing firm chooses  $Y_t(i)$  for all  $i \in [0, 1]$  to maximize its profits  $P_t[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i)Y_t(i)di$ , for all  $t = 0, 1, 2, \dots$ . The first-order conditions for this problem are

$$Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t, \tag{12}$$

for all  $i \in [0, 1]$  and  $t = 0, 1, 2, \dots$ . Competition drives the finished-goods-producing firm's profit to zero at the equilibrium. This zero-profit condition implies that  $P_t = [\int_0^1 P_t(i)^{1-\theta} di]^{\frac{1}{1-\theta}}$  for all  $t = 0, 1, 2, \dots$ .

The representative intermediate-goods-producing firm hires  $h_t(i)$  units of labor from the representative household in order to produce  $Y_t(i)$  units of intermediate good  $i$  according to the constant returns to scale technology

$$Y_t(i) = h_t(i). \tag{13}$$

Since the intermediate goods are not perfect substitutes in the production of the final goods, the intermediate-goods-producing firm faces an imperfectly competitive market. It sets the nominal price  $P_t(i)$  for its output, subject to satisfying the representative finished-goods-producing firm's demand. The intermediate-goods-producing firm faces a quadratic cost to adjusting nominal prices, measured in terms of the finished goods and given by  $(\phi/2)\{P_t(i)/[\pi P_{t-1}(i)] - 1\}^2 Y_t$ , where  $\phi > 0$  is the degree of adjustment cost and  $\pi$  is the steady-state gross inflation rate. This relationship, as stressed in Rotemberg (1982), accounts for the negative effects of price changes on customer-firm relationships. These negative effects increase in magnitude with the size of the price change and with the overall scale of economic activity,  $Y_t$ .

The problem for the intermediate-goods-producing firm is to choose  $\{P_t(i), h_t(i)\}_{t=0}^{\infty}$  to maximize its total market value given by

$$E_0 \sum_{t=0}^{\infty} (\beta^t / C_t) [F_t(i) / P_t], \tag{14}$$

subject to the constraints imposed by equations (12) and (13). In equation (14), the term  $\beta^t / C_t P_t$  measures the marginal utility value to the representative household of an additional dollar in profits received during period  $t$  and

$$F_t(i) = P_t(i)Y_t(i) - h_t(i)W_t - (\phi/2)\{P_t(i)/[\pi P_{t-1}(i)] - 1\}^2 P_t Y_t, \tag{15}$$

for all  $t = 0, 1, 2, \dots$ . Solving the first-order conditions for this problem yields

$$\begin{aligned} \phi \left[ \frac{\pi_t(i)}{\pi} - 1 \right] \frac{\pi_t(i)}{\pi} &= (1 - \theta) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} + \frac{W_t}{P_t} \theta \left[ \frac{P_t(i)}{P_t} \right]^{-(1+\theta)} \\ &+ \beta \phi E_t \left\{ \frac{C_t}{C_{t+1}} \left[ \frac{\pi_{t+1}(i)}{\pi} - 1 \right] \left[ \frac{\pi_{t+1}(i)}{\pi} \frac{Y_{t+1}}{Y_t} \right] \right\}, \end{aligned} \tag{16}$$

where  $\pi_t(i) = P_t(i)/P_{t-1}(i)$  is the gross inflation rate for all  $t = 0, 1, 2, \dots$ . Equation (16) is the New Keynesian Phillips curve in its nonlinearized form, which states that the firm sets prices as a markup over marginal cost, accounting for price adjustment costs.

### 4.3 The Monetary Authority

As estimated in Ireland (2004), the monetary authority conducts monetary policy using the Taylor rule

$$R_t = R_{t-1}(\pi_t)^{\rho_\pi} (g_t)^{\rho_g} e^{\sigma_{rt}} \varepsilon_{rt}, \tag{17}$$

where  $g_t = Y_t/Y_{t-1}$  is the growth rate of output and the zero-mean, serially uncorrelated policy shock  $\varepsilon_{rt}$  is normally distributed, with standard deviation  $\sigma_{\varepsilon r}$ . The parameters  $\rho_\pi$  and  $\rho_g$  capture the degree of reaction of the nominal interest rate to inflation and output growth, respectively. The log of the standard deviation,  $\sigma_{rt}$ , of the innovation to the Taylor rule is not a constant, as commonly assumed, but follows the AR(1) process

$$\sigma_{rt} = \rho_{\sigma_r} \sigma_{rt-1} + \tau \varepsilon_{\sigma_{rt}}, \tag{18}$$

where the zero-mean, serially uncorrelated standard deviation shock  $\varepsilon_{\sigma_{rt}}$  is normally distributed, with unit variance. According to equation (17), the monetary authority gradually adjusts the nominal interest rate in response to movements in inflation and output growth.<sup>11</sup> In addition, the endogenous process for the interest rate allows for stochastic volatility. The parameters  $\sigma_{rt}$  and  $\tau$  control the degree of mean volatility and stochastic volatility in the nominal interest rate: a high  $\sigma_{rt}$  implies a high mean volatility of the nominal interest rate and a high  $\tau$  implies a high degree of stochastic volatility. Hence, in our formulation, two independent innovations affect the nominal interest rate. The first innovation,  $\varepsilon_{rt}$ , changes the instrument itself, while the second innovation,  $\varepsilon_{\sigma_{rt}}$ , determines the spread of values for the nominal interest rate.

### 4.4 Equilibrium, Solution, and Calibration

In equilibrium, all intermediate-goods-producing firms make identical decisions, so that  $Y_t(i) = Y_t$ ,  $h_t(i) = h_t$ ,  $F_t(i) = F_t$ , and  $P_t(i) = P_t$ , for all  $i \in [0, 1]$  and  $t = 0, 1, 2, \dots$ . In addition, the market clearing condition  $B_t = B_{t-1} = 0$  must hold for all  $t = 0, 1, 2, \dots$ . Equations (10), (13), and (15) can be used to solve for the real wage  $W_t/P_t$ , hours worked  $h_t$ , and real profits  $F_t/P_t$ . Consequently, the aggregate resource constraint is

$$Y_t = C_t + (\phi/2)(\pi_t/\pi - 1)^2 Y_t, \tag{19}$$

11. To ensure that the results are robust to alternative monetary policy rules we have replaced the current inflation term in the Taylor rule with lagged or expected inflation and established that the results remain broadly unchanged. Extending the analysis to investigate the affect of different assumptions on the formation of expectations would certainly be a very useful task for future research.

TABLE 1  
PARAMETERS CALIBRATION

Parameter		Value
$\beta$	Discount factor	0.99
$\eta$	Inverse of the Frisch intertemporal elasticity	16.2
$\theta$	Elasticity of substitution among goods	11
$\pi$	Steady-state gross inflation rate	1.0086
$\phi$	Price adjustment cost parameter	1620.7
$\rho_\pi$	Taylor rule parameter, reaction to inflation	0.3597
$\rho_g$	Taylor rule parameter, reaction to output growth	0.2536
$\sigma_{\varepsilon_r}$	Standard deviation of the monetary policy shock	0.0031
$\rho_{\sigma_r}$	Persistence parameter of the volatility shock	0.9846
$\tau$	Stochastic volatility parameter	$(0.6880)^{1/2}$

NOTE. The table shows the parameters calibration used to simulate the theoretical model. The values are the estimates in Ireland (2004), except for the parameters  $\{\rho_{\sigma_r}, \tau\}$ , which are from the estimated SVAR model.

and the first-order condition (16) can be written as

$$\phi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = (1 - \theta) + \frac{C_t}{Z_t} \left( \frac{Y_t}{Z_t} \right)^{\eta-1} \theta + \beta \phi E_t \left[ \frac{C_t}{C_{t+1}} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \left( \frac{\pi_{t+1} Y_{t+1}}{\pi Y_t} \right) \right]. \quad (20)$$

The model describes the behavior of five endogenous variables  $\{Y_t, C_t, g_t, \pi_t, R_t\}$ , and two exogenous shocks  $\{\varepsilon_{r,t}, \varepsilon_{\sigma_r,t}\}$ . The equilibrium is then described by the representative household's Euler equation (11), the price setting equation (20), the monetary authority policy rule (17), the aggregate resource constraint (19) and the definition of the growth rate of output. We solve the model by a third-order perturbation method around the steady state, as in Fernandez-Villaverde et al. (2011). We use a third-order approximation since stochastic volatility shock,  $\varepsilon_{\sigma_r,t}$ , enters as an independent argument in the policy function with a coefficient different from zero. For this reason, cubic terms are important to investigate the direct role of volatility. To calibrate the model we use the parameters' values estimated in Ireland (2004) and summarized in Table 1. In addition, we calibrate the parameters related to the stochastic volatility process in equation (18) by using the mean estimates in the SVAR model. Hence, we calibrate the correlation parameter,  $\rho_{\sigma_r}$ , equal to 0.9846, and the parameter that determines the degree of stochastic volatility,  $\tau$ , equal to  $(0.6880)^{1/2}$ .

#### 4.5 The Effect of Volatility Shocks

To investigate the impact of the volatility shock, Figure 6 plots the impulse responses of selected variables to a 100% increase to the volatility of the nominal interest rate. In reaction to the shock the nominal interest rate, inflation, and output growth fall. These are the same qualitative responses generated by the SVAR model with stochastic volatility, thereby showing that both the empirical and theoretical

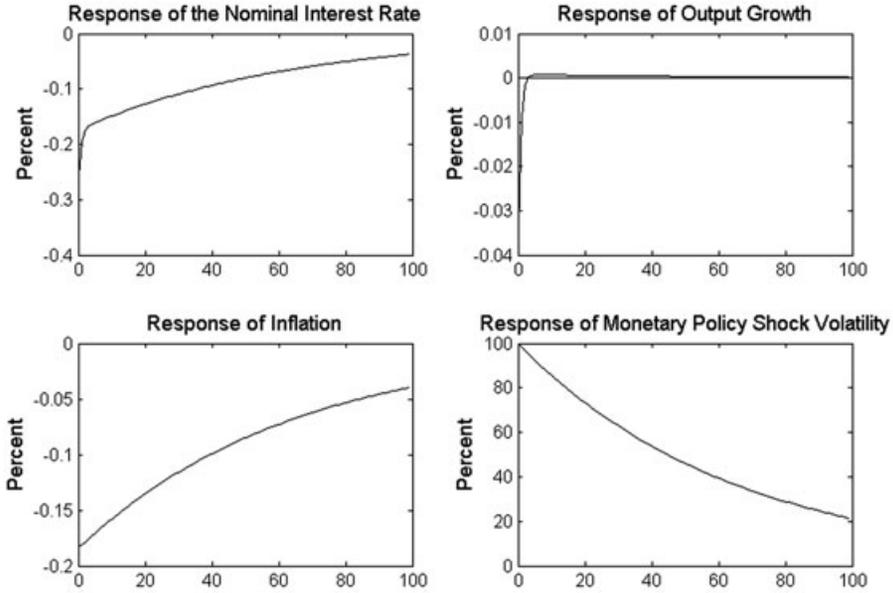


FIG. 6. Impulse Responses to a 100% Increase in the Variance of the Monetary Policy Shock, DSGE Model.  
 NOTE: The entries show the response to a 100% increase in the variance of the monetary policy shock.

models lead to the same qualitative responses to the shock. Interestingly, the theoretical model generates highly persistent responses of the nominal interest rate and inflation to a 100% increase to the volatility of the nominal interest rate, similar to the SVAR model. Output growth shows similar dynamics across the two models, since its reaction is negative on impact and then becomes positive before returning to the equilibrium. However, the persistence of output growth is lower in the theoretical model, as the response reverts to zero after 6 quarters in the DSGE model compared to 25 quarters in the SVAR model. This is an inherent feature of the theoretical framework, since, in order maintain the closest mapping between the theoretical and empirical model and to keep the model simple and tractable, it abstracts from capital accumulation.

We can now use the theoretical model to understand what drives these responses. Consider the Euler equation for consumption (11),

$$1/C_t = \beta R_t E_t[(1/C_{t+1})(1/\pi_{t+1})],$$

which relates the intertemporal changes in consumption with the real return on bonds (defined as the ratio between the nominal interest rate and expected inflation,  $R_t/E_t\pi_{t+1}$ ). The higher interest rate spread implies higher dispersion of

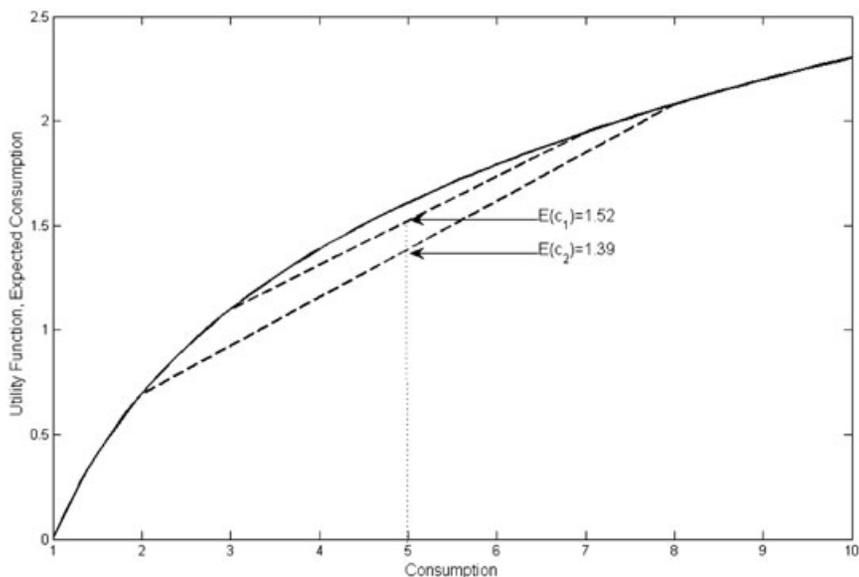


FIG. 7. Expected Utility and the Dispersion of Consumption.

NOTE: The graph relates the utility function and the expected consumption with the level of consumption.

consumption and inflation.<sup>12</sup> Since the utility function (8) is concave in consumption, Jensen's inequality implies that a higher volatility of consumption induces a fall in the level of expected consumption,  $E_t C_{t+1}$ , thereby increasing the term  $1/E_t C_{t+1}$  on the right-hand side (RHS) of equation (11). To see this, consider the graph in Figure 7, which relates the utility function and expected consumption to the level of consumption. The figure shows that for a given level of consumption the expected value of consumption is inversely related with the dispersion of consumption. For instance, if the level consumption is expected to fluctuate between 7 and 3, the expected value of consumption is equal to 1.52, whereas if the shifts in consumption are wider, between 2 and 8, the expected value of consumption is lower and equal to 1.39. Similarly, since the production technology and adjustment costs function are concave, Jensen's inequality implies that a higher volatility of inflation induces a fall in expected inflation,  $E_t \pi_{t+1}$ , thereby increasing the term  $1/E_t \pi_{t+1}$  on the RHS of equation (11). The rise in these two terms must be associated with a fall in the current nominal interest rate  $R_t$ . The intuition is that the rate at which bonds gain value due to movements in inflation increases on average with the level of uncertainty. As a result, holders of nominal assets demand a smaller compensation to keep bonds in their portfolio when uncertainty rises. In turn, if the monetary authority follows the Taylor rule (17), the fall in the interest rate must be associated with a fall in either inflation or output, or both of them. Since fluctuations in inflation

12. Numerical simulations of the model show that a rise in the standard deviation of the monetary policy shock increases the volatility of consumption and inflation.

and output are positively related, as from the New Keynesian Phillips curve (20), both variables fall in response to a decrease in the nominal interest rate. Thus, a rise in monetary policy uncertainty causes the nominal interest rate, inflation, and output growth to fall. It is interesting to note that the dynamics of the nominal interest rate closely follows changes in inflation due to the monetary authority's strong response to fluctuations in inflation, as dictated by equation (17).

#### 4.6 *Parameter Values in the Nonlinear DSGE Model*

In the sections above we work with a calibrated version of the DSGE model. An alternative approach would be to estimate the parameters of the model. The estimation of the model can be implemented in two ways. First, we can use likelihood-based approaches, as described in Fernandez-Villaverde and Rubio-Ramirez (2010). However, since the model is nonlinear, standard techniques such as the Kalman filter cannot be applied to evaluate the likelihood function. Hence, the estimation needs to rely on computationally intensive sequential Monte Carlo methods to compute the likelihood, as described in Fernandez-Villaverde and Rubio-Ramirez (2007). This sequential Monte Carlo step is then embedded within a Markov chain Monte Carlo (MCMC) algorithm to approximate the posterior density of the parameters. Despite their theoretical appeal, likelihood-based methods have a number of practical shortcomings in our application. In particular, combining model solution via third-order perturbation with likelihood computation via sequential Monte Carlo methods increases the computational burden to such a degree as to make a satisfactory and reliable application of likelihood maximization or MCMC infeasible.

An alternative approach to estimate the parameters of the model is to match some moments in the data with the same simulated moments in the model, as described in Ruge-Murcia (2012). Practically, in DSGE models this method minimizes the weighted distance between the empirical moments or impulse response functions and the moments or impulse response functions resulting from the simulated model. This approach is less computationally involved than likelihood-based methods, since it does not rely on the particle filter and sequential Monte Carlo methods. However, despite being computationally convenient, it has an important shortcoming. It is not obvious which moments to select to estimate the parameters of the model, and the experience is that parameter values are sensitive to the chosen moments.<sup>13</sup>

We believe that our analysis based on a calibrated DSGE model is appropriate since we use the model to investigate the theoretical mechanism that generates the empirical results. In order to provide accurate parameter estimates a more detailed theoretical model is needed to match the data closely, as detailed in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Extending the analysis using a more elaborate DSGE model that embeds both nominal and real rigidities as well as a more detailed description of the economy would certainly be a very useful task for future research.

13. Note that since the calibrated model produces impulse responses that are very similar to those from the VAR model, a simulated method of moments estimation that tried to match the VAR impulse responses would be quite likely to produce estimates of the model parameters close to the calibrated values.

## 5. CONCLUSION

This paper develops an SVAR model that allows for time-varying variance of monetary policy via stochastic volatility and also allows for a dynamic interaction between the level of endogenous variables in the VAR and the time-varying volatility. The model is used to estimate the impact of an increase in the volatility of monetary policy. The analysis shows that movements in the volatility of monetary policy have a nontrivial impact on the economy. In particular, the estimation establishes that in the aftermath of an increase in the volatility of monetary policy the nominal interest rate, inflation and output growth fall. These findings are then interpreted through the lens of a nonlinear DSGE model solved by third-order perturbation around the steady state. The theoretical model generates responses that are qualitatively similar to those of the VAR model and provides a consistent explanation for the findings.

The analysis puts forward a number of interesting avenues for future research. In particular, the proposed model is unable to quantify to what extent the responses of the endogenous variables change through time, perhaps due to the different monetary policy regimes. Hence, a natural extension would be to allow for time variation in the responses of endogenous variables to shocks to volatility. In addition, the methodology developed in the paper could also be applied to studying the effect of changes in volatility to a wider set of shocks, which could complement the findings in recent studies based on DSGE models, and could further advance understanding of the role played by uncertainty in macroeconomics.

## APPENDIX A: THE GIBBS SAMPLING ALGORITHM

### A.1 Prior Distributions and Starting Values

*VAR coefficients.* The initial conditions for the VAR coefficients  $\Gamma_0$  (to be used in the Kalman filter as described below) are obtained via an OLS estimate of equation (1) using an initial estimate of the stochastic volatility. The covariance around these initial conditions,  $P_0$ , is set to a diagonal matrix with diagonal elements equal to 0.1. This initial estimate of stochastic volatility is obtained via a simpler version of the benchmark model where the stochastic volatility does not enter the mean equations. This simpler version of the VAR model is estimated using a version of the Gibbs algorithm described in Primiceri (2005). The estimated volatility from this model is added as exogenous regressors to a VAR using the data described in the text to provide a rough guess for initial conditions for the VAR coefficients.

*Elements of  $H_t$ .* The prior for  $\tilde{h}_t$  at  $t = 0$  is defined as  $\tilde{h}_0 \sim N(\ln \mu_0, I_3)$ , where  $\mu_0$  are the first elements of the initial estimate of the stochastic volatility described above.

*Elements of  $A$ .* The prior for the off-diagonal elements  $A$  is  $A_0 \sim N(\hat{a}, V(\hat{a}))$ , where  $\hat{a}$  are the elements of this matrix from the initial estimation described above.  $V(\hat{a})$  is

assumed to be diagonal with the elements set equal to 0.1 times the absolute value of the corresponding element of  $\hat{a}$ .

*Parameters of the transition equation.* Following Cogley and Sargent (2005), we postulate an inverse-gamma distribution for the elements of  $Q_i \sim IG(\frac{Q_{i0}}{2}, \frac{5}{2})$  where  $Q_{i0}$  are the scale matrices obtained from the initial estimation described above. The prior for  $\theta_i$  is given as  $N(\theta_{i,0}, 0.1)$ , where  $\theta_{i,0}$  are the AR(1) coefficients obtained using the initial estimate of stochastic volatility.

### A.2 Simulating the Posterior Distributions

*VAR coefficients.* The distribution of the VAR coefficients  $\Gamma$  conditional on all other parameters  $\Xi$  and the stochastic volatility  $\tilde{h}_t$  is linear and Gaussian:  $\Gamma \setminus Z_t, \tilde{h}_t, \Xi \sim N(\Gamma_{T \setminus T}, P_{T \setminus T})$  where  $\Gamma_{T \setminus T} = E(\Gamma_T \setminus Z_t, \tilde{h}_t, \Xi)$ ,  $P_{T \setminus T} = \text{Cov}(\Gamma_T \setminus Z_t, \tilde{h}_t, \Xi)$ . Following Carter and Kohn (2004), we use the Kalman filter to estimate  $\Gamma_{T \setminus T}$  and  $P_{T \setminus T}$  where we account for the fact that the covariance matrix of the VAR residuals changes through time. The final iteration of the Kalman filter at time  $T$  delivers  $\Gamma_{T \setminus T}$  and  $P_{T \setminus T}$ . The Kalman filter is initialized using the initial conditions  $(\Gamma_0, P_0)$  described above. This application of the Carter and Kohn algorithm to the heteroskedastic VAR model is equivalent to a GLS transformation of the model.

*Element of  $A_t$ .* Given a draw for  $\Gamma$  and  $\tilde{h}_t$  the VAR model can be written as  $A'(\tilde{Z}_t) = e_t$ , where  $\tilde{Z}_t = Z_t - c + \sum_{j=1}^p \beta_j Z_{it-j} + \sum_{j=0}^J \gamma_j \tilde{h}_{it-j} = v_t$  and  $\text{VAR}(e_t) = H_t$ . This is a system of linear equations with known form of heteroskedasticity. The conditional distributions for a linear regression apply to this system after a simple GLS transformation to make the errors homoskedastic. More details on this step can be found in Cogley and Sargent (2005).

*Elements of  $H_t$ .* Conditional on the VAR coefficients and the parameters of the transition equation, the model has a multivariate nonlinear state-space representation. Carlin, Polson, and Stoffer (1992) show that the conditional distribution of the state variables in a general state-space model can be written as the product of three terms:

$$\tilde{h}_t \setminus Z_t, \Xi \propto f(\tilde{h}_t \setminus \tilde{h}_{t-1}) \times f(\tilde{h}_{t+1} \setminus \tilde{h}_t) \times f(Z_t \setminus \tilde{h}_t, \Xi), \tag{A1}$$

where  $\Xi$  denotes all other parameters. In the context of stochastic volatility models, Jacquier, Polson, and Rossi (2004) show that this density is a product of log normal densities for  $\tilde{h}_t$  and  $\tilde{h}_{t+1}$  and a normal density for  $Z_t$  where  $\tilde{h}_t = \exp(\tilde{h}_t)$ . Carlin, Polson, and Stoffer (1992) derive the general form of the mean and variance of the underlying normal density for  $f(\tilde{h}_t \setminus \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi) \propto f(\tilde{h}_t \setminus \tilde{h}_{t-1}) \times f(\tilde{h}_{t+1} \setminus \tilde{h}_t)$  and show that this is given as

$$f(\tilde{h}_t \setminus \tilde{h}_{t-1}, \tilde{h}_{t+1}, \Xi) \sim N(B_{2t} b_{2t}, B_{2t}), \tag{A2}$$

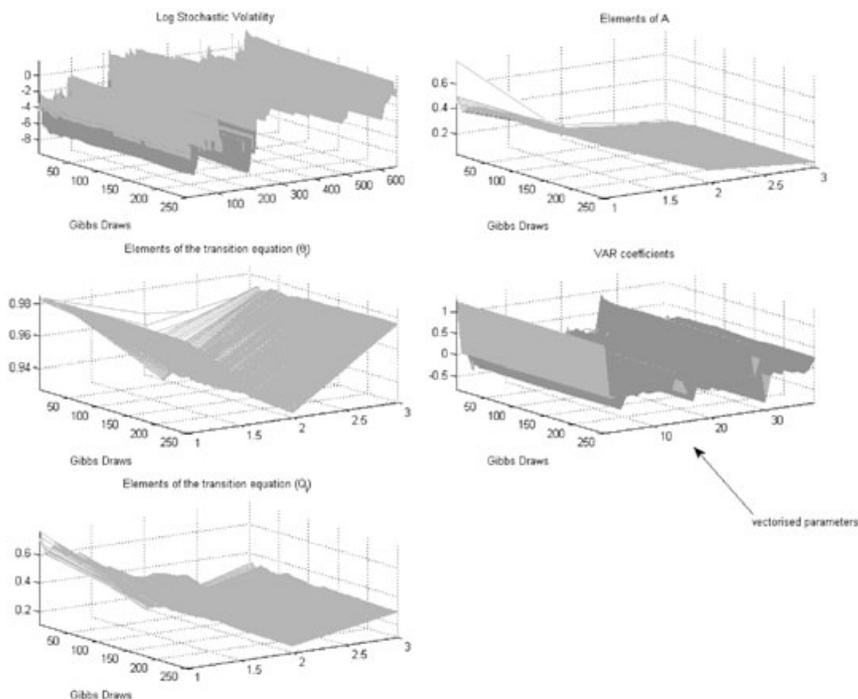
where  $B_{2t}^{-1} = Q^{-1} + F'Q^{-1}F$  and  $b_{2t} = \tilde{h}_{t-1}F'Q^{-1} + \tilde{h}_{t+1}Q^{-1}F$ . Note that due to the nonlinearity of the observation equation of the model an analytical expression for the complete conditional  $\tilde{h}_t \setminus Z_t, \Xi$  is unavailable and a metropolis step is required.

Following Jacquier, Polson, and Rossi (2004), we draw from equation (A1) using a date-by-date independence metropolis step using the density in equation (A2) as the candidate generating density. This choice implies that the acceptance probability is given by the ratio of the conditional likelihood  $f(Z_t \setminus \tilde{h}_t, \Xi)$  at the old and the new draw. In order to take endpoints into account, the algorithm is modified slightly for the initial condition and the last observation. Details of these changes can be found in Jacquier, Polson, and Rossi (2004).

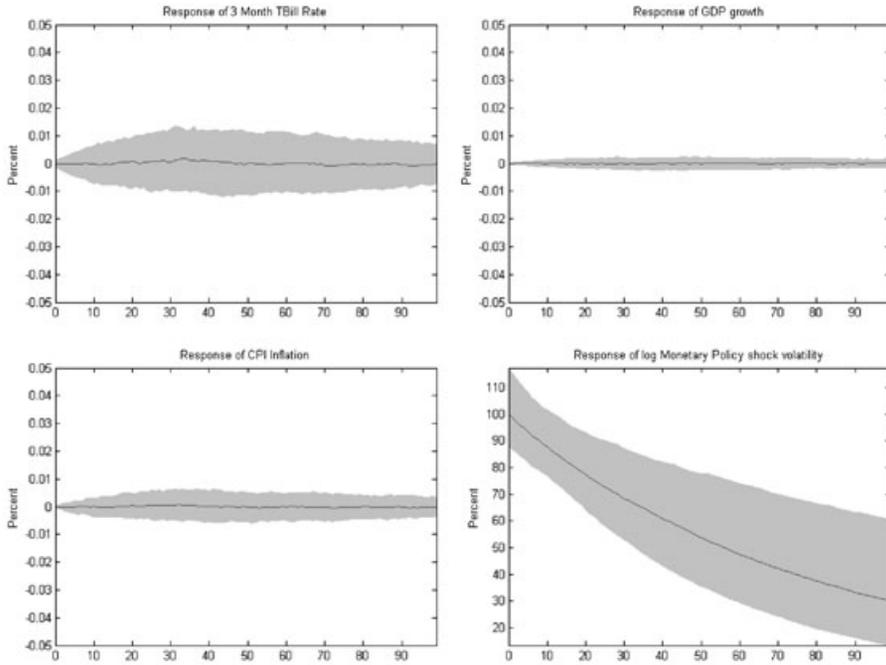
*Parameters of the transition equation.* Conditional on a draw for  $\tilde{h}_t$  the transition equation (4) is simply a sequence of linear regressions where the standard normal and inverse Gamma conditional posteriors apply. Note that when drawing  $\theta_i$  we impose the condition that  $|\theta_i| \leq 1$ .

### A.3 Convergence

The MCMC algorithm is applied using 1,000,000 iterations, discarding the first 990,000 as burn-in. The figure below plots the recursive mean calculated using intervals of 20 draws for the retained draws of the main VAR parameters. The fact that there is very little fluctuation in the recursive mean provides evidence for convergence of the MCMC algorithm.



## APPENDIX B: IMPULSE RESPONSES FROM THE RESTRICTED VAR MODEL



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